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Detection of damaged supports under railway track based on frequency shift

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Abstract

In railway transportation systems, the tracks are usually fastened on sleepers which are supported by the ballast. A lot of research has been conducted to guarantee the safety of railway track because of its importance, and more concern is expressed about monitoring of track itself such as railway level and alignment. The ballast and fasteners which provide strong support to the railway track are important as well whereas the detection of loose or missing fasteners and damaged ballast mainly relies on visual inspection. Although it is reliable when the fastener is missing and the damaged ballast is on the surface, it provides less help if the fastener is only loose and the damaged ballast is under the sleepers, which are however frequently observed in practice. This paper proposes an approach based on frequency shift to identify the damaged supports including the loose or missing fasteners and damaged ballast. In this study, the rail-sleeper-ballast system is modeled as an Euler beam evenly supported by a series of springs, the stiffness of which are reduced when the fastener is loose or missing and the ballast under the sleepers is damaged. An auxiliary mass is utilized herein and when it is mounted on the beam, the natural frequencies of the whole system will change with respect to the location of the auxiliary mass. The auxiliary mass induced frequency shift is analyzed and it is found the natural frequencies change periodically when the supports are undamaged, whereas the periodicity will be broken due to damaged supports. In fact, the natural frequencies drop clearly when the auxiliary mass moves over the damaged support. A special damage index only using the information of the damaged states is proposed and both numerical and experimental examples are carried out to validate the proposed method.

Keywords

Railway track; damaged supports; frequency shift; damage detection

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1. Introduction

Railway inspection plays an important role in ensuring the safe operation of railway transportation. Rail track is usually fastened on the sleepers, which are supported by a compacted ballast layer. The ballast retains the sleeper in its required position and the fasteners ensure the leveling and alignment of rail track. Once the fasteners are loose and even missing, or the ballast is damaged due to reduced size, the rail track will be unevenly supported and furthermore geometrically deteriorated. In fact, several serious accidents have been reported in the last decade due to this reason; therefore it is necessary to develop an effective approach to detect the damaged supports under railway track.

Currently, the most frequently used railway inspection method is visual inspection, either by manual operation or image processing. Although costly and time consuming, it is effective for detection of missing fastener; however, it is difficult to identify the loose fasteners and the damaged ballast under the sleepers. Ultrasonic testing is usually used for inspection of rail track geometry and overhead line, which can be easily conducted through a specific inspection vehicle. Ground Penetrating Radar (GPR) is capable of evaluating the ballasted track bed condition, which is similar to ultrasonic pulse echo technique by using electromagnetic waves instead of ultrasonic waves. However, it may be unsuitable for identifying damaged ballast: the size of damaged ballast changes gradually from small to large, but GPR usually requires layer reflections; on the other hand, GPR focuses on the average thickness and stiffness of the whole layer of ballast, whereas the ballast is always damaged locally.

Because loose or missing fasteners and damaged ballast always result in reduction of local stiffness, which influences the vibration characteristics of the rail-sleeper-ballast system; therefore, it is suitable to identify the damaged supports under the railway track by analyzing the dynamic response. In fact, a lot of research on vibration based damage detection has been carried out for many years [1,2]. Various vibrational characteristics have been investigated and among them, mode shape curvature [3], strain energy [4], flexible matrix [5] and flexibility curvature [6] are very famous, based on which many advanced damage detection methods and indices are developed. Fang and Ricardo [7] proposed a concept of power mode shape which is equivalent to the square of mode shape to detect damages in

linear structures. Grande and Imbimbo [8] presented a fusion approach to detect multiple damage locations based on changes of flexibility. Sakaris et al. [9] used a time series generalized functional model for precise damage localization. Yang et al. [10] proposed a novel curvature calculation method based on Fourier spectral to improve the noise resistance for damage detection. Dervilis et al. [11] considered damage detection as an outlier identification problem and used robust regression to detect multiple outliers from structural health monitoring (SHM) data. Dilella et al. [12] recently used frequency response function interpolation method to localize damage in bridges. Because the incomplete identified modal parameters and uncertainty are frequently met in practice, Chatzieleftheriou and Lagaros [13] developed a two-loop trajectory method to solve the ill-conditioned problem. And Chandrashekhar and Ganguli [14] used mode shape curvature and fuzzy logic to assess structure damages with uncertainty. They [15] further extended this approach to composite plate structures even with material uncertainty. Law et al. [16] also proposed a decentralized damage detection methods for identifying damages with subsets of parameter.

