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2016

Wang, X. K., Wang, C., Li, Y. L., & Tan, S. K. (2016). Flow patterns of a low mass-damping cylinder undergoing vortex-induced vibration: Transition from initial branch and upper branch. *Applied Ocean Research*, 62, 89-99.

<https://hdl.handle.net/10356/83329>

<https://doi.org/10.1016/j.apor.2016.12.004>

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Flow patterns of a low mass-damping cylinder undergoing vortex-induced vibration: Transition from initial branch and upper branch

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This paper presents an experimental study of the structural dynamics and the wake vortex modes of a lightly damped, elastically mounted cylinder undergoing vortex-induced vibration (VIV) in the transverse direction. The cylinder is neutrally buoyant with the mass ratio $m^* = 1.0$, and has a low damping coefficient $\zeta = 0.0173$. The influence of variation in the free-stream velocity (corresponding to the reduced velocity range $1.53 \leq U^* \leq 6.62$, or the Reynolds number range $3000 \leq Re \leq 13,000$) on the structural dynamics of the cylinder is examined, in conjunction with detailed flow measurements around the cylinder using digital Particle Image Velocimetry (PIV). The cylinder is stationary at $U^* \leq 2.55$, and it begins to vibrate at $U^* \geq 3.05$ (or in synchronization regime). The vibrating cylinder exhibits the so-called “soft” synchronization phenomenon that the vortex shedding frequency is synchronous with the structural vibration frequency (or response frequency) of the cylinder, rather than with the natural frequency of the system. The measured data for the cylinder undergoing VIV fall within the initial branch and the upper branches, and agree well with published data on free vibrating cylinders at similar low mass and damping. The transition from initial- to upper-branch is characterized by a switch of vortex formation mode from the classical 2S mode to the newly-discovered 2P₀ mode by Morse and Williamson (2009, Journal of Fluid Mechanics, Vol. 634, pp. 5-39) based on a forced vibrating cylinder. The distinction between the 2S and 2P₀ modes is vivid in terms of length- and velocity-scales of the shed vortices, and is also reflected by quantitative characterization of near-wake flow statistics, including the mean and turbulence properties.

Key words: Vortex-induced vibration (VIV); low mass-damping cylinder; vortex mode;
Particle Image Velocimetry (PIV)

1. Introduction

Vortex-induced vibration (VIV) of cylindrical structures is a challenge in many fields of engineering, for example, transmission cables, heat exchangers, marine pipelines, offshore risers, and bridge piers. The practical significance of cylinder VIV has received much attention in the past few decades, see for example the extensive reviews by Bearman [1], Sarpkaya [2], Williamson and Govardhan [3,4] and Fernandes et al. [5] regarding the state of the art, current debates, as well as the various unresolved problems. Most previous studies focused on the paradigm of an elastically mounted rigid cylinder placed in uniform flow and constrained to move transverse to the flow in one degree of freedom. The principal dynamics giving rise to the cylinder VIV are the periodic vortex shedding from opposite sides of the cylinder. When the cylinder's vibration (or response) frequency (f) approaches the natural frequency of the system (f_N), large-amplitude (which can be up to the order of the cylinder diameter) vibrations take place. This is the so-called 'lock-in' or 'synchronization' phenomenon. Khalak and Williamson [6] showed that for a low combined mass-damping parameter ($m^*\zeta \sim O(10^{-2})$, where m^* is the mass ratio and ζ is the critical damping coefficient, see Table 1 for definitions), three branches of response amplitude are observed as a function of reduced velocity, namely: the 'initial', 'upper' and 'lower' branches. The different vibration states may be associated with different vortex shedding modes from the cylinder per cycle, for example as the '2S' mode (two single vortices), 'C(2S)' mode (coalesce of two single vortices), '2P' mode (two pairs of vortices), and asymmetric 'P+S' mode (a pair of vortices and a single vortex), as originally proposed by Williamson and Roshko [7].

The effects of mass and damping on the cylinder VIVs have been considered in the literature. It is suggested that the response may be determined by the value of $m^*\zeta$ (e.g., Khalak and Williamson [6]; Bahmani and Akbari [8]), or by m^* and ζ independently (Sarpkaya [9]). The classical definition of synchronization, namely, the frequency ratio $f^*(\equiv f / f_N) \approx 1$, is valid only for heavy structures with large mass ratio $m^* \sim O(100)$ or conducted in air. For marine applications with relatively low mass ratio of $m^* \sim O(1)$, f^* may reach higher values [3]. In this regard, Khalak and Williamson [6] suggested that for light structures, the matching between the

1 response frequency and the vortex shedding frequency (called “soft” synchronization) is a more
2 appropriate definition. Further, Govardhan and Williamson [10] demonstrated the existence of a
3 critical mass ratio ($m_{\text{crit}}^* \sim 0.54$), below which the lower branch does not exist at all and the
4 large-amplitude upper response will persist up to infinite reduced velocity.

