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Frequency-domain synthetic aperture focusing for helical ultrasonic imaging

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The synthetic aperture focusing technique (SAFT) is widely used to provide significant improvement in the lateral resolution of ultrasonic images. Frequency-domain SAFT has shown higher accuracy and greater efficiency than time-domain SAFT. However, frequency-domain SAFT should be helix-based for ultrasonic scanning of cylindrical structures such as pipes and axletrees. In this study, a frequency-domain SAFT is proposed for 3D helical ultrasonic imaging applications. This technique adjusts the phase spectra of the images to complete the synthetic aperture focusing process. The focused image is precise because the proposed algorithm is established on the basis of the wave equation in a helical coordinate system. In addition, the algorithm can efficiently separate out point scatterers and present volume scatterers. The experimental results show that the proposed algorithm yields lower side lobes and enhances the angular resolution of the ultrasonic image to approximately 1° – 1.5° , which is much better than the performance of time-domain SAFT. The maximum deviations are only 0.6 mm, 0.5° , and 0.4 mm along the r -axes, θ -axes, and z -axes, respectively, which are appropriate for normal ultrasonic nondestructive testing. *Published by AIP Publishing.* [<http://dx.doi.org/10.1063/1.4979369>]

I. INTRODUCTION

Ultrasonic imaging is important for use in nondestructive testing (NDT) because it provides abundant and quantitative information about the objects under inspection. However, the lateral resolution of the resulting images is degraded by ultrasound diffraction. This degradation intensifies with increasing ultrasound propagation distance. To address this limitation, the synthetic aperture focusing technique (SAFT) was introduced in the 1970s.¹

The initial implementation of the SAFT in ultrasonic NDT was based on delay and sum (DAS) beamforming in the time domain.² The SAFT creates a large synthetic aperture from several overlapping measurements after delayed operation and thus enhances the lateral image resolution. The image resolution and contrast that are achieved using the DAS technique are only limited by the aperture size and operating frequency.^{3,4} Therefore, the frequency domain SAFT was introduced from seismology.

The frequency-domain SAFT achieves synthetic aperture focusing by applying different nonlinear transformations to the image spectrum. Similar to the computationally efficient fast Fourier transform (FFT) algorithm, the frequency-domain SAFT shows less computational complexity than the time-domain SAFT.^{5,6} Several SAFT algorithms in the frequency domain have been developed for plane scanning; these algorithms include phase-shift migration (PSM),^{7–9} split-step Fourier migration,^{10,11} and phase-shift plus interpolation algorithms.¹² Most of these algorithms are adapted for use in multilayer materials or complex structures.

Additionally, the cylindrical PSM algorithm^{13–16} was proposed for cylindrical scanning; using this algorithm, a transducer scans the cylindrical surface under test using a circular motion.

Helical scanning is the most widely used method in ultrasonic NDT for cylindrical structures¹⁷ such as pipes and the axletrees of high-speed rails. Ultrasonic inspection can be implemented continuously and efficiently for cylindrical structures in the helical scan mode. Typical composite motions of a transducer and a cylindrical structure are shown in Fig. 1. In helical scanning, the cylindrical PSM algorithm is not applicable because it is based on a solution to the cylindrical wave equation; a helix-based SAFT is thus required in this case. The time-domain SAFT for helical motion was investigated in 2011 to enhance the resolution of trans-rectal ultrasound.¹⁸ However, only limited studies of the helix-based SAFT have been performed; in particular, the frequency-domain SAFT for helical scanning applications has not been investigated to date.

In this paper, a frequency-domain SAFT algorithm called helical PSM (HPSM) is proposed for helical ultrasonic imaging applications. This algorithm is based on a solution to the helical wave equation¹⁹ and an assumption of the exploding reflector model (ERM).²⁰ The HPSM algorithm uses frequency-phase migration to extrapolate the acoustic field and eliminate the effects of ultrasonic diffraction. The algorithm inherits its high-resolution enhancement capability from PSM and can perform synthetic aperture focusing for 3D imaging applications in the helical coordinate system.