These above mentioned methods highly depend on the quality of identified modal parameters which are extracted from Frequency Response Functions (FRFs) traditionally, and therefore a lot of effort has been carried out on time frequency analysis to improve the identification of dynamics characteristics. Except for conventionally used Short Time Fourier Transform (STFT), wavelet transform is also frequently used because it can overcome the fixed resolution problem occurred in STFT by using adaptive window function. Ruzzene et al. [17] firstly used wavelet transform to extract frequencies and damping. Piombo et al. [18] also proposed a shifted version of wavelet transform to identify modal parameters by using output response. Some researchers have also investigated identification of dynamic characteristics by using measured data from a passing vehicle. Yang et al. [19,20] firstly proposed an idea to extract bridge frequencies by using dynamic response from a passing vehicle. Marchesiello et al. [21] and Bellino et al. [22] identified the dynamic parameters of beam like structures subjected to moving load which are considered nonlinear and time varying.

Several dynamic characteristics can be extracted and used for damage identification; however, it is observed that identification of natural frequencies is more accurate and convenient than other modal

parameters. Therefore Zhong et al. [23] proposed a new damage detection method based on the derivative of natural frequencies instead of mode shape curvature. Zhang and Xiang [24] further analytically investigated the auxiliary mass induced frequency shift for beam like structures. Zhang et al. [25,26] later extended this work to plate and cylinder structures and successfully identified damages like corrosion and crack in CNG tank; however, it requires highly accurate measurement of frequency to guarantee the accuracy of the damage index which uses the curvature of frequency shift curve. Because the auxiliary mass induced frequency shift is proven equivalent to mode shape square and the frequencies can usually be measured more accurately, so it is promising to use it for damage identification instead of other modal parameters. On the other hand, earlier vibration based damage identification methods usually require information from either undamaged structures or finite element models, which is quite difficult in practice. Ratcliffe [3] proposed gapped smoothing method (GSM) for beam like structures by using operating curvature shapes from damaged structure only. Yoon et al. [27,28] extended GSM to 2-D plate like structures and further proposed global fitting method. Limongelli [29] also developed the FRF interpolation method in which the baseline is modeled by the spline shape interpolation.

The dynamic response of rail-sleeper-ballast system has been investigated for a long time [30,31], and it is modeled as an Euler Bernoulli beam supported by a series springs in this study. The stiffness of spring is assumed to decrease if the fastener is loose or the ballast is damaged; moreover, it is considered to be zero if the fastener is missing and there is a gap between the railway track and the sleeper, or the ballast is in poor condition making the sleeper unsupported. An auxiliary mass is used herein to generate the frequency shift. Generally, the natural frequencies will be lower if the auxiliary mass is mounted in the middle of two neighbored supports, whereas they will be higher when the auxiliary mass is put above the supports. When the supports are undamaged, the auxiliary mass induced frequency shift is believed to change periodically with respect to the location of the auxiliary mass because of the periodicity of the supports; and the periodicity will be broken if the supports are damaged. The discontinuity in spatial domain can be identified through spatial Fourier analysis. Reddy and Ganguli [32] and Saipraneeth and Ganguli [33] successfully applied spatial Fourier

analysis to mode shapes to detect damages in beam like structures. In this study, the situation is easier and the natural frequencies are clearly observed lower when the auxiliary mass moves over the damaged supports. Therefore, the auxiliary mass induced frequency shift can be used to evaluate the damage condition, which has an advantage of more accurately and conveniently measurement than other dynamic characteristics. Moreover, an effective damage index is proposed based on the periodicity, which uses the information from the damaged structure only instead of requiring prior information.

