TOA based Localization and Tracking in Indoor Multipath Environment

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SCHOOL OF ELECTRICAL AND ELECTRONIC ENGINEERING

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TOA based Localization and Tracking in Indoor Multipath Environment

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School of Electrical and Electronic Engineering

A thesis submitted to the Nanyang Technological University in partial fulfilment of the requirements for the degree of Doctor of Philosophy

2019
Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

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Zhang Heng
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I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

29 Jan 2019
Date

Tan Soon Yim
Authorship Attribution Statement

This thesis contains material from two papers published in the following peer-reviewed conferences where I was the first author, and one paper submitted to the following peer-reviewed journal where I was the first and/or corresponding author.


The contributions of the co-authors are as follows:

- I discussed with Prof. Tan Soon Yim and Dr. Chee Kiat Seow about the algorithm and performed all simulation and experiment process.
- I prepared the manuscript drafts. The manuscript was revised by Prof. Tan Soon Yim and Dr. Chee Kiat Seow.


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28-Jan-2019

............... Date ................        

                      

Zhang Heng
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Abstract

This thesis addresses the issue of localization and tracking using time-of-arrival (TOA) data for both line-of-sight (LOS) and nonline-of-sight (NLOS) paths measured at multiple reference devices (RDs) in indoor multipath environments. This thesis proposes a novel virtual RD (VRD) based indoor TOA localization algorithm with both LOS and multipath components that can be used when an accurate floor plan is available. By introducing the concept of VRD, multipaths can be considered as virtual LOS paths that originate from mobile device (MD) to VRDs. Due to unknown measurement-to-path correspondence, many possible positions satisfy the localization and tracking equation. A grid-based data association algorithm is proposed to estimate the correct measurement-to-path correspondence. Using the estimated data association result, the MD can be localized with a two-step weighted least squares method. The experimental and simulation results show that the proposed VRD based localization algorithm significantly outperforms conventional LOS based localization algorithms.

When an accurate floor plan is not available, this thesis proposes a novel indoor tracking algorithm for joint estimation of the MD and the map. By modeling the floor plan as a collection of map features, the multiple-RD single-cluster probability hypothesis density (MSC-PHD) filter can be used for joint estimation of the MD and map features. Conventional MSC-PHD filters are developed for outdoor radar-based
scenarios that only consider backscatter paths. For application in indoor localization and tracking, the LOS path and all higher-order reflections that carry information on the MD and map features must be formulated. This thesis proposes two new MSC-PHD filters by incorporating LOS path and higher order reflection paths, which are referred to as a LOS incorporated MSC-PHD (LMSC-PHD) filter and a multi-reflection incorporated MSC-PHD (MRMSC-PHD) filter, respectively. In addition, to mitigate high computation load of the proposed MSC-PHD filters, a computational tractable implementation that combines a new greedy measurement partitioning scheme and a particle-Gaussian mixture filter is presented. Furthermore, a novel mapping error metric is proposed to evaluate the accuracy of estimated map. The experimental and simulation results show that our proposed LMSC-PHD filter and MRMSC-PHD filter outperforms existing MSC-PHD filters by a significant margin in terms of both localization and mapping accuracy.
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List of Abbreviations

ALE       Average localization error
AME       Average mapping error
AOA       Angle of arrival
AWGN      Additive white Gaussian noise
CDF       Cumulative distribution function
EKF       Extended Kalman filter
GNSS      Global navigation satellite system
IMU       Inertial measurement unit
JPDAF     Joint probabilistic data association filter
KF        Kalman filter
$k$NN     $k$-nearest-neighbor
LMSC-PHD  Line-of-sight incorporated multiple reference device single cluster probability hypothesis density
LOS       Line-of-sight
MAP       Maximum a posteriori
MD        Mobile device
MHTF      Multiple hypothesis tracking filter
MMSE      Minimum mean square error
<table>
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<th>Description</th>
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<tr>
<td>MRMSC-PHD</td>
<td>Multi-reflection incorporated multiple reference device single cluster probability hypothesis density</td>
</tr>
<tr>
<td>MSC-PHD</td>
<td>Multiple reference device single cluster probability hypothesis density</td>
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<tr>
<td>NLOS</td>
<td>Non-line-of-sight</td>
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<tr>
<td>OSPA</td>
<td>Optimal subpattern assignment</td>
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<tr>
<td>P-GM</td>
<td>Particle Gaussian mixture</td>
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<tr>
<td>PDF</td>
<td>Probability density function</td>
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<tr>
<td>PDOA</td>
<td>Phase difference of arrival</td>
</tr>
<tr>
<td>PF</td>
<td>Particle filter</td>
</tr>
<tr>
<td>PGFL</td>
<td>Probability generating functional</td>
</tr>
<tr>
<td>PHD</td>
<td>Probability hypothesis density</td>
</tr>
<tr>
<td>RB-PHD</td>
<td>Rao-Blackwellized probability hypothesis density</td>
</tr>
<tr>
<td>RD</td>
<td>Reference device</td>
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<tr>
<td>RFID</td>
<td>Radio frequency identification</td>
</tr>
<tr>
<td>RFS</td>
<td>Random finite set</td>
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<tr>
<td>RMSE</td>
<td>Root mean square error</td>
</tr>
<tr>
<td>RSS</td>
<td>Received signal strength</td>
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<tr>
<td>SC-PHD</td>
<td>Single cluster probability hypothesis density</td>
</tr>
<tr>
<td>SMP</td>
<td>Smallest M-vertex polygon</td>
</tr>
<tr>
<td>SVM</td>
<td>Support vector machine</td>
</tr>
<tr>
<td>TDOA</td>
<td>Time difference of arrival</td>
</tr>
<tr>
<td>TOA</td>
<td>Time of arrival</td>
</tr>
<tr>
<td>UKF</td>
<td>Unscented Kalman filter</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Full Form</td>
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<td>--------------</td>
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<tr>
<td>UWB</td>
<td>Ultra-wideband</td>
</tr>
<tr>
<td>VMD</td>
<td>Virtual mobile device</td>
</tr>
<tr>
<td>VRD</td>
<td>Virtual reference device</td>
</tr>
<tr>
<td>WLANs</td>
<td>Wireless local area networks</td>
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<tr>
<td>WSNs</td>
<td>Wireless sensor networks</td>
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List of Symbols

Throughout this thesis, bold letters denote vectors and matrices; lower case letters denote variables and parameters.

<table>
<thead>
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<tr>
<td>$</td>
<td></td>
</tr>
<tr>
<td>$(\cdot)^T$</td>
<td>Transpose operator</td>
</tr>
<tr>
<td>$(\cdot)^{-1}$</td>
<td>Inverse of a matrix</td>
</tr>
<tr>
<td>$\oplus$</td>
<td>Vertical concatenation</td>
</tr>
<tr>
<td>$(d \leftrightarrow z)$</td>
<td>Associate a measurement $z$ to a path with length $d$</td>
</tr>
<tr>
<td>$0_{N \times N}$</td>
<td>$N \times N$ Zero matrix</td>
</tr>
<tr>
<td>$\text{diag}(\cdot)$</td>
<td>Diagonal elements of a diagonal matrix</td>
</tr>
<tr>
<td>$d_p^{(c)}(X, Y)$</td>
<td>OSPA distance between set $X$ and set $Y$</td>
</tr>
<tr>
<td>$d^{(c)}(x, y)$</td>
<td>An arbitrary distance between $x$ and $y$ cut off at $c$</td>
</tr>
<tr>
<td>$\mathbb{E}(\cdot)$</td>
<td>Expectation of a random variable</td>
</tr>
<tr>
<td>$f(\cdot)$</td>
<td>MD movement model</td>
</tr>
<tr>
<td>$h(\cdot)$</td>
<td>Measurement model</td>
</tr>
<tr>
<td>$I_{N \times N}$</td>
<td>$N \times N$ Identity matrix</td>
</tr>
<tr>
<td>$m$</td>
<td>Number of reflection</td>
</tr>
<tr>
<td>$mr$</td>
<td>$m^{th}$ order reflection path</td>
</tr>
</tbody>
</table>
\( m^i \)  \( \) The \( i \)th map feature

\( M \)  \( \) RFS of map features

\( \mathcal{N}(\cdot) \)  \( \) Gaussian distributed random variable

\( RD_s \)  \( \) The \( s \)th RD

\( v_{k\mid k-1} \)  \( \) Predicted PHD at time \( k \)

\( v_k \)  \( \) Updated PHD at time \( k \)

\( VRD^i_s \)  \( \) First order VRD reflected from \( m^i \) of \( RD_s \)

\( VRD^{i,j}_s \)  \( \) Second order VRD reflected from \((m^i, m^j)\) of \( RD_s \)

\( VRD^{i,j,l,...}_s \)  \( \) \( m \)th order VRD reflected from \((m^i, m^j, m^l, \ldots)\) of \( RD_s \)

\( x_k \)  \( \) State of the MD at time \( k \)
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Chapter 1

Introduction

Nowadays, people spend more than 80% of their time indoors, so the requirement for indoor localization and tracking services has significantly increased in applications such as localizing customers in a shopping mall for personal recommendation of nearby stores, tracking criminals in a train station or airport for arrest, and helping firemen navigate burning buildings [1–6]. The existing Global Navigation Satellite System (GNSS) does not work indoors due to severe signal attenuation and multipath fading caused by the walls and furniture in indoor environments. Therefore, the design of accurate indoor localization and tracking algorithms has become an extremely demanding topic. The remainder of this chapter is organized as follows. The background of indoor localization and tracking technologies and techniques is introduced in Section 1.1. Section 1.2 gives the motivations and objectives of this thesis. The main contributions are summarized in Section 1.3 and the last section outlines this thesis.
1.1 Background

The objective of indoor localization and tracking algorithms is to provide accurate state information (i.e., position and velocity) of an unknown mobile device (MD) in an indoor environment using measurements from reference devices (RDs) with known positions. Therefore, the indoor localization and tracking problem typically involves two steps: collecting position-related measurements and processing the measurements to estimate the state of the MD. This section includes an overview of widely used measurements and processing methods. According to the collected measurements, the indoor localization techniques can be divided into four main types: triangulation, fingerprinting, proximity, and self-measurement.

1. Triangulation: This kind of technique conveys the geometric relationship, including distance (or range) and angle information, between an MD and the RDs. The received signal strength (RSS), time of arrival (TOA), time difference of arrival (TDOA) and phase difference of arrival (PDOA) provide the distance information, and the angle of arrival (AOA) provides the angle information.

(a) RSS: RSS-based techniques measure the path loss caused by signal propagation to estimate the distance between an MD and an RD with theoretical or empirical path loss models. A widely used path loss model that describes the relationship between the path loss $L$ (in dB) and the distance $d$ can be expressed as

$$L = P - P_0 = -10k \log_{10} \frac{d}{d_0} + \epsilon$$  \hspace{1cm} (1.1)

where $P$ and $P_0$ represent the RSS measured at RD and at unit distance.
1.1. BACKGROUND

d_0 \text{ from the MD, respectively. } k \text{ is the path loss exponent, whose value usually varies between 2.0 and 4.0 and can be measured when necessary. The last term } \epsilon \text{ is the additive white Gaussian noise (AWGN), with zero mean and standard deviation } \sigma. \text{ An RSS measurement can be interpreted geometrically as a circle with its center at the RD with the estimated distance as the radius. With at least three RDs in a two-dimensional (2-D) plane, the MD can be localized at the intersection of the circles, as shown in Fig. 1.1(a). The RSS-based technique is commonly used in low-cost systems such as wireless sensor networks (WSNs) and wireless local area networks (WLANs). The advantage is that the RSS information is easy to measure using existing wireless devices without the need for time synchronization. However, due to severe multipath overlapping in indoor environments, the path loss model in (1.1) does not always hold, which makes it difficult to achieve high-ranging accuracy.}

(b) TOA: TOA-based techniques measure the elapsed time for signal transmission from an MD to an RD. TOA has a linear relationship with distance as 
\[ d = c \cdot t \]
where \( t \) is the TOA measured at the RD and \( c \) is the speed of light. Like RSS-based techniques, a TOA measurement can also be treated as a circle, and the MD can be localized at the intersection of circles with at least three RDs in a 2-D plane, as shown in Fig. 1.1(a). The TOA-based technique can achieve great distance accuracy using an ultra-wideband (UWB) system, but time synchronization is required between the MD and the RDs. To resolve the synchronization issue, a scheme based on time-difference-of-arrival (TDOA) can be used at the expense of using one RD.
1.1. BACKGROUND

Figure 1.1: Distance-based techniques: (a) Localization based on distance, where $RD_s$ denotes the $s^{th}$ RD and $d_s$ represents the distance from the MD to the $RD_s$. (b) Localization based on distance difference, where $d_{s0}$ represents the measured distance difference between the MD to $RD_s$ and to the reference.

as a reference. Alternatively, a two-way ranging method can be used to resolve this issue $[2][36]$. In the two-way ranging method, the MD transmits a signal to an RD, which then loops the signal back to the MD after a fixed time delay $t_d$. Using the two-way ranging method, the time information is measured on the MD side, which resolves the synchronization issue. In this case, the distance between the MD and the RD can be expressed as $d = c \cdot (t - t_d)/2$, where $t$ is the measured two-way TOA.

(c) TDOA: TDOA-based techniques use an RD as the time reference and measure the distance differences between an MD to the other RDs and to the reference RD. In this case, a TDOA measurement can be interpreted geometrically as a hyperbola with points such that the absolute distance difference to the RD and the reference is the same as the TDOA measurement. The MD can then be localized at the intersection point of hyperbolas...
1.1. BACKGROUND

Figure 1.2: Angle-based technique: (a) AOA estimation, in which $\Delta d$ is the interval between adjacent array elements. (b) Localization based on AOA measurement, in which $\theta_s$ represents the AOA between the MD and $RD_s$.

with at least four RDs in a 2-D plane, as shown in Fig. 1.1(b). The TDOA scheme resolves the time synchronization issue in the TOA method at the expense of using one RD as a reference.

(d) PDOA: PDOA-based techniques measure the phase difference between two continuous waves at different frequencies to estimate the distance between an MD and an RD. This technique is widely used in radio frequency identification (RFID) and WSNs. Suppose the frequencies of the two continuous waves are $f_1$ and $f_2$, respectively; the relationship between the phase difference $\Delta \phi$ and the distance $d$ can be expressed as $\Delta \phi = 2\pi \Delta f d / c$. Usually, this technique requires only simple transceivers with a small bandwidth. However, estimation ambiguity occurs when the phase difference exceeds $2\pi$. In addition, the phase information may be corrupt due to multipath overlapping in indoor environments.
1.1. BACKGROUND

(e) AOA: Unlike the ranging-based techniques mentioned above, the AOA-based techniques measure the angle information between an MD and an RD. To estimate the angle information, each RD requires an antenna array, as shown in Fig. 1.2. The interval between adjacent array elements, $\Delta d$, is usually set as half of the carrier signal wavelength $\lambda$, which is $\Delta d = \lambda/2$, as shown in Fig. 1.2(a). The AOA calculation is based on the far-field assumption, in which case the signal is assumed to be parallel incident to each antenna. The phase difference between adjacent antenna elements $\Delta \phi$ can be expressed as $\Delta \phi = k \Delta d \sin \theta$, where $k = 2\pi/\lambda$ is the wave number. It should be noted that the phase difference measured with the AOA technique differs from the phase difference measured with the PDOA technique. The AOA techniques measure the phase difference of the same incident signal between two adjacent antennas, whereas the PDOA techniques measure the phase difference between two signals with different frequencies received at the same antenna. With at least two RDs in a 2-D plane, the position of the MD can be localized as the intersection point of lines along respective AOA directions, as shown in Fig. 1.2(b). However, in indoor environments, the phase information may be corrupt due to severe path overlapping. Even if a technique such as the matrix pencil is used to deal with the path overlapping issue, the accuracy of the estimated AOA information still suffers from the antenna array size $[2]$.

After measuring the geometric parameters, the MD can be localized using Bayesian estimators such as least squares and maximum likelihood estimators. If prior information of the MD is known, then the MD can be localized using
Bayesian estimators such as the minimum mean square error (MMSE) and the maximum a posteriori (MAP) estimators \[37\]. The tracking process can be viewed as a concessive localization process with an entire history of measurements and estimated states for the MD. By introducing other state information such as velocity and accelerations, the concessive localized MD positions can be connected with a movement model. The MD can then be tracked using more efficient Bayesian filters such as Kalman filter (KF), extended KF (EKF), unscented KF (UKF), and the more versatile particle filter (PF) \[38\].

2. Fingerprinting: In an indoor environment, the line-of-sight (LOS) path may be blocked, which makes the straightforward expression of the geometric relationship between an MD and an RD difficult. In this case, the fingerprinting method, which measures position-dependent fingerprints, can be used to localize and track the MD. This kind of technique usually involves two steps. The first is to collect one or more position-dependent fingerprints of an environment at known positions to construct the database of fingerprints. The second involves locating and tracking the MD by matching the measured fingerprints with the database using a pattern-recognition algorithm, such as the probabilistic method, \(k\)-nearest-neighbor (kNN) method, neural network, support vector machine (SVM), and smallest M-vertex polygon (SMP). The most commonly used fingerprint is the RSS because it requires very simple receiver architecture and can be measured using existing infrastructure such as Wi-Fi access points \[39\]-\[42\]. Recently, the geomagnetic field is also been proposed as an alternative fingerprints \[43\]. It should be noted that this kind of technique requires an offline fingerprints collection step to build a database. To achieve high accuracy,
1.1. BACKGROUND

the first step usually requires collection of fingerprints at dense grids, which
leads to a large work load. In the meantime, this type of method is sensitive to
the changing environment.

3. Proximity: Proximity-based techniques rely on a dense grid of RDs previously
deployed at known positions in an environment. Each RD has its own detection
radius. Once the MD enters an RD’s detection region, the MD and this RD are
connected. According to the connectivity between the MD and RDs, the MD
can be localized [44–47]. For example, when MD is detected by a single RD,
the MD will be localized at the position of the detected RD. When multiple
RDs detect the MD, the MD can be localized at the position of the RD that
receives the strongest signal, or at the weighted average position of the RDs that
are receiving signals, or based on a more sophisticated smoothing method [46].
RFID is the most common realization of this technique. This kind of technique
requires a simple receiver architecture, but it requires predeployment of dense
grids of RDs.

4. Self-measurements: Unlike the types of measurements mentioned above that
rely on infrastructure, self-measurements can be collected by a stand-alone MD
with an inertial measurement unit (IMU). Typically, an IMU is composed of
a three-dimensional (3-D) gyroscope and a 3-D accelerometer, which provide
angular velocity and linear acceleration, respectively [48]. Given the initial
position and the MD’s velocity and orientation information, the standalone
MD can be localized and tracked by integrating the measured angular velocity
and linear acceleration with the Bayesian filters. However, the measurement
errors of the IMU will be integrated and propagate unbounded over time, which
limits stand-alone inertial localization and tracking with a low-cost IMU.

The TOA-based indoor positioning method provides accurate ranging information, does not require a large infrastructure of RDs and antenna arrays, and does not require premeasurement of the dense fingerprints, so the thesis is focused on the design of TOA-based indoor localization and tracking algorithms.

1.2 Motivation and Objectives

As illustrated in the last section, conventional TOA-based techniques use the geometric relationship of the LOS paths between an MD and RDs to localize the MD. However, LOS paths are not always available in indoor environments. The estimation accuracy will degrade if an NLOS path is treated as an LOS path and used to localize the MD [49, 50]. To overcome this issue, conventional algorithms attempt to identify and either mitigate or discard the NLOS paths [51–59]. However, if multipath components can be properly used to localize and track the MD instead of being discarded or mitigated, the accuracy and stability of indoor localization and tracking algorithms will be greatly improved because multipath components carry information on the MD. In addition, the multipath components can be used in situations in which several LOS paths are blocked, leading to an insufficient number of LOS paths for conventional LOS-based methods to localize the MD. This study was performed to determine how to use multipath components for indoor localization and tracking.

When an accurate floor plan is available, multipaths can be considered LOS paths originating from virtual RDs (VRDs) to localize and track an MD [60, 64]. The VRDs can be precalculated with signal propagation modes. In indoor environments, the signal propagation modes can be considered to be the fusion of an LOS path,
1.2. MOTIVATION AND OBJECTIVES

reflection paths, and diffraction paths. Some studies also consider random diffusion modelled by a random process \[62,65\], which is treated as noise in this thesis. No VRD is needed for the LOS path; in other words, the position of the VRD is same as that of the RD. For a single reflection path, the position of the VRD is the image of the RD reflecting from the reflector, as shown in Fig. 1.3(a). As can be seen, a TOA of the reflection path can be interpreted geometrically as a circle with its center at the VRD, with the TOA as the radius. The MD can be localized at the intersection of several circles. For a point scattering path from a corner, the position of VRD is same as the corner, as shown in Fig. 1.3(b). Suppose that the distance between the corner and the RD is \(d\), a TOA of the point scattering path can be treated geometrically as a circle with its center at the VRD, with the difference between the TOA and \(d\) as the radius. For a multipath that involves multiple propagation modes, it can be considered a fusion of several single propagation modes, and the VRD can be calculated mode by mode.

As can be seen, multipath components can be traced to their corresponding VRDs and considered virtual LOS paths by introduction of the VRD. If measurements can be associated with their corresponding VRDs, the MD can be localized and tracked using conventional algorithms such as MMSE and EKF. The issue is the association of measurements with their corresponding VRDs. Conventional data association methods, such as the joint probabilistic data association filter (JPDAF) \[38\] and multiple hypothesis tracking filter (MHTF) \[66\], rely on prior information regarding the MD position. However, when no prior MD information is available, it cannot perform data association. The performance of data association with no prior MD information and the design of an accurate TOA-based indoor localization algorithm using an accurate
1.2. MOTIVATION AND OBJECTIVES

Figure 1.3: Example of VRDs corresponding to different path models. (a) VRD corresponding to a single reflection path. (b) VRD corresponding to a diffraction path.

floor plan is the first objective of this thesis.

When an accurate floor plan is unavailable, the position of VRDs cannot be calculated accurately, which means that it is difficult to correctly associate multipath components with their corresponding VRDs. This necessitates joint estimation of the MD and updating the inaccurate floor plan. By modeling the map as a collection of map features, such as lines, corners, and points, it can be estimated jointly with the MD \cite{67, 68}. As mentioned, the data association result is unreliable because neither accurate MD nor map information are available. Because the localization and tracking result is very sensitive to the data association uncertainty \cite{69}, the data association uncertainty should be integrated into the formulation of the tracking algorithm.

To handle this issue, the concept of a random finite set (RFS) is introduced \cite{70, 72}. An RFS is a collection of all permutations of all elements in a random vector. By
modeling the collection of map features as an RFS, the indoor tracking with inaccurate floor plan can be performed using filters developed based on the RFS theory, such as the Rao-Blackwellized probability hypothesis density (RB-PHD) filter \cite{71} and the single cluster probability hypothesis density (SC-PHD) filter \cite{72}. These methods involve recursive propagation of the PHD of map features (first-order statistics of the RFS-modeled map features) and full distribution of the MD to track the MD and update the map \cite{70}. Compared to algorithms based on data association, the RFS-based framework has better accuracy and stability because RFS based algorithms incorporate data association uncertainty and an unknown number of map features \cite{73,74}. However, to the best of our knowledge, these filters were developed for outdoor radar-based scenarios, in which only backscattering paths (single reflection paths in our case) are considered. For indoor tracking, the LOS path should be also considered because the LOS path carries information on the MD, which can improve the localization accuracy and results to improve the mapping accuracy. Therefore, extension of the RFS-based filters for indoor tracking to incorporate LOS paths is the second objective of this thesis.