The rest of this paper is organized as follows. In Section II, the helical coordinate system is established and the HPSM algorithm is proposed based on a solution to wave equation in the system. In Section III, a numerical simulation

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FIG. 1. Composite motions of a transducer and a cylindrical structure: (a) circularly moving transducer and horizontally moving cylindrical structure; (b) horizontally moving transducer and circularly moving cylindrical structure; (c) helically moving cylindrical structure.

of nine point scatterers is used to compare the performance of the HPSM algorithm with that of a DAS algorithm. In Section IV, six rods and a letter pattern are measured using an ultrasonic pulse-echo system and the measured data are used to evaluate the proposed algorithm. In Section V, a discussion of the results is presented and conclusions are drawn.

II. THEORY

A. Helical coordinate system

A cylindrical coordinate system constructed based on (r, θ, z) coordinates is shown in Fig. 2. Based on this cylindrical coordinate system, a nonorthogonal right-handed helical coordinate system that is constructed based on (r, ϕ, ξ) coordinates is established using the following transformation:

$$\begin{cases} r = r, \\ |\phi| = |\theta|, \\ \xi = z - \bar{p}\theta, \end{cases} \quad (1)$$

where $\bar{p} = p/2\pi > 0$, and p is the helical pitch. ϕ and θ are equal numerically but differ in their physical meanings. In the correlation between ξ and ϕ , nonunique representations

only exist for one point. Therefore, the limitation of ξ is defined as follows:

$$0 \leq \xi < p. \quad (2)$$

In helical scanning, a transducer moves along the helix, and the ultrasound beam is aimed towards the central axis. A 3D helical scan contains a set of helical scans with different offsets in their ξ axes or with transducer arrays distributed along their ξ axes.

B. Solution to wave equation in the helical coordinate system

The wave equation is

$$\left(\nabla^2 - \frac{1}{v^2} \frac{\partial^2}{\partial t^2} \right) \psi = 0, \quad (3)$$

where v is the ultrasonic wave velocity. In the helical coordinate system, the Laplacian operator can be expressed as follows:¹⁹

$$\begin{aligned} \nabla^2 = & \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + \left(1 + \left(\frac{p}{2\pi r} \right)^2 \right) \frac{\partial^2}{\partial \xi^2} \\ & - 2 \left(\frac{p}{2\pi r^2} \right) \frac{\partial^2}{\partial \phi \partial \xi}. \end{aligned} \quad (4)$$

Based on this helical Laplacian operator, the solution to the helical wave equation is given as

$$\psi(t, \phi, \xi, r) \propto H_q^{(1)(2)}(k_r r) e^{\pm j n \phi} e^{\pm j \beta \xi} e^{-j \omega t}, \quad (5)$$

where $H_q^{(1)(2)}$ denotes the q -th order Hankel functions of the first and second kinds; k_r , n , and β are the wave numbers in r , ϕ , and ξ direction, respectively; and ω represents the angular frequency. The order q in the Hankel function is calculated using

$$q = n - \frac{p\beta}{2\pi}. \quad (6)$$

The relationship among k_r , ω , and β is

$$k_r = \sqrt{\left(\frac{\omega}{v} \right)^2 - \beta^2}. \quad (7)$$

FIG. 2. Cylindrical and right-handed helical coordinate systems.

Based on the asymptotic behavior of the Hankel function, $H_q^{(1)}(k_r r)$ and $H_q^{(2)}(k_r r)$ in (5) correspond to the outgoing diverging wave and the incoming converging wave, respectively. In the exterior 3D helical ultrasonic scan, only the outgoing waves could be received by the transducers. Therefore, the general solution to the wave equation in the helical coordinate system should be composed of a combination of all possible outgoing wave solutions

$$\psi(t, \phi, \xi, r) = \sum_{n=-\infty}^{\infty} e^{in\phi} \int \int A_q(\omega, \beta) H_q^{(1)}(k_r r) e^{i\beta\xi} e^{-j\omega t} d\beta d\omega, \quad (8)$$

where $A_q(\omega, \beta)$ is the coefficient for each outgoing wave solution.