The paper is organized into sections: Section 2 presents the dynamic analysis of the interactive system including an auxiliary mass and an Euler beam with periodically supports, and proposes a new damage index; Section 3 shows several numerical examples to verify the proposed approach; Section 4 describes the experimental tests used to validate the new method and finally, conclusions and discussions are presented in Section 5.

2. Theoretical dynamic analysis

The rail track is modeled as an infinite Euler-Bernoulli beam supported by a series of springs in this study, which can be seen in Fig. 1. An auxiliary mass is mounted on the beam and it is considered as a spring mass system.

Fig. 1 Interaction system of an auxiliary mass and rail track

Because it is difficult to obtain explicit solutions of the interactive system including infinite beam and auxiliary mass, therefore the infinite beam needs to be simplified to a beam with finite length first. In fact, the rail track can be considered as an infinite beam lying on Winkler foundation (Fig. 2), and the governing equations can be obtained easily:

$$\frac{d^4 u}{dx^4} + 4\alpha^4 u = \frac{q(x)}{EI}$$

1 (1)

$$\alpha = \sqrt[4]{\frac{k}{4EI}}$$

2 (2)

3 where EI and u are the bending stiffness and vertical displacement of the rail track, respectively;
 4 $q(x)$ is the distributed force, α is the characteristic of the system and k is the constant of the
 5 foundation.

6

Fig. 2 Infinite beam lying on Winkler foundation

7 The general solution of Eqs. (1-2) can be easily obtained as:

$$u(x) = e^{\alpha x}(C_1 \cos \alpha x + C_2 \sin \alpha x) + e^{-\alpha x}(C_3 \cos \alpha x + C_4 \sin \alpha x)$$

8 (3)

9 where C_1, C_2, C_3 and C_4 are constants of integration and can be determined by boundary conditions.

10 Especially, for a concentrated force P at $x = 0$, it must have $C_1 = C_2 = 0$ to make sure $u(\infty) = 0$,

11 then the static displacement can be obtained as:

$$u(x) = 2 \frac{\alpha P}{k} e^{-|\alpha x|} \cos \alpha x \leq 2 \frac{\alpha P}{k} e^{-|\alpha x|}$$

12 (4)

13 It is observed from Eq. (4) that the displacement value gradually decreases with the increase of x , and
 14 the maximum value is $2\alpha P/k$ at $x = 0$. The effective length of the infinite beam is proposed based on
 15 the fact that the displacement value can be ignored if x is large enough. Considering that $e^{-4} = 0.018$,
 16 the displacement $u(x)$ is less than two percent of the maximum value which can be ignored if

$|\alpha x| > 4$. As a consequence, the effective length of the infinite beam resting on the elastic foundation is determined in this study as:

$$l = 2 \times \frac{4}{\alpha} = \frac{8}{\alpha} \quad (5)$$

That is, the infinite beam lying on the Winkler foundation is simplified as a beam with effective length of l supported by N discrete springs with stiffness of k_i and locations of x_i , which can be seen in Fig. 3. In this study, the springs are evenly distributed and the distance between two adjacent springs is $\Delta x = l/(N + 1)$. The auxiliary mass is still modeled as a spring mass system mounted on the beam. For further simplification, the two ends of the beam are simply supported.

Fig. 3 The simplified model of the interactive system including auxiliary mass and simply supported beam

Based on the experience of previous studies [24], the governing equations of this interactive system can be written as:

$$m \frac{\partial^2 u}{\partial t^2} + EI \frac{\partial^4 u}{\partial x^2} = f(t) \delta(x - x_a) - \sum_{i=1}^N k_i u(x, t) \delta(x - x_i) \quad (6)$$

$$f(t) = -M(g + \ddot{q}) \quad (7)$$

$$M \frac{\partial^2 q}{\partial t^2} + Kq(t) = Ku(x_a, t) \quad (8)$$

where u and q are the vertical displacement of the beam and auxiliary mass, respectively; EI is the bending stiffness of the beam and m is the mass per unit length; M and K indicates the mass and stiffness of the auxiliary mass, respectively; x_a is the location of the auxiliary mass.