Furthermore, to the best of our knowledge, conventional RFS-based filters all assume that each map feature generates only one measurement. Thus, if single reflections are considered measurements generated by map features, higher-order reflections can only be considered clutter. However, higher-order reflections carry information on both the MD and map features that will improve the accuracy of localization and mapping. Indoor multipath environments contain many double and higher-order reflections \cite{75}. If such reflection paths can be modeled in the RFS-based filter to track the MD and update the inaccurate map, the mapping and localization accuracy will
both be improved. Therefore, extension of the RFS-based filters for indoor tracking

to incorporate higher-order reflection paths is the third objective of this thesis.

1.3 Contributions of the Thesis

The main contributions of the thesis are summarized as follows:

- A novel TOA-based indoor localization algorithm is proposed that uses multi-
  path components with an accurate floor plan. By dividing the given floor
  plan into grid points, the noiseless path length between each grid point to RDs
  and VRDs can be calculated. At each grid point, the calculated path lengths
  can be associated with the measurements, and the data association cost can
  be estimated. The MD will then be considered near the grids with the mini-
  mum data association cost. Using the average position of several grids with
  the minimum data association cost, the final data association result can be es-
  timated to localize the MD. By introducing the data association matrix, the
  data association process is integrated with the two-step weighted least squares
  method. The experimental and simulation results show that the proposed mul-
  tipath based localization algorithm outperforms the conventional LOS based
  localization algorithm in terms of localization accuracy.

- The proposed data association method is integrated with a conventional track-
  ing filter, such as EKF and PF, for indoor tracking using multipath components.
  The data association matrix can be estimated in a straightforward manner based
  on the MD’s prior information. Based on the estimated data association results,
  the MD can be tracked using conventional EKF and PF. The experimental and
simulation results show that the proposed multipath based tracking algorithm outperforms the conventional LOS based tracking filters in terms of localization accuracy.

- A novel multiple-RD SC-PHD (MSC-PHD) filter that incorporates the LOS path, which is referred to as a LOS incorporated MSC-PHD (LMSC-PHD) filter, is proposed for TOA-based indoor tracking and updating inaccurate map. The single reflection paths are modeled as a Poisson point process and used to update the map features and MD, whereas the LOS path is modeled as a Bernoulli point process and used to update the MD because the number of LOS can only be one or zero and depends only on the MD. In addition, MSC-PHD filters are computationally intractable because of the combinatorial complexity. To mitigate the computational load, a computational trackable implementation that combines a new greedy measurement partition process and a particle Gaussian mixture (P-GM) filter is presented. Furthermore, a novel mapping error metric for planar scatterers is proposed to evaluate the accuracy of estimated map features. The experimental and simulation results show that the proposed LMSC-PHD filter outperforms the conventional MSC-PHD filter in terms of both localization and mapping accuracy.

- A novel MSC-PHD filter that incorporates higher-order reflection paths, which is referred to as a multi-reflection incorporated MSC-PHD (MRMSC-PHD) filter, is proposed for TOA-based indoor tracking and updating inaccurate map. To incorporate the multiple reflection paths, it is necessary to decouple the map features involved in multiple reflections. By considering each multiple reflection as a virtual single reflection generated by a single map feature, the associations
among map features can be decoupled, and the MRMSC-PHD filter can be de-

rived. The MRMSC-PHD filter is implemented by the proposed P-GM filter. 
The experimental and simulation results show that the proposed MRMSC-PHD 
filter outperforms the conventional MSC-PHD filter in terms of both localization 
and mapping accuracy.

1.4 Outline of the Thesis

This thesis is organized as follows. In Chapter 2, the state of the art is reviewed,
including the data association based indoor localization schemes and the RFS-based 
indoor tracking algorithms. In Chapter 3, the proposed TOA-based indoor localiza-
tion algorithm that uses multipath components with accurate knowledge of the floor 
plan is presented. The localization accuracy is analyzed using simulation and exper-
imental data collected in a typical indoor environment. In Chapter 4, a TOA-based 
indoor tracking algorithm using multipath components with an accurate floor plan is 
presented. The tracking performance is presented using simulation and experimental 
data collected in a typical indoor environment. In Chapter 5, the proposed LMSC-
PHD filter for TOA-based indoor tracking is derived. The tracking performance is 
shown using the simulation and experimental data collected in a typical indoor en-
vironment. In Chapter 6, the proposed MRMSC-PHD filter for TOA-based indoor 
tracking is presented. The tracking performance is given based on simulation and 
experimental data collected in a typical indoor environment. Chapter 7 gives the 
conclusions of this thesis and discusses work for future studies. Finally, the proofs of 
some analytical expressions are presented in the Appendices.
This chapter presents a review of indoor localization and tracking algorithms using multipath components. When an accurate floor plan is available, the MD can be localized and tracked using both LOS and NLOS paths by conventional algorithms if the data association result can be estimated. Therefore, the indoor localization and tracking algorithms are reviewed from the perspective of handling the data association process. When no accurate floor plan is available, the map and the MD must be jointly estimated. As mentioned, this thesis is focused on RFS-based filters because they are robust to data association uncertainty. In this chapter, the background of the RFS and its first-order statistics PHD is recapped, and the existing RFS-based filters are reviewed.
2.1 Data Association based Indoor Localization and Tracking Algorithms

The previous chapter showed that the NLOS paths can be considered virtual LOS paths that originate from MD to their corresponding VRDs. If measurements can be correctly associated with their corresponding VRDs, the multipath components can be used to localize and track the MD with conventional methods. The literature includes two typical classes of data association frameworks: the joint probabilistic data association filter (JPDAF) \[38, 62, 65, 76\] and the multiple hypothesis tracking filter (MHTF) \[61, 66, 77, 78\].

Given an MD movement model, the location of the MD at time \(k\) can be predicted based on the estimated location of the MD at time \(k - 1\). Based on the predicted MD position, the path lengths from the predicted MD to all RD and VRDs can be calculated. Suppose that at time \(k\) the calculated path length vector and measurement vector are denoted as \(d_k = (d_1, \ldots, d_n)\) and \(z_k = (z_1, \ldots, z_m)\), respectively. Then, for each association pair \((d \leftrightarrow z)\), the corresponding likelihood \(l_k(z|d)\) can be calculated, where \(d \in d_k\) and \(z \in z_k\). It should be noted that several paths may be blocked, which means that no measurement corresponds to such paths. The corresponding elements in \(d_k\) should then be associated with \(\emptyset\), which can be denoted as \((d \leftrightarrow \emptyset)\).

Suppose that the detection probability of a path is \(P_d\); the probability of the blocked path blocked can be denoted as \(1 - P_d\), which means that the likelihood of each pair of \((d \leftrightarrow \emptyset)\) is \(1 - P_d\). The association pairs between the calculated path length and measurements then take the following form

\[
(d_i \leftrightarrow z_{\theta(i)}) \tag{2.1}
\]
2.1. DATA ASSOCIATION BASED INDOOR LOCALIZATION AND TRACKING ALGORITHMS

where $\theta: \{1, \ldots, n\} \rightarrow \{0, 1, \ldots, m\}$ is a one-to-one function to map the $\{1, \ldots, n\}$ to $\{0, 1, \ldots, m\}$. When $\theta(i) > 0$, it means that the path $d_i$ is not blocked and associated with $z_{\theta(i)}$. When $\theta(i) = 0$, it means that the path $d_i$ is blocked and associated with $\emptyset$. It should be noted that association sequence $\theta$ is a one-to-one function that ensures that each path can be associated to only one measurement and vice versa. The likelihood of an association sequence $\theta_k$ at time $k$ can then be defined as

$$L(\theta_k) = \prod_{\theta_k(i) = 0} (1 - P_d) \prod_{\theta_k(i) > 0} P_d l_k(z_{\theta_k(i)}|d_i)$$ (2.2)

For JPDAF, the association sequence $\theta_k^{opt}$ that maximizes the association likelihood $L(\theta_k)$ is treated as the optimal data association result and is used to localize the MD at time $k$. The advantage of JPDAF is memoryless and easy to implement because the association result depends only on the MD’s current state. However, if the data association result is incorrect in one step, it may affect the performance of the following localization of the MD. In [65], a TOA-based indoor tracking algorithm is proposed that uses JPDAF to associate measurements with their corresponding VRDs and then uses conventional EKF to track the MD. In [62], a TOA-based indoor tracking algorithm extended from [65] is proposed by integrating the process of covariance estimation of each path. In [60, 79], joint TOA- and AOA-based indoor localization algorithms are proposed that use JPDAF to associate measurements with their corresponding VRDs and then use the conventional weighted least squares method to localize the MD. Without prior MD information, these algorithms require both TOA and AOA information to eliminate incorrect association sequences.

For MHTF, the filter records all possible association sequences and corresponding association likelihoods from the initial time to the current time. The MD is then
localized using the association sequence with maximum $\prod_k L(\theta_k)$. The advantage of MHTF is its greater resistance to the incorrect association at a single step because the MD is localized using the entire past association history. However, because the MHTF requires that every association sequence be recorded, the computational load grows exponentially. Algorithms usually set a maximum number of hypotheses, and a hypothesis with a low possibility will be truncated when the total number of hypotheses exceeds the threshold. Compared to JPDAF, the MHTF achieves greater accuracy but has a larger computational load. [61] proposed a joint TOA- and AOA-based indoor tracking algorithm that uses MHTF to associate measurements and then uses conventional PF to track the MD.

For the TOA-based indoor localization algorithm, the existing data association framework cannot be applied in a straightforward manner because the prior information of the MD is unknown. Therefore, a new data association framework should be proposed to associate measurements with their corresponding paths for indoor localization using multipath components.

### 2.2 RFS based Indoor Tracking Filter

Conventional tracking and mapping algorithms model both the MD and the map features as a random vector, and the MD and map features can then be jointly estimated [80,82]. However, each of these filters requires additional processes such as data association, clutter filtering, and map management [61,83]. [83] proposed an indoor tracking scheme by modeling both LOS and reflections and using JPDAF to associate measurements with corresponding path origins; this approach has the
potential to construct the surrounding map during the tracking process. Proposed an indoor tracking and mapping algorithm by modeling LOS, reflections, and point scattering paths and using MHTF to associate measurements with their path origins. As mentioned, the results of these methods are sensitive to data association uncertainty. The RFS-based Bayesian filters, such as the Rao-Blackwellized probability hypothesis density (RB-PHD) filter and single cluster PHD (SC-PHD) filter, are robust to data association uncertainty and more accurate and stable. Therefore, this thesis focuses on RFS-based tracking algorithms.

2.2.1 Background of RFS and PHD

The concept of RFS was first proposed by [87], whose elements and cardinality are random variables. For an RFS, $M$, with random cardinality $|M|$, it can be denoted as

$$M = \{m^1, m^2, \ldots, m^{|M|}\}$$

(2.3)

where $m^j$ is the $j^{th}$ element in $M$. The probability density function (PDF) of the RFS, $M$, can then be expressed as

$$f(M) = \begin{cases} f(\emptyset), & \text{if } |M| = 0 \\ \frac{1}{|M|!} f(\{m^1, \ldots, m^{|M|}\}), & \text{otherwise.} \end{cases}$$

(2.4)

where $f(\emptyset)$ and $f(\{m^1, \ldots, m^{|M|}\})/|M|!$ represents the probability of the empty set and the set $\{m^1, \ldots, m^{|M|}\}$ with cardinality as $|M|$, respectively. $1/|M|!$ comes from the fact that all permutations of $\{m^1, \ldots, m^{|M|}\}$ represent the same set, the probability should then be divided by $|M|!$. $f(M)$ also satisfies the unity equation as
\[ \int_M f(M)\delta M = f(\emptyset) + \sum_{|M|=1}^{\infty} \frac{1}{|M|!} \int f(\{m^1, \ldots, m^{|M|}\}) dm^1 \ldots dm^{|M|} = 1 \quad (2.5) \]

where \( \delta M \) represents the infinitesimal region on the finite set space \( M \). The RFS-based multitarget Bayesian filter can be derived based on RFS theory. However, it is computationally intractable because the calculation relies on the set integral, which involves the integration of an infinite number of elements as shown in (2.5) \[73\]. To resolve this issue, the concept of PHD is proposed.

The PHD is the first-order statistical moment of an RFS, whose integral on any region of the state space is the expected number of targets in that region \[70\]. The PHD of \( M \) can be denoted as \( D_M(m) \), which represents the expected target density at \( m \), and is given as

\[ D_M(m) = \int \delta_M(m) f(M) \delta M \quad (2.6) \]

where \( \delta_M(m) = \sum_{m' \in M} \delta(m - m') \), and \( \delta(\cdot) \) is the Dirac function. The PHD \( D_M(m) \) is defined on vector space instead of set space, which means that it is computationally tractable. If we assume that the RFS, \( M \), is a Poisson point process, the relationship between \( f(M) \) and its PHD \( D_M(m) \) can be given as

\[ f(M) = \exp^{-N} D_M(m^1) \ldots D_M(m^{|M|}) \quad (2.7) \]

for any \( M = \{m^1, \ldots, m^{|M|}\} \) with \( m^1, \ldots, m^{|M|} \) distinct and \( N = \int D_M(m) dm \).

By modeling the multiple targets as an RFS, the PHD of the RFS can be propagated recursively to simultaneously estimate the number and state of the multitarget,
2.2. RFS BASED INDOOR TRACKING FILTER

which is called PHD filter. The PHD filter was first proposed by [70] for outdoor radar-based multitarget tracking. An extension of the PHD filter considering higher-order statistics in the target number of the RFS was proposed by [88], which is called the cardinalized PHD (CPHD) filter. The CPHD filter has better accuracy than the PHD filter because it accounts for the second-order statistics, but it is less computationally efficient. Neither the PHD filter nor the CPHD filter have the theoretical solution. The computationally efficient implementation of the PHD filter and CPHD filter is via the use of Gaussian mixture (GM) filters, which are called GM-PHD [89] filters and GM-CPHD filters [90], respectively.

For indoor tracking with inaccurate floor plan map application, the number of map features detected by the MD varies over time. Therefore, the map features can be modeled as an RFS. Assuming that one MD is used, the MD can be modeled as a random vector. The MD and the map features can then be jointly estimated by recursive propagation of the distribution of the MD and PHD of the map features, which are referred as PHD-based tracking and mapping filters. The first PHD-based tracking and mapping filter was proposed by [71] by assuming that the prior and posterior distributions of map features are both Poisson processes, which is called a RB-PHD filter. In [72], a PHD-based tracking and mapping filter only assumes that the prior distribution of map features is a Poisson process, which is called an SC-PHD filter. Because the SC-PHD filter removes the assumption that the posterior distribution of map features is a Poisson process; this thesis focuses on the SC-PHD filter.
2.2. SC-PHD filter

Given measurement history $Z_{1:k-1}$, the joint posterior probability distribution of the MD and map features at time $k - 1$ can be denoted as $p_{k-1}(x_{k-1}, M_{k-1}|Z_{1:k-1})$. The prediction and update for the joint probability density of the MD and map can then be written as

$$p_{k|k-1}(x_k, M_k|Z_{1:k-1}) = \int \int f_{k|k-1}(x_k, M_k|x_{k-1}, M_{k-1})$$

$$\times p_{k-1}(x_{k-1}, M_{k-1}|Z_{1:k-1})dx_{k-1}\delta M_{k-1}$$

(2.8)

$$p_{k|k}(x_k, M_k|Z_{1:k}) = \frac{p_{k|k-1}(x_k, M_k|Z_{1:k-1})L_k(Z_k|x_k, M_k)}{\int p_{k|k-1}(x_k, M_k|Z_{1:k-1})L_k(Z_k|x_k, M_k)dx_k}\delta M_k$$

where $f_{k|k-1}(x_k, M_k|x_{k-1}, M_{k-1})$ is the joint Markov transition density at time $k$. $L_k(Z_k|x_k, M_k)$ is the measurement likelihood at time $k$. The prediction and update formulas involve a set integral that is computationally intractable. Therefore, the SC-PHD filter is introduced and can be given as [72]:

- **Prediction equation**

$$v_{k|k-1}(x_k, m) = \int f_{k|k-1}(x_k|x_{k-1})p_{k-1}(x_{k-1}|Z_{1:k-1})v_{k|k-1}(m|x_{k-1})dx_{k-1}$$

$$v_{k|k-1}(m|x_{k-1}) = b_k(m|x_k) + v_{k-1}(m|x_{k-1})$$

(2.9)

where $v_{k|k-1}(x_k, m)$ and $v_{k|k-1}(m|x_{k-1})$ represent the predicted joint PHD of the MD and map features at time $k$ and the predicted conditional PHD of the map features at time $k$, respectively. $f_{k|k-1}(x_k|x_{k-1})$ and $p_{k-1}(x_{k-1}|Z_{1:k-1})$ denote the marginal Markov transition density of the MD at time $k$ and the posterior distribution of the MD at time $k-1$, respectively. $b_k(m|x_k)$ denotes the conditional PHD of new birth map features at time $k$. $v_{k-1}(m|x_{k-1})$ represents
2.2. RFS BASED INDOOR TRACKING FILTER

the conditional posterior PHD of the map features at time $k - 1$.

- Update equation

$$v_k(x_k, m) = \frac{p_{k|k-1}(x_k|Z_{1:k-1}) L_{Z_k}(x_k)}{\int p_{k|k-1}(x_k|Z_{1:k-1}) L_{Z_k}(x_k) dx_k} v_k(m|x_k)$$

$$v_k(m|x_k) = \left(1 - P_d(m|x_k) + \sum_{z \in Z_k} \frac{P_d(m|x_k) l_z(m|x_k)}{\Gamma_z(m|x_k)}\right) v_{k|k-1}(m|x_k) \tag{2.10}$$

$$\Gamma_z(m|x_k) = c(z) + \int P_d(m|x_k) l_z(m|x_k) v_{k|k-1}(m|x_k) dm$$

where $v_k(x_k, m)$ and $v_k(m|x_k)$ denote the updated joint PHD of the MD and map features at time $k$ and the updated conditionally PHD of the map features at time $k$, respectively. $p_{k|k-1}(x_k|Z_{1:k-1})$ represents the predicted distribution of the MD at time $k$. $P_d(m|x_k)$ and $l_z(m|x_k)$ denote the detection probability of map feature $m$ and the single-object measurement likelihood of $z$, respectively. $L_{Z_k}(x_k)$ is the multi-object likelihood function of the measurement set $Z_k$, which is given as

$$L_{Z_k}(x_k) = \exp^{-P_d(m|x_k) v_{k|k-1}(m|x_k)} \prod_{z \in Z_k} \Gamma_z(m|x_k)$$

The SC-PHD filter was first proposed by \[86\] for outdoor radar-based scenario. In \[86\], the SC-PHD filter only works for static environments, constant detection probabilities, and known clutter statistics. By modeling the movement model of dynamic map features, the SC-PHD filter proposed by \[72\] works in more complex environments that contain both dynamic and static map features. In \[91\], the SC-PHD filter is applied for underwater vehicle tracking and mapping, in which the environment is more dynamic and noisy. The SC-PHD filter assumes that only one MD is present, but it can be extended to cases with multiple MDs. \[92\] proposed a
collaborative multiple MD tracking and mapping algorithm by modeling the MD as an RFS, in which the number and state of the detected MD are estimated simultaneously. In practice, the detection probability is not constant. [93, 94] modeled the statistics of the detection probability of laser sensors, which showed better accuracy. Also, the clutter rate is usually unknown and should be estimated. [95] developed the SC-PHD filter for joint estimation of the clutter statistics and the state of the MD and map features. [96] applied the SC-PHD filter to a scenario in which the measurement noise is dependent upon the environment dynamics. By modeling the measurement noise as dependent upon the environmental dynamics, the derived SC-PHD filter is more robust to the dynamic environment, as with underwater tracking and mapping. The SC-PHD filter can be implemented using the P-GM filter by modeling the MD as a particle filter and the map features as a GM filter [72]; however, it has been reported that the conventional importance-weighting method used in the P-GM filter may be tend to diverge. [97] proposed a new importance weighting method to overcome this issue and found it to be more robust than conventional methods.

The multiple RD version of the SC-PHD filter (MSC-PHD filter), is computationally intractable because the number of partitions of measurements and associations of measurements in each partition increases combinatorically [98]. [99] proposed an iterated-corrector approximation method to address this issue. The iterated-corrector approximation iteratively updates the PHD of the MD and map features using measurements from a single RD each time. The updated PHD of the MD and map features from the previous RD provide the predicted PHD for the next RD, and it is updated again using the measurements from the next RD. However, it has been
2.2. **RFS BASED INDOOR TRACKING FILTER**

reported that the estimation results depend upon the order in which the RDs are processed \[100\]. An alternative method to mitigate the computational load is to directly reduce the number of partitions and associations in each partition \[98, 101, 104\]. \[101\] proposed a distance partition method for extended target tracking. It groups closed measurements into the same cell and then partitions the cells. \[85, 105\] incorporated an MHTF into the SC-PHD filter to truncate the partitions with low hypothesis. \[98\] proposed a \(l_{\text{max}}\) partition method for multi-detection tracking. It restricts the number of elements in each measurement subset to no more than the number of propagation paths \(l\). \[102\] proposed a two-step partition algorithm that includes a distance partition and a \(l_{\text{max}}\) partition. Measurements can be initially partitioned by using the distance partition with a self-defined multi-detection distance. By applying the \(l_{\text{max}}\) partition to the partitioned measurement set, the number of resulting partitions can then be further reduced. \[103, 104\] proposed a greedy partition method to greatly reduce the number of partitions. It considers only a certain number of partitions with the highest weight at each step and discards other partitions. However, these methods were proposed for tracking, which only involves uncertainties of the MD. If they are extended to indoor tracking and mapping, the uncertainties of the map features must also be taken into account.

To the best of our knowledge, almost all SC-PHD filters were developed for outdoor radar-based scenario in which the measurements are backscattering, which can be considered a single reflection in indoor. To extend it for indoor scenario, the LOS path should be taken into consideration because the LOS path carries information regarding the MD. Therefore, if the LOS path can be formulated in the SC-PHD filter, the localization accuracy will be improved, which will result in better mapping
accuracy. As mentioned, the SC-PHD filter assumes that the map feature is modeled as a Poisson point process, so the generated single reflection path can also be characterized as a Poisson point process. proposed an SC-PHD filter to incorporate the LOS path, which is modeled as a Poisson process. However, the number of LOS paths detected by each RD can only be one or zero, which means that it should be modeled as a Bernoulli point process.

In addition, to the best of our knowledge, all SC-PHD filters assume that each map feature generates no more than one measurement. Thus, if single reflections are considered effective measurements, then higher-order reflections can only be considered clutter. However, higher-order reflections carry information regarding the MD and map features that will improve the accuracy of localization and mapping. Many double and higher-order reflections exist in an indoor multipath environment. If such reflection paths can be modeled in the SC-PHD filter to perform indoor tracking and mapping, both the mapping and the localization accuracy will be improved. To incorporate multiple reflections into the SC-PHD filter, the map features involved in multiple reflections must be decoupled. By considering each multiple reflection as a virtual single reflection generated by a single map feature, the associations among map features can be decoupled. The multiple reflections issue can then be considered a virtual multi-detection problem. The concept of multi-detection is widely used in the topic of extended object tracking. In extended object tracking, the extended object is modeled as a collection of point scatterers, and the detected multiple measurements are mainly backscattering and are spatially close to each other. In this thesis, the map features are modeled as separate plane reflectors. The multiple measurements refer to multipath propagation involving more than one object. In this
case, the detected multiple measurements are not spatially close to each other. For example, the single and double reflections from the same reflector may be far apart.

In short, to extend conventional SC-PHD filters for use in indoor tracking, the LOS and multiple reflection paths should be formulated with the single reflection paths. In addition, the computational intractable issue of the MSC-PHD filter should be handled to account for the uncertainties of both MD and map features. Furthermore, the mapping accuracy of planar scatterers should be evaluated.