C. Acoustic field extrapolation in the helical coordinate system

Equation (8) is actually a Fourier transform in t , ϕ , and ξ coordinates, and the helical acoustic field in the frequency domain $\Psi_q(\omega, \beta, r)$ could therefore be defined as

$$\Psi_q(\omega, \beta, r) = A_q(\omega, \beta) H_q^{(1)}(k_r r). \quad (9)$$

Based on the principles of angular spectrum transmission, the helical acoustic field at $r = R$ can be extrapolated to give the acoustic field at an arbitrary r

$$\Psi_q(\omega, \beta, r) = \Psi_q(\omega, \beta, R) W_q(R, r, v, \omega, \beta), \quad r < R, \quad (10)$$

where the complex transfer function $W_q(R, r, v, \omega, \beta)$ from R to r is

$$W_q(R, r, v, \omega, \beta) = \frac{H_q^{(1)}(k_r r)}{H_q^{(1)}(k_r R)} = \frac{H_{n-\frac{p\beta}{2\pi}}^{(1)}\left(\sqrt{\left(\frac{\omega}{v}\right)^2 - \beta^2} r\right)}{H_{n-\frac{p\beta}{2\pi}}^{(1)}\left(\sqrt{\left(\frac{\omega}{v}\right)^2 - \beta^2} R\right)}. \quad (11)$$

To ensure that all received waves are nonevanescant waves, the following equation must be satisfied:²¹

$$\frac{(2\pi n - p\beta)^2}{4\pi r^2} + \beta^2 \leq \left(\frac{\omega}{v}\right)^2. \quad (12)$$

The nonevanescant wave condition thus becomes flexible because r decreases with the acoustic field extrapolation.

D. Imaging

The acoustic field $\Psi_q(\omega, \beta, r)$ in (10) is transformed using a 3D inverse Fourier transform. The results at $t = 0$ are then used for imaging in the following form:

$$I(r, \phi, \xi) = F_{\phi, \xi}^{-1} \left\{ \int_{-\infty}^{\infty} \Psi_q(\omega, \beta, R) W_q(R, r, v, \omega, \beta) d\omega \right\}, \quad (13)$$

where $F_{\phi, \xi}^{-1}$ is an inverse Fourier transform in terms of ϕ and ξ . At $t = 0$, $e^{i\omega t} = 1$ and the inverse Fourier transform in time becomes an integral over ω . The imaging condition $t = 0$ is derived from the ERM,²² which assumes that the scatterers explode at $t = 0$ and thus generate the ultrasonic waves. At $t = 0$, all ultrasonic waves are focused on their sources and are not diffused. Therefore, the lateral image resolution is enhanced. When the ERM is used to simulate pulse-echo waves, the ERM velocity should be considered in the form $v_e = c/2$, where c is the actual acoustic velocity of the material. Thus, the time delay of the waves that originated from the explosion is equivalent to that of the pulse-echo waves.

A flowchart for the HPSM algorithm is shown in Fig. 3. The ultrasonic signal in $\psi(t, \phi, \xi, R)$ is initially acquired from the exterior helical scan at a radius R and is then converted into $\Psi_q(\omega, \beta, r)$ using a Fourier transformation, where $F_{t, \phi, \xi}\{\cdot\}$ represents the 3D Fourier transformation. $\Psi_q(\omega, \beta, R)$ can be extrapolated to $\Psi_q(\omega, \beta, r)$ at an arbitrary r using (10). Finally, $\Psi_q(\omega, \beta, r)$ is 3D inverse Fourier transformed under the imaging condition that $t = 0$. The acoustic field should then be extrapolated and inversely Fourier transformed layer by layer with steps of Δr to obtain the focused images at the different radii. Note that the acoustic velocity is defined by the ERM velocity $v = v_e$.

E. Left-handed and right-handed coordinate system

The HPSM algorithm is based on the right-handed helical coordinate system shown in Fig. 2; however, the signs of

FIG. 3. Flowchart for the HPSM.

TABLE I. Sign table for p and β .

Parameter	Outgoing diverging wave		Incoming converging wave	
	Right-handed	Left-handed	Right-handed	Left-handed
p	+	−	+	−
β	−	−	+	+

p and β are different in the solution to the wave equation when using the left-handed helical coordinate system. Thus, the formula used to calculate q must also be altered. The sign table is listed below.