The displacement of the beam, $u(x, t)$, can be presented by modal superposition method:

$$u(x, t) = \sum_j \varphi_j(x) q_j(t) \quad (9)$$

where $\varphi_j(x)$ is the j th mode shape and q_j is the corresponding modal coordinate.

When the stiffness of the auxiliary mass is large enough (e.g. stiffness making the frequency of the mass-spring system ten times larger than the fundamental frequency of the beam), the displacement of the auxiliary mass is approximated as:

$$q(t) = u(x_a, t) = \sum_j \varphi_j(x_a) q_j(t) \quad (10)$$

Using the similar derivations described in Ref. [24], the square of j th natural frequency of the interactive system, ω_j^2 , can be approximately obtained as:

$$\omega_j^2(x_a) = \frac{\omega_{bj}^2 + \frac{\sum_{i=1}^N k_i \varphi_j^2(x_i)}{m \Phi_j}}{1 + \frac{M \varphi_j^2(x_a)}{m \Phi_j}} \quad (11)$$

where ω_{bj} is the j th natural frequency of the beam, $\Phi_j = \int_0^l \varphi_j^2(x) dx$ is a constant of integration.

It is observed from Eq. (11) that the j th natural frequency of the interactive system is a function of the location of auxiliary mass, x_a , and it only depends on the mode shape of the beam. For better illustration, a particular case is taken into consideration where $m = 0.936 \text{ kg/m}$, $l = 5 \text{ m}$, $M =$

0.35kg, $K = 1 \times 10^8 \text{N/m}$, $EI = 32 \text{N/m}^2$, $N = 9$, $k_i = 1 \times 10^5 \text{N/m}$. Fig. 4 shows numerical result of the first order auxiliary mass induced frequency shift which is calculated through ABAQUS 6.14. It is observed from Fig. 4 that the auxiliary mass induced frequency shift is periodic, which is because the mode shapes ($\varphi_j(x)$) of the beam are periodic. Moreover, it is found that the frequency drops slightly heavier near two ends, which is because the simplification of infinite beam to simply supported beam with finite length. Only the first mode is used in this study because it can be measure most accurately and conveniently in practice.

Fig. 4 Numerical result of the fundamental auxiliary mass induced frequency shift

Then the following damage index can be proposed. Since the damaged supports are always local and they will not influence large area, so the average of auxiliary mass induced frequency shift of all periods can be considered as a baseline:

$$\bar{f}_1 = \text{mean}(f_1(x)), \quad 0 \leq x \leq l \quad (12)$$

where 1 indicates the first mode and $f_1 = \omega_1/2\pi$.

Then the damage index is proposed as following which can be seen in Fig. 5 for better illustration:

$$D_{1,i} = \frac{|\bar{f}_{1,i} - \bar{f}_1|}{\max(f_1)}$$

(13)

$$\bar{f}_{1,i} = \text{mean}(f_1(x)), \quad \left(i - \frac{1}{2}\right) \Delta x \leq x \leq \left(i + \frac{1}{2}\right) \Delta x$$

(14)

Fig. 5 Illustration of the proposed damaged index

Compared with the damage index in Ref. [25-26], the one proposed in this study (Eqs. (12-14)) does not require calculation of curvature which usually brings inevitable numerical errors in practice; therefore it is less sensitive to noise.

Because this method highly depends on the frequency, therefore the extraction of natural frequency is important in this study. The procedure of extracting frequency used in Ref. [34] is used in this study and it is briefly reviewed herein. Firstly, the Fast Fourier Transform is applied to the acceleration to obtain a rough frequency; then the estimation of the frequency is carried out through an optimization algorithm to search the peak location in the discrete time Fourier transform of the original signal. This procedure is validated in Ref. [34] to be highly noise resistant and accurate.