2.3 Conclusions

This chapter reviews state-of-the-art data association based indoor localization and tracking algorithms. It shows that conventional data association frameworks require prior information on the MD to associate measurements with their corresponding paths. For TOA-based indoor localization without prior information of the MD, a new data association scheme should be designed. In addition, the SC-PHD filter and its multiple RD version are reviewed. It shows that conventional SC-PHD filters are designed for outdoor radar-based frameworks. To extend them for indoor tracking purposes, the LOS path and the multiple reflection paths should be modeled.
Chapter 3

TOA based Indoor Localization
Algorithm with Accurate Map

As presented in the last chapter, the localization accuracy can be improved if the multipath components can be correctly associated with their corresponding VRDs. Conventional data association methods such as joint probabilistic data association filter (JPDAF) \cite{38, 62, 65, 76} and multiple hypothesis tracking filter (MHTF) \cite{61, 66, 77, 78} require prior information regarding the MD to estimate the association result. In this chapter, a new TOA-based indoor localization algorithm with the knowledge of floor plan is proposed. According to the floor plan, the data association is performed using a new grid-based scheme. By introducing the concept of a data association matrix, the data association process is integrated with the conventional two-step weighted least squares localization method. The remainder of this chapter is organized as follows. The problem formulation, including the multipath propagation model and the measurement model, is given in Section 3.1. Section 3.2 gives the two-step weighed least squares localization algorithm, the proposed data association
3.1 Problem Formulation

3.1.1 Multipath Propagation Model

By introducing the concept of VRD, all reflection paths can be considered to be a virtual LOS path that originates from the MD to the corresponding VRD. Fig. 3.1 shows the example of traced single and double reflection paths to their corresponding VRD. As mentioned, the planar reflector is expressed as \( m = (\varphi, \rho) \) based on Hessian normal form \( x \cos \varphi + y \sin \varphi = \rho \), where \( \varphi \) denotes the angle between the planar reflector and the y-axis, and \( \rho \) is the distance from the origin to the planar reflector.

The \( s^{th} \) RD is denoted as \( RD_s \) which is placed at known location \( p_0^s = (x_0^s, y_0^s) \). As shown in Fig. 3.1, there are two planar reflectors, plane \( i \) and plane \( j \), that are denoted as \( m^i = (\varphi^i, \rho^i) \) and \( m^j = (\varphi^j, \rho^j) \), respectively. The \( VRD_s^i \) is the image position.

\[ \text{Figure 3.1: Single and double reflection paths traced to their corresponding VRD.} \]
3.1 PROBLEM FORMULATION

The VRD
d of \( RD_s \) reflected from \( m^i \), which is referred to as a first-order VRD with position \( p^i = (x^i_s, y^i_s) \). The \( p^i \) can be calculated using image theory by reflecting \( RD_s \) via \( m^i \) as

\[
\begin{align*}
x^i_s &= -x_s \cos 2\varphi^i - y_s \sin 2\varphi^i + 2\rho^i \cos \varphi^i \\
y^i_s &= -x_s \sin 2\varphi^i + y_s \cos 2\varphi^i + 2\rho^i \sin \varphi^i
\end{align*}
\] (3.1)

The \( VRD^{i,j}_s \) is the image position of \( RD_s \) reflected from \((m^i, m^j)\) at time \( k \), which is referred to as a second-order VRD with position \( p^{i,j}_s = (x^{i,j}_s, y^{i,j}_s) \). The \( p^{i,j}_s \) can be calculated using (3.1) by reflecting \( VRD^i_s \) via \( m^j \). In general, the \( m^{th} \) order VRD of \( RD_s \) reflected from a vector of map features \((m^i, m^j, m^l, \ldots)\), where the number of map features in the vector is \( m \), is denoted as \( VRD^{i,j,l,\ldots}_s \) with position \( p^{i,j,l,\ldots}_s = (x^{i,j,l,\ldots}_s, y^{i,j,l,\ldots}_s) \). The \( p^{i,j,l,\ldots}_s \) can be calculated using (3.1) by reflecting \( RD_s \) through the vector of map features \((m^i, m^j, m^l, \ldots)\) one by one. It should be noted that the RD can be considered as a zero-order VRD.

Fig. 3.2 shows an example of multipath propagation in a typical indoor environment consisting of an LOS path, a single reflection path, and a double reflection path.
3.1. PROBLEM FORMULATION

As shown, all reflection paths can be treated as virtual LOS paths that originate from an MD to their corresponding VRDs. For example, the single reflection path that originates from MD reflected from $m^2$ to $RD_s$ is treated as a virtual LOS path that originates from MD to $VRD^2_s$, which is the first-order VRD of $RD_s$ reflected from $m^2$. The double reflection path that originates from MD reflected from $(m^2, m^3)$ to $RD_s$ is treated as a virtual LOS path that originates from MD to $VRD^{2,3}_s$, which is the second-order VRD of $RD_s$ reflected from $(m^2, m^3)$ one by one.

In the remainder of this chapter, for ease of presentation, the superscript of VRD that represents the collection of reflected map features is omitted. The collection of VRDs of $RD_s$ is denoted as

$$VRD_s = \{VRD^t_s, t = 0, \ldots, N_s\}. \quad (3.2)$$

where $N_s$ is the number of VRDs, or in other words, the number of reflection paths considered. It should be noted that when $t = 0$, the $VRD^t_s$ is just the $RD_s$. When $t > 0$, the position of $VRD^t_s$, denoted as $p^t_s = (x^t_s, y^t_s)$, can be calculated using (3.1).

3.1.2 Measurement Model

By introducing the VRD, all reflection paths can be considered as virtual LOS paths. Suppose that the location of MD is $p = (x, y)$, then the measured TOA by $RD_s$ which is traced to $VRD^t_s$, can be expressed as

$$z^t_s = d^t_s + \varepsilon^t_s = \sqrt{(x - x^t_s)^2 + (y - y^t_s)^2} + \varepsilon^t_s \quad (3.3)$$
3.2. PROPOSED TOA BASED INDOOR LOCALIZATION ALGORITHM

where \(\varepsilon_s^t\) represents the Gaussian measurement noise with zero mean and standard deviation of \(\sigma_s^t\). \(d_s^t\) denotes the noiseless path length. When \(t = 0\), (3.3) represents the measurement of the LOS paths. When \(t > 0\), (3.3) represents measurements of reflection paths.

Some measurements for each RD cannot be traced to any VRD and may come from other scatterers such as ceilings and floors. These scatterers are not considered in the model, and the generated measurements are treated as clutter, which is denoted as \(C_s\) for clutter received at \(RD_s\). It should also be noted that some LOS or reflection paths in the real environment may be blocked by obstructers, which means that some VRDs cannot be assigned any measurement. Therefore, the TOA measured at \(RD_s\), denoted as \(\tilde{z}_s\), contains two types of measurements: effective measurements (unblocked LOS and reflection paths) and clutter. To localize the MD, the data association process should be performed to filter out the clutter and estimate the correct association between the measurements and the corresponding VRDs.

3.2 Proposed TOA based Indoor Localization Algorithm

3.2.1 Two-step weighted least squares localization algorithm

Suppose that the data association has been performed, the clutter has been filtered out, and the correct association result is estimated. Suppose that the number of associated measurements of \(RD_s\) is \(M_s\). It is reasonable to assume that \(M_s \leq 1 + N_s\) because of the path blockage. The MD can be localized with a two-step weighted least squares algorithm using associated paths, which will be summarized as follows. 

\[ \text{106} \]
By squaring both sides of (3.3) and rearranging it, we can get

\[(z_t^s)^2 - k^t_s = -2x_t^s x - 2y_t^s y + x^2 + y^2 + 2z_t^s \varepsilon_s + \varepsilon_s^2 \]  

(3.4)

where \(k^t_s = (x_t^s)^2 + (y_t^s)^2\) is a constant related to the position of \(VRD_t^s\). It should be noted that \(t = \{1, \ldots, M_s\}\) now. Defining \(h^t_s = (z_t^s)^2 - k^t_s\) and \(R^2 = x^2 + y^2\), then (3.4) can be rewritten as

\[n^t_s = h^t_s - (-2x_t^s x - 2y_t^s y + R^2) \]  

(3.5)

where \(n^t_s = 2z_t^s \varepsilon_s\) represents the noise term, and the higher-order noise \(\varepsilon_s^2\) is neglected. It should be noted that (3.5) holds for every \(s = \{1, \ldots, S\}\) and \(t = \{1, \ldots, M_s\}\). By defining \(p_a = [x, y, R^2]^T\) and assuming that \(R^2\) is independent of \(x\) and \(y\), (3.5) can be arranged in matrix form as

\[\mathbf{n} = \mathbf{h} - \mathbf{Gp}_a \]  

(3.6)

where \(\mathbf{G} = [\mathbf{G}_1^T, \ldots, \mathbf{G}_S^T]^T\), \(\mathbf{h} = [\mathbf{h}_1^T, \ldots, \mathbf{h}_S^T]^T\) and \(\mathbf{n} = [\mathbf{n}_1^T, \ldots, \mathbf{n}_S^T]^T\), where \([\cdot]^T\) indicates transposition of a matrix. The submatrices \(\mathbf{G}_s\), \(\mathbf{h}_s\), and \(\mathbf{n}_s\) can be expressed as

\[
\mathbf{G}_s = \begin{bmatrix}
-2x_0^s & -2y_0^s & 1 \\
-2x_1^s & -2y_1^s & 1 \\
\vdots & \vdots & \vdots \\
-2x_{M_s}^s & -2y_{M_s}^s & 1
\end{bmatrix}, \quad \mathbf{h}_s = \begin{bmatrix}
(z_0^s)^2 - (k_0^s)^2 \\
(z_1^s)^2 - (k_1^s)^2 \\
\vdots \\
(z_{M_s}^s)^2 - (k_{M_s}^s)^2
\end{bmatrix}, \quad \mathbf{n}_s = \begin{bmatrix}
n_0^s \\
n_1^s \\
\vdots \\
n_{M_s}^s
\end{bmatrix}
\]
\( p_a \) can then be estimated using the weighted least squares method by minimizing the mean square residual \( \mathbb{E}(n^T \Psi^{-1} n) \), which is given as

\[
p_a = \arg \min_{p_a} (h - G p_a)^T \Psi^{-1} (h - G p_a)
\]

\[
= (G^T \Psi^{-1} G)^{-1} G^T \Psi^{-1} h
\]  

(3.7)

where \( \mathbb{E}(\cdot) \) and \( (\cdot)^{-1} \) represent the expectation of a random variable and the inverse of a matrix, respectively. \( \Psi \) is the covariance matrix of the noise term, which is a diagonal matrix whose principal diagonal is \( \text{diag}(\Psi) = [\text{diag}(\Psi_1), \ldots, \text{diag}(\Psi_S)] \), where \( \text{diag}(\cdot) \) represents the main diagonal of the given diagonal matrix. The submatrix \( \Psi_s \) can be expressed as

\[
\Psi_s = \mathbb{E}(n_s n_s^T) = 4B_s Q_s B_s
\]

\[
\text{diag}(B_s) = [d_0^s, \ldots, d^s_{M_s}], \quad Q_s = \sigma_z^2 I_{M_s}
\]  

(3.8)

where \( I_{M_s} \) represents \( M_s \times M_s \) identity matrix. The covariance matrix of \( p_a \) is given as \( \Psi_a = (G^T \Psi^{-1} G)^{-1} \). It should be noted that \( B_s \) contains the noiseless path length information, \( [d_0^s, \ldots, d^s_{M_s}] \), which is currently unknown. It is solved with the use of associated measurements, \( [z_0^s, \ldots, z^s_{M_s}] \), as the noiseless path length to estimate \( p_a \). After estimating an initial \( p_a \), the noiseless path lengths can be calculated. The \( p_a \) can then be re-estimated using the new calculated noiseless path lengths. After several iterations, we obtain the final \( p_a \). It should be noted that \( p_a \) assumes that \( R^2 \) is independent of \( x \) and \( y \). The dependency relationship between them can be imposed via \( R^2 = x^2 + y^2 \). By defining \( p_b = [x^2, y^2]^T \), the error of the previous step
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estimation can be represented as

\[ n' = h' - G' p_b \]

\[ h' = \begin{bmatrix} [p_a]_1^2 \\ [p_a]_2^2 \\ [p_a]_3^2 \end{bmatrix}, \quad G' = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \]  \hspace{1cm} (3.9)

where \([p_a]_i\) represents the \(i^{th}\) element of \(p_a\). \(p_b\) can also be estimated with the weighted least squares method, which is given as

\[ p_b = (G'^T \Psi'^{-1} G')^{-1} G'^T \Psi'^{-1} h' \]  \hspace{1cm} (3.10)

where

\[ \Psi' = E(n'n'^T) = 4B'\Psi B' \]

\[ \text{diag}(B') = [x, y, 0.5] \]

The final estimated MD location \(p = \sqrt{p_b}\) or \(p = -\sqrt{p_b}\). The sign of \(p\) coincides with first two elements of \(p_a\).

### 3.2.2 Proposed Grid-based Data Association Method

To localize the MD using the two-step weighted least squares method, all clutter should be filtered out, and the correct associations between the measurements and the corresponding VRD should be estimated, which means that data association should be performed without any prior location information regarding the MD. In this section, the grid-based data association algorithm is proposed to handle this
issue. A given accurate floor plan can be divided into grid points. At each grid point, a set of noiseless path lengths to each RD and VRD can be calculated. Suppose that the calculated noiseless path lengths at grid \( i \) are denoted as \( d_{s,i} \) whose element is denoted as \( d_{s,i,t} \) where \( t = \{0, \ldots, N_s\} \). Suppose that the measured data set is denoted as \( \tilde{z}_s \) whose element is denoted as \( \tilde{z}_{s,r} \) where \( r = \{1, \ldots, R_s\} \). Then, at each grid point, the data association process is to assign the elements in \( \tilde{z}_s \) to the elements in \( d_{s,i} \) and make the overall difference minimum. However, because \( R_s \) is usually nonequal to \( N_s \), comparing two vectors with different sizes is not straightforward. To handle this issue, the optimal subpattern assignment (OSPA) metric is introduced.

The OSPA metric proposed in \([107]\) is used to define the difference between two sets. For two sets \( X \) and \( Y \) with cardinality \( |X| \leq |Y| \), the OSPA distance \( \overline{d}_p^{(c)}(X, Y) \) between \( X \) and \( Y \) with some \( c > 0 \) and \( 1 \leq p < \infty \) is defined as

\[
\overline{d}_p^{(c)}(X, Y) = \left( \frac{1}{|Y|} \left( \min_{\pi \in \Pi_{|Y|}} \sum_{i=1}^{|X|} d^{(c)}(x_i, y_{\pi(i)})^p + c^p(|X| - |Y|) \right) \right)^{1/p} \tag{3.11}
\]

where \( d^{(c)}(x, y) = \min(c, d(x, y)) \) denotes an arbitrary distance between \( x \) and \( y \) cut off at \( c \). \( \Pi_{|Y|} \) represents the set of permutations of \( \{1, 2, \ldots, |Y|\} \) for any \( |Y| \in \mathbb{N} \). The first part of the OSPA distance involves assigning elements in set \( X \) to elements in set \( Y \) and determining the assignment with the minimum difference. The second part of the OSPA distance accounts for the difference in cardinality.

Therefore, the first step of data association is to determine the permutation of \( \tilde{z}_s \) using the first part of the OSPA metric, which minimizes the difference between \( \tilde{z}_s \)
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and $d_s$. Assuming $1 + N_s \leq R_s$, this process can be expressed as

$$\pi = \arg \min_{\pi \in \Pi_{R_s}} \sum_{t=0}^{N_s} d_t^{(c)} (d_{s,i}^{\pi(t)}, z_t^{\pi(t)})^p$$

(3.12)

where $c$ is usually selected around three times of measurement noise $\sigma_z$ [108]. After this process, the first $N_s$ elements in the permutation $\pi$ represent indices of $\tilde{z}_s$ that have been assigned to elements in $d_{s,i}$. However, (3.12) assigns a measurement to every path, even if the path is blocked, in which case the distance difference $|d_{s,i}^{\pi(t)} - \tilde{z}_s^{\pi(t)}| > c$, where $|\cdot|$ represents the absolute value. To avoid this issue and integrate the data association process with the two-step weighted least squares method, the concept of a data association matrix is proposed. At each grid $i$, define a $(1 + N_s) \times (1 + N_s)$ path association matrix $P^p_{s,i}$ and a $(1 + N_s) \times R_s$ measurement association matrix $P^m_{s,i}$. Initially, all elements in both matrices are zero. To account for the data association result, some elements will be assigned to 1 as

$$P^p_{s,i}(t, t) = \begin{cases} 
1, & \text{if } |d_{s,i}^{t} - \tilde{z}_s^{\pi(t)}| < c \\
0, & \text{otherwise.}
\end{cases}$$

(3.13)

$$P^m_{s,i}(t, r) = \begin{cases} 
1, & \text{if } \pi(t) = r \text{ and } |d_{s,i}^{t} - \tilde{z}_s^{\pi(t)}| < c \\
0, & \text{otherwise.}
\end{cases}$$

(3.14)

It should be noted that each measurement and path can only be associated once, then each row and column in both data association matrices have at most a single 1. Some rows in both data association matrices may contain only 0. A row of all zeros in $P^p_{s,i}$ means that the corresponding path is blocked so that no measurement is associated with it. Similarly, a row of all zeros in $P^m_{s,i}$ indicates that the corresponding
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measurement is a clutter, so no path is associated with it.

To integrate the data association process in the two-step weighted least squares method, the (3.6) can be reformulated as

\[ n_i = b_i - A_i p_a \]  \hspace{1cm} (3.15)

where \( A_i = [A_{i,1}^T, \ldots, A_{i,S}^T]^T \) and \( b_i = [b_{i,1}^T, \ldots, b_{i,S}^T]^T \). The definition of \( n_i \) is similar to (3.6) which represents the noise term at grid point \( i \). The submatrix \( A_{s,i} \) and \( b_{s,i} \) can be expressed as

\[ A_{s,i} = P_{s,i}^p G_{s,i} \]  \hspace{1cm} (3.16)

\[ b_{s,i} = (P_{s,i}^m \tilde{z})^2 - P_{s,i}^p k_s \]

where \((\cdot)^2\) indicates the square of each element of a given vector. It should be noted that several rows in both data association matrices may be zero, in which case the corresponding rows in the noise term \( n_i \) are zero, which means that the errors of the unassociated paths are zeros, which is unreasonable. To solve this issue, the rows in \( n_i \) that correspond to all-zero rows in the data association matrices would be assigned a sufficiently large value to penalize the unassociated paths.

At each grid point \( i \), we can calculate the mean square residual \( E(n_i^T \Psi^{-1} n_i) \) using (3.15). The possible MD position will then be considered near the grids with the minimum square residual. The final estimation of \( P_p \) and \( P_m \) can then be determined by performing data association at the centroid of \( H \) grids with the minimum square residual. Then \( p_a \) could be estimated using the weighted least squares method, which is given as

\[ \hat{p}_a = \arg \min_{p_a, P_p, P_m} E(n^T \Psi^{-1} n) \]  \hspace{1cm} (3.17)
3.3. EXPERIMENTAL AND SIMULATION RESULTS

where $\Psi$ is the covariance matrix of the noise term. Estimation of $\hat{\mathbf{p}}_b$ and the final MD position follow (3.9) and (3.10) in the last section.

3.3 Experimental and Simulation Results

In this section, the simulation and experimental results of the proposed TOA-based indoor localization algorithm were examined in various situations. The first situation, which considers only LOS paths with perfect data association results, is called LOS-PDA. The second situation, which considers both LOS and NLOS paths with perfect data association results, is called multipath-PDA. The LOS-PDA and the multipath-PDA are both used as benchmarks. The third situation, which considers only the LOS path but assumes that the shortest path is the LOS path, is called LOS-DA. The fourth situation, which considers both LOS and NLOS paths but requires data association to associate multipath components with their corresponding VRDs, is called multipath-DA. The number of grids, $H$, with the minimum square residual used
3.3. EXPERIMENTAL AND SIMULATION RESULTS

Figure 3.4: Comparison of localization performance: (a) Average localization error (ALE) with different levels of measurement noise $\sigma_z^0$, (b) Cumulative distribution function (CDF) of the localization error (LE) when measurement noise $\sigma_z^0 = 0.2m$.

...to estimate the final association matrices is set to 6. By using four RDs, localization using multipaths become unimodal problem which means the weighted average of $H$ grids will be almost same no matter how $H$ varies. For less RDs situation, it requires further studying. The simulated and experimental environments are shown in Fig. 3.3. The environment was a closed meeting room environment with dimensions of $8.3m \times 7.3m$ in the INFINITUS laboratory at the School of EEE, Nanyang Technological University (NTU). Four RDs are placed at the corners of the meeting room with coordinates of (1.4, 1), (1.4, 6.3), (7.1, 6.1), and (7.3, 1) respectively. The MD is placed at a $4 \times 4$ rectangular grid with 1-m intervals between each grid, for a total of 16 positions, with coordinates from (2.4, 2) to (5.4, 5).

3.3.1 Simulation Results

The performance comparison of the proposed algorithms with different levels of measurement noise $\sigma_z^t$ was given. The measurement noise of LOS path $\sigma_z^0$ varied from
0.1m to 0.4m. To account for the reflection loss, the measurement noises of the reflection paths are doubled for each reflection, which means that the measurement noises of the single and double reflection paths were $2\sigma_z^0$ and $4\sigma_z^0$, respectively. Each algorithm was reiterated 25 times using different random sequences to generate measurements. Following references [71, 72, 103], the detection probability of the LOS and reflections were set as 0.9. The pillar shown in Fig. 3.3 was considered to be a point scatterer that generates clutter. The average localization error (ALE) of the MD are presented in Fig. 3.4(a). The multipath-DA achieved ALE between 0.15m and 0.4m when $\sigma_z^0$ was varied from 0.1m to 0.4m. The multipath-DA performs even better than the LOS-PDA because the number of LOS paths is insufficient to localize the MD at some points. This result shows the ability of the proposed multipath-DA to work in situations with an insufficient number of LOS paths. When $\sigma_z^0 = 0.2m$, the cumulative distribution function (CDF) of the localization error (LE) of the MD are presented in Fig. 3.4(b). The proposed multipath-DA algorithm achieved performance in LE around 0.4m, for 90% of the time, which outperformed LOS-DA by around 60% in terms of LE. Suppose that the measurement noises of the reflection paths $\sigma_z^t$ for $t > 0$ are equal to the measurement noise of the LOS path $\sigma_z^0$. The localization results show a larger performance margin; the details are omitted to save space.

Also, we compared the proposed multipath-DA with another multipath based single RD localization algorithm proposed in [109], which is called maximum likelihood function with hard-mapping decision denoted as ML-HD. To make fair comparison, both multipath-DA and ML-HD were simulated using single RD, which is $RD_1$, with LOS measurement noise $\sigma_z^0 = 0.2m$. Other parameters are kept same and both
3.3. EXPERIMENTAL AND SIMULATION RESULTS

Figure 3.5: Comparison of localization performance using $RD_1$ when measurement noise $\sigma_z^0 = 0.2m$.

algorithms were reiterated 25 times using different random sequences to generate measurements. The CDF of the LE of the MD are presented in Fig. 3.5. The proposed multipath-DA algorithm achieved performance in LE around 1.3m, for 90% of the time by using single RD, which outperformed ML-HD by around 30% in terms of LE.

3.3.2 Experimental Results

A measurement campaign was carried out in the environment, as shown in Fig. 3.3. The channel response between the MD and the four RDs was measured using a single-input single-output (SISO) system (Agilent Technologies PNA-X Network Analyzer N5244A) in a frequency domain with a 500MHz bandwidth centered at 2.4GHz over 201 frequency points [110]. It should be noted that because of the limitation in the number of ports in the PNA, the measurements between the MD point to the four RDs are not performed at the same time. Assuming that the channel is static during
3.3. EXPERIMENTAL AND SIMULATION RESULTS

Figure 3.6: Comparison of the cumulative distribution function (CDF) of the localization error (LE).

The experiment, the channel response is measured one RD by one RD for each MD point. For the remainder of the thesis, all channel responses between an MD and multiple RDs are measured in this manner. The collected channel responses were then used to estimate the TOA using an expectation maximization algorithm [111]. For each MD point, the number of multipaths estimated at each RD is 10. The estimated measurement noise of the LOS path is around 0.19m.