5 Morse and Williamson [11] suggested that the response and vortex modes of a cylinder under
6 forced sinusoidal vibration could reasonably predict those of a freely (self-excited) vibrating
7 cylinder prescribed in the transverse direction, once the parameters are correctly matched in the
8 normalized amplitude-wavelength ($A^* - \lambda^*$) space (see Table 1 for definitions of A^* and λ^*), as
9 will be shown in Section 3.2. This is due to the fact that the results of free vibrations have shown
10 that the cylinder motion and the lift force in the synchronization regime are well represented by
11 sinusoidal functions. For a low mass-damping system, the 2S and 2P modes pertain to the initial
12 and lower branches, respectively. Furthermore, Morse and Williamson [11] identified a new
13 mode responsible for peak amplitude vibration in the upper branch, termed as the “2P_{OVERLAP}”
14 or “2P_O” mode because its regime in $A^* - \lambda^*$ plane overlaps with other regions. This mode
15 comprises two pairs of vortices in each cycle, but the second vortex of each pair is distinctly
16 smaller than the first vortex. Of course, VIV in multiple degrees of freedom would be more
17 complicated. For instance, Fernandes et al. [5] showed that for a cylinder that is free to vibrate
18 transversely but is forced longitudinally, several new shapes of cylinder trajectory are observed,
19 including the ‘S’, ‘epsilon’, ‘three rings’, ‘four rings’ and ‘five rings’.

20 The most immediate and obvious concern for Morse and Williamson’s highly resolved data
21 set is to what extent they can be applied to the free vibration case, as commented by Bearman
22 [12]. Firstly, the data set were obtained for constant Reynolds numbers ($\text{Re} = 4000$ and $12,000$),
23 whereas in the case of free vibration it is normal to fix the structural parameters and vary the
24 flow velocity U , and corresponding the Re . On the other hand, as shown by Govardhan and
25 Williamson [13] on a freely vibrating cylinder at low mass and damping, the maximum
26 transverse amplitude is basically Re -invariant in the laminar regime (up to $\text{Re} = 200$), but
27 continues to rise almost linearly with Re in the subcritical regime (up to $\text{Re} = 10^4$). The Re -
28 dependent nature of VIV was recently reported by Narendran et al. [14] for high Reynolds
29 numbers ($\text{Re} \sim 10^5$). Secondly, although the free and forced vibration cases exhibit many
30 similarities in the wake modes and fluid forcing, there still exist major differences in energy
31 transfer between the fluid and the structure. For instance, Morse and Williamson [11] showed
32 that in the P+S mode, energy is transferred from the fluid to the structure and hence this mode is

not expected in the wake of a freely vibrating cylinder. Then, the question arises for the newly-discovered “2P_O” mode – does it exist for freely vibrating cylinder?

The above concerns motivate the present work on a freely vibrating cylinder with low mass and damping. As illustrated by Morse and Williamson [15], at the higher end of the synchronization regime, the overlapping phenomenon may lead to some interesting behaviors of VIV, such as the intermittent switching between the upper and lower branches (between 2P_O and 2P modes). However, emphasis of the present study is focused on the initial- to upper-branch transition region (between 2S and 2P_O modes), or the lower end of the synchronization regime. Detailed flow measurements around the cylinder were conducted using digital Particle Image Velocimetry (PIV).

Unlike most previous studies where the cylinder was in vertical surface-piercing arrangement, the cylinder was aligned horizontally and fully submerged in water body to eliminate the possible effects of free surface. The cylinder was neutrally buoyant ($m^* = 1.0$). During the experiments, the cylinder was free to vibrate in the transverse direction with the structural parameters maintained constant. The present study arises from our earlier study – Wang et al. [16] whereby the cylinder was placed at various heights above a plane wall to investigate the effects of boundary layer on the vortex dynamics and structural response of the cylinder. The present study considers the pure effects of varying the incoming flow velocity by removing the near-wall constraints.

2. Experimental Set-up and Methodology

The experiments were performed in the re-circulating rectangular water channel located at Maritime Research Centre, Nanyang Technological University. The water channel consisted of a pipeline with a centrifugal pump and a flow meter, and a contraction section with a contraction ratio of 5:3 leading to a test section (length 5 m, width 0.3 m, height 0.45 m). The channel floor and the two side walls of the test section were made of glass to allow for optical access. The flow rate was controlled using the pump fitted with a variable speed controller, and the flow velocity in the test section could be adjusted. The free-stream velocity was uniform to within 1.5% across the test section, and the turbulence intensity was well below 2%.

Table 1 Key dimensionless parameters used in this study.

