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Acoustic valley edge states in a graphene-like resonator system

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The concept of valley physics, as inspired by the recent development in valleytronic materials, has been extended to acoustic crystals for manipulation of air-borne sound. Many valleytronic materials follow the model of a gapped graphene. Yet the previously demonstrated valley acoustic crystal adopted a mirror-symmetry-breaking mechanism, lacking a direct counterpart in condensed matter systems. In this paper, we investigate a two-dimensional (2D) periodic acoustic resonator system with inversion symmetry breaking, as an analogue of a gapped graphene monolayer. It demonstrates the quantum valley Hall topological phase for sound waves. Similar to a gapped graphene, gapless topological valley edge states can be found at a zigzag domain wall separating different domains with opposite valley Chern numbers, while an armchair domain wall hosts no gapless edge states. Our study offers a route to simulate novel valley phenomena predicted in gapped graphene and other 2D materials with classical acoustic waves. *Published by AIP Publishing.* <https://doi.org/10.1063/1.5009626>

I. INTRODUCTION

The discovery of topological ordered states, including the quantum Hall effect (QHE),^{1,2} quantum spin Hall effect (QSHE),^{3,4} and topological insulators,^{5,6} is a remarkable development with profound impact in condensed matter physics. Such systems host robust unidirectional edge states immune to backscattering from local defects or disorders. Recent studies have demonstrated that analogous states can arise in the context of classical wave systems including photonic^{7–13} and acoustic systems.^{14–22} Some early attempts to construct topological phases in acoustic systems aimed at the QHE and relied on breaking time-reversal (T) symmetry.^{14–16} Due to the difficulty in breaking T symmetry, later studies largely emphasized on the analogue of the QSHE for acoustic waves, which support two effective “spin” states over a topological band gap with intact T symmetry.¹⁹

Recently, the valley degree of freedom (DOF) has attracted intense research interest because of its potential applications in storing and delivering information. The conservation of the valley DOF is protected by the large separation between two valleys in momentum space that leads to negligible intervalley interaction. This concept of valley manipulation has inspired a novel field of valleytronics,^{23,24} similar to the use of spin in spintronics. Inspired by the exciting achievements in valleytronics, the valley DOF has recently been extended to acoustic crystals for novel manipulation of air-borne sound.^{25,26} However, the previously demonstrated acoustic crystal adopted a mirror-symmetry-breaking mechanism,²⁶ thus lacking a direct counterpart in condensed matter systems.

In this paper, we investigate a two-dimensional (2D) periodic acoustic resonator system that can be directly

described with a tight-binding model, as an acoustic counterpart of a gapped graphene monolayer. Similar to a gapped graphene, gapless topological valley edge states can be found at a zigzag domain wall separating different domains with opposite valley Chern numbers, while an armchair domain wall hosts no gapless edge states. Besides, the valley vortex states in the previously demonstrated valley acoustic crystal exist in the space among adjacent sites (artificial atoms), but in our resonator system, the valley vortex states rotate around the six sites as a π orbit in graphene.

II. ACOUSTIC MODEL

We start with a simple nearest-neighbor (NN) tight-binding model for a honeycomb acoustic lattice which consists of acoustic resonators connected by thin cylindrical coupling waveguides with radius $r_c = 0.1a$, as shown in Fig. 1(a). The nearest-neighbor distance is $a = 0.01$ m in the calculation, and the lattice constant is $a_0 = \sqrt{3}a$. Each unit cell consists of two acoustic resonators (cylinders A and B of the same radius $r = 0.3a$, but with different heights $h_A = h + \Delta h$, $h_B = h - \Delta h$, where $h = \frac{5}{6}a$ and $\Delta h = 0.15h$). All blue surfaces in Fig. 1(a) are treated as sound hard boundaries in the numerical simulation, while the white surfaces are applied with periodic boundary conditions. The Bloch Hamiltonian H of this system can be written as

$$\begin{aligned}
 H &= \begin{pmatrix} V_a & H_{12}(\vec{k}) \\ H_{12}(\vec{k})^* & V_b \end{pmatrix} \\
 &= \begin{pmatrix} 0 & H_{12}(\vec{k}) \\ H_{12}(\vec{k})^* & 0 \end{pmatrix} + \frac{V_a + V_b}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
 &\quad + \frac{V_a - V_b}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (1)
 \end{aligned}$$

where

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FIG. 1. (a) Top and side views of the unit cell of acoustic honeycomb lattice. Two resonators playing the role of artificial atoms are connected by thin coupling waveguides. (b) Band diagram when a unit cell contains identical resonators (blue lines) and when the resonators are of different heights with $\Delta h = 0.15h$ (red lines). (c) Pressure field distribution for eigenstates K_1 and K_2 . (d) Topological transition as the height difference Δh changes from negative to positive.