For the experimental results, the correct data association results are unknown and thus required estimation for each MD point. Therefore, the performance compared only the multipath-DA and LOS-DA for experimental data. It should be noted that the LOS paths existed for all MDs and RDs during the experiment which means that the detection probability of LOS path is 1. The cumulative distribution function (CDF) performance of the localization error (LE) is shown in Fig. 3.6. As shown,
3.3. EXPERIMENTAL AND SIMULATION RESULTS

![Localization error distribution](image)

Figure 3.7: Comparison of localization performance using $RD_1$ using measurement data when detection probability $P_d = 1$.

When the $P_d = 1$, the proposed multipath-DA achieved performance in LE around 0.5$m$, for 90% of the time, while the LOS-DA achieved performance in LE around 0.4$m$, for 90% of the time. This is because that the multipath-DA may use wrong multipath to localize the MD, while the LOS-DA always use the correct LOS path to localize the MD because the detection probability of LOS is 1. We randomly removed the estimated multipaths components in the measurement data to decrease the detection probability of each path. As shown in Fig. 3.6, the proposed multipath-DA and LOS-DA achieved similar performance in LE around 0.7$m$, for 90% of the time when $P_d = 0.9$. When $P_d = 0.8$, the proposed multipath-DA achieved performance in LE around 0.9$m$, for 90% of the time, which outperformed the LOS-DA by around 30%. The experimental results show that as the detection probability decreasing, the proposed multipath-DA performs quite stable compared to LOS-DA. It means that the proposed multipath-DA is able to handle situations with insufficient number of LOS paths.
3.4. CONCLUSIONS

Again, we compared the proposed multipath-DA with the ML-HD proposed in [109] using single RD, which is $RD_1$ with $P_d = 1$. Other parameters are kept same and both algorithms were reiterated 25 times using different random sequences to generate measurements. The CDF of the LE of the MD are presented in Fig. 3.5. The proposed multipath-DA algorithm achieved performance in LE around 2.3m, for 90% of the time by using single RD, which outperformed ML-HD by around 30% in terms of LE.

3.4 Conclusions

In this chapter, a novel TOA-based indoor localization algorithm is proposed that uses multipath components with accurate knowledge of the floor plan. The NLOS paths are associated with their corresponding VRDs with the proposed grid-based data association method. The data association process is integrated with the two-step weighted least squares method by the proposed data association matrix. The experimental and simulation results show that the proposed TOA-based indoor localization algorithm using multipath components outperformed the conventional TOA-based indoor localization algorithm using LOS only in terms of localization accuracy.
Chapter 4

TOA based Indoor Tracking
Algorithm with Accurate Map

The last chapter proposed a TOA-based indoor localization algorithm with an accurate floor plan. To use multipath components, a grid-based data association algorithm is proposed to associate measurements with the corresponding VRDs. For indoor tracking, prior information regarding the MD is available, which means that data association can be performed in a straightforward manner using the previous MD position. Because the literature contains many indoor tracking algorithms that use multipath components, this chapter discusses two typical algorithms and provides a transition between the indoor localization with accurate floor plan presented in the last chapter and the indoor tracking with inaccurate floor plan, which is presented in next two chapters. The remainder of this chapter is organized as follows. The problem formulation, including the MD movement model and measurement model, are given in Section 4.1. Section 4.2 gives the indoor tracking algorithms, including EKF with data association and PF with data association. The experimental and
4.1 Problem Formulation

Tracking the MD can be considered as localizing a set of related MD positions. The relationship is called the state transition model (or movement model), i.e.

\[ x_k = f(x_{k-1}, u_{k-1}) \] (4.1)

where \( f(\cdot) \) is the MD movement model. \( x_k \) and \( x_{k-1} \) are the states of the MD at time \( k \) and time \( k - 1 \), respectively. \( u_{k-1} \) is the movement noise. In the 2-D space, the state of the MD at time \( k \) can be denoted as \( x_k = [p_k^T, v_k^T]^T \), where \( p_k = [x_k, y_k]^T \) and \( v_k = [\dot{x}_k, \dot{y}_k]^T \) represent the position and velocity of the MD at time \( k \), respectively. Assuming that \( f(\cdot) \) is a linear Gaussian constant-velocity model, the state transition model of the MD can be expressed as

\[ x_k = Fx_{k-1} + Gu_{k-1} = \begin{bmatrix} I_2 & \Delta T \cdot I_2 \\ 0_2 & I_2 \end{bmatrix} x_{k-1} + \begin{bmatrix} \frac{\Delta T^2}{2} \cdot I_2 \\ \Delta T \cdot I_2 \end{bmatrix} u_{k-1} \] (4.2)

where \( \Delta T \) denotes the sampling interval. \( u_{k-1} \) is the Gaussian distributed movement noise with zero mean and covariance matrix \( \sigma_a^2 I_2 \), where \( \sigma_a \) is the acceleration noise in either direction. \( I_N \) and \( 0_N \) represent the \( N \times N \) identity and the zero matrix, respectively.

The measurement models for each time \( k \) are the same as those presented in Section 3.1.2. For each RD, the measurements \( \tilde{z}_{k,s} \) contain two parts that are effective measurements \( z_{k,s} \) including the unblocked LOS path and reflection paths, which can
be traced to the corresponding VRDs, and clutter. Suppose the collection of VRDs of \( RD_s \) is denoted as

\[
VRD_s = \{VRD^t_s, t = 0, \ldots, N_s\}. \tag{4.3}
\]

where \( N_s \) is the number of VRDs (or the number of reflection paths) considered. It should be noted that when \( t = 0 \), the \( VRD^t_s \) is just the \( RD_s \). When \( t > 0 \), the position of \( VRD^t_s \), denoted as \( p^t_s = (x^t_s, y^t_s) \), can be calculated using image theory, as shown in (3.1). The effective measurements collected at \( RD_s \) traced to \( VRD^t_s \) can then be expressed as

\[
z^t_{k,s} = h^t_s(x_k, n^t_{k,s}) = \sqrt{(x_k - x^t_s)^2 + (y_k - y^t_s)^2} + n^t_{k,s} \tag{4.4}
\]

where \( h^t_s(\cdot) \) denotes the measurement model, where measurements are traced to \( VRD^t_s \), \( n^t_{k,s} \) is the Gaussian-distributed measurement noise with zero mean and standard deviation of \( \sigma^t_z \). (4.4) represents the measurement of the LOS path when \( t = 0 \) and the reflection paths when \( t > 0 \).

Assuming that data association has been done, the associated measurements from all RDs at time \( k \) can be concatenated and denoted as \( z_k \). Suppose that the associated measurement history from the initial time to time \( k - 1 \) is denoted as \( z_{1:k-1} \); the posterior density of the MD at time \( k - 1 \) can be expressed as \( p_{k-1}(x_{k-1}|z_{1:k-1}) \). The predicted density of the MD at time \( k \), denoted as \( p_{k|k-1}(x_k|z_{1:k-1}) \), can then be expressed as

\[
p_{k|k-1}(x_k|z_{1:k-1}) = \int p_{k-1}(x_{k-1}|z_{1:k-1}) f(x_k|x_{k-1}) dx_{k-1} \tag{4.5}
\]
4.2 TOA BASED INDOOR TRACKING ALGORITHM

where \( f(x_k|x_{k-1}) \) is the Markov state transition function, which is given by movement model (4.2). At time \( k \), when the RDs collect new associated measurements \( z_k \), the updated distribution of the MD at time \( k \), denoted as \( p_k(x_k|z_{1:k}) \) can then be expressed as

\[
p_k(x_k|z_{1:k}) = \frac{l(z_k|x_k)p_{k|k-1}(x_k|z_{1:k-1})}{\int l(z_k|x_k)p_{k|k-1}(x_k|z_{1:k-1})dx_k}
\]

where \( l(z_k|x_k) \) is the measurement likelihood function, which is given by the measurement model (4.4). (4.5) and (4.6) represent the Bayesian tracking filter. The Bayesian filter propagates the full distribution of the MD, which is computationally inefficient. One must resort to conventional methods to propagate the first- and second-order moments such as the EKF or to use particles to approximate the distribution of the MD such as the PF.

4.2 TOA based Indoor Tracking Algorithm

According to the movement model (4.2), we can predict the MD at time \( k \) based on the estimated MD at time \( k - 1 \). The noiseless path length can be calculated from the predicted MD to all RDs and VRDs. Data association can then be performed based on the first part of the OSPA metric, as shown in Section 3.2.2. After data association, the MD can be tracked with the conventional EKF or PF.

4.2.1 EKF with Data Association

It should be noted that the EKF with data association has been proposed in [65]. Here, we quickly recap it using the representation of data association matrix. The EKF assumes that the state of the MD is a random variable with an approximately
4.2. TOA BASED INDOOR TRACKING ALGORITHM

Gaussian distribution that can be characterized by its mean and covariance matrix. In addition, to linearize the nonlinear measurement function, it is approximated by the first-order Taylor expansion. The state of the MD can then be recursively estimated by propagating the mean and covariance matrix of the MD. Suppose that the estimated MD at time $k - 1$ is $\mathbf{x}_{k-1}$ with mean $\bar{x}_{k-1}$ and covariance $\mathbf{P}_{k-1}$. The mean and covariance of the predicted MD can then be calculated using the movement model (4.2) as

$$
\begin{align*}
\bar{x}_{k|k-1} &= F \bar{x}_{k-1} \\
\mathbf{P}_{k|k-1} &= F \mathbf{P}_{k-1} F^T + G \mathbf{Q} \mathbf{P}_{k-1} G^T
\end{align*}
$$

where $\mathbf{Q}_{k-1}$ is the covariance matrix of the acceleration noise, which can be expressed as $\mathbb{E}(\mathbf{u}_{k-1} \mathbf{u}_{k-1}^T)$. $\mathbf{F}$ and $\mathbf{G}$ are given in the movement model (4.2). Based on the predicted MD, the predicted path length can be calculated and denoted as $d_k$. Data association can then be performed between the path length $d_k$ and the measurements $\tilde{z}_k$ using (3.12) and (3.13). Suppose that the estimated path association matrix and measurement association matrix are given as $\mathbf{P}^p$ and $\mathbf{P}^m$, respectively. By removing the all-zero rows from both association matrices, the mean and covariance of the updated MD can be expressed as

$$
\begin{align*}
\mathbf{x}_k &= \bar{x}_{k|k-1} + \mathbf{K}_k (\mathbf{P}^m \tilde{z}_k - \mathbf{P}^p d_k) \\
\mathbf{P}_k &= (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k|k-1}
\end{align*}
$$

where

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4.2. TOA BASED INDOOR TRACKING ALGORITHM

where $K_k$ and $H_k$ are the Kalman gain and the Jacobian matrix of the measurement model at time $k$, respectively, which are given as

$$K_k = \mathbf{P}_{k|k-1}^T H_k^T (\mathbf{P}_p R_k + H_k \mathbf{P}_{k|k-1} H_k^T)^{-1}$$

$$H_k = \mathbf{P}_p \bigoplus_{s,t} \frac{\partial h^s_t}{\partial x_{k|k-1}}_{x_k = \bar{x}_k|k-1}$$

where $\mathbf{R}_k$ is the diagonal covariance matrix of measurement noise with diagonal elements as $(\sigma_z^2)^2$. $\bigoplus_{s,t}$ represents vertical concatenation for all $s$ and $t$. $\mathbf{I}$ in (4.8) represents the identity matrix. The state of the MD at time $k$ can then be estimated as $\bar{x}_k$. By recursive propagation of the mean and covariance, the state of the MD can be estimated at every time $k$.

4.2.2 PF with Data Association

EKF assumes that the MD has a Gaussian distribution and uses first-order Taylor expansion to approximate the nonlinear measurement function, whereas the PF does not need these assumptions. The PF represents the distribution of the MD, either Gaussian or non-Gaussian, with a set of particles with associated weights and estimates the MD using the particles and corresponding weights. As the number of particles increases, the particles and their associated weights may characterize the distribution of the MD without any loss of information, so the PF is approximately the optimal Bayesian tracking filter. Suppose that the particles and associated weights at time $k - 1$ are given as $\{x_{k-1}^{(i)} \}$ and $\{\omega_{k-1}^{(i)} \}$, respectively, where $N_p$ is the number of particles. The posterior density of the MD at time $k - 1$ can
4.2. TOA BASED INDOOR TRACKING ALGORITHM

then be expressed as

$$p_{k-1}(x_{k-1}|z_{1:k-1}) = \sum_{i=1}^{N_p} \omega_{k-1}^{(i)} \delta(x_{k-1} - x_{k-1}^{(i)}) \quad (4.9)$$

where $\delta(\cdot)$ is the Dirac function. Suppose that other $N_p$ particles are sampled at time $k$, denoted as $x_k^{(i)}$, according to some importance density $q(x_k|x_{k-1}^{(i)}, z_k)$. The particles can then be expressed as

$$x_k^{(i)} \sim q(x_k|x_{k-1}^{(i)}, z_k) \quad (4.10)$$

According to (4.5) and (4.6), the weight of particles at time $k$ can be calculated as

$$\omega_k^{(i)} = \omega_{k-1}^{(i)} \frac{f(x_k^{(i)}|x_{k-1}^{(i)})l(z_k|x_k^{(i)})}{q(x_k^{(i)}|x_{k-1}^{(i)}, z_k)} \quad (4.11)$$

where $f(x_k^{(i)}|x_{k-1}^{(i)})$ and $l(z_k|x_k^{(i)})$ are the Markov transition function and the measurement likelihood function given in (4.5) and (4.6), respectively. The most commonly used importance function to sample $x_k^{(i)}$ is the Markov transition function $f(x_k^{(i)}|x_{k-1}^{(i)})$.

In this case, the sampled particles and the weight of the particles at time $k$ can be expressed as

$$x_k^{(i)} \sim f(x_k^{(i)}|x_{k-1}^{(i)}), \quad \omega_k^{(i)} = \omega_{k-1}^{(i)} l(z_k|x_k^{(i)}) \quad (4.12)$$

The posterior density of the MD at time $k$ can be approximated by new particles and associated weights as

$$p_k(x_k|z_{1:k}) = \sum_{i=1}^{N_p} \omega_k^{(i)} \delta(x_k - x_k^{(i)}) \quad (4.13)$$

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4.2. TOA BASED INDOOR TRACKING ALGORITHM

For each particle $x_k^{(i)}$, the noiseless path length to all RDs and VRDs can be calculated and denoted as $d_k^{(i)}$. Suppose that data association is performed between $d_k^{(i)}$ and measurement $z_k$, and the estimated data association matrices are denoted as $P^m_i$ and $P^p_i$. Suppose that the number of associated paths of $RD_s$ is $M_s^{(i)}$, then the total number of associated and blocked paths are $M^{(i)} = \sum_s M_s^{(i)}$ and $N - M^{(i)}$, respectively, where $N = \sum_s (1 + N_s)$ is the total number of paths considered. By removing the all-zero rows in both data association matrices, the likelihood function $l(z_k|x_k^{(i)})$ can then be given as

$$l(z_k|x_k^{(i)}) = ((1 - P_d)^{(N-M^{(i)})} + (P_d)^{M^{(i)}} l_{n_z}(P^m_i z_k|P^p_i d_k^{(i)}))^{1/M^{(i)}}$$

(4.14)

where $P_d$ is the path detection probability. Here we assume that the path detection probability is the same for all paths. $l_{n_z}(\cdot)$ is a distribution characterizing measurement noise that is given by the measurement model (4.4). By recursive propagation of the particles and associated weights, the state of the MD can be estimated at every time $k$.

The particle filter may cause a degeneracy problem, which means that after a few iterations, all particles but a few have negligible weights. This issue implies that most computations are used for particles that make a negligible contribution to approximate the posterior distribution of $p_k(x_k|z_{1:k})$. It has been shown that the degeneracy problem is unavoidable [112]. To mitigate this issue, the resampling strategy can be used whenever the degeneracy issue is observed. Resampling involves sampling new $N_s$ particles from the estimated posterior of the MD at time $k$ according to (4.13). Many efficient resampling schemes have been developed, such as stratified sampling, residual sampling, and systematic resampling [112].
4.3 Experimental and Simulation Results

In this section, the simulation and experimental results of the presented TOA-based indoor TOA tracking filters were examined in various situations. The first situation considers only LOS paths with perfect data association results using EKF and PF filters to track the MD, which are called EKF-LOS-PDA and PF-LOS-PDA, respectively. The second situation considers both LOS and NLOS paths with perfect data association results using EKF and PF filters to track the MD, which are called EKF-multipath-PDA and PF-multipath-PDA, respectively. With either EKF or PF, both LOS-PDA and multipath-PDA are used as benchmarks. The third situation considers only the LOS path but assumes that the shortest path is the LOS path using EKF and PF to track the MD, which are called EKF-LOS-DA and PF-LOS-DA, respectively. The fourth situation considers both LOS and NLOS paths using EKF and PF to track the MD, but it requires data association to associate multipath components with their corresponding VRDs, which are called EKF-multipath-DA and PF-multipath-DA, respectively.

To simulate the particle filters, the number of particles used to model the MD was 200. The simulated and experimental environment is shown in Fig. 4.1. The environment was a half-enclosed experimental area in the INFINITUS laboratory at the School of EEE, Nanyang Technological University (NTU), with dimensions of 14.5$m \times 5.7$m. Three RDs were placed at known positions with coordinates (3,3), (3,6.7), and (8.9,6.7). The trajectory of the MD consisted of 49 points spaced by 20cm, starting from the upper right corner. The MD movement model and the TOA measurement model (4.2) and (4.4) were used for all filters.
4.3. EXPERIMENTAL AND SIMULATION RESULTS

Table 4.1: Reflector Trajectory of MD

<table>
<thead>
<tr>
<th>x (m)</th>
<th>y (m)</th>
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<tbody>
<tr>
<td>0</td>
<td>-2</td>
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</tbody>
</table>

Metallic Cabinet Wall Window Pillar

Figure 4.1: Experimental and simulation environment: (a) Vertical view of the environment layout. (b) Snapshot of the experimental environment.

4.3.1 Simulation Results

The performance comparison of the proposed filters with various levels of measurement noise \( \sigma_z^i \) was given. The measurement noise of the LOS path \( \sigma_z^0 \) varied from 0.1m to 0.4m. To account for reflection loss, the measurement noise of the reflection paths is doubled for each reflection, which means that the measurement noise values of single and double reflection paths were \( 2\sigma_z^0 \) and \( 4\sigma_z^0 \), respectively. Each algorithm was reiterated 25 times using different random sequences to generate measurements. Following references [71, 72, 103], the detection probabilities of LOS and reflections were set to 0.9. The pillar shown in Fig. 4.1 was considered as a point scatterer that would generate clutter. The MD is initialized using an accurate MD position. The ALE of the MD is presented in Fig. 4.2(a). EKF-multipath-DA and PF-multipath-DA achieved similar ALE ranging from 0.1m to 0.35m when \( \sigma_z^0 \) varied from 0.1m to 0.4m. When \( \sigma_z^0 = 0.2m \), the CDF of the LE of the MD are presented in Fig. 4.2(b). The EKF-multipath-DA and PF-multipath-DA achieved similar performance in LE...
4.3. EXPERIMENTAL AND SIMULATION RESULTS

Figure 4.2: Comparison of tracking performance: (a) Average localization error (ALE) with different levels of measurement noise $\sigma_z^0$, (b) Cumulative distribution function (CDF) of the localization error (LE) when measurement noise $\sigma_z^0 = 0.2m$.

around 0.3m, for 90% of the time, which outperformed EKF-LOS-DA and PF-LOS-DA by around 60%. With the use of $\sigma_z^{mr} = \sigma_z^l$, the localization and mapping results show a larger performance margin; the details are omitted for brevity.

Compared the results presented in Fig. 4.2 with results in Fig. 3.4, there were some similarities and differences. As shown, the multipath-PDA for localization and tracking achieved similar ALE ranging from 0.05m to 0.2m when $\sigma_z^0$ varied from 0.1m to 0.4m. This is because that the data association results are known then the multipath-PDA localization process does not rely on the prior information of the MD to estimate the data association result. The LOS-PDA for tracking achieved ALE ranging from 0.1m to 0.3m when $\sigma_z^0$ varied from 0.1m to 0.4m, which outperformed the LOS-PDA for localization with ALE ranging from 0.2m to 0.45m. This is because that sometimes there is not enough LOS path to localize the MD due to the LOS path blockage, while the EKF-LOS-PDA and PF-LOS-PDA still can utilize the
prior information of the MD to track the position of the MD. The multipath-DA for tracking achieved ALE ranging from 0.1m to 0.35m when $\sigma_z^0$ varied from 0.1m to 0.4m, which outperformed the multipath-DA for localization with ALE ranging from 0.15m to 0.4m. This is because that the prior information of the MD improves the data association accuracy for EKF-multipath-DA and PF-multipath-DA. The LOS-DA for tracking achieved ALE ranging from 0.35m to 0.4m when $\sigma_z^0$ varied from 0.1m to 0.4m, which outperformed the LOS-DA for localization with ALE ranging from 0.35m to 0.55m. This is because that sometimes LOS-DA treats reflection paths as LOS paths and utilized to localize the MD when LOS path is blocked, while the EKF-LOS-PDA and PF-LOS-PDA can utilize the prior information of the MD to help to mitigate the error caused by considering reflection paths as LOS paths to track the MD.

The performance of the proposed filters was examined with an inaccurate initial MD position. In the simulation, the measurement noise of the LOS path $\sigma_z^0 = 0.2m$ and the other parameters remained the same as before. Each filter was reiterated 25 times using different random sequences, and the initial MD position was uniformly distributed in a $1m \times 1m$ area around the exact MD position. The cumulative distribution function (CDF) performance of the localization error (LE) is shown in Fig 4.3. EKF-multipath-DA and PF-multipath-DA achieved similar performance in LE of around 0.4m 90% of the time, which outperformed EKF-LOS-DA and PF-LOS-DA by around 55% in terms of localization accuracy.
4.3 EXPERIMENTAL AND SIMULATION RESULTS

Figure 4.3: Comparison of cumulative distribution function (CDF) with uniformly distributed initial MD.

4.3.2 Experimental Results

A measurement campaign was carried out in the environment, as shown in Fig. 4.1. The channel response between the MD and the three RDs was measured with a single-input single-output (SISO) system, the Agilent Technologies PNA-X Network Analyzer N5244A, in a frequency domain with 500MHz bandwidth centered at 2.4GHz over 201 frequency points [110]. These measurements were then used to estimate the TOA using an expectation maximization algorithm [111]. For each snapshot, the number of multipaths estimated at each RD is 10. Because LOS paths existed for all MDs and RDs during the experiment, the shortest path for each estimation can be treated as LOS path. Comparing the estimated shortest path measurement and the real LOS path length, the measurement noise of LOS path $\sigma_z^0$ is around 0.19m. And the measurement noise of the reflection paths is doubled for each reflection.
4.3. EXPERIMENTAL AND SIMULATION RESULTS

The performance of the proposed filters was examined using measurement data. The correct data association results for the experimental results are unknown and must be estimated at each time $k$. Therefore, the performance was only compared between EKF-multipath-DA, PF-multipath-DA, EKF-LOS-DA, and PF-LOS-DA for the experimental data. It should be noted that the LOS paths existed for all MDs and RDs during the experiment which means that the detection probability of LOS path is 1. Each filter was reiterated 25 times using different random sequences and initialization with perfect knowledge of MD. The CDF performance of LE is shown in Fig. 4.4. As shown, when $P_d = 1$, the EKF-multipath-DA and PF-multipath-DA both achieved similar performance in LE of around $0.6m$ for 90% of the time, while the LOS-DA achieved performance in LE around $0.3m$, for 90% of the time. This performance gap comes from the reason that the multipath-DA may use wrong multipath
4.3. EXPERIMENTAL AND SIMULATION RESULTS

Figure 4.5: Comparison of cumulative distribution function (CDF) with uniformly distributed initial MD using measurement data.

to track the MD, while the LOS-DA always use the correct LOS path to track the MD because the detection probability of LOS is 1. Similar with last chapter, we randomly removed the estimated multipaths components in the measurement data to decrease the detection probability of each path. When $P_d = 0.7$, the proposed multipath-DA achieved performance in LE around 0.8m, for 90% of the time, which outperformed the LOS-DA by around 55%. The experimental results show that as the detection probability decreasing, the proposed multipath-DA performs quite stable compared to LOS-DA. It means that the proposed multipath-DA is able to handle situations with insufficient number of LOS paths.

