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2019

Guo, J., Hu, X., Song, Y. F., Shao, G. D., Zhao, L. M., & Tang, D. Y. (2019). Observation of vector solitons supported by third-order dispersion. *Physical Review A*, 99(6), 061802-. doi:10.1103/PhysRevA.99.061802

<https://hdl.handle.net/10356/83526>

<https://doi.org/10.1103/PhysRevA.99.061802>

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Observation of vector solitons supported by third-order dispersionJ. Guo,¹ X. Hu,² Y. F. Song,¹ G. D. Shao,² L. M. Zhao,¹ and D. Y. Tang^{1,2,*}¹*Jiangsu Key Laboratory of Advanced Laser Materials and Devices, School of Physics and Electronic Engineering, Jiangsu Normal University, Xuzhou 221116, China*²*School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798*

(Received 27 March 2019; published 24 June 2019)

We report on the experimental observation of dark-bright and antidark-bright vector solitons supported by third-order dispersion in a weakly birefringent cavity fiber laser. Using a cavity dispersion and birefringence management technique we constructed a weakly birefringent cavity fiber laser with near-zero net cavity dispersion. Operating the fiber laser in either the net normal or net anomalous dispersion regime, we found that both dark-bright and antidark-bright vector solitons could be simultaneously formed as a result of incoherent cross-polarization phase modulation. Our experimental results confirm the existence of antidark-bright vector solitons and the coexistence of dark-bright and antidark-bright vector solitons in a real physical system.

DOI: [10.1103/PhysRevA.99.061802](https://doi.org/10.1103/PhysRevA.99.061802)**I. INTRODUCTION**

Solitons as a nonlinear localized wave have been observed in diverse fields of physics such as fluid dynamics [1], plasma physics [2], and Bose-Einstein condensates [3,4], and are extensively investigated [5]. In nonlinear optics both the temporal and spatial solitons formed as a result of the balance between dispersion (diffraction) and nonlinearity have been demonstrated [6–9]. The formation of solitons in single-mode fibers (SMFs) was first theoretically predicted by Hasegawa *et al.* [10]. It was shown that light propagation in SMFs is governed by the nonlinear Schrödinger equation (NLSE), a paradigm equation that admits both bright and dark soliton solutions. The bright soliton formation in SMFs was first experimentally demonstrated by Mollenauer *et al.* in 1980 [6]. Later, Emplit *et al.* further confirmed the formation of dark solitons in SMFs in 1987 [7]. As light propagation in SMFs is confined in a fiber core that has a small core diameter ($\sim 9\ \mu\text{m}$) and negligible losses, a very long nonlinear interaction length between the light and fiber can be achieved. Therefore, it is a desired system for experimental studies on the NLSE solitons and their dynamics.

Due to the fiber bending and/or manufacturing imperfections, etc., a SMF in practice always possesses weak birefringence, hence supporting two orthogonal polarization modes. Menyuk had first theoretically shown that light propagation in a weakly birefringent SMF is described by coupled NLSEs [11]. Comparing with the scalar NLSE where only light self-action effects exist, the cross-polarization coupling of lights in a weakly birefringent SMF could further result in a number of different forms of optical solitons, such as trapped solitons [12], phase-locked bright-bright solitons [13], polarization-rotation solitons [14], etc. As the solitons consist of two mutually coupled orthogonal linearly polarized components, they are also called vector solitons. The majority of the

theoretically predicted vector solitons have been experimentally confirmed [15–17]. Studies on the vector solitons have greatly enriched soliton and nonlinear wave theories.