For further study, a passing vehicle can be used as the auxiliary mass and the frequency can be extracted from the dynamic response of the vehicle, which will make the proposed method more convenient and time saving. However, this makes the problem time varying and therefore the extraction of frequency will more difficult than the procedure used in this study. Fortunately, several

researchers have attempted to identify the modal parameters for time varying system. Except for the approaches proposed in Ref. [21,22], Ghanem and Romeo [35] proposed a wavelet based approach to identify modal parameters of linear time varying systems, Feldman [36] tried to use Hilbert transform to extract the modal features of nonlinear time varying vibration systems. Instead of using non-parametric technique, Yang et al. [37] recently developed a parameterized time frequency approach to evaluate the modal parameters and it shows better performance.

3. Numerical Examples

Except for the undamaged case which is presented in the previous section, three more numerical examples including damaged supports are presented in this section to check the validity of the proposed method. The interactive system including auxiliary mass and beam is modeled in ABAQUS 6.14 and the fundamental natural frequency is calculated by linear perturbation step. All parameters are the same as those used in the previous section.

Fig. 6(a) shows the application of the proposed damage index on the undamaged case. Two peaks indicating the first and last supports can be observed, which is considered as the boundary effect; the damage indices of the fasteners in the middle are larger than 0.005. On the other hand, a model with identical parameters but the length of which is extended to 10 m is calculated and the damage index is shown in Fig. 6(b). It is found that the boundary effect is also obvious; however the damage indices of the fasteners in the middle decrease to 0.003. Therefore the proposed damage index shows better performance if the beam is sufficiently long; fortunately the rail track is always long enough in actual engineering application.

Fig. 6 Damage index when all supports are undamaged (a) 5 m model, (b) 10 m model

3.1 Slight decrease of local support stiffness

The situations that the fastener is loose and the ballast is damaged are considered first, which can be seen in Figs. 7 and 8, respectively. In this study, these two cases are regarded identical and the corresponding dynamic model is shown in Fig. 9 where the damage is simulated as the decrease of stiffness of the fifth spring. In this example, k_5 decreases from $1 \times 10^5 \text{ N/m}$ to $5 \times 10^4 \text{ N/m}$. One thing should be noted here that even the fastener is loose slightly, the effective support stiffness reduces quite a lot; therefore in this example, although the stiffness of the support is halved, the fastener may be just slightly loose.

Fig. 7 The situation that the fastener is loose

Fig. 8 The situation that the ballast is damaged

Fig. 9 The dynamic model of slight decrease of local support stiffness

The first order auxiliary mass induced frequency shift is shown in Fig. 10(a). It is clearly seen that the fundamental frequency of the interactive system changes periodically. At the fifth support having a lower stiffness, the frequency shift pattern is different: the frequency changes sharply near the support whereas the frequency changes smoothly near other undamaged supports and moreover, the frequency dropping near the damaged support is slightly heavier. When applying the proposed damaged index, it is observed from Fig. 10(b) that the damaged support is clearly indicated by a sharp peak, which is red while others are green. Because the damage in this example is slight, therefore there are still two peaks indicating the first and last supports due to the boundary effect.

Fig. 10 Results from model where local support stiffness slightly decreases

(a) the fundamental frequency shift, (b) damage index

3.2 Severe decrease of local support stiffness

When the damage shown in either Fig. 7 or Fig. 8 further develops, it becomes more severe and therefore it is considered to further reduce the local support stiffness. Fig. 11 shows the corresponding dynamic model where the stiffness of the spring, k_5 decreases from $1 \times 10^5 \text{ N/m}$ to $2.5 \times 10^4 \text{ N/m}$.

Fig. 11 The finite element model of severe decrease of local support stiffness

Fig. 12(a) represents the fundamental auxiliary mass induced frequency shift; the frequencies drop apparently at the damaged support so that it can be identified directly even without applying the proposed damage index. Fig. 12(b) shows the application of proposed damaged index to this case, where a more apparent peak indicating the damaged support can be observed. Compared with the damage index shown in Fig. 10(b), it is found the value of damage index for this case is much larger (above 0.1), indicating the damage is more severe. Moreover, the boundary effect can be eliminated because the damage is severe.