Parameter name	Symbol	Definition	Value
Mass ratio	m^*	$m/(\pi\rho D^2 L/4)$	1.0
Damping coefficient	ζ	$c/(2\sqrt{k(m+m_A)})$	0.0173
Reduced velocity	U^*	$U/(f_N D)$	1.53 – 6.62
Reynolds number	Re	UD/ν	3000 – 13,000
Strouhal number	St	fD/U	
Normalized wavelength	λ^*	$\lambda/D = U/(fD)$	
Normalized amplitude	A^*	A/D	
Frequency ratio	f^*	f/f_N	

Note: m = mass of the cylinder; ρ = fluid density; D = cylinder diameter; L = cylinder length; c = structural damping coefficient; k = spring stiffness; $m_A = C_A m_d$, where m_d is the displaced-fluid mass and C_A is the potential added-mass coefficient ($C_A = 1.0$ for circular cylinder); U = free-stream velocity; f_N = natural frequency of the system in still water; ν = kinematic viscosity of fluid; f = vibration (response) frequency; λ = vibration (response) wavelength ($= U/f$); A = vibration (response) amplitude.

The cylinder was made of smooth, hollow, acrylic tube with an outer diameter of $D = 20$ mm. The free-stream velocity U was varied from 0.15 to 0.65 m/s, corresponding to $U^* = 1.53 - 6.62$ or $\text{Re} (\equiv U^* f_N / \nu) = 3000 - 13,000$. The cylinder was mounted in the test section with its axis arranged horizontal and perpendicular to the flow direction, as shown schematically in Fig. 1. The origin of the coordinate system is fixed at the centre of the cylinder at rest. The x -axis is along the flow direction and pointed downstream (streamwise), the y -axis is perpendicular to the flow direction and to the cylinder axis (transverse), and the z -axis is along the cylinder axis (spanwise). During the experiments, the water depth in the test section was maintained at 220 mm, resulting in a blockage ratio of 9.1%, which is comparable to those in the literature, e.g., 7.5% in [5] and 10% in [11]. Through decay tests in still water, the natural frequency of the system was found to be $f_N = 4.91$ Hz, which was used to obtain the dimensional parameters as shown in Table 1. For each case the flow was started from rest, thus hysteresis effect was not considered in the present study. The span (length) of the cylinder was 280 mm, resulting in an aspect ratio of 14, which was large enough to ensure 2-dimensional (2D) flow in the near wake. The PIV measurements were carried out at the mid-span plane. Details of the experimental set-up are available in Wang et al. [16].

1 Velocity measurements were performed using a digital PIV system (LaVision model). The
2 flow field was illuminated with a double cavity Nd:YAG laser light sheet at 532 nm wavelength
3 (Litron model, power ~ 135 mJ per pulse, duration ~ 5 ns). Spherichel[®] 110P8 hollow glass
4 spheres (neutrally buoyant with a mean diameter of 13 μm) were seeded in the flow as tracer
5 particles. The particle images were recorded using a 12-bit charge-coupled device (CCD)
6 camera, which had a resolution of 1600×1200 pixels and a frame rate of 15 Hz. This apparatus
7 has been used to study the VIVs of a tethered cylinder (Wang et al. [17]).

8 The LaVision Davis package was used to process the particle images and determine the flow
9 vector field. Particle displacement was calculated using the fast-Fourier-transform (FFT) based
10 cross-correlation algorithm with the standard Gaussian sub-pixel fit structured as an iterative
11 multi-grid method. The processing procedure included two passes, starting with a grid size of 64
12 $\times 64$ pixels, stepping down to 32×32 pixels overlapping by 50%, which resulted in a set of 7500
13 (100×75) vectors for a typical field. In between passes, the vector maps were filtered by using
14 a 3×3 median filter in order to remove possible outliers. The field of view was $190 \text{ mm} \times 142$
15 mm, therefore the spatial resolution for the present set-up was $1.9 \text{ mm} \times 1.9 \text{ mm}$ (i.e., $0.095 D \times$
16 $0.095 D$). For each case, a series of 1260 instantaneous snapshots was acquired at the sampling
17 frequency of 15 Hz (or 84s recordings), in order to achieve reasonably converged statistics of
18 the flow (such as mean and turbulence properties). The uncertainty in the instantaneous velocities
19 (u and v) was estimated to be about 2%. Based on the velocity vector distribution, the
20 instantaneous spanwise vorticity ($\omega_z = \Delta v / \Delta x - \Delta u / \Delta y$) was calculated using the least
21 operations extrapolation scheme. The uncertainty in ω_z was about 10%. A more detailed
22 description of the PIV post-processing procedure and uncertainty analysis is given in Wang and
23 Tan [18].

Fig. 1. Schematics of the experimental set-up and coordinate system.

The motions of the cylinder were recorded using a Canon video camera with a recording frequency of 25 Hz. The normalized amplitude (A^*), defined as the peak displacement away from its mean position (A) normalized by the cylinder diameter, was calculated by taking the average of 10% highest peaks recorded for each case.