The performance of the proposed filters with an inaccurate initial MD position was examined using measurement data. In this case, we also simulated an multi-path based tracking algorithm proposed in [65], which is called maximum likelihood
function algorithm and denoted as ML. The correct data association results for the experimental results are unknown and must be estimated at each time \( k \). Each filter was reiterated 25 times using different random sequences, and the initial MD position was uniformly distributed in a \( 1m \times 1m \) area around the exact MD position. The cumulative distribution function (CDF) performance of the localization error (LE) with different detection probability is shown in Fig 4.3. Again, there is gap between the proposed multipath-DA and LOS-DA situation when \( P_d = 1 \), which comes from the reason that the multipath-DA may use wrong multipath to track the MD, while the LOS-DA always use the correct LOS path to track the MD because the detection probability of LOS is 1. However, when there are some paths not detected and \( P_d = 0.7 \), the proposed multipath-DA achieved performance in LE around 0.9m, for 90% of the time, which outperformed the LOS-DA by around 50%. The results show that both the ML and proposed multipath-DA outperformed the LOS-DA when some of the LOS paths are blocked, which express the necessity of incorporating multipath components for indoor tracking.

4.4 Conclusions

This chapter presents two TOA-based indoor tracking algorithms that use multipath components with the help of a floor plan. Data association can be performed based on the prior information regarding the MD provided by the movement model. The proposed data association matrices can be used to combine the data association process with conventional tracking filters such as EKF and PF. The simulation and experimental results show that the proposed TOA-based indoor tracking algorithms that use multipath components outperform conventional TOA-based indoor tracking.
filters that use only the LOS path in terms of the localization accuracy.
Chapter 5

TOA based Indoor Tracking via LMSC-PHD Filtering

The capability of localizing and tracking an MD using multipath components when an accurate floor plan is available is shown in last two chapters. In this chapter, localization and tracking of the MD using multipath components when the accurate floor plan is unavailable will be discussed. In this case, the MD and map should be jointly estimated. By modeling the map as a collection of map features, such as lines, corners, and points, it can be jointly estimated with the MD via a multi-RD single-cluster probability hypothesis density (MSC-PHD) filter. Conventional MSC-PHD filters are used for outdoor radar based scenario where only backscattering (or single reflection) paths are taken into account. In indoor scenario, the LOS path should also be modeled because LOS path carries information of the MD which will improve the localization and mapping accuracy. In this chapter, a new MSC-PHD filter incorporates LOS path is proposed, which is called LOS incorporated MSC-PHD (LMSC-PHD) filter. The remainder of this chapter is arranged as follows.
5.1. Problem Formulation

The problem formulation including the MD and map feature movement model and multipath components measurement model are proposed in Section 5.1. Section 5.2 gives the proposed LMSC-PHD filter and implementation details. The experimental and simulation results are given in Section 5.4. The last section gives the conclusion.

5.1 Problem Formulation

Considering the scenario of performing tracking using TOA in an indoor multipath environment. Considering a two-dimensional (2-D) space, suppose RDs are placed at known position with coordinates $p_s = [x_s, y_s]^T$, where $s = \{1, \ldots, S\}$ and $S$ represents the number of RDs. The state of the MD at time $k$, can be expressed as $x_k = [p_k^T, v_k^T]^T$, where $p_k = [x_k, y_k]^T$ and $v_k = [\dot{x}_k, \dot{y}_k]^T$ represent the position and velocity of MD at time step $k$, respectively. Considering reflectors only, the reflectors can be modeled as straight lines based on Hessian normal form: $x \cos \varphi_k + y \sin \varphi_k = \rho_k$, with map features $m_k = [\varphi_k, \rho_k]^T$, where $\varphi_k$ denotes the angle between the line and y-axis at time $k$ and $\rho_k$ represents the distance between the line and origin at time $k$.

5.1.1 Dynamic Model of MD and Map Feature

Assuming a linear Gaussian constant-velocity model, then the state transition model for the MD is

$$x_k = F x_{k-1} + G u_{k-1} = \begin{bmatrix} I_2 & \Delta T \cdot I_2 \\ 0_2 & I_2 \end{bmatrix} x_{k-1} + \begin{bmatrix} \Delta T^2 \cdot I_2 \\ \Delta T \cdot I_2 \end{bmatrix} u_{k-1} \quad (5.1)$$
5.1. PROBLEM FORMULATION

where $u_{k-1}$ is the driving acceleration noise with zero mean and covariance matrix $\sigma_a^2 \mathbf{I}_2$, $\sigma_a$ is the acceleration noise in either direction. $\mathbf{I}_N$ and $\mathbf{0}_N$ represents $N \times N$ identity and zero matrix, respectively. $\Delta T$ denotes the sampling interval.

Due to path blockage, the number of detected map features, denoted as $|M_k|$, is time variant. Therefore, the collection of detected map features should be modeled as an RFS [70]. The $i^{th}$ map feature at time $k$ is denoted as $m_i^k = [\varphi_i^k, \rho_i^k]^T$. Assuming all of the map features are static, then the dynamical model of the map features can be expressed as

$$M_k = M_{k-1} \cup B_k = \left\{m_1^k, \ldots, m_{|M_k|}^k\right\}$$  \hspace{1cm} (5.2)

where $M_{k-1}$ represents the map features estimated at time $k-1$. $B_k$ representing the new birth map features detected at time $k$, which can be modeled as a Poisson point process with intensity $b_k = \mu_k I(\cdot)$ [70], where $\mu_k$ is the new birth rate, which means the average number of new birth map features. $I(\cdot)$ is the distribution of new birth map features. The actual distribution of $I(\cdot)$ is problem dependent, which can be modeled as Gaussian distribution with either fixed [89] or measurement dependent [72] mean and covariance.

5.1.2 Measurement Model

Considering LOS and single reflection paths only, then each map feature can generate at most one measurement. Given the $RD_s$ with position $p_s = (x_s, y_s)$ and $MD_k$ with position $p_k = (x_k, y_k)$, the measured LOS TOA by $RD_s$ at time $k$ can be defined as

$$z_{k,s}^l = h^l(p_k, p_s) + n_{k,s}^l = \sqrt{(x_k - x_s)^2 + (y_k - y_s)^2} + n_{k,s}^l$$  \hspace{1cm} (5.3)
where $n_{l,s}^k$ represents the Gaussian measurement noise of the LOS path with zero mean and standard deviation of $\sigma_z^l$. The measured TOA of single reflection paths originated from $MD_k$ reflected from $i^{th}$ map feature $m_i^k$ to $RD_s$ can be expressed as

$$z_{k,s}^i = h^l(p_k, m_i^k, p_s) + n_{l,s}^k = h^l(p_k, p_s^i) + n_{l,s}^i$$  \hspace{1cm} (5.4)

where $n_{l,s}^i$ represents the Gaussian measurement noise of single reflections, with zero mean and standard deviation of $\sigma_z^{lr}$. $p_s^i = (x_s^i, y_s^i)$ denotes the VRD position by reflecting RD through $m_i^k$, which can be calculated according to image theory as

$$x_s^i = -x_s \cos 2\varphi^i - y_s \sin 2\varphi^i + 2\rho^i \cos \varphi^i$$

$$y_s^i = -x_s \sin 2\varphi^i + y_s \cos 2\varphi^i + 2\rho^i \sin \varphi^i$$  \hspace{1cm} (5.5)

The number of measurements received at each RD depends on the detected map features and clutter, which is time variant. Therefore, the collection of measurements should be modeled as an RFS:

$$Z_{k,s} = \mathcal{D}(x_k) \cup \mathcal{D}(x_k, M_{k,s}) \cup C_{k,s}$$  \hspace{1cm} (5.6)
expressed as $Z_k = Z_{k,1} \cup \cdots \cup Z_{k,S}$.

Therefore, the prediction and update for the joint probability density of the MD and map can be written as

$$p_{k|k-1}(x_k, M_k | Z_{1:k-1}) = \int f_{k|k-1}(x_k, M_k | x_{k-1}, M_{k-1}) \times p_{k-1}(x_{k-1}, M_{k-1} | Z_{1:k-1}) dx_{k-1} \delta M_{k-1}$$

$$p_k(x_k, M_k | Z_{1:k}) = \frac{\int p_{k|k-1}(x_k, M_k | Z_{1:k-1}) L_k(Z_k | x_k, M_k) dx_k \delta M_k}{\int \int p_{k|k-1}(x_k, M_k | Z_{1:k-1}) L_k(Z_k | x_k, M_k) dx_k \delta M_k}$$

where $f_{k|k-1}(x_k, M_k | x_{k-1}, M_{k-1})$ and $L_k(Z_k | x_k, M_k)$ are joint Markov transition density at time $k$ and multi-object measurement likelihood function at time $k$, respectively. $p_{k-1}(x_{k-1}, M_{k-1} | Z_{1:k-1})$ is the joint posterior density of the MD and map at time $k - 1$. It should be noted that the integrals of map features are set integrals, so the recursion of joint probability density is computationally intractable.

### 5.1.3 Likelihood Model

As stated, the measurements are modeled as an RFS, including the LOS path, single reflections, and clutter. For any nonempty measurement subset $W \in Z_k$, given the MD state $x_k$ and map feature $m_i^k$, the multi-object likelihood function can be calculated as follows. If subset $W$ represents a collection of LOS measurements, the multi-object likelihood function can be expressed as

$$L_W^l(x_k) = \begin{cases} \prod_{s \notin W} (1 - P_d^{l,s}) \prod_{z_s \in W} P_d^{l,s} p_{z_s}^{l,s}, & \text{if } W_s = \{z_s\} \\ 0, & \text{otherwise.} \end{cases}$$
where \( W_s \) denotes the collection of elements in \( W \) measured at \( RD_s \). \( P_{d,s}^{l,s} \) and \( l_{z_s}^{l,s} \) are abbreviations of \( P_{d,s}^{l,s}(x_k) \) and \( l_{z_s}^{l,s}(x_k) \), which represent the detection probability and single-object measurement likelihood function of LOS path at \( RD_s \), respectively. According to (5.3), the LOS measurement likelihood can be modeled as \( \mathcal{N}(z; h^r(p_k, p_s), (\sigma_z^r)^2) \), where \( \mathcal{N}(\cdot; \mu, \Sigma) \) represents a Gaussian distribution with mean \( \mu \) and covariance \( \Sigma \). \( s \notin W \) means the collection of RDs whose measurements are not contained in \( W \).

If subset \( W \) represents a collection of single reflection measurements, the multi-object likelihood function can be expressed as

\[
L_W^r = \begin{cases} 
\prod_{s \in W}(1 - P_{d,s}^{l,s}) \prod_{z_s \in W} P_{d,s}^{l,s} l_{z_s}^{l,s}, & \text{if } W_s = \{z_s\} \\
0, & \text{otherwise}.
\end{cases}
\]  

(5.9)

where \( P_{d,s}^{l,s} \) and \( l_{z_s}^{l,s} \) are abbreviations of \( P_{d,s}^{l,s}(m|x_k) \) and \( l_{z_s}^{l,s}(m|x_k) \), which represent the detection probability and measurement likelihood of the single reflection path at \( RD_s \), respectively. According to (5.4), the measurement likelihood of a single reflection can be modeled as \( \mathcal{N}(z; h^r(p_k, p_s^i), (\sigma_z^r)^2) \) given map feature \( m_k^i \).

If subset \( W \) represents clutter, the multi-object likelihood function can be expressed as

\[
c_W = \begin{cases} 
c_s(z_s), & \text{if } W = \{z_s\} \\
0, & \text{otherwise}.
\end{cases}
\]  

(5.10)

where \( c_s(z) \) is intensity of clutter at \( RD_s \) as defined in (5.6).
5.2 Proposed LMSC-PHD Filter

5.2.1 Prediction of LMSC-PHD Filter

Based on the assumption of a static map, the joint prediction of the MD and map features can be expressed as

\[
v_{k|k-1}(x_k, m) = \int f_{k|k-1}(x_k|x_{k-1}) p_{k-1}(x_{k-1}|Z_{1:k-1}) v_{k|k-1}(m|x_{k-1}) \, dx_{k-1}
\]

\[
v_{k|k-1}(m|x_{k-1}) = b_k(m|x_k) + v_{k-1}(m|x_{k-1})
\]

(5.11)

where \( f_{k|k-1}(x_k|x_{k-1}) \) and \( p_{k-1}(x_{k-1}|Z_{1:k-1}) \) represent marginal Markov transition density at time \( k \) and posterior distribution of MD at time \( k-1 \), respectively. \( b_k(m|x_k) \) denotes the conditional PHD of new birth map features at time \( k \). \( v_{k-1}(m|x_{k-1}) \) and \( v_{k|k-1}(m|x_{k-1}) \) represent the conditional posterior PHD of map features at time \( k-1 \) and predicted PHD of map features at time \( k \), respectively.

5.2.2 Update of LMSC-PHD Filter

The joint update of the LMSC-PHD filter is

\[
v_k(x_k, m) = \sum_{\mathcal{P} \subseteq Z_k} \omega_{\mathcal{P}} \frac{p_{k|k-1}(x_k|Z_{1:k-1}) L_{\mathcal{P}}(x_k)}{p_{k|k-1}([L_{\mathcal{P}}])} v_k(m|x_k)
\]

\[
v_k(m|x_k) = \left( 1 - \tilde{P}_d^r + \sum_{W \in \mathcal{P}} (1 - \Omega_W) \frac{L_W}{\Gamma_W} \right) v_{k|k-1}(m|x_k)
\]

(5.12)
5.2. PROPOSED LMSC-PHD FILTER

where

\[
\omega_p = \frac{p_{k|k-1}[L_p]}{\sum_{\mathcal{Q} \subseteq \mathcal{Z}_k} p_{k|k-1}[L_Q]}, \quad L_p(x_k) = \frac{(1 - \tilde{P}_d + \sum_{W \in \mathcal{P}} (L_{W}^{l}/\Gamma_{W})) \prod_{W \in \mathcal{P}} \Gamma_{W}}{\exp^{v_{k|k-1}[P_d]|x_k]} \\
\Omega_W = \frac{L_{W}^{l}/\Gamma_{W}}{1 - \tilde{P}_d + \sum_{W \in \mathcal{P}} (L_{W}^{l}/\Gamma_{W})}, \quad \Gamma_{W} = c_{W} + v_{k|k-1}[L_{W}^{r}|x_k]
\]

\(p_{k|k-1}(x_k|Z_{1:k-1})\) and \(v_k(m|x_k)\) denote the predicted marginal distribution of MD and updated conditional PHD of map features at time \(k\), respectively. \(\omega_p\) and \(L_p(x_k)\) represent the weight and the multi-RD multiobject measurement likelihood of partition \(\mathcal{P}\), respectively. \(\Omega_W\) is the coefficient of LOS path. To update map features, \(\Omega_W\) is subtracted, because the LOS path is independent of map features. \(L_{W}^{l}, L_{W}^{r}, c_{W}\) are likelihood functions defined in (5.8), (5.9), (5.10), respectively. \(1 - \tilde{P}_d = \prod_s (1 - P_{d}^{l,s})\) and \(1 - \tilde{P}_d = \prod_s (1 - P_{d}^{r,s})\) denote the probability that all LOS paths are blocked and all reflection paths are blocked, respectively. \(\mathcal{P} \subseteq \mathcal{Z}_k\) represents the partition of the measurement set \(Z_k\). The number of elements from the same RD cannot be greater than 1, either LOS path or single reflection, otherwise the likelihood function will be zero as shown in (5.8) and (5.9). Thus for each subset \(W \in \mathcal{P}\), the number of elements in \(W\) is no greater than \(l = S\), where \(S\) is the number of RDs, which is referred to as the \(l_{\text{max}}\) partition method \([98]\). The following example shows the partition method with \(Z_k = \{z_{k,1}^1, z_{k,1}^2, z_{k,2}^1\}\) and \(l = 2\):

\[
\mathcal{P}_1 = \left\{ \{z_{k,1}^1\}, \{z_{k,1}^2\}, \{z_{k,2}^1\} \right\}, \quad \mathcal{P}_2 = \left\{ \{z_{k,1}^1, z_{k,2}^1\}, \{z_{k,2}^2\} \right\}, \quad \mathcal{P}_3 = \left\{ \{z_{k,1}^1, z_{k,2}^1\}, \{z_{k,1}^2\} \right\}
\]

(5.13)

The detailed derivation of (5.12) is shown in the Appendix. [A]
5.3 Implementation of LMSC-PHD Filter

The implementation of the proposed LMSC-PHD filter is realized using a particle filter to model the MD and a Gaussian-mixture (GM) filter to characterize map features. Assume the posterior density of the MD and PHD of map features at time $k-1$ are

$$p_{k-1}(x_{k-1}|Z_{1:k-1}) = \sum_{i=1}^{N_{k-1}} \omega_{k-1}^{(i)} \delta(x - x_{k-1}^{(i)})$$

(5.14)

$$v_{k-1}(m|x_{k-1}^{(i)}) = \sum_{j=1}^{J_{k-1}^{(i)}} \nu_{k-1}^{(j,i)} N(m; m_{k-1}^{(j,i)}, P_{k-1}^{(j,i)})$$

(5.15)

where $\delta(\cdot)$ is the Dirac function. $\omega_{k-1}^{(i)}$ and $\nu_{k-1}^{(j,i)}$ are the corresponding weight of particle and GM components, respectively. $N_{k-1}$ and $J_{k-1}^{(i)}$ are the number of particles and number of GM components of the $i^{th}$ particle, respectively.

5.3.1 Implementation of Prediction

According to [5.11], the predicted density of MD can be expressed as

$$p_{k|k-1}(x|Z_{1:k-1}) = \sum_{i=1}^{N_{k|k-1}} \omega_{k|k-1}^{(i)} \delta(x - x_{k}^{(i)})$$

(5.16)

where $N_{k|k-1} = N_{k-1}$, $\omega_{k|k-1}^{(i)} = \omega_{k-1}^{(i)}$ and $x_{k}^{(i)} = Fx_{k-1}^{(i)} + Gn_{k-1,a}$. The predicted PHD of map features can be expressed as

$$v_{k|k-1}(m|x_{k}^{(i)}) = v_{k-1}(m|x_{k-1}^{(i)}) + v_{k}^{b}(m|x_{k})$$

(5.16)
5.3. IMPLEMENTATION OF LMSC-PHD FILTER

where \( v_b^k(m|x_k) \) is the PHD of new birth components, which is assumed with form
\[
v_b^k(m|x_k) = \sum_{j=1}^{f^b_k} \eta^b_j \mathcal{N}(m; m_{bj}^k, P_{bj}^k).
\]
For a TOA based indoor tracking with inaccurate map scheme, the new birth term can be calculated as in [67], using a management process to decide whether a new map feature has been detected. Another way to solve the new birth term follows [113], by utilizing the information from an inaccurate initial map, inputting all possible map features at the initial step, and then removing the new birth term. In this paper, an inaccurate initial map is utilized. Hence the predicted PHD of map features \( v_k|_{k-1}(m|x_k^{(i)}) \) becomes the same as \( v_{k-1}(m|x_{k-1}^{(i)}) \).

5.3.2 Greedy Partition based on OSPA Metric

According to (5.12), to update MD and map features, all possible partitions of measurements should be considered. To avoid the combinational increase of the computation load, a greedy partition method based on the OSPA metric is presented. As shown in Chapter. [3] the OSPA metric is proposed is proposed in [107] and used to define the difference between two sets. For two sets \( X \) and \( Y \) with \( |X| > |Y| \), then
the OSPA distance $\bar{d}_p^c(X,Y)$ between $X$ and $Y$ with some $c > 0$ and $1 \leq p < \infty$ is defined as

$$
\bar{d}_p^c(X,Y) = \left( \frac{1}{|X|} \left( \min_{\pi \in \Pi_k} \sum_{i=1}^{|Y|} d^c(x_i, y_{\pi(i)}) + c^p(|X| - |Y|) \right) \right)^{1/p}
$$

(5.17)

where $d^c(x, y) = \min(c, d(x, y))$ denotes an arbitrary distance between $x$ and $y$ cut off at $c$. $\Pi_k$ represents the set of permutations of $\{1, 2, \ldots, k\}$ for any $k \in \mathbb{N}$.

For each particle $x_k^{(i)}$, a set of noiseless path lengths $Z_{k,s}^o$ including the LOS path and single reflections can be calculated based on the estimated map $\hat{M}_{k-1}$. Then for each particle $x_k^{(i)}$, we can obtain a set of closest measurements $\hat{Z}_{k,s}$ and corresponding cost function $C_k^{(i)} = \bar{d}_1^c(Z_{k,s}^o, Z_{k,s})$ by associating the measurement set $Z_{k,s}$ with the path length set $Z_{k,s}^o$, as shown in Fig. 5.1. As can be seen, the path associated with $\emptyset$ indicates that the corresponding path is not detected. If the measurement is not associated with any path, it is considered clutter. After estimating $\hat{Z}_{k,s}$ for each RD, the measurements generated by the LOS path and each map feature can be concatenated, respectively. Then the partition based on current particle and corresponding cost function $C^{(i)} = \sum_s C_k^{(i)}$ can be constructed. After calculating the cost $C^{(i)}$ of all particles, the $N_{max}^P$ partitions with minimum $C^{(i)}$ are selected as possible partitions.