However, all the above-mentioned solitons share the same characteristic that their formation and properties are mainly determined by second-order material (fiber) dispersion. Extensive theoretical studies on soliton formation in SMFs have shown that optical solitons could even be formed under the effect of third-order dispersion (TOD). Kivshar had shown the existence of antidark solitons, i.e., a bright soliton on top of a continuous-wave background, in scalar NLSE systems near the zero second-order dispersion point [18]. Frantzeskakis predicted a novel form of antidark-bright vector solitons supported by TOD in incoherently coupled NLSEs [19]. It was shown that under a small group-velocity mismatch, the coupled NLSEs could be reduced to a Korteweg–de Vries (KdV) equation coupled with a self-consistent source satisfying a stationary Schrödinger equation. Both small-amplitude dark-bright, and antidark-bright vector solitons could be formed in the systems. As antidark solitons are only supported by the presence of TOD, they are also regarded as a signature of solitons sustained by TOD. Despite antidark-bright vector solitons having been theoretically predicted for more than a decade, an experimental confirmation of the vector solitons is missing. In our opinion, the main challenge for the experimental demonstration of the vector solitons lies in finding a suitable SMF that not only has near-zero second-order dispersion, but also uniform birefringence over a long distance. In addition, how to experimentally prepare the required initial condition so that it could eventually be shaped into dark-bright or antidark-bright vector solitons in a SMF could be another challenge.

We have found an innovative way to solve the problems. A significant feature of the light circulation in a fiber laser is that under certain conditions, its averaged dynamics can be well described by scalar or coupled NLSEs with parameters determined by the corresponding values averaged over the laser cavity. Taking advantage of the feature, we had

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FIG. 1. A schematic of the erbium-doped fiber ring laser. EDF: Erbium-doped fiber. SMF: Single-mode fiber. DCF: Dispersion compensation fiber. WDM: Wavelength division multiplexer. PC: Polarization controller. ISO: Isolator. OC: Output coupler. OSA: Optical spectrum analyzer.

successfully demonstrated a number of solitons that are theoretically predicted for the NLSE or the coupled NLSEs but are difficult to realize in conventional SMFs. These include phase-locked bright-bright vector solitons [20], polarization rotation vector solitons [21], high-order vector solitons [22], as well as incoherently coupled dark-bright vector solitons [23]. All these previous experiments were conducted in fiber lasers with relatively large net cavity dispersions, and the observed soliton phenomena could be well explained by coupled NLSEs with ignorable TOD.

In this Rapid Communication we further report on vector soliton formation in fiber lasers with near-zero net second-order cavity dispersion. We show experimentally that, even at close to zero dispersion, dark-bright vector solitons could still be obtained under incoherent cross-polarization coupling. Moreover, an unconventional form of coupled bright-antidark solitons could also be obtained. Antidark-bright vector solitons appear simultaneously with dark-bright vector solitons, indicating that they are formed under exactly the same cavity conditions. As antidark solitons are only supported by TOD, the experimental result also confirms in return that the net second-order cavity dispersion of our fiber lasers is close to zero, and the dark-bright vector solitons observed are sustained by TOD rather than second-order dispersion.

II. EXPERIMENTAL SETUP

We have conducted the experiment using an erbium-doped fiber ring laser as schematically shown in Fig. 1. Briefly, it has a ring cavity consisting of a 3-m erbium-doped fiber (EDF) with a group velocity dispersion (GVD) coefficient of $\beta_2 = 63.4 \text{ ps}^2/\text{km}$, 9.05-m single-mode fiber (SMF) with a GVD coefficient of $\beta_2 = -23.8 \text{ ps}^2/\text{km}$, and 4.7-m dispersion compensation fiber (DCF) with a GVD coefficient of $\beta_2 = 5.3 \text{ ps}^2/\text{km}$. The fiber laser is pumped by a 1480-nm single-mode Raman fiber laser. A maximum of about 2 W pump light can be injected into the laser cavity. A polarization-independent isolator is inserted into the cavity to force the unidirectional circulation of light in the cavity. In addition, an intracavity polarization controller (PC) is used to fine tune the linear cavity birefringence. A wavelength division multiplexer (WDM) is used to couple the pumping light into

the cavity, and a 10% fiber output coupler is used to output the light. We note that there is no mode-locking element in our fiber laser cavity. Special care has also been taken to ensure that all intracavity components used have ignorable polarization-dependent losses. No mode locking could occur in our fiber laser. Therefore, different from conventional soliton fiber lasers where the solitons are formed initially through the pulse shaping of the mode-locked pulses, the solitons in our fiber laser are formed through nonlinear cross-polarization coupling. Experimentally, to separate the two orthogonal polarization components of the laser emission, the laser output is first sent to a fiber pigtailed polarization beam splitter and then monitored with a high-speed electronic detection system consisting of 40-GHz photodetectors and a 33-GHz bandwidth real-time oscilloscope. A polarization controller is inserted between the laser output and the polarization beam splitter to balance the linear polarization change caused by the lead fibers. An optical spectrum analyzer is used in our experiment to monitor the optical spectrum of the laser emission.