3. Results and Discussion

3.1 Structural response

The experimental results show that when the flow velocity is low (at $U^* \leq 2.55$ or $Re \leq 5000$), the cylinder is essentially stationary (or $A^* \approx 0$). Examples of the time histories of transverse displacement (y/D) of the cylinder within $3.05 \leq U^* \leq 5.6$ ($6000 \leq Re \leq 11000$) are presented in Fig. 2. As U^* increases, the vibrations become more periodic with an increase in the amplitude. At $U^* \leq 3.56$ the amplitude is extremely low and the vibrations are irregular, whereas for $U^* \geq 5.1$ the vibrations are highly periodic and nearly sinusoidal. At $U^* = 4.07$ or 4.58 , the motion is characterized by intermittent bursts of large-amplitude vibration. Take $U^* = 4.58$ as example. Apart from the fundamental frequency of about 3.75 Hz, a low-frequency unsteadiness can also be observed to be superimposed, which appears to occur with a period of about 15 ~ 20 times the fundamental period. In other words, the motion comprises of two states: the small-amplitude state (e.g., $A^* \sim 0.25$ at $t = 3 \sim 5$ s or $9 \sim 11$ s, which are highlighted in the figure) and large-amplitude state (e.g., $A^* \sim 0.5$ at $t = 1 \sim 3$ s or $6 \sim 8$ s). This suggests the possibility of

mode competition between two (or more) modes of vortex shedding, as will be elucidated by the PIV results.

Fig. 2. Time history of the cylinder displacement (y/D) for different reduced velocities.

Fig. 3(a) and 3(b) show the response amplitude and frequency as a function of U^* , respectively. Data in Govardhan and Williamson [10] on a freely vibrating cylinder with similar mass ratio and damping coefficient ($m^* \approx 1.19$ and $\zeta = 0.00502$) are included for comparison. Fig. 3(a) illustrates that these two sets of data collapse very closely, with the present data points falling in the initial branch ($3.05 \leq U^* \leq 4.58$) and the upper branch ($5.1 \leq U^* \leq 6.62$). The value to distinguish the two branches is at $U^* \approx 5$ (a value that is consistent with reported findings), as indicated by a sharp jump in A^* , that is, from 0.53 at $U^* \approx 4.58$ to 0.84 at $U^* \approx 5.1$. The maximum amplitude is $A^*_{\max} \approx 1$, which agrees well with that reported by Govardhan and Williamson [10]. The frequency response data ($f^* = f/f_N$) are shown in Fig. 3(b), where

1 the thick straight line of $St = 0.19$ represents the vortex shedding frequency from a stationary
 2 cylinder. Except for $U^* = 3.05$ that corresponds to the beginning of the initial branch whereby
 3 $f^* \approx 1$ (Yang et al. [19]), the vibrating cylinder locks on to a frequency that is defined by the
 4 Strouhal number function of $St = f_{St}D/U = 0.167$. This kind of lock-in is different from that of
 5 the classical definition as the regime where the response frequency is close to the natural
 6 frequency of the system (or $f^* = 1$). This is a consistent phenomenon for very “light” structure,
 7 see reviews by Williamson and Govardhan [3]. The values of f^* in the present study are slightly
 8 lower (variation within 10%) than those in [10], but the overall trend for these two sets of data
 9 is similar. In particular, f^* increases with U^* over the measurement range and passes across
 10 the line of $f^* = 1$. An examination of the PIV results indicates that the response frequency f_{St}
 11 always synchronizes with the vortex shedding frequency from the cylinder undergoing VIV (f_V
 12), i.e., $f_{St} = f_V$. This is the so-called “soft” synchronization proposed by Khalak and Williamson
 13 [6]. Owing to the constraints of the water channel (its maximum flow speed was limited by the
 14 capacity of the recirculating pump), however, it is unable to predict VIV behavior at even higher
 15 U^* . However, it can be confirmed that the cylinder would vibrate at a frequency other than f_N
 16 if U^* is further increased. This is consistent with the conclusion of Govardhan and Williamson
 17 [10] on light structures.

Fig. 3. Response data, together with those reported by Govardhan and Williamson [10] on a freely vibrating cylinder ($m^* = 1.19$, $\zeta = 0.00502$): (a) amplitude; (b) frequency.

3.2 Vortex shedding patterns

Fig. 4 plots the measurement results of the cylinder in the normalized amplitude-wavelength ($A^* - \lambda^*$) plane, overlaid on the vortex mode map proposed by Morse and Williamson [11] based on a forced vibrating cylinder at $Re = 12,000$. As U^* increases, A^* increases monotonically and sharply, but λ^* slightly increases and is nearly constant at $\lambda^* \approx 5.4 \sim 6.0$. The first four cases (i.e., $3.05 \leq U^* \leq 4.58$) lie in the 2S regime, while the last three cases ($5.1 \leq U^* \leq 6.62$) in the

2P₀ regime. As will be illustrated hereafter, this is consistent with the present results, indicating the validity of predicting a freely vibrating cylinder using the forced vibration data.

Fig. 4. Locations of the cylinder undergoing VIV in the normalized amplitude-wavelength plane, superimposed on the map of regimes for vortex shedding modes of a cylinder under forced vibrations at $Re = 12,000$ by Morse and Williamson [11].