### 5.3.3 Implementation of Update

After performing the greedy partition, the update of the MD and map features based on each partition can be implemented as follows:

$$
v_k(m|x_k^{(i)}) = v_k^{md}(m|x_k^{(i)}) + \sum_{W \in P} v_k^d(m,W|x_k^{(i)})
$$

(5.18)
where $v_k^{md}$ and $v_k^d$ represent the mis-detected and detected parts of the map features at time $k$, respectively, which can be expressed as

$$v_k^{md}(m|x_k^{(i)}) = (1 - \tilde{P}_d) v_{k|k-1}(m|x_k^{(i)})$$

$$v_k^d(m, W|x_k^{(i)}) = \sum_{j=1}^{J_k^{(i)}} \eta_k^{(j,i)}(W) \mathcal{N}(m; \bar{m}_k^{(j,i)}(W), P_k^{(j,i)}(W))$$

The weight of each updated map feature using subset $W$ can be expressed as

$$\eta_k^{(j,i)}(W) = (1 - \Omega_W^{(i)}) \frac{\eta_k^{(j,i)}(W)}{\Gamma_W^{(i)}}$$

where

$$\Omega_W^{(i)} = \frac{L_W^{(i)}}{1 - \tilde{P}_d + \sum_{W \in P} (L_W^{(i)}/\Gamma_W^{(i)})}$$

$$\Gamma_W^{(i)} = c_W + \sum_j \eta_k^{(j,i)} q^{(j,i)}(W)$$

$$q^{(j,i)} = P_W^r \mathcal{N}(z_W, \mathbf{H}_W \bar{m}_k^{(j,i)} - \mathbf{H}_W \bar{m}_{k|k-1}^{(j,i)} \mathbf{H}_W^T + \mathbf{R}_W)$$

$$P_W^r = \prod_{s \notin W} (1 - P_d^{1r,s}) \prod_{z_s \in W} P_d^{1r,s}$$

$P_W^r$ represents the detection probability of the subset $W$ by treating it as a reflection measurement set. $z_W$ is a vector generated by vertical vectorial concatenation using elements in $W$. $\mathbf{H}_W$ is a Jacobian matrix of nonlinear measurement function $h^1r(\cdot)$ evaluated at $x_k^{(i)}$ and $m_{k|k-1}^{(j,i)}$. $\mathbf{R}_W$ is a diagonal covariance matrix with diagonal elements equal to $(\sigma_z^{1r})^2$. The Gaussian components in (5.19) are

$$\bar{m}_k^{(j,i)}(W) = \bar{m}_{k|k-1}^{(j,i)} + K_k^{(j,i)} [z_W - \mathbf{H}_W \bar{m}_{k|k-1}^{(j,i)}]$$
5.3. IMPLEMENTATION OF LMSC-PHD FILTER

and

\[
P^{(j,i)}_k(W) = [I - K^{(j,i)}_k(W)H_W]P^{(j,i)}_{k|k-1}
\]

\[
K^{(j,i)}_k(W) = P^{(j,i)}_{k|k-1}H^T_W(H_WP^{(j,i)}_{k|k-1}H^T_W + R_W)^{-1}
\]

\[
L_P(x_k) \text{ and } \omega_P \text{ can be calculated as}
\]

\[
\omega_P = \frac{\sum_i L_P^{(i)}\omega_{k|k-1}^{(i)}}{\sum_{\Omega \subseteq \mathcal{Z}_k} \sum_i L_P^{(i)}\omega_{k|k-1}^{(i)}}, \quad L_P^{(i)} = \left(1 - \tilde{P}_d^i + \sum_{W \in \mathcal{P}}(L_W^i/\Gamma_W^{(i)})\right)\prod_{W \in \mathcal{P}}\Gamma_W^{(i)} \exp^{P_r^i \sum_j n^{(j,i)}_{k|k-1}}
\]

Then the updated weight of each particle is

\[
\omega_k^{(i)} = \frac{\sum_{\mathcal{P}} \omega_P L_P^{(i)}\omega_{k|k-1}^{(i)}}{\sum_i L_P^{(i)}\omega_{k|k-1}^{(i)}}
\]

Finally, the state of MD is estimated using the weighted average and the corresponding posterior PHD of map features can be estimated using the weighted average of trajectory conditioned PHD, which can be expressed as

\[
\hat{x}_k = \frac{\sum_i \omega_k^{(i)}x_k^{(i)}}{\sum_i \omega_k^{(i)}}, \quad v_k(m|x_k) = \frac{\sum_i \omega_k^{(i)}v_k(m|x_k^{(i)})}{\sum_i \omega_k^{(i)}}
\]

Then the expected map features are estimated as the local maxima of posterior PHD \(v_k(m|x_k)\) with weight greater than a threshold, which is 0.5 in this paper [89]. It should be noted that the number of GM terms grows exponentially during the recursion, so pruning should be performed to reduce the number of GM components after each update, which includes truncating GM components with low weights, merging GM components within a close vicinity, and retaining at most \(N_{\max}^{GM}\) GM components of each particle with maximum weights [89].
5.3. IMPLEMENTATION OF LMSC-PHD FILTER

5.3.4 Error Metric

After estimating the MD and map features, it is important to evaluate the performance of the proposed LMSC-PHD filter. For MD position $p_k$, the localization error can be calculated as $|p_k - \hat{p}_k|$, where $\hat{p}_k$ denotes the estimated position of the MD and $|| \cdot ||$ represents the Euclidean norm. For the collection of map features $M_k$, the mapping error is defined using the OSPA metric $\bar{d}_p(M_k, \hat{M}_k)$ as shown in (5.17), where $\hat{M}_k$ is the collection of estimated map features. However, to calculate it, the error metric $d_p(c)(m, \hat{m})$ should be defined as in (5.17). For outdoor radar-based scenario, where map features are treated as points, the $d_p(c)(m, \hat{m})$ can be defined using Euclidean distance [71, 72]. However, for indoor environment, where map features are treated as planar reflectors, using Euclidean distance to define the error in $\varphi - \rho$ space gives rise to two issues. First, $\varphi$ and $\rho$ involve different units. Second, the Euclidean distance is dependent on choice of origin.

Therefore, we propose a new error metric $d_1(c)(m, \hat{m})$ that provides the average distance separation between the estimated planar map feature and true planar map
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feature, as shown in Fig. 5.2. Suppose the distances of two perpendicular lines from
the two end points of the true map feature to the estimated map feature are $a_1$ and
$a_2$, and the resulting enclosed area for case (a) is $A$ and case (b) is $A_1 + A_2$; the error
metric can then be defined as

$$\begin{cases} 
\frac{A}{l}, & \text{case (a)} \\
\frac{A_1 + A_2}{\ell_1 + \ell_2}, & \text{case (b)}.
\end{cases} \quad (5.26)$$

where $l$ is the length of the true map feature for case (a). $\ell_1$ and $\ell_2$ are the lengths
of two segments of the true map feature split by the estimated map feature for case
(b). The $\varrho_{21}$ is the ratio between $\ell_2$ and $\ell_1$, which is denoted as $\ell_2/\ell_1$. $\Delta \varphi$ is the
intersection angle between the estimated map feature and true map feature. As
shown in (5.26), the proposed error metric is independent of the origin position and
length of the reflector. In addition, it resolves the issue of different units for $\varphi$ and $\rho$.
In other words, the proposed error metric only depends on the relative position and
intersection angle between the estimated map feature and true map feature.

5.4 Experimental and Simulation Results

In this section, a performance comparison is made between the proposed LMSC-
PHD filter and the multi-hypothesis SC-PHD (MH-SC-PHD) filter proposed in [85].
It should be noted that the [85] was applied for AOA based tracking and mapping
with multiple RD. However, the MH-SC-PHD filter was proposed for any type of mea-
urement. In this research, we applied the MH-SC-PHD filter for TOA data. It also
should be noted that the MH-SC-PHD filter also incorporates LOS path. However,
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The MH-SC-PHD filter models the LOS path together with single reflection paths as a Poisson process. Whereas, the proposed LMSC-PHD models the LOS path separately with the single reflection paths as a Bernoulli process. For fair comparison, conventional MSC-PHD filter without LOS path are not taken into account. To simulate the LMSC-PHD filter, the number of particles used to model the MD was 200 and the maximum number of GM components for each particle was set as $N_{GM}^{max} = 12$. The greedy partition approach proposed in Section 5.3.2 was implemented, with the maximum number of partitions set as $N_{p}^{max} = 5\, 103$. To allow for fair comparison with the MH-SC-PHD filter, the number of hypotheses retained at each step was 35, and each hypothesis could generate 35 new hypotheses at the next step. For both filters, only LOS and single reflection paths are taken into account. The simulation and experimental environment is shown in Fig. 5.3, which is same with the environment used in last chapter with dimensions of $14.5m \times 5.7m$. Three RDs were placed at known positions with coordinates (3, 3), (3, 6.7) and (8.9, 6.7). The trajectory of the MD consisted of 49 points spaced by 20cm, starting from the upper right corner.

Figure 5.3: Experimental and simulation environment: (a) Vertical view of the environment layout, (b) A snapshot of the experimental environment.
The MD movement model and TOA measurement model shown in Section 5.1 were used for all filters. The clutter were generated by the paths diffracted from the pillar shown in Fig. 5.3. It should be noted that the simulated measurements contain LOS paths, single reflection paths and clutters.

5.4.1 Simulation Results

The performance of the proposed filters under different measurement noise $\sigma_z^l$ was examined, where $\sigma_z^l$ is the measurement noise of the LOS path. To account for reflection loss, the measurement noise of single reflection is set as double of $\sigma_z^l$. In simulation, $\sigma_z^l$ varied from 0.1$m$ to 0.4$m$. Following references [71, 72, 103], the detection probability of LOS and reflections were set as 0.9. Each filter was reiterated 25 times using different random sequences to generate measurements. The pillar shown in Fig. 5.3 was considered a point scatterer and would generate clutter. The MD and map features are initialized at known position. The average localization error (ALE) of MD and average OSPA mapping error (OSPA-AME) are presented in Fig. 5.4. As can be seen, the proposed LMSC-PHD filter achieved ALE ranging from 0.1$m$ to 0.4$m$ and OSPA-AME ranging from 0.2$m$ to 0.5$m$ when $\sigma_z^l$ varied from 0.1$m$ to 0.4$m$. When $\sigma_z^l = 0.4$m, the proposed LMSC-PHD filter outperformed the MH-SC-PHD filter by around 75% and 44% in terms of localization and mapping accuracy, respectively.

By using $\sigma_z^{par} = \sigma_z^l$, the localization and mapping results show a larger performance margin; the details are omitted for compactness.

The performance of proposed LMSC-PHD filter under inaccurate initial MD position was simulated. In the simulation, the measurement noise of LOS path $\sigma_z^l = 0.2$m and other parameters remained the same as before. Each filter was reiterated 25 times
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Figure 5.4: Comparison of tracking and mapping performance with different measurement noise $\sigma_z^2$: (a) Average localization error (ALE) of the MD, (b) Average OSPA mapping error (OSPA-AME) of the map, (c) Cumulative distribution function (CDF) of localization error (LE) when measurement noise $\sigma_z^2 = 0.2m$, (d) CDF of OSPA mapping error (OSPA-ME) when measurement noise $\sigma_z^2 = 0.2m$. 
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Figure 5.5: Comparison of tracking and mapping performance with uniformly distributed initial MD: (a) CDF of localization error, (b) CDF of OSPA mapping error.

using different random sequences, and initial MD position was uniformly distributed in a $1m \times 1m$ area around the exact MD position. The cumulative distribution function (CDF) performance of the localization error (LE) and OSPA-mapping error (OSPA-ME) are shown in Fig 5.5. As can be seen, the proposed LMSC-PHD filter achieved performance in LE and OSPA-ME of around $0.3m$ and $0.7m$, respectively, for 90% of the time. The results show that the proposed LMSC-PHD filter outperformed the MH-SC-PHD filter by around 80% and 45% in terms of localization and mapping accuracy, respectively.

The performance of the proposed LMSC-PHD filter under an inaccurate initial map was simulated. In simulation, the measurement noise of LOS path $\sigma_z = 0.2m$ and other parameters remained the same as before. Each filter was reiterated 25 times using different random sequences, and each time the initial map was generated randomly according to Gaussian distribution with $\sigma_\phi = 5^\circ$ and $\sigma_\rho = 0.1m$. The estimated trajectory and map are shown in Fig. 5.6. As can be seen, the map features
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Figure 5.6: The estimated trajectory and map features at different time step.

Figure 5.7: Comparison of tracking and mapping performance with $\sigma_{\varphi} = 5^\circ$ and $\sigma_{\rho} = 0.1m$: (a) CDF of localization error, (b) CDF of OSPA mapping error.
Figure 5.8: Comparison of tracking and mapping performance using measurement data: (a) CDF of localization error, (b) CDF of OSPA mapping error.

can be estimated correctly based on an inaccurate initialization. The cumulative distribution function (CDF) performance of the localization error (LE) and OSPA-mapping error (OSPA-ME) are shown in Fig. 5.7. As can be seen, the proposed LMSC-PHD filter achieved performance in LE and OSPA-ME of around 0.3m and 0.8m, respectively, for 90% of the time. The results show that the proposed LMSC-PHD filter outperformed the MH-SC-PHD filter by around 80% and 40% in terms of localization and mapping accuracy, respectively. The estimated trajectory and map features are

5.4.2 Experimental Results

As presented in last chapter, a measurement campaign was carried in the given environment. For each snapshot, the number of multipaths estimated at each RD is 5 to incorporate most LOS path and single reflection paths and least higher order
reflection paths. The performance of the proposed LMSC-PHD filter using measurement data was examined. Each filter was reiterated 25 times using different random sequences and initialization with perfect knowledge of MD and map features. The CDF performance of LE and OSPA-ME are shown in Fig. 5.8. As can be seen, the proposed LMSC-PHD filters achieved performance in LE and OSPA-ME of around 0.5 m and 0.3 m, respectively, for 90% of the time. The proposed filter outperformed the MH-SC-PHD filters by around 70% and 75%, in terms of localization and mapping accuracy, respectively. More experimental results will be shown in next chapter.

5.5 Conclusions

This chapter presents a new LMSC-PHD filter incorporating the LOS path that greatly enhances the accuracy of localization and mapping in indoor environments. A new greedy measurement partition scheme is designed to implement the LMSC-PHD filter. In addition, a novel error metric applicable to non-point scatterers is proposed to evaluate the error of estimated map features. Based on simulation and experimental results, the proposed LMSC-PHD filter outperforms MH-SC-PHD filter in terms of LE and OSPA-ME by a significant margin. These results show the critical importance of including the LOS path and modeling as a Bernoulli process in indoor tracking with inaccurate map.
Chapter 6

TOA based Indoor Tracking via
MRMSC-PHD Filtering

In last chapter, the LMSC-PHD filter was discussed to perform indoor tracking with inaccurate map. Both LMSC-PHD filter and conventional MSC-PHD filters assume each map feature generates at most one measurement from each RD. Thus, if single reflections are considered measurements generated by map features, then all higher-order reflections must be treated as clutter. However, higher order reflections carry information on the MD and map features that will improve the accuracy of localization and mapping. In an indoor multipath environment, there are many double and higher order reflections [75]. If such reflection paths can be modeled in the SC-PHD filter in performing tracking and updating inaccurate map, both mapping and localization accuracy will be improved. This chapter proposes a new MSC-PHD filter that incorporates associations among map features due to multiple reflections; it is thus called a multi-reflection incorporated MSC-PHD (MRMSC-PHD) filter. The remainder of this chapter is organized as follows. Section 6.1 formulates the multipath
propagation model, the statistical model for MD, map features and measurements. The proposed MRMSC-PHD filter and implementation details are presented in Section 6.2. Section 6.4 presents the experimental and simulation results, which show that our proposed filter outperforms existing MSC-PHD filters by a significant margin in terms of average localization and mapping accuracy. Conclusions are given in last section.

6.1 Problem Formulation

6.1.1 Multipath Propagation Model

In Chapter 3, a multipath propagation model that considers all NLOS paths as virtual LOS paths is presented. In this section, a new multipath propagation model is going to be presented. Again, considering an indoor environment, RDs are placed at known position with coordinates \( p_s = (x_s, y_s) \), where \( s = \{1, \ldots, S\} \) and \( S = 3 \) represents the number of RDs. The position of the MD at time \( k \) is denoted as \( p_k = (x_k, y_k) \). Considering reflectors only, the map features can be expressed as \( m = (\varphi, \rho) \). The definition of a planar reflector is based on Hessian normal form: \( x \cos \varphi + y \sin \varphi = \rho \), where \( \varphi \) denotes the angle between the planar reflector and the y-axis and \( \rho \) represents the distance from the origin to the planar reflector, as shown in Fig. 6.1(a).

To incorporate multiple reflections in the MSC-PHD filter, it is necessary to decouple the map features involved in multiple reflections. By considering each multiple reflection as a virtual single reflection generated by a single map feature, the associations among map features can be decoupled. Therefore, in this chapter, all higher order reflection paths are treated as virtual single reflection paths originating from
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Figure 6.1: Modeling of map features and multipath propagation: (a) Single and double reflection paths and their corresponding VRD and VMD, (b) Illustration of multipath propagation including LOS and multiple reflection paths.

$MD_k$ to their corresponding RD or VRD. Fig. 6.1(a) shows the single and double reflection paths and their corresponding VRD and virtual MD (VMD) \[110\]. The concept of VRD is proposed in Chapter 3 so the detailed expression of VRD position can be referred to Section 3.1.1. The VMD of $MD_k$ reflected from $m_j^i$ at time $k$ can be denoted as $MD_j^k$ with position $p_j^k = (x_j^k, y_j^k)$. Similar with VRD, the position of $MD_j^k$ can be calculated using image theory as

$$
\begin{align*}
    x_j^k &= -x_k \cos 2\varphi_k^j - y_k \sin 2\varphi_k^j + 2\rho_k^j \cos \varphi_k^j \\
    y_j^k &= -x_k \sin 2\varphi_k^j + y_k \cos 2\varphi_k^j + 2\rho_k^j \sin \varphi_k^j
\end{align*}
$$

(6.1)

Fig. 6.1(b) shows an example of multipath propagation in a typical indoor environment consisting of an LOS path, a single reflection path, a double reflection path, and a triple reflection path. As can be seen, the LOS path is independent of map features and can be used to track the MD, which will help to estimate map features. All reflection paths can be considered virtual single reflection paths originating from
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MD_\_k reflected from m_\_1^k to their corresponding RD or VRD. For example, the double reflection path originating from MD_\_k reflected from (m_\_1^k, m_\_2^k) to RD_\_s is considered a virtual single reflection path originating from MD_\_k reflected from m_\_1^k to VRD_\_2^{m_\_1^k, s}, which is the first order VRD of RD_\_s reflected from m_\_2^k. In the remainder of this chapter, the collection of m^{th} order VRDs is denoted as \{VRD_\_m^{m_\_1^k, s}, VRD_\_m^{m_\_2^k, s}, \ldots\}.

6.1.2 Dynamic Model of MD and Map Feature

Similar with last chapter, the state vector of the MD can be expressed as x_k = [p_k^T, v_k^T]^T, where p_k = [x_k, y_k]^T and v_k = [\dot{x}_k, \dot{y}_k]^T represent the position and velocity of MD at time k, respectively. Assuming a linear Gaussian constant-velocity model, the dynamical model of the MD can be expressed as

$$x_k = Fx_{k-1} + Gu_{k-1} = \begin{bmatrix} I_2 & \Delta T \cdot I_2 \\ 0_2 & I_2 \end{bmatrix} x_{k-1} + \begin{bmatrix} \frac{\Delta T^2}{2} \cdot I_2 \\ \Delta T \cdot I_2 \end{bmatrix} u_{k-1}$$

(6.2)

where u_{k-1} is the driving acceleration noise with zero mean and covariance matrix \(\sigma_a^2 I_2\), \(\sigma_a\) is the acceleration noise in either direction. \(I_N\) and \(0_N\) represents \(N \times N\) identity and zero matrix, respectively. \(\Delta T\) denotes the sampling interval.

The collection of detected map features is modeled as an RFS. Assuming all of the map features are static, then the dynamical model of the map features can be expressed as

$$M_k = M_{k-1} \cup B_k = \left\{ m_1^k, \ldots, m_{|M_k|}^k \right\}$$

(6.3)

where \(m_i^k = [\varphi_i^k, \rho_i^k]^T\) is \(i^{th}\) map feature at time \(k\). \(M_{k-1}\) and \(B_k\) represents estimated map features at time \(k - 1\) and new birth map features at time \(k\), respectively.
### 6.1.3 Measurement Model

By given the $RD_s$ with position $p_s = (x_s, y_s)$ and $MD_k$ with position $p_k = (x_k, y_k)$, the measured LOS TOA by $RD_s$ at time $k$ can be defined as

$$z_{k,s}^l = h_l(p_k, p_s) + n_{k,s}^l = \sqrt{(x_k - x_s)^2 + (y_k - y_s)^2} + n_{k,s}^l \quad (6.4)$$

where $n_{k,s}^l$ represents the Gaussian measurement noise of the LOS path with zero mean and standard deviation (std) of $\sigma_z^l$.

The measured TOA of reflection paths can be calculated as follows. The $m^{th}$ order reflection paths generated by $j^{th}$ map feature $m_j^k$ to $VRD_{k,s}^{m-1,t}$ can be expressed as

$$z_{k,s}^{mr,t} = h_{mr}(p_k, m_j^k, p_{m-1,t,k,s}) + n_{k,s}^{mr} = h_l(p_k, p_{m-1,t,k,s}) + n_{k,s}^{mr} \quad (6.5)$$

where $n_{k,s}^{mr}$ represents the Gaussian measurement noise of $m^{th}$ order reflections, with zero mean and std of $\sigma_{z}^{mr}$. The superscript $mr$ means this path is multiple reflection path with $m^{th}$ order reflection. $p_j^k = (x_j^k, y_j^k)$ denotes the VMD position by reflecting MD through $m_j^k$, which can be calculated according to image theory as in (6.1). $p_{m-1,t,k,s}$ is the position of $VRD_{k,s}^{m-1,t}$, which can be calculated using image theory as in (3.1) and (6.1).

Also, the collection of measurements is modeled as an RFS:

$$Z_{k,s} = \mathcal{D}(x_k) \cup \mathcal{D}(x_k, M_{k,s}) \cup \mathcal{C}_{k,s} \in \mathcal{F}(Z) \quad (6.6)$$

where $\mathcal{F}(Z)$ denotes the collection of all finite subsets of $Z$. $\mathcal{D}(x_k)$ is the collection of measurements from the LOS path whose element is given in (6.4). $\mathcal{D}(x_k, M_{k,s})$
is the collection of measurements from reflections whose element is shown in (6.5). \( C_{k,s} \) is an RFS denoting clutter received by \( RD_s \) such as diffractions, which can be formulated as a Poisson point process with intensity \( c_s(z) = \lambda_s^c U(z) \), where \( \lambda_s^c \) is the clutter rate, which means the average amount of clutter measured by \( RD_s \) [89].

### 6.1.4 Likelihood Model

For any nonempty measurement subset \( W \in Z_k \), given the MD state \( x_k \) and map feature \( m_j^k \), the multi-object likelihood function can be calculated as follows. If subset \( W \) represents LOS measurements, the multi-object likelihood function can be expressed as

\[
L^l_W(x_k) = \begin{cases} 
\prod_{s \notin W} (1 - P_d^{l,s}) \prod_{z_s \in W} P_d^{l,s} l_z^{l,s}, & \text{if } W_s = \{z_s\} \\
0, & \text{otherwise.} 
\end{cases}
\]  

(6.7)

where \( W_s \) denotes the collection of elements in \( W \) from \( RD_s \). \( P_d^{l,s} \) and \( l_z^{l,s} \) are abbreviations of \( P_d^{l,s}(x_k) \) and \( l_z^{l,s}(x_k) \), which represent the LOS detection probability and measurement likelihood function of \( RD_s \). According to (6.4), the LOS measurement likelihood can be modeled as \( \mathcal{N}(z; h'(p_k, p_s), (\sigma_z)^2) \), where \( \mathcal{N}(\cdot; \mu, \Sigma) \) represents a Gaussian distribution with mean \( \mu \) and covariance \( \Sigma \). \( s \notin W \) means the collection of RDs whose measurements are not contained in \( W \).
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If subset $W$ represents reflection measurements, the multi-object likelihood function can be expressed as

$$L_{rW}^r(m_k^j|x_k) = \prod_{s \notin W} \prod_{m} \prod_{t=1}^{n_{mr}^m} (1 - P_{d_{mr,s,t}}) \prod_{z_s \in W} \left( \sum_{\theta_s} \left( \prod_{\theta_s(t)=0} (1 - P_{d_{mr,s,t}}) \prod_{\theta_s(t)>0} P_{d_{mr,s,t}} \right) \right)$$

(6.8)

where $n_{mr}^m$ represents the number of $m^{th}$ order reflection paths. $\theta_s$ represents all possible associations between reflection paths from $RD_s$ and $W_s$. $\theta_s(t) = 0$ indicates that the associated path is not detected, while $\theta_s(t) > 0$ indicates that the associated path is detected. The detailed definition of association $\theta_s$ can be found in [98]. $P_{d_{mr,s,t}}$ and $l_{mr,s,t}$ are abbreviations of $P_{d_{mr,s,t}}(m|x_k, p_{m1,t}^{k,s})$ and $l_{mr,s,t}(m|x_k, p_{m1,t}^{k,s})$, which represent the detection probability and measurement likelihood of the $m^{th}$ order reflection path traced to $VRD_{m1,t}^{k,s}$. $p_{m1,t}^{k,s}$ is the position of $VRD_{m1,t}^{k,s}$, which is the $t^{th}$ VRD of $RD_s$ in the collection of $(m - 1)^{th}$ order VRDs. Same with last chapter, the measurement likelihood of a single reflection can be modeled as $N(z; h_l(p_{j,k}^r, p_{s}^r), (\sigma_{z}^{1r})^2)$ by given map feature $m_k^j$. However, the modeling of higher order reflection is more complex because it involves uncertainties of VRD position, as shown in (6.5).