$$H_{12}(\vec{k}) = -t \left[\exp(-i\vec{k} \cdot \vec{e}_1) + \exp(-i\vec{k} \cdot \vec{e}_2) + \exp(-i\vec{k} \cdot \vec{e}_3) \right]. \quad (2)$$

Here, $V_a(V_b)$ is on site energy for resonator A(B) and t represents the hopping terms. $\vec{e}_1 = (0, a)$, $\vec{e}_2 = \left(-\frac{\sqrt{3}}{2}a, -\frac{a}{2}\right)$, and $\vec{e}_3 = \left(\frac{\sqrt{3}}{2}a, -\frac{a}{2}\right)$ define the position of NN bound atoms and $\mathbf{k} = (k_x, k_y, k_z)$ is the Bloch wave vector. When $V_a = V_b$, which means the two acoustic resonators are of identical height ($\Delta h = 0$), the system exhibits space inversion symmetry.

III. RESULTS AND DISCUSSION

The band structure of the system obtained through full-wave simulation (all simulations are performed with COMSOL Multiphysics and simulation details are provided in the [supplementary material](#)) exhibits a Dirac cone at the K point for the simple lattice, as the blue lines shown in Fig. 1(b). This agrees well with the analytical model. If $V_a \neq V_b$, the space inversion symmetry of the honeycomb lattice is broken. Red dotted lines in Fig. 1(b) present the band structure for such a lattice ($\Delta h = 0.15h$) in which the band gap opens at the K point in the range from 4273 Hz to 5045 Hz. We further show the pressure field for K_1 and K_2 states [marked by black dots in Fig. 1(b)] in Fig. 1(c). The top panel displays the distribution of real and imaginary parts of pressure for the K_1 state. For the lower band, the higher

resonators [marked as B in Fig. 1(c)] coupled to each other through the shorter resonators [marked as A in Fig. 1(c)] which remain static. The distribution of the real part and the imaginary part of time-harmonic acoustic pressure reveals that a vortex state circulates counterclockwise around the six sites as a π orbit in graphene. The K_2 state at the higher band behaves similarly, but this time, the shorter resonators couples to each other through the static higher resonators and the vortex state rotates clockwise. Note that the vortex here circulates along the hexagon ring, being different from previous studies where the vortices exist among adjacent sites.²⁵ Then, we further investigate the evolution of eigenfrequencies of these two bands at the K point by varying Δh (while keeping other parameters constant). The eigenfrequencies as a function of Δh are shown in Fig. 1(d). It is shown that these two states will cross each other when Δh transits from negative to positive, with the degeneracy occurring at $\Delta h = 0$. When the acoustic resonators A and B interchange their heights (i.e., when Δh changes its sign), the bandgap appears unchanged, although a band inversion has occurred.

We can expand the above Hamiltonian in Eq. (1) near the K point, which can be expressed as

$$H_k = \delta k_x \sigma_x \chi_z + \delta k_y \sigma_y \chi_0 + \frac{V_a - V_b}{2} \sigma_z \chi_0 + \frac{V_a + V_b}{2} \sigma_0 \chi_0, \quad (3)$$

where χ_i and σ_i (i can be 0, x, y, z) are Pauli matrices related to degrees of freedom for the valley and the unit cell, respectively. The topological nature of the bands can be characterized by

computing the topological invariant, that is, the valley Chern number C_v in our case. C_v of the n th band is calculated by integrating the Berry curvature $\Omega_n(k) = \nabla_k \times \langle u_n(k) | i \nabla_k | u_n(k) \rangle$, where $u_n(k)$ is the eigenstate in k space and $\nabla_k = (\partial_{k_x}, \partial_{k_y}, \partial_{k_z})$, throughout half of the first Brillouin zone. The Berry curvature of the upper band (that of the lower band is of opposite sign) is displayed in Fig. 2, which shows distributions with opposite signs in the vicinity of the K and K' points. The valley Chern number is then obtained by integrating the Berry curvature around the valley

$$2\pi C_v = \int \Omega_n(k) d^2k. \quad (4)$$

The Chern numbers of the upper band are $\frac{1}{2}(-\frac{1}{2})$ at valley K (K') for a domain with $\Delta h > 0$ (denoted as “domain A”) and $-\frac{1}{2}(\frac{1}{2})$ at valley K (K') for a domain with $\Delta h < 0$ (denoted as “domain B”). If a proper domain wall is constructed between domain A and domain B, the difference of the valley-projected Chern numbers is equal to ± 1 , indicating the existence of a topological edge state per valley at the interface between two domains as a result of the bulk-edge correspondence.