Fig. 5(a) – 5(d) show the phased-averaged flow field in the near-wake region at $U^* = 2.55$ – 6.62 when the cylinder is at the mid-point of upward stroke. The readers are referred to Perrin et al. [20] for phase-averaging technique applied to PIV results. The vector maps have been Galilean decomposed by subtracting the convection velocity of $U_c = 0.7U$ (which is estimated by trial and error) in order to visualize the vortex motions as also identified by contours of the spanwise vorticity (Kolář [21]). One can observe alternating vortex shedding from the cylinder, but the flow structure exhibits appreciable variation with respect to U^* . One apparent trend is that the vortex formation length, which is defined as the streamwise distance from the cylinder to the core of fully formed vortex [22], decreases with the increase in U^* . At $U^* = 2.55$ or 3.56, the wake is characterized as the classical von Kármán vortex street, or the so-called 2S mode (two single vortices per cycle). This is consistent with the previous literature that the initial branch corresponds to the 2S mode. At $U^* = 4.58$ (the last point in the initial branch), however,

the vortex mode is somewhat modified in that the transverse spacing between the opposite-signed vortices become larger. This increasing trend in transverse spacing is more obvious in the upper branch, resulting in a completely different vortex mode from 2S. For example at $U^* = 6.62$ as shown in Fig. 5(d), the wake is composed of two rows of opposite-signed vortices that are widely separated in the transverse (y) direction with the spacing of the order of $3D$. Similarly, the cylinder motion results in an increase in the streamwise spacing between the vortices, although not as sharp as that of the transverse spacing. This wake mode is actually the newly discovered $2P_0$ mode by Morse and Williamson [11], as will be discussed in detail hereafter.

Fig. 5. An example of the phase-averaged flow field measured by PIV at $U^* =$: (a) 2.55; (b) 3.56; (c) 4.58; and (d) 6.62. Each plot is a mean of 20 snapshots corresponding to the correct phase $\pm 5^\circ$. Velocity vector plot is with a reference-frame velocity of $0.7U$, superimposed with flood contours of spanwise vorticity $\omega_z D/U$: blue – clockwise vorticity; red – anticlockwise vorticity. The cylinder’s position and movement are highlighted, with the dashed lines indicating the extent of the transverse motion.

A time sequence showing the vortex dynamics at $U^* = 5.1$ during a cycle of vibration is given in Fig. 6. The phase-averaged flow fields are separated by a quarter-period of the vibration

cycle, where vortices are numbered, as V1 – V8, in the order of their genesis in the wake. It is evident that the cylinder motion synchronizes with the vortex shedding with a pair of counter-rotating vortices shed per cycle, namely, V6 and V7 shed along with the upward and downward motions, respectively. At the top of the vibration cycle (Fig. 6(b)), the negative vortex immediately adjacent to the cylinder base (V6) is stretched in the transverse direction and extends well into the lower half of the wake. The same is the case of the positive vortex, V7, at the bottom of vibration cycle (Fig. 6(d)). However, once the vortices are shed as discrete entities, they are nearly circular in shape, forming two rows of opposite-signed vortices, i.e., the upper row (V2 – V6) and the lower row (V1 – V7). These two rows of vortices are considerably separated in the transverse direction and propagate downstream almost parallel to each other. Consequently, the sweeping across the wake and the interaction between the two shear layers are less vigorous than those for the 2S mode.

Fig. 6. A set of phase-averaged flow fields showing the vortex shedding process during one vibration cycle for $U^* = 5.1$. Each of these plots is separated by a quarter period, i.e., $\Delta t / T = 1/4$.

In order to quantify the vortex structures, here we follow the work by Perry et al. [23] on the flow topology analysis in the wake of bluff bodies using critical-point theory. For 2D flows, two non-degenerate critical (zero velocity or stagnation) points may develop in the flow field: (i) centres, characterized by closed streamlines; and (ii) saddles, characterized by intersection of streamlines. The principal features of the phase-averaged topology are foci of the initially formed Kármán vortices and a saddle point between them. A sketch of the critical points in an idealized von Kármán vortex street consisting of point vortices all moving rectilinearly with the same velocity is shown in Fig. 7.

Fig. 7. (a) Definition sketch of flow around a circular cylinder; (b) sketch of streamlines of the Kármán vortex street seen from a reference frame moving with the vortices (adapted from Perry et al. [23]): C – centre; S – saddle.

An example of the phase-averaged streamline patterns is shown in Fig. 8(a) for $U^* = 5.6$ at the upper-most position of the vibration cycle. An instantaneous vorticity field at the same flow condition is shown in Fig. 8(b). Comparison between these two figures shows that each centre-saddle system corresponds to a well-confined, large-scale vortex structure, although in the instantaneous vorticity field some additional, small-scale vortices exist besides the large-scale ones. Each small-scale vortex is due to the stretching and bifurcation of the elongated region of concentrated vorticity in the transverse direction (e.g., the region with negative vorticity to be shed in Fig. 8(b)). The leading part is shed and pairs up with a preceding vortex with positive vorticity to become one vortex pair in lower side of the shear layer, whereas the trailing part of the stretched vortex is shed at a later stage with the breaking up of positive vorticity and forms the upper row of vortices. Such stretching and bifurcation of vorticity immediately downstream of the cylinder is evident for vortices V6 in Fig. 6(b) and V7 in Fig. 6(d).