To obtain the likelihood function of higher order reflections, we assume map features $m_k$ are Gaussian distributed random variables. Therefore, $h_l(p_{j,k}^{r}, p_{m1,t}^{k,s})$ represents a non-linear transformation of a Gaussian distributed random variable. In this paper, we assume that $h_l(p_{j,k}^{r}, p_{m1,t}^{k,s})$ is still a Gaussian distribution for ease of implementation, and use a numerical method to find its mean and std. Fig. 6.2 gives an example of using Gaussian distribution to fit the likelihood of double reflections. As shown in Fig. 6.2(a), there are two map features with mean positions of $m_1^k = (\pi/2, 2)$ and $m_2^k = (0, 2)$, respectively, and the same std $\sigma_{\varphi} = 5^\circ$ and $\sigma_{\rho} = 0.1m$. 

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Figure 6.2: Gaussian fit of a double reflection between MD and RD through two map features with $\sigma_\phi = 5^\circ$ and $\sigma_p = 0.1m$: (a) Calculated VRD position corresponding to 5000 realizations of $m_2$, (b) Gaussian fit of noiseless double reflections.

The calculated VRDs corresponding to 5000 realizations of $m_k^1$ are also shown. The noiseless path length of double reflections for each generated VRD can be calculated and denoted as $z_{2r}^{1:5000}$. A Gaussian distribution with mean $\mu_{k,s}^{2r} = h'(p_k^2, p_{k,s}^{1,1})$ and $\sigma_{k,s}^{2r} = \text{std}(z_{1:5000}^{2r})$ is used to fit the calculated noiseless double reflections, as shown in Fig. 6.2(b), where $P_{k,s}^{1,1}$ means the VRD position calculated using $m_k^1$. As can be seen, the noiseless double reflections are fitted quite well. The likelihood functions of higher order reflections can be estimated similarly. It should be noted that $\sigma_{k,s}^{mr}$ must be numerically estimated for each particle, each reflection path, and each time $k$, because the std depends on the relative position of MD and VRD. After estimating the std $\sigma_{k,s}^{mr}$, $t_{z}^{mr,s,t}$ can be approximated using $\mathcal{N}(z; \mu_{k,s}^{mr}, (\sigma_{k,s}^{mr})^2 + (\sigma_{z}^{mr})^2)$.

If subset $W$ represents clutter, the multi-object likelihood function can be expressed as

$$c_W = c_s(z_s)$$  \hspace{1cm} (6.9)

if $W = \{z_s\}$ and 0 otherwise. The $c_s(z)$ is intensity of clutter as defined in (6.6).
6.2 Proposed MRMSC-PHD Filter

Before we present the derived MRMSC-PHD filter, we first recap the assumptions made in the conventional MSC-PHD filter \[86, 114\]: (1) The map features are modeled as a Poisson point process whose realization is conditioned on the MD position, which is also a stochastic process; (2) There is no LOS path; (3) Each map feature generates at most one measurement, which is a single reflection in indoor scenario; (4) Measurements generated by map features are conditionally independent of each other given the MD position; (5) The clutter measurement is modeled as a Poisson process and independent of measurements generated by map features. For the indoor MRMSC-PHD filter, assumptions (1), (4), and (5) are kept, while assumption (2) is removed, so that the LOS path is either detected or blocked. More importantly, assumption (3) is relaxed, so that each map feature can generate more than one measurement through multiple reflections.

The prediction of MRMSC-PHD filter is the same with the LMSC-PHD filter, which can be referred to as Section 5.2.1. The joint update of the MRMSC-PHD filter \[113\] is

\[
v_k(x_k, m) = \sum_{\mathcal{P} \subseteq Z_k} \omega_{\mathcal{P}} \frac{p_{k|k-1}(x_k|Z_{1:k-1}) L_p(x_k)}{p_{k|k-1}[L_p]} v_k(m|x_k)
\]

\[
v_k(m|x_k) = \left(1 - \tilde{P}_d^r + \sum_{W \in \mathcal{P}} (1 - \Omega_W) \frac{L_w^r}{\Gamma_W} \right) v_{k|k-1}(m|x_k)
\]

(6.10)
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where

\[ \omega_p = \frac{p_{k|k-1} [L_p]}{\sum_{q \in Z_k} p_{k|k-1} [L_q]}, \quad L_p(x_k) = \frac{(1 - \tilde{P}_d^l + \sum_{W \in \mathcal{P}} (L_{W}^l/\Gamma_W)) \prod_{W \in \mathcal{P}} \Gamma_W}{\exp^{n_{k|k-1} [P_d^l|x_k]}}, \]

\[ \Omega_W = \frac{L_W^l/\Gamma_W}{1 - \tilde{P}_d^l + \sum_{W \in \mathcal{P}} (L_W^l/\Gamma_W)}, \quad \Gamma_W = c_W + v_{k|k-1} [L_W^l|x_k] \]

\[ p_{k|k-1}(x_k|Z_{1:k-1}) \text{ and } v_k(m|x_k) \] denote the predicted marginal distribution of MD and updated conditional PHD of map features at time \( k \), respectively. \( \omega_p \) and \( L_p(x_k) \) represent the weight and the multi-RD multiobject measurement likelihood of partition \( \mathcal{P} \), respectively. \( \Omega_W \) is the coefficient of LOS path. To update map features, \( \Omega_W \) is subtracted, because the LOS path is independent of map features. \( L_W^l, L_W^c, c_W \) are likelihood functions defined in (6.7), (6.8), (6.9), respectively. 

\[ 1 - \tilde{P}_d^l = \prod_s (1 - P_{d,s}^l) \]

and \( 1 - \tilde{P}_d^r = \prod_s \prod_m \prod_{t=1}^{l_{mr}} (1 - P_{d,s}^{mr,s,t}) \) denote the probability that all LOS paths are blocked and all reflection paths are blocked, respectively. \( \mathcal{P} \sqsubset Z_k \) represents the partition of the measurement set \( Z_k \). The number of elements from the same RD cannot be greater than \( \sum_m n_{mr}^m \), which is \( n_{mr}^m \) number of \( m \)th order reflections generated from the same map feature, otherwise the likelihood function will be zero. Thus for each subset \( W \in \mathcal{P} \), the number of elements in \( W \) is no greater than \( l_{max} = S \sum_m n_{mr}^m \), where \( S \) is the number of RDs, which is referred to as the \( l_{max} \) partition method \[98\]. The following example shows the partition method with \( Z_k = \{z_{k,1}, z_{k,2}, z_{k,1} \} \) and \( l = 3 \):

\[ \mathcal{P}_1 = \{\{z_{k,1}^1\}, \{z_{k,1}^2\}, \{z_{k,2}^1\}\}, \quad \mathcal{P}_2 = \{\{z_{k,1}^1, z_{k,2}^1\}, \{z_{k,2}^2\}\} \]

\[ \mathcal{P}_3 = \{\{z_{k,1}^2, z_{k,2}^1\}, \{z_{k,1}^1\}\}, \quad \mathcal{P}_4 = \{\{z_{k,1}^1, z_{k,2}^2\}, \{z_{k,2}^1\}\}, \quad \mathcal{P}_5 = \{\{z_{k,1}^1, z_{k,1}^2, z_{k,2}^2\}\} \]

(6.11)

It should be noted that although the expression of the proposed MRMSC-PHD filter
6.3. IMPLMENTATION OF MRMSC-PHD FILTER

is the same with the LMSC-PHD filter proposed in last chapter, the expression of each parameter is different. The detailed derivation of (6.10) is shown in the Appendix A.

6.2.1 LMSC-PHD Filter as a Special Case

When $P_{d}^{mr,s,t} = 0$ for all double and higher reflections, which means there are only single reflections, then the MRMSC-PHD filter simplifies to the LMSC-PHD filter proposed in last chapter [113]. The reflections’ related terms $\tilde{P}_{d}^{r}$ and $L_{W}^{r}$ are reduced to

$$\tilde{P}_{d}^{r} = 1 - \prod_{s}(1 - P_{d}^{lr,s}), \quad L_{W}^{r} = \prod_{s \in W}(1 - P_{d}^{lr,s}) \prod_{z_{s} \in W} P_{d}^{lr,s} l_{rs}^{lz_{s}}$$

(6.12)

The expression of other terms remains the same, then the resulting formulas denote the explicit form of the LMSC-PHD filter. If it is further assumed that $P_{d}^{lr,s} = 0$ in the LMSC-PHD filter, which means there is no LOS path, then the filter simplifies to the conventional MSC-PHD filter [72].

6.3 Implementation of MRMSC-PHD Filter

The prediction of the MRMSC-PHD filter is the same with the LMSC-PHD filter, which can be referred to as Section. 5.3.1. Therefore, the predicted density of MD and PHD of map features can be expressed as

$$p_{k|k-1}(x|Z_{1:k-1}) = \sum_{i=1}^{N_{k|k-1}} \omega_{k|k-1}^{(i)} \delta(x - x_{k}^{(i)})$$

$$v_{k|k-1}(m|x_{k}^{(i)}) = v_{k-1}(m|x_{k-1}^{(i)})$$

(6.13)
6.3. IMPLEMENTATION OF MRMSC-PHD FILTER

where $\delta(\cdot)$ is the Dirac function. $\omega^i_{k|k-1}$ and $\eta_{(j,i)}^{(j,i)}_{k|k-1}$ are the predicted weight of particle and GM components, respectively. $N^i_{k|k-1}$ and $J^i_{k|k-1}$ are the predicted number of particles and number of GM components of the $i^{th}$ particle, respectively.

6.3.1 Greedy Partition based on OSPA Metric

Similar with the LMSC-PHD filter, the MRMSC-PHD filter also consider all possible partitions of measurements to update the MD , as shown in (6.10). To avoid the combinatorial increase of the computation load, a greedy partition method similar with the one presented in last chapter is proposed. For LMSC-PHD filter, the greedy partition only consider LOS and single reflection paths. To do greedy partition for MRMSC-PHD filter, the higher order reflections also should be taken into account as shown in Fig. 6.3.

For each particle $x^{(i)}_k$ at time $k$, a set of noiseless path lengths $Z^0_{k,s}$ including the LOS path and reflections to $RD_s$ can be calculated based on the estimated map $\hat{M}_{k-1}$. Then for each particle $x^{(i)}_k$, we can obtain a set of closest measurements $\hat{Z}_{k,s}$.
and corresponding cost function $C_s^{(i)} = \tilde{d}_1^c(Z_{k,s}^o, Z_{k,s})$ by associating the measurement set $Z_{k,s}$ with the noiseless path length set $Z_{k,s}^o$, as shown in Fig. 6.3. As can be seen, the path associated with $\emptyset$ indicates that the corresponding path is not detected. If the measurement is not associated with any path, it is considered clutter. After estimating $\hat{Z}_{k,s}$ for each RD, the measurements generated by the LOS path and each map feature can be concatenated, respectively. Then the partition based on current particle and corresponding cost function $C^{(i)} = \sum_s C_s^{(i)}$ can be constructed. After calculating the cost $C^{(i)}$ of all particles, the $N_{max}^P$ partitions with minimum $C^{(i)}$ are selected as possible partitions.

### 6.3.2 Implementation of Update

After performing the greedy partition, the update of the MD and map features based on each partition can be implemented as follows:

$$v_k(m|x_k^{(i)}) = v_k^{md}(m|x_k^{(i)}) + \sum_{W \in P} v_k^d(m, W|x_k^{(i)})$$  \hspace{1cm} (6.14)

where $v_k^{md}$ and $v_k^d$ represent the mis-detected and detected parts of the map features at time $k$, respectively, which can be expressed as

$$v_k^{md}(m|x_k^{(i)}) = (1 - \tilde{P}_d) v_{k|k-1}(m|x_k^{(i)})$$

$$v_k^d(m, W|x_k^{(i)}) = \sum_{j=1}^{J_k^{(i)}} n^{(j,i)}_k(W) \mathcal{N}(m; m_k^{(j,i)}(W), P_k^{(j,i)}(W))$$  \hspace{1cm} (6.15)
6.3. IMPLEMENTATION OF MRMSC-PHD FILTER

The weight of each updated map feature using subset $W$ can be expressed as

$$
\eta_k^{(j,i)}(W) = (1 - \Omega_W^{(i)} \frac{\eta^{(j,i)}_{k|k-1}}{\Gamma^{(i)}_W})
$$

(6.16)

where

$$
\Omega_W^{(i)} = \frac{L_W^i/\Gamma_W^{(i)}}{1 - \bar{P}_d + \sum_{W \in \mathcal{P}} (L_W^i/\Gamma_W^{(i)})}
$$

$$
\Gamma_W^{(i)} = c_W + \sum_j \eta_{j,k|k-1}^{(i)} q^{(j,i)}(W)
$$

$$
q^{(j,i)}(W) = P_{W}^r \mathcal{N}(z_W, H_W m_k^{(j,i)}_{k|k-1}, H_W P_k^{(j,i)}_{k|k-1} H_W^T + R_W)
$$

$$
P_{W}^r = \prod_{s \in W} \prod_{m} \prod_{t=1}^{n_{mr}} (1 - P_d^{mr,s,t}) \prod_{s \in W} \left( \sum_{\theta_s} \left( \prod_{\theta_s(t)=0} (1 - P_d^{mr,s,t}) \prod_{\theta_s(t)>0} P_d^{mr,s,t} \right) \right)
$$

$P_{W}^r$ represents the detection probability of the subset $W$ by treating it as a reflection measurement set. $z_W$ is a vector generated by vertical vectorial concatenation using elements in $W$. $H_W$ is a Jacobian matrix of nonlinear measurement function $h^{lr}(\cdot)$ evaluated at $x_k^{(i)}$ and $m_k^{(j,i)}_{k|k-1}$. $R_W$ is a diagonal covariance matrix with diagonal elements equal to $(\sigma_{z}^{lr})^2$ for single reflections and $(\sigma_{z}^{mr})^2 + (\sigma_{z}^{mr})^2$ for higher order reflections. The Gaussian components in (6.15) are

$$
m_k^{(j,i)}(W) = m_k^{(j,i)}_{k|k-1} + K_k^{(j,i)} [z_W - H_W m_k^{(j,i)}_{k|k-1}]
$$

$$
P_k^{(j,i)}(W) = [I - K_k^{(j,i)}(W)H_W]P_{k|k-1}^{(j,i)}
$$

$$
K_k^{(j,i)}(W) = P_k^{(j,i)}_{k|k-1} H_W^T (H_W P_k^{(j,i)}_{k|k-1} H_W^T + R_W)^{-1}
$$

(6.17)
6.3. IMPLEMENTATION OF MRMSC-PHD FILTER

$L_P(x_k)$ and $\omega_P$ can be calculated as

$$\omega_P = \frac{\sum_i L_P^{(i)}(x_i)^{k|k-1}}{\sum_{q \in Z_k} \sum_i L_q \omega_q^{(i)}(x_i)^{k|k-1}}$$

Then the updated weight of each particle is

$$\omega_k^{(i)} = \frac{\omega_P L_P^{(i)}(x_i)^{k|k-1}}{\sum_i \omega_k^{(i)} L_P^{(i)}(x_i)^{k|k-1}}.$$

Then the expected map features are estimated as the local maxima of posterior PHD $v_k(m|x_k)$ with weight greater than a threshold, which is 0.5 in this paper [89]. It should be noted that the number of GM terms grows exponentially during the recursion, so pruning should be performed to reduce the number of GM components after each update, which includes truncating GM components with low weights, merging GM components within a close vicinity, and retaining at most $N_{max}^{GM}$ GM components of each particle with maximum weights [89].

6.3.3 Identifiability

In [61], the identifiability of the joint MD and multipath parameter estimation with both TOA and AOA measurements is analyzed by studying CRLB, which provides a bound for estimated parameters. In [115], the identifiability of a network localization and mapping with only TOA measurements is discussed. By using at least three LOS paths, the position of an MD can be unambiguously localized. If there are at least
three single reflection paths reflected from the same map feature, the map feature can be unambiguously estimated [115]. If each map feature satisfies the condition and the correct data association is known, then the whole map can be constructed unambiguously. With the estimated map and correct data association result, the MD can be localized with three paths that are either the LOS or reflections. Although the correct data association result is usually unknown, it still can be estimated by utilizing the prior position information of the MD. To date, the literature lacks a theoretic analysis of the identifiability of indoor tracking with inaccurate floor plan using only TOA measurements. According to the simulation, after the MD and map features are unambiguously initialized, the MD will remain tracked with at least three paths (either LOS or reflection paths).

6.3.4 Complexity Analysis

Suppose the number of particles is denoted as $N_p$, the maximum number of GM components of each particle is denoted as $N_{max}^{GM}$, the number of maximum partitions is denoted as $N_{max}^P$, the number of $m^{th}$ order reflection paths is denoted as $n_{mr}^m$ where $m$ is the order of reflection, and the number of RDs is denoted as $S$. Then the complexity of the prediction part of the proposed MRMSC-PHD filter is $O(N_p N_{max}^{GM})$, the complexity of the proposed greedy partition algorithm is $O(N_p S (1 + \sum_m n_{mr}^m))$, and the complexity of the update part of the proposed MRMSC-PHD filter is $O(N_p N_{max}^P N_{max}^{GM} (1 + m))$. Therefore, the complexity of the proposed MRMSC-PHD filter can be considered to be $O(N_p N_{max}^P N_{max}^{GM} (1 + m))$, which is linear with the number of reflections.
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Table R

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Figure 6.4: Experimental and simulation environment: (a) Vertical view of the environment layout, (b) A snapshot of the experimental environment.

6.4 Experimental and Simulation Results

In this section, a performance comparison is made between the proposed MRMSC-PHD filter and the LMSC-PHD filter proposed in last chapter and the multi-hypothesis SC-PHD filter (MH-SC-PHD filter) proposed in [85]. To simulate the MRMSC-PHD filter and LMSC-PHD filter, the number of particles used to model the MD was 200 and the maximum number of GM components for each particle was set at \( N_{GM}^{max} = 12 \). The greedy partition approach proposed in Section 6.3.1 was implemented, with the maximum number of partitions set as \( N_{P}^{max} = 5 \) [103]. For MRMSC-PHD filters, we considered two cases featuring double reflections and triple reflections, which are referred to as the double reflection incorporated MSC-PHD (DRMSC-PHD) filter and triple reflection incorporated MSC-PHD (TRMSC-PHD) filter, respectively. To allow for fair comparison with the MH-SC-PHD filter, the number of hypotheses retained at each step was 200, and each hypothesis could generate 5 new hypotheses at the next step. The experimental and simulation environment is shown in Fig. 6.4 which...
is the same with the environment used in Chapter 4 and Chapter 5 with dimensions of $14.5m \times 5.7m$. Three RDs were placed at known positions with coordinates (3, 3), (3, 6.7) and (8.9, 6.7). The trajectory of the MD consisted of 49 points spaced by 20cm, starting from the upper right corner. The MD movement model and TOA measurement model shown in Section 6.1 were used for all filters. The clutter were generated by the paths diffracted from the pillar shown in Fig. 6.4. It should be noted that the simulated measurements contain LOS paths, single reflection paths, double reflection paths, triple reflection paths and clutters.

6.4.1 Simulation Results

The performance of the proposed filters under different measurement noise $\sigma_z^l$ and $\sigma_z^{mr}$ was examined, where $\sigma_z^l$ and $\sigma_z^{mr}$ are defined in (6.5) representing the std of the LOS path and multi-reflections, respectively, and $m$ is the order of reflection. In simulation, $\sigma_z^l$ varied from 0.1m to 0.4m. To account for reflection loss, we assume $\sigma_z^{mr}$ to be double for each reflection, which means $\sigma_z^{mr} = 2^m\sigma_z^l$. Following references [71,72,103], the detection probability of LOS and reflections were set as 0.9. Each filter was reiterated 25 times using different random sequences with perfect initialization of MD and map features. The average localization error (ALE) of MD and average OSPA mapping error (OSPA-AME) are presented in Fig. 6.5. As can be seen, the proposed TRMSC-PHD filter achieved ALE ranging from 0.1m to 0.5m and OSPA-AME ranging from 0.04m to 0.17m when $\sigma_z^l$ varied from 0.1m to 0.4m. In addition, the proposed DRMSC-PHD filter achieved ALE ranging from 0.1m to 0.8m and OSPA-AME ranging from 0.04m to 0.20m when $\sigma_z^l$ varied from 0.1m to 0.4m. When $\sigma_z^l = 0.4m$, the proposed TRMSC-PHD filter outperformed the LMSC-PHD and
Figure 6.5: Comparison of tracking and mapping performance with different measurement noise $\sigma_z^2$: (a) Average localization error (ALE) of MD, (b) Average OSPA mapping error (OSPA-AME) of map, (c) Cumulative distribution function (CDF) of localization error (LE) when measurement noise $\sigma_z^2 = 0.2 m$, (d) CDF of OSPA mapping error (OSPA-ME) when measurement noise $\sigma_z^2 = 0.2 m$. 

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### 6.4. EXPERIMENTAL AND SIMULATION RESULTS

MH-MSC-PHD filters by around 85% and 90%, respectively, in terms of localization accuracy. The proposed TRMSC-PHD filter also outperformed the LMSC-PHD and MH-MSC-PHD filters by around 40% and 85%, respectively, in terms of mapping accuracy. By using $\sigma_{z_{mr}}^2 = \sigma_{z_{r}}^2$, the localization and mapping results show a larger performance margin; the details are omitted for compactness.

The performance of proposed filters under inaccurate initial MD position was simulated. In the simulation, the measurement noise of LOS path $\sigma_{z_{l}}^2 = 0.2m$ and other parameters remained the same as before. Each filter was reiterated 25 times using different random sequences, and initial MD position was uniformly distributed in a $1m \times 1m$ area around the exact MD position. The cumulative distribution function (CDF) performance of the localization error (LE) and OSPA-mapping error (OSPA-ME) are shown in Fig 6.6. As can be seen, the proposed TRMSC-PHD and DRMSC-PHD filters achieved similar performance in LE and OSPA-ME of around 0.4m and 0.2m, respectively, for 90% of the time. The results show that our proposed filter

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**Figure 6.6:** Comparison of tracking and mapping performance with uniformly distributed initial MD: (a) CDF of localization error, (b) CDF of OSPA mapping error.
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![Figure 6.7](image)

Figure 6.7: Comparison of tracking and mapping performance with $\sigma_{\phi} = 5^\circ$ and $\sigma_{P} = 0.1m$, (a) CDF of localization error, (b) CDF of OSPA mapping error, (c) Average cardinality of estimated map features along trajectory, (d) RMS mapping error along trajectory.

outperformed the LMSC-PHD and MH-MSC-PHD filters by around 90% and 95%, respectively, in terms of localization accuracy. Our proposed filter also outperformed the LMSC-PHD and MH-MSC-PHD filters by around 70% and 90%, respectively, in terms of mapping accuracy.
The performance of the proposed filters under an inaccurate initial map was simulated. In simulation, the measurement noise of LOS path $\sigma_z^l = 0.2m$ and other parameters remained the same as before. Each filter was reiterated 25 times using different random sequences, and each time the initial map was generated randomly according to Gaussian distribution with $\sigma_\varphi = 5^\circ$ and $\sigma_\rho = 0.1m$. The cumulative distribution function (CDF) performance of the localization error (LE) and OSPA-mapping error (OSPA-ME) are shown in Fig. 6.7. As can be seen, the proposed TRMSC-PHD and DRMSC-PHD filters achieved similar performance in LE and OSPA-ME of around 0.7m and 0.3m, respectively, for 90\% of the time. The results show that our proposed filter outperformed the LMSC-PHD and MH-MSC-PHD filters by around 70\% and 90\%, respectively, in terms of localization accuracy. Our proposed filter also outperformed the LMSC-PHD and MH-MSC-PHD filters by around 55\% and 85\%, respectively, in terms of mapping accuracy. Fig. 6.7(c) shows the changes in averaged cardinality of estimated map features along the trajectory. As can be seen, the proposed TRMSC-PHD and DRMSC-PHD filters correctly estimated the number of map features 99.9\% and 98.5\% of the time, respectively. The LMSC-PHD and MH-MSC-PHD filters only correctly estimated the number of map features 87.7\% and 6.5\% of the time. We also studied how the root mean square OSPA-ME (RMS-OSPA-ME) changed along the trajectory, as shown in Fig. 6.7(d). As can be seen, the RMS-OSPA-ME of the proposed TRMSC-PHD and DRMSC-PHD filters decreased continually from 0.3m to 0.1m, showing stability and convergence. However, the RMS-OSPA-ME of other filters either diverged or remained the same.
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Figure 6.8: Comparison between measurements and simulated reflection paths from walls and other reflectors such as ceiling, floor, and pillar at MD1, numbers next to the measurements represent signal to noise ratio (SNR) of corresponding measurement.