III. EXPERIMENTAL RESULTS AND DISCUSSIONS

Using a fiber laser with exactly the same cavity configuration but relatively larger net cavity dispersion, we had previously obtained coherently coupled bright-bright vector solitons in the anomalous dispersion regime, and incoherently coupled dark-dark vector solitons in the normal dispersion regime [24], respectively. Moreover, operating such a fiber laser with a net cavity dispersion in the range of about $0.15 \text{ ps}^2/\text{km} < |\beta_2| < 1.5 \text{ ps}^2/\text{km}$, we have also experimentally demonstrated a kind of incoherently coupled dark-bright vector soliton. Typical examples of the dark-bright vector solitons obtained in the net normal and net anomalous cavity dispersion regime are shown in Fig. 2. Under incoherent cross-polarization coupling between the two orthogonal linearly polarized laser modes of the fiber laser, polarization domains as reported in Ref. [25] are always formed. When the cross-polarization coupling strength is continuously increased, the polarization domains break up constantly. After a certain threshold the domains would become so narrow that they are automatically shaped into coupled dark-bright solitons. Hence, many dark-bright vector solitons are always simultaneously formed in the laser cavity, and different pairs of coupled dark-bright solitons could have different soliton pulse widths, pulse heights, and pulse energies, as shown in Fig. 2. The feature of the dark-bright vector solitons is in good agreement of those theoretically derived from incoherently coupled NLSEs [14], but they are different from those of the bright-bright vector solitons formed in a mode-locked fiber laser, where the formed vector solitons always have exactly the same soliton parameters [17].

The difference could be traced back to the different formation mechanisms between the coupled bright-bright vector solitons and the dark-bright vector solitons. In an anomalous dispersion fiber laser, even without cross-phase modulation, bright solitons could still be formed. The function of cross-phase modulation is to introduce the interaction between the bright solitons. Since the bright scalar solitons formed in mode-locked fiber lasers are quantized [26,27], the formed bright-bright vector solitons will also be quantized. However,

FIG. 2. Typical coupled dark-bright solitons observed in a fiber laser with relatively large net cavity second-order dispersion. The horizontal axis (upper trace) and vertical axis (lower trace) shows the laser emission measured along the two orthogonal polarization directions, respectively. (a) With an estimated net average second-order cavity dispersion of $\bar{\beta}_2 \sim 0.5 \text{ ps}^2/\text{km}$. (b) With an estimated net average second-order cavity dispersion of $\bar{\beta}_2 \sim -0.6 \text{ ps}^2/\text{km}$.

in the normal (anomalous) dispersion regime no bright (dark) solitons could be naturally formed by self-phase modulation in the NLSE. Therefore, the formation of dark-bright vector solitons is purely a result of cross-phase modulation. Here, cross-phase modulation plays a similar role as self-phase modulation on the scalar bright or dark soliton formation in anomalous or normal dispersion regimes. Each of the observed dark-bright pulse pairs is in a sense a fundamental soliton of the coupled NLSEs and the formation of the solitons is independent on the sign of the material dispersion.