Fig. 8. Flow around the cylinder for $U^* = 5.6$ at its upper-most position: (a) phase-averaged velocity vector plot; (b) instantaneous vorticity plot. Streamlines are included in the velocity vector plot to highlight centre-saddle and vortex structures.

The present results confirm the finding of Morse and Williamson [11] that the initial and upper branches correspond to the 2S and 2P_O modes, respectively. The 2P_O mode is similar to the 2P mode in that two pairs of vortices are shed per cycle of cylinder vibration, but the

secondary vortex of each the vortex pair is significantly weaker. The present studies indicate that the secondary vortices (as highlighted by black arrows in Fig. 8(b)) are indeed rather weak and incoherent, such that they be evident in the instantaneous flow field, but may not be observable in the phase-averaged flow field (Fig. 8(a)). This mode is equivalent to the “intermediate wake state” identified by Carberry et al. [24], or the “2P_{upper}” mode found by Govardhan and Williamson [10] in the upper branch of a free vibration response at low mass and damping ($m^* = 8.63$, $\zeta = 0.00151$). In the case of 2P₀ mode, the maximum value of spanwise vorticity is observed in the immediate vicinity of the cylinder, thereafter the vortices decay rapidly with downstream distance. This trend is opposed to the 2S mode, where the vortices are shed after a certain formation length (which is about $4.8D$ for natural vortex formation, as suggested by Jeon and Gharib [25]) with their strength generally kept constant over a substantial streamwise distance.

In addition, special attention is paid to the case of $U^* = 4.58$, the response amplitude of which is varying over time as shown in Fig. 2. By observing the instantaneous wake snapshots in excess of 300 shedding cycles for this case, it is found that the vortex shedding exhibits a feature of mode competition: it can switch between the 2S and 2P₀ modes periodically with different response amplitudes. The small-amplitude state tends to be in the 2S mode, whereas the large-amplitude state is more likely in the 2P₀ mode that corresponds to a wider wake, as depicted in Fig. 9.

Fig. 9. Instantaneous flow field for $U^* = 4.58$ vibrating at: (a) small-amplitude; (b) large-amplitude. The cylinder is at the mid-point of the upward stroke.

3.3 Statistical characterization of the flow

The vorticity field shown in Fig. 8(b) shows that the secondary vortices are indeed rather weak in strength and small in scale, so the 2P₀ mode can be easily mistaken as the 2S mode if the measurement resolution is not fine enough. In this section, a detailed flow analysis is conducted to highlight the distinction between these two modes.

The streamwise (L_x) and transverse (L_y) spacings between the vortices as shown in Fig. 7(b) can be calculated. A frame-by-frame inspection of the instantaneous snapshots which are of the order $O(10^3)$ obtained for each case shows that both L_x and L_y generally keep constant within the streamwise range of $x/D = 3 - 10$. The normalized streamwise (L_x/D) and transverse (L_y/D) spacings for different cases are shown in Fig. 10(a). In the initial branch ($U^* \leq 4.58$) that corresponds to the 2S mode, L_x/D is virtually invariant (≈ 4.1) with the free-stream velocity. This value agrees well with the reported value of $L_x/D \approx 4.3$ by Perry et al. [23] on a stationary cylinder. The same trend is for L_y/D in the initial branch, which keeps nearly constant at about 1.0 but slightly increases within the range $3.05 \leq U^* \leq 4.58$ (in the initial branch). This leads to $L_y/L_x = 0.244$, which is slightly lower than the predicted value of 0.28 for classical von Kármán vortices. In the upper branch ($U^* \geq 5.1$), on the other hand, both L_x/D and L_y/D increase sharply with U^* . The extraordinarily sharp increase in L_y/D is consistent with the observed wider wake associated with 2P₀ mode. Moreover, some velocity scales can be obtained by analyzing the flow field. Since L_x is the distance travelled by a vortex in one cycle and f_v is the vortex shedding frequency, the convection velocity of the shed vortex along the streamwise direction (U_c) can be calculated as

$$U_c = f L_x = \frac{St U}{D} L_x$$

therefore,

$$\frac{U_c}{U} = \frac{St L_x}{D}.$$

This relationship indicates that once L_x/D and St keep constant (as in the present study), so does U_c/U . Fig. 10(b) suggests distinct characteristics of the variation trend of U_c/U with U^* depending on the cylinder's vibration state. At $U^* \leq 2.55$ when the cylinder is stationary,

the convection velocity is nearly constant with $U_c/U \approx 0.76$, a value agrees with previous publications on stationary cylinder. In the initial branch ($3.05 \leq U^* \leq 4.58$), U_c/U decreases almost linearly with velocity; the converse trend holds for the upper branch ($5.1 \leq U^* \leq 6.62$).

Fig. 10. Properties of vortex structures as a function of U^* based on PIV results: (a) length scales (L_x/D and L_y/D); (b) velocity scale (U_c/U).