Fig. 12 Results from model where local support stiffness severely decreases

(a) the fundamental frequency shift, (b) damage index

3.3 Missing of local support

Two situations are considered herein: One presents that the fastener is missing and there is a small gap between the bottom surface of rail and the sleeper (Fig. 13), the other one indicates that the ballast is severely damaged and the sleeper is unsupported (Fig. 14). For these two situations, the dynamic models are regarded as the same (Fig. 15), where the spring is removed.

Fig. 13 The situation that the fastener is missing

Fig. 14 The situation that the sleeper is unsupported

Fig. 15 The finite element model of missing of local support

The numerical results including the fundamental natural frequency and damage index are presented in Figs. 16(a) and 16(b), respectively. It is found in Fig. 16(a) that the fundamental frequency suddenly drops when the auxiliary mass just passes through the fourth support. It looks like the continuous beam splits into three parts: part one is from the left end to the fourth support, part two is from the fourth support to the sixth support and part three is from the sixth support to the right end. For the first and third parts, the frequency shift pattern is almost identical to the intact case, while the second part has a quite different performance. In fact, the second part performs like a separate beam with a length of $2\Delta x = 1\text{m}$, which has elastic supports at the two ends; and the auxiliary mass induced frequency drops to nearly 10Hz. For these two situations where the stiffness of the damaged supports is ignored, the auxiliary mass induced frequency shift can be used directly to identify the damaged support. When applying the proposed damage index, a peak which has a higher value than 0.4 can be observed.

The damage indices corresponding to undamaged supports also have higher values than those in other cases, which is because the dramatic decrease of the frequency near the damaged support significantly lowers the baseline, $\bar{f}_1$ (Eq. 12). It may be eliminated if the rail track is quite long and the number of supports is large enough.

Fig. 16 Results from model where local support is missing

(a) The fundamental frequency shift (b) damage index

The effect of noise is also investigated: different levels of white noise are added to the acceleration and the traditionally used procedures of measuring FRFs are carried out to extract the frequency. The results are not displayed herein because they are almost identical to those shown in Figs. 10, 12, and 16, which indicate the proposed method is robust to noise. It is easy to be understood because the noise is assumed as white noise and it almost does not influence the frequency extraction (Ref. [34]). In actual engineering application, there should be a lot of noise which may not be white noise and it may influence the accuracy of frequency identification. However, because the proposed damage index (Eqs. (12-14)) uses the summation of the frequency instead of curvature, so it is more invulnerable to noise.

4. Experimental Verification

Several experimental examples are designed and carried out herein to further validate the new proposed method.

4.1 Experimental setup and description

The setup of experiment is shown in Fig. 17: a steel beam with dimensions of $1800\text{ mm} \times 30\text{ mm} \times 3\text{ mm}$ (effective length 1440 mm) is evenly supported by six pairs of fasteners, under which small nylon plastic plates with dimensions of $100\text{ mm} \times 30\text{ mm} \times 5\text{ mm}$ are used to perform as sleepers. Under the sleeper, there is a larger piece of wooden layer to simulate the ballast. A shaker (Wilcoxon Model F3/Z602WA, mass of 0.3 kg) which is regarded as auxiliary mass is mounted on the top surface of the beam with a magnetic base. Between any two neighbored supports, twelve testing points are labeled, on which the accelerometer is positioned. Because this model of shaker contains a coaxial accelerometer together, when it is placed on one testing point, it generates a pulse impact to excite the beam and the corresponding acceleration is recorded accordingly. Although Fast Fourier Transform is usually used to extract the natural frequencies, it is not used herein because it has low frequency resolution if the duration time of the measured acceleration is short. Instead, the approach described in Ref. [34] is used to extract the frequencies more accurately. Once a measurement is completed, the accelerometer is positioned to next testing point and another measurement is carried out again. Then the auxiliary mass induced frequency shift along the whole beam can be obtained.