According to Table I, the formula for q for an outgoing diverging wave in the left-handed helical coordinate system is defined as

$$q = n + \frac{p\beta}{2\pi}. \quad (14)$$

When the transducer moves along the helix during the helical scan, the physical scan trajectory is confirmed using the helical coordinate system. However, the formula for q that is used in the HPSM algorithm cannot simply be determined based on the physical scan trajectory because it is also related to the arrangement of the image (Fig. 4).

When the directions of the ξ and ϕ axes are drawn on the right-handed helical scan trajectory [Figs. 4(a) and 4(c)], q can be calculated using (6). In Figs. 4(b) and 4(d), the formula for q is replaced with that of (14). The opposite cases occur on the left-handed helical scan trajectory. The positive scan direction of ϕ in the physical space is clockwise; therefore, the complex transfer function in the HPSM algorithm should be revised based on both the physical helical scan system and the image arrangement.

FIG. 4. Formulas for q in right-handed helical scan system using (6) for (a) and (c), and (14) for (b) and (d).

III. NUMERICAL SIMULATION

A. Simulation model

The geometrical positions of the scatterer and the transducer are set in the cylindrical coordinate system (Fig. 5). The distance between the transducer and the scatterer is

$$\begin{cases} dis = \sqrt{dz^2 + ds^2} \\ dz = z_s - z_d \\ ds = \sqrt{r_s^2 + r_d^2 - 2r_s r_d \cos \beta} \\ \beta = \theta_s - \theta_d, \end{cases} \quad (15)$$

where (r_s, θ_s, z_s) and (r_d, θ_d, z_d) represent the positions of the scatterer and the transducer in the cylindrical coordinate system, respectively.

The angle α shown in Fig. 5 is calculated using the cosine theorem as

$$\alpha = \arccos[(dis^2 - r_d^2 - m^2)/r_d m], \quad (16)$$

where $m = \sqrt{r_s^2 + dz^2}$.

Using the angular spectrum theorem, the back-scattered signal that is received by the transducers can be formulated in the cylindrical coordinate system as follows:

$$s(r_d, \theta_d, z_d, t) = \left(\frac{1}{dis}\right)^2 \int_{-\infty}^{\infty} S(\omega) e^{i2\omega \frac{dis}{v}} D^2(\omega, \alpha) e^{-i\omega t} d\omega, \quad (17)$$

where $S(\omega)$ is the Fourier transform of the Gaussian-modulated sinusoidal pulse, $e^{i2\omega \frac{dis}{v}}$ is the phase factor, and $D(\omega, \alpha)$ denotes the directivity function, which is defined as

$$D(\omega, \alpha) = jinc\left(\omega a \frac{\sin(\alpha)}{2v}\right). \quad (18)$$

Here, a represents the transducer diameter and $jinc(x)$ is defined as $J_1(x)/x$. $J_1(x)$ is a Bessel function of the first kind. $D^2(\omega, \alpha)$ applies to the directivity functions of both transmission and reception, and a Gaussian-modulated sinusoidal pulse is used to express the electronic impulse responses of the transducer. An inverse Fourier transform is used to replace the integration in (17) because the backscattered signal should be integrated over all frequencies

$$s(r_d, \theta_d, z_d, t) = \left(\frac{1}{dis}\right)^2 F_{\omega}^{-1} \left\{ S(\omega) e^{i2\omega \frac{dis}{v}} D^2(\omega, \alpha) \right\}, \quad (19)$$

FIG. 5. Geometrical positions of the scatterer and the transducer.

where $F_{\omega}^{-1}\{\cdot\}$ is the inverse Fourier transform in ω .

This simulation model could easily be established in the helical coordinate system using the transformation given in (1), and thus

$$s(r_d, \phi_d, \xi_d, t) = \left(\frac{1}{dis}\right)^2 F_{\omega}^{-1}\left\{S(\omega)e^{i2\omega\frac{dis}{v}}D^2(\omega, \alpha)\right\}. \quad (20)$$

The simulated backscattered signal of the 3D helical scan is then acquired by changing the transducer locations along the helices.

B. Simulation of scattering points

A simulation using nine point scatterers is presented using the model established above. The positions of all nine point scatterers in the helical coordinate system are listed in Table II.