In addition, the flow statistics, such as mean velocity and vorticity, root-mean-square (r.m.s.) velocity and Reynolds shear stress, are calculated by ensemble-averaging the 1260 instantaneous flow fields obtained for each case. The sampling period is 84 s, which typically amounts to a

range of $630 - 2730$ convective time units (D/U), or about $120 - 430$ vortex shedding cycles, depending on the value of flow velocity. The difference between the initial and upper branches is also evident in the mean velocity vector plots, as shown in [Fig. 11](#). One obvious observation is that for each case in the initial branch ($U^* \leq 4.58$) there exists a recirculation zone in near-wake region. As U^* increases within this range, the streamwise length of the recirculation zone generally decreases, but the magnitude of the reverse flow velocity increases. A similar observation is for the case of a stationary cylinder, where the length of the recirculation zone is found to decrease slightly from about $2.5 D$ to $2.2 D$ over the range $3000 \leq Re \leq 13,000$. The recirculation zone, however, disappears in the upper branch ($U^* \geq 5.1$) where all vectors are instead positive in the streamwise direction and the magnitude of mean streamwise velocity immediately downstream of the cylinder base increases with U^* . The disappearance of the recirculation zone, also reported by Govardhan and Williamson [\[26\]](#) for cases corresponding to the upper branch, is explainable, considering that the response amplitude for these three cases lies in the range $0.8 < A^* < 1.0$. The response amplitude of $A^* > 0.5$ implies that the cylinder's lower (or upper) face will periodically move above (or below) the cylinder's neutral position ($y = 0$) during one vibrating cycle. This is as opposed to the initial branch where $A^* \leq 0.5$, so that a certain region is permanently blocked by some portion of the cylinder.

Fig. 11. Plot of the mean velocity vectors ($\bar{u}$, $\bar{v}$) in the near wake of the cylinder at: (a) $U^* = 2.55$ ($\text{Re} = 5000$); (b) $U^* = 3.56$ ($\text{Re} = 7000$); (c) $U^* = 4.58$ ($\text{Re} = 9000$); (d) $U^* = 5.1$ ($\text{Re} = 10,000$); (e) $U^* = 5.6$ ($\text{Re} = 11,000$); (f) $U^* = 6.62$ ($\text{Re} = 13,000$).

The contour plots of the normalized Reynolds shear stress $-\overline{u'v'}/U^2$ for different cases are shown in Fig. 12, where u' and v' are the fluctuating streamwise and transverse velocities, respectively. When the cylinder is stationary (e.g., at $U^* = 2.55$ or $\text{Re} = 5000$), the Reynolds stress distribution is characterized as the so-called four distinct “lobes”: besides the two large-scale lobes, there are two additional small-scale ones located immediately downstream of the cylinder base. This feature agrees with both the PIV measurements and direct numerical

simulation (DNS) results reported by Dong et al. [27] on a stationary cylinder at comparable Reynolds numbers ($Re = 4000$ and $10,000$). A similar 4-lobe structure is also evident at $Re = 7000$ ($U^* = 3.56$) that corresponds to the initial branch (2S mode), but the entire pattern has moved a substantial distance upstream toward the cylinder base. In the upper branch (2P_O mode), on the other hand, the smaller pair of lobes disappear and the Reynolds shear stress distribution becomes dramatically more compact in the streamwise direction, but much wider in the transverse direction. This is in accordance with the observation on the 2S and 2P_O modes that the latter is characterized by vortex formation much closer the cylinder base but with a much larger transverse spacing between the two rows of vortices. The distribution shown in Fig. 12(c) for $U^* = 4.58$ (upper limit of the initial branch) is somewhat peculiar, which comprises 3 pairs of distinct “lobes” that are anti-symmetric about the cylinder centerline. This type of distribution is believed to be associated with the periodic switching between the two modes – 2S and 2P_O – for this case.

Fig. 12. Contour plot of the normalized Reynolds shear stress $(-\overline{u'v'})/U^2$ at: (a) $U^* = 2.55$ (Re = 5000); (b) $U^* = 3.56$ (Re = 7000); (c) $U^* = 4.58$ (Re = 9000); (d) $U^* = 5.1$ (Re = 10,000); (e) $U^* = 5.6$ (Re = 11,000); (f) $U^* = 6.62$ (Re = 13,000). Positive – solid lines; negative – dashed lines. Δ is the contour interval.