At a fixed pump power and cavity birefringence, it is found that the smaller the net cavity dispersion, the easier it is to obtain dark-bright vector solitons. We attribute the phenomenon to the fact that at a smaller second-order dispersion, both the bright and dark solitons would have a smaller pulse energy [28]. Therefore, their formation would become relatively

FIG. 3. A typical state of dark-bright vector soliton emission of the fiber laser. (a) The polarization-resolved oscilloscope traces. (b) The corresponding polarization-resolved optical spectra. The net average second-order cavity dispersion is estimated as $\bar{\beta}_2 = 0.04 \text{ ps}^2/\text{km}$.

easier. Using the fiber length cutting back method, we also operated the fiber laser with a close to zero net cavity dispersion coefficient of $|\beta_2| < 0.1 \text{ ps}^2/\text{km}$, where we expect that the effects of TOD would become non-negligible for the fiber laser. We found experimentally that even with an estimated zero net second-order cavity dispersion, dark-bright vector solitons could still be obtained in our laser. Figure 3 shows, for example, a result experimentally obtained at an estimated net second-order cavity dispersion of $\beta_2 = 0.04 \text{ ps}^2/\text{km}$. As the net cavity dispersion approaches zero, the average strength of the dark-bright vector soliton pulses becomes weaker and weaker, and their pulse width also becomes broader.

Our fiber laser has a cavity round-trip time of about 84 ns. As the dark-bright vector solitons formed in a relatively larger net cavity dispersion coefficient of $|\beta_2| > 0.15 \text{ ps}^2/\text{km}$, many dark-bright vector solitons are simultaneously formed in the cavity. They are randomly distributed in the cavity, as shown in Fig. 3. We used an intracavity polarization controller (PC) to fine tune the net cavity birefringence. Adjusting the

FIG. 4. A state of laser emission with the coexistence of dark-bright and antidark-bright vector solitons in the cavity. The net average second-order cavity dispersion is estimated as $\bar{\beta}_2 \sim 0.04 \text{ ps}^2/\text{km}$.

orientations of the paddles of the intracavity PC, the wavelength separation and relative strength of the two orthogonal linearly polarized laser emissions can be altered. In this way the incoherent coupling strength and the group velocity mismatching between them can be slightly varied. It is experimentally identified that by increasing the pump strength, or decreasing the net cavity birefringence, more and more dark-bright vector solitons can be formed. In particular, in the close to zero net cavity dispersion range of $|\beta_2| < 0.1 \text{ ps}^2/\text{km}$, occasionally a state as shown in Fig. 4 is also obtained, where, in addition to dark-bright vector solitons, so-called antidark-bright vector solitons also appear in the laser and coexist with the dark-bright vector solitons. In our experiment both the coupled dark-bright and antidark-bright solitons repeat themselves with the cavity round-trip time, indicating that they are stable in the cavity. We note that antidark-bright vector solitons were first theoretical predicted by Frantzeskakis [19] in 2001. As antidark-bright vector solitons could only be supported by third-order dispersion, the appearance of the vector solitons clearly shows that our fiber laser is indeed operating at a near net zero cavity dispersion, and the observed dark-bright solitons are supported by third-order dispersion.

We point out that the simultaneous occurrence of both dark-bright and antidark-bright vector solitons in a system was also theoretically predicted for vector solitons supported by TOD [19]. Moreover, theoretical studies have shown that in small-amplitude limits the vector solitons are isomorphic to the Mel'nikov solitons [29]. To check this, we operated the laser under different pump powers and cavity birefringences in the $|\beta_2| < 0.1 \text{ ps}^2/\text{km}$ net cavity dispersion region. Figure 5 shows another interesting state that is frequently obtained. In a state apart from the coupled dark-bright solitons, scalar dark solitons are also obtained. We suspect that the absence of bright solitons at the position of the dark solitons could be due to the wave absorption feature associated with the Mel'nikov solitons. The Mel'nikov system, comprising a KdV equation and a self-consistent source satisfying a stationary

FIG. 5. A state of laser emission with the coexistence of scalar dark solitons and dark-bright vector solitons in the cavity. The net average second-order cavity dispersion is estimated as $\bar{\beta}_2 \sim 0.02 \text{ ps}^2/\text{km}$.

Schrödinger equation, is a completely integrable nonlinear system. Theoretically it was shown that under specific conditions there are creation and annihilation of waves in the system [30]. Although limited by our experimental conditions in that we could not monitor the soliton evolution for a long time, therefore, we could not confirm that asymptotically the missing bright solitons will be created or the visible bright solitons will eventually be annihilated, given the fact that under a relatively larger cavity dispersion as shown in Fig. 2, dark solitons are always paired with bright solitons and no exceptions have been observed, we believe the possibility of wave annihilation could not be excluded.