All point scatterers are immersed in water (where the acoustic velocity is 1500 m/s). A 5 MHz planar transducer with a diameter of 6 mm is used to record the backscattered signals using a 100 MHz sampling frequency. The transducer scans along the helices within a radius of 150 mm using an angular step of $\Delta\phi = 0.5^\circ$ and an axial step of $\Delta\xi = 1$ mm. The helical pitch is set at 144 mm. The scan ranges of ϕ , ξ , and t are set at $[-50^\circ, 50^\circ]$, $[-24 \text{ mm}, 24 \text{ mm}]$, and $[129 \mu\text{s}, 169 \mu\text{s}]$, respectively.

The simulated backscattered signal is in the form of a 3D data matrix and is presented in the angle-R image and in the ξ -angle image by plotting the maximum envelope amplitude along the projection axis [Figs. 6(a) and 6(b), respectively]. In the angle-R image, the point scatterers are presented as parabolic curves. The parabolic curve is asymmetric because the helical ultrasonic scan trajectory is asymmetric relative to the scatterers. Additionally, the scatterers shown in Fig. 6(b) could not be regarded as points in this case.

Images that were reconstructed using the conventional DAS algorithm are displayed in Figs. 6(c) and 6(d). The radius could be calculated using (21) based on the propagation time. While the effects of ultrasonic diffraction are restricted, the point scatterers could still not be separated adequately. Additionally, the DAS algorithm for 3D data processing is time-consuming

$$r = R - tc/2, \quad R = 150 \text{ mm}, \quad c = 1500 \text{ m/s}, \quad (21)$$

where r is the radius and t is the time.

TABLE II. Positions of the point scatterers.

Point Number	r (mm)	ϕ (deg)	ξ (mm)
1	50	27	-10.8
2	50	30	-14.0
3	50	33	-17.2
4	40	-3	0.12
5	40	0	0
6	40	3	-0.12
7	30	-27	10.8
8	30	-30	10.0
9	30	-33	9.2

IV. EXPERIMENTAL

A. Experimental setup

The experimental setup is composed of a scanning stage and the ultrasonic test system (Fig. 7). The helical scanning path of the transducer is controlled using the scanning stage, and the ultrasonic test system is used to transmit, receive, digitize, and process the ultrasonic signals. A plane transducer with a 5 MHz operating frequency and diameter of 6 mm (I5P6NF, Doppler Co., Guangzhou, China) is used in the ultrasonic test system. The transducer's output signal is digitized at a sampling rate of 100 MHz using a 16-bit analog-to-digital converter (A/D) card.

B. Experiment on rods

Six stainless steel rods that were immersed in water were used as scatterers to evaluate the performance of the HPSM algorithm. The rods, which were 2 mm in diameter, were stood upright on the aluminum disc at various radii from the center and the length information of rods is shown in Fig. 8. The center of the frame disc coincides with the rotational axis of the scanning stage. Pairs of rods were located at the same radius to form groups. The angular intervals within each group and between the groups were both 10° . The lengths of the rods were increased as the helix moved upwards to obtain a suitable scanned image.

The transducer was located at the 90 mm radius (which was limited by the size of the tank) and was used to perform clockwise helical scans around the rods with a helical scan pitch of 108 mm. The angular and axial step sizes of the scan trajectory were $\Delta\phi = 0.5^\circ$ and $\Delta\xi = 0.2$ mm, respectively. The angular scan range was set at 70° , and the entire offset on axis ξ was 15 mm. Valid signals were selected in the range from $54 \mu\text{s}$ to $86 \mu\text{s}$ for imaging purposes.

The 3D raw data are presented in the angle-R and ξ -angle images [Figs. 9(a) and 9(b), respectively]. Although the data are not as seriously diffracted as they were in the simulation, the scatterers can still be detected as a diffused parabolic curve in the angle-R image. The parabolic curve varies with radius, and some overlaps occur within the 30 and 40 mm radii. The weak echo signals that tailed after the main reflections probably occurred as a result of internal reverberation within the rods. In the ξ -angle image, the diffraction may cause distortion of the lengths and diameters of the rods used for inspection and overlaps are thus evident. Notably, rods of different sizes may appear to be similar in length while the helix is moving.