The above discussions provide evidence that the topology of the vortices in the wake sheds lights on possible explanation of wake transition in accordance with the VIV behavior. Fig. 13 shows the contour plot of the normalized r.m.s. streamwise velocity $\sqrt{u'^2}/U$ (left column) and mean spanwise vorticity $\overline{\omega_z}D/U$ (right column) at $U^* = 2.55, 4.07$ and 5.6 , which correspond to the three distinct states of the cylinder motion, namely, stationary, the initial and upper branches, respectively. As U^* increases, the peak value of $\sqrt{u'^2}/U$ increases from 0.24 to 0.56, and the overall distribution of $\sqrt{u'^2}/U$ moves substantially toward the cylinder together with a

broadening in the transverse direction. The distribution of $\overline{\omega}_z D/U$, in contrast, exhibits a decreasing trend in magnitude as U^* increases. As compared to the case of $U^* = 2.55$ (stationary, 2S mode), the $\overline{\omega}_z D/U$ distribution at $U^* = 4.07$ (initial branch, 2S mode) shrinks significantly in the streamwise direction, but expands slightly in the transverse direction. In the case of $U^* = 5.6$ (upper branch, 2P₀ mode), zone of significant vorticity becomes extraordinarily large in both the streamwise and transverse directions, but with a much lower magnitude (less than half of $U^* = 2.55$). Also, it is noted that in Fig. 13(f) a pair of small-scale vortices (as highlighted by two arrows) with opposite-signed vorticity exist immediately downstream of the cylinder. This is attributed to the aforementioned stretching and bifurcation of initially formed vorticity in 2P₀ mode, with its leading part shed as a separate weak vortex from either side of the cylinder per half cycle of motion, and periodically fed into the opposite side of the shear layer.

Fig. 13. Contour plot of the normalized r.m.s. streamwise velocity $\sqrt{u'^2}/U$ (left column) and mean spanwise vorticity $\bar{\omega}_z D/U$ (right column) at $U^* = 2.55$ (a) and (c); 4.07 (b) and (e); 5.6 (c) and (f).

4. Concluding Remarks

This study investigates the structural dynamics and the vortex modes of a low mass-damping cylinder ($m^* = 1.0$, $\zeta = 0.0173$) that is placed in a uniform flow and subject to self-excited VIV in the transverse direction. With the structural parameters held constant, the objective is to examine the effect of variation in the free-stream velocity (corresponding to $1.51 \leq U^* \leq 6.73$, or $3000 \leq \text{Re} \leq 13,000$).

The results demonstrate the strong dependence of cylinder VIV on the free-stream velocity. Three distinct states of the structural dynamics are categorized: (i) stationary ($U^* \leq 2.55$, or $\text{Re} \leq 5000$); (ii) small-amplitude vibration in the initial branch ($3.05 \leq U^* \leq 4.58$, or $6000 \leq \text{Re} < 9000$); and (iii) large-amplitude vibration in the upper branch ($U^* \geq 5.09$, or $\text{Re} \geq 10,000$). There is a switch of vortex shedding frequency from the cylinder (f_v), from $\text{St} = 0.19$ when the cylinder is essentially stationary, to $\text{St} = 0.167$ when the cylinder is vibrating in the synchronization regime. The response amplitude and frequency are in close agreement with published studies at similar mass and damping. In the synchronization regime, the response frequency always locks on to f_v except for $U^* = 3.05$ at which the cylinder just starts to vibrate. This kind of synchronization, instead of synchronizing with the natural frequency of the system, seems to be a consistent phenomenon for light structures, as reported by previous researchers.

The results also depict appreciable difference in the near-wake flow structure depending on the cylinder's VIV characteristics. Velocity and vorticity field measurements using PIV enable quantitative understanding of the vortex dynamics. The vortex formation mode in the initial branch is the same as that of a stationary cylinder, namely, the 2S mode. On the other hand, the upper branch of the free vibrating cylinder corresponds to the 2P₀ mode, consistent with the conclusion drawn by Morse and Williamson [11] based on forced vibrating cylinder. With the aid of topology analysis of the vortices for the time series of instantaneous PIV snapshots obtained for each case, it appears that there exist certain length- and velocity-scales to quantitatively demarcate the two vortex modes. The transition from 2S mode to 2P₀ mode is characterized by a dramatic reduction in vortex formation length and simultaneously a considerable increase in the transverse spacing between the two rows of vortices. Depending the

state of the cylinder vibration, the normalized convection velocity U_c/U of the shed vortices exhibit different trends with respect to U^* : that is, nearly constant when the cylinder is essentially stationary, decreasing linearly in the initial branch, and increasing linearly in the upper branch. Unlike the case of forced vibration where the cylinder motions are always regular, the response amplitude of free vibration at a given reduced velocity particularly in the initial- to upper-branch transition region may vary over time. This would result in an intermittent switching of vortex shedding mode between 2S and 2P_O depending on the response amplitude. The difference in the vortex shedding process also leads to a quantitative distinction in flow statistics around the cylinder, including distributions of the mean velocity and vorticity, r.m.s. velocity and Reynolds shear stress. Using various flow visualization techniques (including centre-saddle topology and Galilean decomposition), it is shown that for the 2P_O mode, the secondary vortex of each vortex pair is indeed weak and incoherent, and hence may not be observable in the phase-averaged flow fields. Depending on the reduced velocity, the Reynolds shear stress distributions in the wake of the cylinder exhibit distinct patterns, including the ‘4-lobe’, ‘6-lobe’ and ‘2-lobe’ structures.

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