To verify the existence of such topological edge states at the domain wall, we consider a supercell which consists of 6 unit cells for each of the two domains ($\Delta h = \pm 0.15h$) separated by a domain wall. At first, we consider a zigzag domain wall as shown in Fig. 3(a). The domain wall is formed with the higher resonators along the interface between domain A (upper domain) and domain B (upper domain). The x -projection of valley K' and K onto this domain wall is $\frac{k_x a_0}{\pi} = -\frac{2}{3}$ and $\frac{k_x a_0}{\pi} = \frac{2}{3}$, respectively. Simulation results in Fig. 3(b) exhibit a gapless edge state at valley K (K') within the bulk bandgap. The group velocity of the edge state at valley K' is positive because the change of valley Chern number from domain B to domain A is equal to 1. In contrast, this change of valley Chern number is -1 at valley K , indicating the negative propagation direction of the edge state. Valley pseudospin of the state is locked to the propagation direction. The pressure distribution of this edge state at the K point is shown in Fig. 3(c).

FIG. 2. Berry curvature of the upper band over the first Brillouin zone.

FIG. 3. (a) Zigzag domain wall between domain A as the upper domain and domain B as the lower domain. Red circles represent shorter resonators, while blue are higher resonators. (b) Band structure of acoustic valley edge states at the domain wall. (c) Pressure field of the edge state shown in (b) at K .

Next, we consider an armchair domain wall along the y direction [Fig. 4(a)]. For this domain wall, the y -projection of valley K' coincides with that of valley K , which means that the topological charge annihilation will occur at $k_y = 0$. Figure 4(b) presents the band diagram for a supercell with this domain wall. The bulk bandgap is consistent with that in Fig. 1(b). However, there are two trivial edge states within the bulk bandgap. The small gap at $k_y = 0$ means that these two states tend to have zero group velocity at valley K (K'). The pressure fields for these two states are shown in Fig. 4(c). Then, we can conclude that the propagation of edge states along the domain wall is only supported at a zigzag domain wall, as predicted in gapped graphene.²⁷

FIG. 4. (a) Armchair domain wall between domain A as the left domain and domain B as the right domain. Red circles represent shorter resonators, while blue are higher resonators. (b) Band structure for this structure. (c) Pressure fields for two states within the bandgap, as marked by red points in (b).

FIG. 5. Acoustic pressure field for a finite honeycomb lattice comprising 26×15 unit cells with a Z-shaped zigzag interface. The red arrow indicates the position of the point source.

FIG. 6. Acoustic pressure field for a finite honeycomb lattice comprising 1794 unit cells. A domain wall with 90° bends separates the two domains, as indicated by a blue dotted line. The vertical parts of the domain wall are zigzag, while the horizontal part is armchair. The red arrow indicates the position of a point source.

We then performed analysis on a finite lattice comprising 26×15 unit cells. As shown in Fig. 5, the lattice consists of domains A (left domain) and B (right domain) separated by a Z-shaped zigzag domain wall (marked by a blue dotted line). We excite the edge state by a point source operating at 4600 Hz. The obtained acoustic pressure distribution is plotted. The domain wall behaves as a waveguide with sharp bends and the edge state propagates along the domain wall with no scattering at the corners.

Then, we consider a domain wall with bending angles of 90° , as shown in Fig. 6. Different from the previous case, the domain wall here contains two vertical interfaces which are zigzag, while the middle vertical part is armchair. The operating frequency is 4500 Hz, located within the small gap of edge states in Fig. 4(b). The simulated field pattern in Fig. 6 shows that, when the domain wall transits from zigzag to armchair, the edge state stops propagating. This result is consistent with the previous band structure analysis which has shown that the armchair domain wall does not support the gapless topological edge state.

IV. CONCLUSIONS

We presented the design of an acoustic resonator system that works as the acoustic analogue of the quantum valley

Hall topological insulator phase. Different from the previously demonstrated valley acoustic crystal which lacks a direct counterpart in condensed matter systems, our acoustic resonator system directly simulates a gapped graphene monolayer. Gapless topological valley edge states can be found at a zigzag domain wall separating different domains with opposite valley Chern numbers, while an armchair domain wall hosts no gapless edge states. The similarity between our system and gapped graphene provides a unique platform to simulate novel valley phenomena predicted in gapped graphene and other 2D materials with classical acoustic waves.

SUPPLEMENTARY MATERIAL

See [supplementary material](#) for simulation details.

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