Figures 9(c) and 9(d) show the imaging results obtained using the DAS algorithm. In the angle-R image, the diffused parabolic curves are focused into ellipses. The lateral resolutions of the reflection signals from the scatterers located within the 30 and 40 mm radii are enhanced, but the resolution of the corresponding signals from within the 50 mm radius is not improved efficiently. Nevertheless, the reconstructed image in the ξ -angle domain had sufficient resolution to separate the six rods. The image remains blurred, however, and would be difficult to use for quantitative evaluation of the dimensions of the six rods. The images for

FIG. 6. Simulation images. (a) Angle-R image and (b) ξ -angle image formed using the raw data; (c) angle-R image and (d) ξ -angle image reconstructed using the DAS algorithm; (e) angle-R image and (f) ξ -angle image reconstructed using the HPSM algorithm.

$\xi \in (0 \text{ mm}, 2 \text{ mm})$ are weakened because of the zero padding used when $\xi < 0$ for the DAS computations.

The images that were reconstructed using the HPSM algorithm are shown in Figs. 9(e) and 9(f). The scatterers are focused as points and pencil-shaped rods in the angle-R

and ξ -angle images, respectively. Although some pseudo-morphic images are present in the angle-R image, the amplitudes of these pseudomorphic images are lower than those of the scatterers and could easily be eliminated using a threshold filter. To quantitatively compare the resolution

FIG. 7. Experimental set-up.

FIG. 8. Dimensions and distributions of the rod pairs.

FIG. 9. Experimental images of the rods. (a) Angle-R image and (b) ξ -angle image using raw data; (c) angle-R image and (d) ξ -angle image reconstructed using the DAS algorithm; (e) angle-R image and (f) ξ -angle image reconstructed using the HPSM algorithm.

of imaging methods, i.e., the DAS and HPSM, we recorded the maximum amplitudes along the Angle axis, as shown in Fig. 10(a). Then, the resolution is calculated based on full width half maximum (FWHM) and shown in Fig. 10(b). From the raw data, we can see that the resolution is increasing with the wave propagation distance. After applying DAS, the resolution is improved to 6–8 pixels. The resolution is further improved when the raw data are reconstructed using HPSM and the resolution is 2–3 pixels, which is much better than that of the DAS method. Since the angle pitch is 0.5° , the ultimate resolution of HPSM is about 1° – 1.5° .

The rod lengths were also quantitatively measured based on the high-resolution images, and the results are as presented in Table III. The geometries and positional dimensions of the rods that were measured using a vernier

caliper are listed in the column labelled as “Actual values in cylindrical coordinate system” in Table III. The rod dimensions that were acquired from the ultrasound images are listed in the column of “Measured in helical coordinate system”. For ease of comparison, the data based on the helical coordinates were transformed into cylindrical coordinate data. The equation used for the transformation from helical coordinates to cylindrical coordinates is defined as

$$\begin{cases} r = r, \\ |\theta| = |\phi|, \\ z = \xi + p(\phi/360) + z_0. \end{cases} \quad (22)$$

Here, the helical pitch p is 108 mm and the starting offset z_0 is 39 mm in this experiment. Based on the results, the

FIG. 10. Comparison of the imaging resolution. (a) The recorded maximum amplitude along the Angle axis. The raw data (dot dash), data reconstructed using DAS (dash) and HPSM (solid) are presented, respectively; (b) the imaging resolution calculated based on FWHM.

maximum deviations are 0.6 mm, 0.5° , and 0.4 mm in the r -axes, the θ -axes and the z -axes, respectively.

C. Experiment on letter pattern

A letter pattern of “UT,” which was prepared using a 3D printer, is used to validate the HPSM algorithm. The letter pattern structure is shown in Fig. 11. The pattern is located within the 35 mm radius, and its surface is bent along with the circular arc surface.

The helical ultrasonic scan parameters were set at $\Delta\phi = 0.5^\circ$ and $\Delta\xi = 0.1$ mm. The scan trajectory ranges were set at $\phi = [0^\circ, 40^\circ]$, $\xi = [0, 20 \text{ mm}]$, and $t = [68 \mu\text{s}, 73 \mu\text{s}]$. The helical pitch was set at 90 mm to match the slope of the letter pattern.