6.4.2 Experimental Results

A measurement campaign was carried out in the environment, as shown in Fig. 5.3. The channel response between MD and three RDs was measured using a single input single output (SISO) system, the Agilent Technologies PNA-X Network Analyzer N5244A, in a frequency domain with 500MHz bandwidth centered at 2.4GHz over 201 frequency points [110]. These measurements were then used to estimate TOA using an expectation maximization (EM) algorithm [111]. For each snapshot, the number of multipaths estimated at each RD is 10. Fig. 6.8 shows the comparison between measurements at MD1 and simulated reflection paths from walls, ceiling, floor, and pillar. It should be noted that in our model, we only include three wall reflectors. This means that the simulated reflections from walls are considered simulated paths, which are denoted as $R_{i,j,l}$ in Fig. 6.8 where the subscript represents the collection of reflected walls. The simulated reflections from ceiling, floor, and pillar are considered
simulated clutter. The measurements associated with simulated paths are denoted as associated measurements, and the rest are referred to as unassociated measurements. The associated and unassociated measurements can be considered true measurements and clutter, respectively. As can be seen in Fig. 6.8, some of the clutter was close to the simulated clutter from ceiling, floor, and pillar, which can be considered to be generated by the ceiling, floor, and pillar. It should be noted that in Chapter 3 and Chapter 4 only up to double reflection paths are incorporated, the number of multipaths estimated at each RD is also 10. The reason is that some double reflection paths are longer than triple reflections. For example, the simulated double reflection path $R_{3,1}$ at $RD_1$ is longer than the simulated triple reflection path $R_{1,2,3}$ at $RD_1$. If we decrease the number of estimated multipaths, then some double reflection paths may be removed. The localization and tracking results presented in Chapter 3 and Chapter 4 show that the proposed algorithms were able to filter out the triple reflection paths and achieved a good performance. On the other hand, the association results show that in the estimated 10 multipaths, there are around 10% of triple reflection paths and 30% of clutters. If we estimated more paths, perhaps we can get more triple reflection paths but definitely we will get more clutters. To avoid incorporating more clutters, we still keep the number of multipaths estimated each RD is 10.

Each filter was reiterated 25 times using different random sequences and initialization with perfect knowledge of MD and map features. The CDF performance of LE and OSPA-ME are shown in Fig. 6.9. As can be seen, the proposed TRMSC-PHD and DRMSC-PHD filters achieved similar performance in LE and OSPA-ME of around 1m and 0.3m, respectively, for 90% of the time. Our proposed filter outperformed
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Figure 6.9: Comparison of tracking and mapping performance based on collected measurement data using all three RDs: (a) CDF performance of localization error, (b) CDF performance of OSPA mapping error.

the LMSC-PHD filter by 30% for LE. However, there was little performance gain in OSPA-ME with the proposed filter over the LMSC-PHD filter, because in this semi-open cubical environment, there is a greater number of single reflections than higher order reflections. Therefore, there are only a few higher order reflections to further improve the OSPA-ME margin over the LMSC-PHD filter. However, in many indoor environments, there are often more higher order reflections than single reflections [75]. This situation can be emulated by decreasing the number of RDs in the environment, and Fig. 6.10 shows the performance of our proposed filter with fewer RDs. As can be seen, using only $RD_1$ and $RD_2$, the proposed TRMSC-PHD and DRMSC-PHD filters achieve similar performance in LE and OSPA-ME of around $1.2m$ and $0.5m$, respectively, for 90% of the time. The results show that our proposed filter outperformed the LMSC-PHD and MH-MSC-PHD filters by around 65% and 70%, respectively, in terms of localization accuracy. Our proposed filter also outperformed the LMSC-PHD
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Figure 6.10: Comparison of tracking and mapping performance using collected measurement data using $RD_1$ and $RD_2$: (a) CDF performance of localization error, (b) CDF performance of OSPA mapping error.

and MH-MSC-PHD filters by around 45% and 65%, respectively, in terms of mapping accuracy. Thus as the number of RD decreases, which results in the number of detected single reflections decreasing, the performance margin between the proposed MRMSC-PHD filter and other filters increases. A performance comparison using only a single RD was also made, and it showed a larger performance margin; details are omitted here for compactness.

The performance of the proposed filters under an inaccurate initial map using measurement data was also examined. Each filter was reiterated 25 times using different random sequences, and each time an initial map was generated randomly according to Gaussian distribution with $\sigma_\phi = 5^\circ$ and $\sigma_\rho = 0.1\, m$. The CDF performance for LE and OSPA-ME are shown in Fig. 6.11. As can be seen, the proposed TRMSC-PHD and DRMSC-PHD filters achieved similar performance in LE and OSPA-ME of around 1.4\, m and 0.5\, m, respectively, for 90% of the time. The results show that our proposed filter outperformed the LMSC-PHD and MH-MSC-PHD filters by around
6.4. EXPERIMENTAL AND SIMULATION RESULTS

Figure 6.11: Comparison of tracking and mapping performance with $\sigma_{\rho} = 5^\circ$ and $\sigma_{\rho} = 0.1m$ using all three RDs: (a) CDF of localization error, (b) CDF of OSPA mapping error, (c) Average cardinality of estimated map features along trajectory, (d) RMS mapping error along trajectory.

55% and 80%, respectively, in terms of localization accuracy. Our proposed filter also outperformed the LMSC-PHD and MH-MSC-PHD filters by around 30% and 50%, respectively, in terms of mapping accuracy. Fig. 6.11(c) shows the changes in averaged cardinality of estimated map features along the trajectory. As can be seen, the proposed TRMSC-PHD and DRMSC-PHD filters correctly estimated the number of map features 99.4% and 98.1% of the time, respectively. The LMSC-PHD and MH-MSC-PHD filters correctly estimated the number of map features only 95.4%
and 40.5% of the time. We also examined how the RMS-OSPA-ME changed along the trajectory, as shown in Fig. 6.11(d). As can be seen, the RMS-OSPA-ME of the proposed TRMSC-PHD and DRMSC-PHD filters decreased continually from 0.37m to 0.30m, showing stability and convergence. However, the RMS-OSPA-ME of other filters either diverged or remained the same throughout.

The method proposed in this and last chapter uses both the LOS and NLOS paths to localize the MD and update an inaccurate map. By using the proposed filters, we can improve map feature accuracy, which results in better localization accuracy. To construct map features without any prior map, the initial MD should be estimated or localized using LOS paths. Without prior map, the map features can be constructed if there are at least three single reflection paths from each map feature, as presented in [115].

6.5 Conclusions

In this chapter, a new MRMSC-PHD filter incorporating both LOS and multiple reflections is proposed to greatly enhance the accuracy of localization and mapping in indoor environment. Based on both simulation and experimental results, the proposed MRMSC-PHD filter outperforms existing RFS-based filters in terms of LE and OSPA-ME by significant margin. This shows that the importance and criticalness of including higher order reflections for indoor tracking with inaccurate floor plan.
Chapter 7

Conclusions and Future Work

7.1 Conclusions

In this thesis, several TOA based algorithms are proposed to improve the performance of indoor localization and tracking schemes.

This thesis begins from the TOA-based indoor localization algorithm that uses both the LOS and NLOS paths with the help of accurate floor plan information. Based on the known floor plan, an NLOS path can be considered a virtual LOS path that originates from the MD to a precalculated VRD. To associate the NLOS path with the corresponding VRD with no prior information on the MD, a grid-based data association algorithm is proposed. By introducing the path association matrix and the measurement association matrix, the data association process can be integrated with the two-step weighted least squares method to localize the MD. The simulation and experimental results show that the proposed TOA-based indoor localization algorithm that uses multipath components outperforms conventional TOA-based indoor localization algorithms that use only LOS paths by around 40% in terms of
localization accuracy. This means that the localization accuracy will be improved by considering both LOS and NLOS paths.

TOA-based indoor tracking algorithms that use multipath components with the help of accurate floor plan information is then presented. Based on the prior information on the MD, data association can be performed and combined with a conventional tracking algorithm such as EKF or PF. The simulation and experimental results show that the TOA-based indoor tracking algorithm using multipath components outperformed conventional TOA-based indoor tracking algorithms that use only LOS paths by around 40% in terms of localization accuracy. This means that the tracking accuracy will be improved by considering both LOS and NLOS paths.

In addition, this thesis proposes an LMSC-PHD filter to perform indoor tracking when no accurate floor plan is available. Conventional MSC-PHD filters are designed for outdoor radar-based scenarios in which only backscattering (or single reflection) paths are considered. For extension to indoor tracking with inaccurate floor plan, the LOS path is considered. Because the LOS path is independent on the map, it is modeled separately from the single reflection paths. By modeling the LOS path as a Bernoulli process, because the number of LOS paths can only be zero or one and assuming that the single reflection path follows a Poisson process, the LMSC-PHD filter that incorporates the LOS path is proposed. The simulation and experimental results show that the proposed LMSC-PHD filter outperformed the MH-MSC-PHD filter that modeled the LOS path as a Poisson process by around 65% in terms of localization accuracy and by around 75% in terms of mapping accuracy. These results mean that both the localization and mapping accuracy will be improved by incorporation of the Bernoulli LOS path in the conventional MSC-PHD filter.
Finally, this thesis proposes an MRMSC-PHD filter that incorporates both the LOS and multiple reflection paths to perform indoor tracking with inaccurate floor plan. The conventional MSC-PHD filter assumes that each map feature can generate only a single measurement, which means that only single reflection paths can be considered effective measurements and that higher-order reflection paths must be treated as clutter. By treating all higher-order reflection paths as virtual single reflection paths originated from the MD to corresponding VRDs, the MRMSC-PHD filter that incorporates both the LOS and multiple reflection paths is derived to perform indoor tracking with inaccurate floor plan. The simulation and experimental results show that the proposed MRMSC-PHD filter outperformed conventional MSC-PHD filters by around 90% in terms of localization accuracy and by around 75% in terms of mapping accuracy. These results mean that both the localization and mapping accuracy will be improved by incorporation of multiple reflection paths in the conventional MSC-PHD filter.

7.2 Future Work

The indoor localization and tracking algorithms presented in this thesis could be extended in several directions. First, the data association method proposed in Chapter 3 and Chapter 4 is based on an OSPA metric that minimizes the overall difference between the measurement set and the noiseless path length set. In practice, it is reasonable to give priority to minimizing the difference between the LOS path length and the associated measurements because the LOS path has greater accuracy. One possible way to solve this issue is to design a systematic weight for different paths
to penalize the difference between the path length and associated measurement according to measurement accuracy. The design of a systematic weight is one possible investigation direction. In addition, the cutoff distance used in the OSPA metric is a self-defined parameter. The relationship between the cutoff distance and the localization performance is another possible topic for research.

Second, the proposed LMSC-PHD filter in Chapter 5 and the proposed MRMSC-PHD filter in Chapter 6 for indoor tracking with inaccurate floor plan assume a known and constant detection profile and clutter rate. In practice, the detection profile and clutter rate are unknown and should be jointly estimated with the state of the MD and map features. Several studies in the literature present joint estimation of the detection probability or clutter profile and the state of the MD for tracking \[93, 95, 116\]. However, for indoor tracking with inaccurate floor plan, the uncertainty of the map feature also should be handled. The indoor tracking filter with an unknown detection profile and clutter rate is a possible direction for investigation.

Third, more measurement can be conducted in environments with different construction materials, furniture and equipment. Applying the proposed algorithms to these different environments to test the universality of accuracy and stability.

Fourth, the proposed algorithms can be extended for 3-dimensional (3D) scenario to estimate ceiling, floor and even to track the devices flying inside buildings.
Appendix A

Proof of Equation (5.12) and (6.10)

The joint update formula involves set integral which is computation intractable. Define the two-variable probability generating functionals (PGFL)

\[ F[g, h] = \int \int h^{M_k} p_k | z_{1:k-1} \rangle p_k | z_{1:k-1} \rangle G_{s} | x_k, M_k \rangle \delta M_k \]

where \( G_{s} | x_k, M_k \rangle \) is the PGFL of \( L_k(\delta Z_{k,s}|x_{k}, M_{k}) \), which is defined as \( G_{s} | x_k, M_k \rangle = \int g_s Z_{k,s} L_k(\delta Z_{k,s}|x_{k}, M_{k}) \delta Z_{k,s} \). The measurements from each RD consist of three classes which are LOS, single and double reflections, and clutter measurements, with corresponding PGFL as \( G_{l}^{s}|g_{s}|x_{k}, M_{k}\rangle \), \( G_{r}^{s}|g_{s}|x_{k}, M_{k}\rangle \), and \( G_{c}^{s}|g_{s}\rangle \), respectively. According to properties of PGFL, then the \( G_{k}^{s}|g_{s}|x_{k}, M_{k}\rangle \) can be extended from \( [98] \) by adding in LOS component as

\[ G_{k}^{s}|g_{s}|x_{k}, M_{k}\rangle = G_{k}^{l}|g_{s}|\cdot(G_{k}^{r}|g_{s}|\cdot)^{M_k}G_{k}^{c}|g_{s}\rangle \]

(A.2)
where

\[ G_k^l[g_s|\cdot] = 1 - P^{l_s} + P^{l_s} P^g, \quad G_k^r[g_s|\cdot] = \prod_m \prod_t G_k^{mr}[g_{s,t}|\cdot] \]  

\[ G_k^{mr}[g_s|\cdot] = 1 - P^{mr,s,t} + P^{mr,s,t} P_g^{mr,s,t}, \quad G_k^c[g_s] = \exp^{\lambda_s c^s[g_s]} \]  

(A.3)

The \( G_k^l[g_s|\cdot] \) and \( G_k^r[g_s|\cdot] \) are abbreviations of \( G_k^l[g_s|x_k] \) and \( G_k^r[g_s|x_k, \mathbf{m}] \), respectively. The \( G_k^{mr}[g_{s,t}|\cdot] \) is an abbreviation of \( G_k^{mr}[g_{s,t}|x_k, \mathbf{m}, p_{k,s}^{-1,t}] \). And

\[ p_g^{l_s} = \int g_s(z) l_z^{l_s} \, dz, \quad p_g^{mr,s,t} = \int g_s(z) l_z^{mr,s,t} \, dz, \quad c^s[g_s] = \int g_s(z) c_s(z) \, dz \]  

(A.4)

The clutter is modelled as a Poisson process. Substituting (A.2), (A.3) into (A.1), we can get

\[ F[g,h] = p_x \left[ G_k^c[g_s] \prod_s G_k^l[g_s|\cdot] G_m[h \prod_s G_k^r[g_s|\cdot]] \right] \]  

\[ p_x[\Phi[h]] = \int \Phi[h] p_{k,k-1}(x_k|Z_{1:k-1}) \, dx_k \]  

(A.5)

The \( G_m[h|\cdot] \) is an abbreviation of \( G_m[h|x_k] \) which is the PGFL of map features, which is defined based on Poisson process assumption as

\[ G_m[h|\cdot] = \int h^k p_{k,k-1}(M_k|x_k, Z_{1:k-1}) \, dM_k = \exp^{p_m[h|x_k] - \mu} \]  

\[ p_m[h|x_k] = \int h(\mathbf{m}) p_{k,k-1}(\mathbf{m}_k|x_k, Z_{1:k-1}) \, d\mathbf{m} \]  

(A.6)
where $\mu$ is the average number of predicted map features which is defined as $\mu = \int v_{k|k-1}(m|x_k)dm$. Substituting (A.6) and (A.4) into (A.5), we can get

$$F[g, h] = p[x] \prod_s G^t_k[g_s|\cdot] \exp^{\Phi[g,h]}$$

$$\Phi[g, h] = \mu p_m \left[ h \prod_s G^t_k[g_s|\cdot] \cdot \right] - \mu + \sum_s \left( c^s[g_s] - \lambda^s \right)$$

(A.7)

Then the functional derivative of $F[g, h]$ to set $Z_k$ is

$$\frac{\delta F}{\delta Z_k}[g, h] = p[x] \left[ \sum_{\mathcal{P} \subseteq Z_k} \exp^{\Phi[g,h]} \prod_{W \in \mathcal{P}} \Phi[\cdot]_W \left( \prod_s G^t_k[g_s|\cdot] + \sum_{W \in \mathcal{P}} L^t_W[g] \right) \right]$$

(A.8)

where

$$L^t_W[g] = \begin{cases} \prod_{s \in W} P^{l,s}_d l^s_z \prod_{s \notin W} G^t_k[g_s|\cdot], & \text{if } W_s = \{z^s\} \\ 0, & \text{otherwise.} \end{cases}$$

(A.9)

$$\Phi[\cdot]_W = \frac{\delta \Phi[g,h]}{\delta W} = c_W + \mu p_m \left[ h L^r_W[g] \cdot \right]$$

(A.10)

$$L^r_W[g] = \prod_{s \notin W} G^r_k[g_s|\cdot] \prod_{s \in W} \sum_{\theta_s} \left( \prod_{\theta_s(t) = 0} G^r_{k,s,t}[g_{s,t}|\cdot] \prod_{\theta_s(t) > 0} P^{mr,s,t}_d \right)$$

(A.11)

We will use the mathematical induction method to prove (A.8) - (A.11). Initially, considering the case when $Z_k = \{z^1\}$, then there is only single partition $\mathcal{P} = z^1$. In this case, taking the functional derivation of $F[g, h]$ to $Z_k = \{z^1\}$, we can get

$$\frac{\delta F}{\delta z^1}[g, h] = p[x] \left[ \exp^{\Phi[g,h]} \Phi[\cdot]_{z^1} \left( \prod_s G^t_k[g_s|\cdot] + \frac{P^{l}_d L^t_{z^1}}{\Phi[\cdot]_{z^1}} \right) \right]$$

(A.12)

where

$$\Phi[\cdot]_{z^1} = c_s(z^1) + \mu p_m \left[ h P^{r}_d L^r_{z^1}[g] \cdot \right]$$

(A.13)
\[ P_{z_1}^r L_{z_1}^r[g] = \prod_{s \neq 1} G_k^r[g_{s1}] \left( P_d^{1r,1} l_{z_1}^{1r,1} \prod_t G_k^{2r}[g_{1t}] + G_k^{1r}[g_{1}] \prod_t G_k^{2r}[g_{1t}] \right) \]

The result in (A.12) and (A.13) is as claimed in (A.8)-(A.11) when \( Z_k = \{ z_1 \} \) and also hold for \( Z_k = \{ z_s \} \) by symmetry. Thus formulas hold for the initial step. Suppose results hold for the case \( |Z_k| > 1 \), then for case \( Z_k = \{ z_1 \} \), we have

\[
\frac{\delta}{\delta z_1} \frac{\delta F}{\delta Z_k}[g, h] = p_x \left[ \sum_{\mathcal{P} \subset Z_k} \exp^{\phi[g, h]} \left( \prod_{W \in \mathcal{P} \cup \{ z_1 \}} \Phi[\cdot]_W \left( \prod_s G_k^l[g_{s}] + \sum_{W \in \mathcal{P} \cup \{ z_1 \}} \frac{P^l_W L_W^r[g]}{\Phi[\cdot]_W} \right) \right) \right. \\
\left. + \prod_{W \in \mathcal{P}} \Phi[\cdot]_W \sum_{W \in \mathcal{P}} \frac{\Phi[\cdot]_{W \cup \{ z_1 \}}}{\Phi[\cdot]_W} \left( \prod_s G_k^l[g_{s}] + \sum_{V \in \mathcal{P} \setminus \{ W \cup \{ z_1 \} \}} \frac{P^l_V L_V^r[g]}{\Phi[\cdot]_V} \right) \right]
\]

(A.14)

It should be noted that the partitions of \( Z_k \cup \{ z_1 \} \) have two forms. The first form is for each partition \( \mathcal{P} \) of \( Z_k \), append the set \( \{ z_1 \} \) to \( \mathcal{P} \) results in the partition \( \mathcal{P} \cup \{ z_1 \} \), which is mathematically represented by the first two lines of (A.14). The alternative form is for each partition \( \mathcal{P} \) of \( Z_k \), replace the subset \( W \) with \( \{ W \cup z_1 \} \) results in the partition \( \mathcal{P} \setminus W \cup \{ W \cup z_1 \} \), which is mathematically represented by the last two lines of (A.14). Therefore, the functional derivative of \( F[g, h] \) to \( Z_k \cup \{ z_1 \} \) is

\[
\frac{\delta F}{\delta (Z_k \cup \{ z_1 \})}[g, h] = p_x \left[ \sum_{\mathcal{P} \subset Z_k \cup \{ z_1 \}} \exp^{\phi[g, h]} \left( \prod_{W \in \mathcal{P}} \Phi[\cdot]_W \left( \prod_s G_k^l[g_{s}] + \sum_{W \in \mathcal{P}} \frac{P^l_W L_W^r[g]}{\Phi[\cdot]_W} \right) \right) \right]
\]

(A.15)

Therefore, the formulas hold for \( Z_k \cup \{ z_1 \} \) and also hold for \( Z_k \cup \{ z_s \} \) by symmetry, which concludes the inductive step. Therefore, (A.8)-(A.11) are proved. Based on
the relationships between PGFL and PHD that

\[ v_k(x_k, m) = \frac{\delta F}{\delta(x_k, m)} \frac{\delta Z_k}{\delta Z_k} [0, 1] / \frac{\delta F}{\delta Z_k} [0, 1] \] (A.16)

Taking functional derivative of \( \frac{\delta F}{\delta Z_k} [0, h] \) to \((x_k, m)\), we can get

\[
\frac{\delta F}{\delta(x_k, m)} \frac{\delta Z_k}{\delta Z_k} [0, h] = p_k|k-1(x_k|Z_{1:k-1}) \sum_{P \in Z_k} \exp^\Phi[0,h] \left( 1 - \tilde{P}_d^t + \sum_{W \in \mathcal{P}} \frac{L_W^t}{\Gamma_W[h]} \right) \\
\times \prod_{W \in \mathcal{P}} \Gamma_W[h] \left( 1 - \tilde{P}_d^t + \sum_{W \in \mathcal{P}} (1 - \Omega_W[h]) \frac{L_W^r}{\Gamma_W[h]} \right) v_{k|k-1}(m|x_k)
\] (A.17)

where

\[ \Omega_W[h] = \frac{L_W^t/\Gamma_W[h]}{1 - \tilde{P}_d^t + \sum_{W \in \mathcal{P}} (L_W^t/\Gamma_W[h])} \]

\[ \Phi[0, h] = v_{k|k-1}[h(1 - \tilde{P}_d^t)|x_k] - \mu - \sum_s \lambda_c^s \]

\[ \Gamma_W[h] = \Phi[\cdot]|_{g=0} + c_W + v_{k|k-1}[hL_W^r|x_k] \]

Dividing (A.17) by \( \frac{\delta F}{\delta Z_k} [0, h] \), and setting \( h = 1 \), we can get the update formulas of MRMSC-PHD filter shown in (6.10). By setting \( P_{d}^{mr,s,t} = 0 \) for all \( m > 1 \), then we can get the update formulas of LMSC-PHD filter shown in (5.12).
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