Fig. 17 Experimental setup

There are totally four experimental cases in this study which are shown in Fig. 18: one is undamaged case and the other three are damaged cases. For the first case, all supports are undamaged and each fastener is fastened by a torque of 8 Nm ; for Cases 2, the fourth pair of fasteners are loosen to 2 Nm ;

- 1 for Case 3, the piece of wooden layer is replaced by soft rubber to simulate the damaged ballast; for
2 Case 4, the fourth pair of fasteners are removed and there is actually no support at the damaged area.

(a) Case 1

(b) Cases 2

(c) Case 3

(d) Case 4

Fig. 18 Experimental examples (a) Case 1 (b) Case 2 (c) Case 3 (d) Case 4

4.2 Experimental results

When all supports are undamaged, the first order auxiliary mass induced frequency shift is shown in Fig. 19(a). It is seen that the fundamental frequency shift shows almost perfect periodicity which coincides with the numerical example very well. Fig. 19(b) presents the damage indices of the six supports which have small values and no obvious peak indicating damaged support can be observed.

For the second case that the fourth pair of fasteners is loosen, it observes from Fig. 19(b) that the first order auxiliary mass induced frequency shift presents regular periodicity as well but the frequency slightly drops when the auxiliary mass is placed near the damaged support. The damaged support is identified when the proposed damaged index is applied (Fig. 19(d)). Because the length of the beam is quite limited in experiment, the boundary effect is more obvious than that in numerical examples.

In the third case, the wooden layer under the fourth sleeper is replaced by a soft rubber layer to simulate the damaged ballast. The dynamic model is considered the same as the previous case but with more severe damage. It can be seen from Fig. 19(e) that the fundamental natural frequency drops apparently when the auxiliary mass pass through the third pair of fasteners; it is quite similar to the pattern shown in Fig. 12 which presents the fundamental frequency shift when local support stiffness severely decreases. Therefore, it can be concluded that the simplified dynamic model is promising.

The damage index of the fourth support is larger than 0.15, which is nearly twice of that in Case 2. It also indicates that the damage in this case is more severe. Moreover, it is observed from Fig. 19(f) that the boundary effect is less obvious because the damage is more severe.

1 For the last case where the fourth support is removed to simulate that the fastener is missing and the
2 sleeper is unsupported, the experimental results coincide with the numerical examples quite well. The
3 periodicity of the first order auxiliary mass induced frequency shift is broken at the damaged support
4 by emerging two intervals into one, which can be seen in Fig. 16 as well. The damage index
5 indicating the damaged support is as large as 0.22. This observation also proves that the proposed
6 damage index is capable of evaluating the damage severity. At the same time, the boundary effect is
7 eliminated in this case because the damage is sufficiently severe.

Fig. 19 Fundamental frequency shift for (a) Case 1 (b) Case 2 (c) Case 3 (d) Case 4 and damage index for (e) Case 1 (f) Case 2 (g) Case 3 (h) Case 4

5. Conclusions

This paper describes a new method for detection of damaged supports under railway track based on auxiliary mass induced frequency shift. Based on the analysis of the auxiliary mass induced frequency shift, it is observed that the fundamental frequency of interactive system including auxiliary mass and rail track with evenly distributed supports changes periodically with respect to the auxiliary mass location, which is because the fundamental mode shape of the rail track with evenly distributed supports are periodical (Eq. (11)). Once the support is damaged, the stiffness decreases accordingly; and therefore the periodicity will be broken. A new damage index based on this periodicity is proposed to eliminate the requirement of prior information of undamaged structure. In fact, since the damaged supports only influence locally, the average of frequencies measured on all points can be considered as a baseline. Both numerical and experimental verifications show that the proposed method is sensitive to damaged supports especially for the cases that there is a small gap between the track and the sleeper due to missing fastener and the sleeper is unsupported due to severely damaged ballast.

Compared with traditional damage detection methods, this method is more sensitive because it uses the frequency which can be measured more accurately, and it is more practical because it does not require the prior information from undamaged structure or finite element model.

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