FIG. 11. Letter pattern “UT.”

The projected images from the helical ultrasonic scan are shown in Figs. 12(a) and 12(b). The outline of the “UT” pattern is blurred, and the letter “T” cannot be identified in the images. The images that were reconstructed using the DAS algorithm are shown in Figs. 12(c) and 12(d). While the shapes of the letters could be distinguished, the letter boundaries are not distinct.

The images that were reconstructed using the HPSM algorithm are shown in Figs. 12(e) and 12(f). The diffraction is again restricted, and the shapes of the letter pattern can be seen clearly. The uneven distribution of the energy in the ξ -angle domain was probably due to the rough surface of the letter pattern, which resulted in unstable reflection signals.

V. DISCUSSION AND CONCLUSIONS

In this paper, we have proposed the HPSM algorithm for use in synthetic aperture focusing of pulse-echo measurements taken over helical surfaces in the frequency domain. The Fourier domain description of the algorithm is efficiently implemented using the FFT. The simulated and experimental results indicate that the HPSM algorithm can separate point scatterers and can be used to accurately reconstruct images of complex scatterer patterns by eliminating

TABLE III. Quantitative analysis of HPSM results.

Rod	Measured in helical coordinate system			Measured in cylindrical coordinate system			Actual values in cylindrical coordinate system		
	r^a	ϕ^a	ξ^a	r^a	θ^a	z^a	r^a	θ^a	z^a
1	30.20	-24.5	12	30.20	-24.5	43.65	30.00	-24	44.0
2	30.18	-14.5	9.6	30.18	-14.5	44.25	30.00	-14	44.4
3	39.73	-4	9.2	39.73	-4	47.00	40.00	-4	47.0
4	39.93	6	6.2	39.93	6	47.00	40.00	6	46.9
5	49.39	16	7.6	49.39	16	51.4	50.00	16	51.2
6	49.36	26	6.4	49.36	26	53.2	50.00	26	53.6

^aThe units for r , ξ , and z are mm, and the units for ϕ and θ are degrees ($^\circ$).

FIG. 12. Experimental images of letter pattern “UT”. (a) Angle-R image and (b) ξ -angle image using raw data; (c) angle-R image and (d) ξ -angle image reconstructed using the DAS algorithm; (e) angle-R image and (f) ξ -angle image reconstructed using the HPSM algorithm.

diffraction. The proposed algorithm also demonstrates higher resolution than the DAS algorithm.

In most cases, because the test samples are generally solids that are covered by a coupling layer for practical ultrasonic NDT, the detection object becomes a multi-layer structure. The refraction that occurs at the boundary between the different materials makes the use of the time domain SAFT more difficult. Fortunately, many PSM-type algorithms have been shown to maintain their performance levels for multilayer materials without any increase in the computational burden. Because the basic principle of the HPSM algorithm is similar to that of PSM, it therefore holds the potential to be applied to multilayer structures. Another point that must be researched in future is the scan direction. In this paper, we mainly focused on exterior helical scanning. Interior helical scanning is also widely used in ultrasonic NDT and also in ultrasonic medicine. Our future work will address the application of the HPSM

algorithm, including interior helical scanning methods and the detection of multilayer structures.

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APPENDIX: EXPLODING REFLECTOR MODEL (ERM)

As shown in Fig. 13(a), the ultrasound wave propagates back and forth in the pulse-echo ultrasonic system. Because the round trip satisfies the reciprocity theorem, the exploding reflector model (ERM) is introduced to simplify the pulse-echo system and represent this system as a single upward wave scenario, which assumes that all the scatterers in the medium explode simultaneously and thus become upward-emitting acoustic sources, as shown in Fig. 13(b). The resulting ultrasonic waves are

FIG. 13. Ultrasonic wave propagation model. (a) Pulse-echo system, and (b) exploding reflector model (ERM).

then received by a transducer array. In the ERM, the acoustic velocity is half of the actual acoustic velocity, and the time delay of the received ultrasound wave will be therefore be equivalent to that of the pulse-echo system. According to the ERM, the scattered wave is focused at $t = 0$, where all waves are focused on their sources and are not diffused. As a result, the lateral image resolution is enhanced.

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