IV. NUMERICAL SIMULATIONS

To confirm that incoherently coupled dark-bright vector solitons could be formed in a dispersion-managed cavity fiber laser with near-zero net cavity dispersion, we further numerically simulated the operation of our fiber laser. We modeled the light propagation in the cavity fibers with the coupled Ginzburg-Landau equations (GLEs),

$$\begin{aligned} \frac{\partial u}{\partial z} &= i\beta u - \delta \frac{\partial u}{\partial t} - i \frac{\beta_{2u}}{2} \frac{\partial^2 u}{\partial \tau^2} + \frac{\beta_{3u}}{6} \frac{\partial^3 u}{\partial \tau^3} \\ &\quad + i\gamma \left(|u|^2 + \frac{2}{3} |v|^2 \right) u + \frac{g}{2} u + \frac{g}{2\Omega_g^2} \frac{\partial^2 u}{\partial \tau^2}, \\ \frac{\partial v}{\partial z} &= -i\beta v + \delta \frac{\partial v}{\partial t} - i \frac{\beta_{2v}}{2} \frac{\partial^2 v}{\partial \tau^2} + \frac{\beta_{3v}}{6} \frac{\partial^3 v}{\partial \tau^3} \\ &\quad + i\gamma \left(|v|^2 + \frac{2}{3} |u|^2 \right) v + \frac{g}{2} v + \frac{g}{2\Omega_g^2} \frac{\partial^2 v}{\partial \tau^2}. \end{aligned} \quad (1)$$

Here, u and v are the two normalized slowly varying pulse envelopes along the slow and fast axes of the cavity, respectively; $2\beta = 2\pi \frac{\Delta n}{\lambda}$ is the wave-number difference, and $2\delta = \frac{2\beta\lambda}{2\pi c}$ is the group-velocity difference. β_{2u} , β_{3u} and β_{2v} , β_{3v} are the second- and third-order dispersion coefficients of

FIG. 6. Coupled dark-bright vector solitons numerically simulated. (a) and (b) are the evolution of the vector solitons in a fiber laser cavity; (c) is the snapshot at the round trip of 10^5 . In the simulation, $L_{\text{edf}} = 3$ m, $b_{2\text{edf}} = -48$ ps/nm/km; $L_{\text{smf}} = 9$ m, $b_{2\text{smf}} = 18$ ps/nm/km; $L_{\text{dsf}} = 5$ m, $b_{2\text{dsf}} = -4$ ps/nm/km, so the averaged $\beta_{2u} = \beta_{2v} = 0.15$ ps²/km; $\delta = 0.006$ ps/km, $\beta_{3u} = \beta_{3v} = 0.1$ ps³/km, $g_0 = 200$ km⁻¹, $E_s = 10$ pJ, and the gain bandwidth limitation is ignored.

the fibers for the pulses along the slow and the fast axes, respectively, and γ is the nonlinearity of the fibers. g is the laser gain coefficient and Ω_g is the bandwidth of the laser gain. Gain saturation of the gain laser is considered as

$$g = \frac{g_0}{1 + \int (|u|^2 + |v|^2) dt / E_s}, \quad (2)$$

where g_0 is the small signal gain and E_s is the saturation energy. Our simulations were based on a fiber ring cavity as shown in Fig. 1. We start the simulations with a weak arbitrary dark-bright pulse pair and let them circulate in the cavity; for the undoped fibers, $g_0 = 0$. Whenever the light meets the cavity output, we reduce the light power by a factor of the cavity output coupling. The detailed numerical

simulation method and techniques were reported in previous articles [20,26].

Numerically, we found that exactly as the experimental observations, independent on the sign of the net cavity dispersion, even when the net cavity dispersion is close to zero where the effects of the third-order dispersion of the cavity need to be considered, the dark-bright vector solitons can still be easily formed in the lasers. Figure 6 shows as an example a typical case numerically obtained, which shows the formation of multiple dark-bright vector solitons in the laser. Finally, we note that although in our numerical simulations we have used the coupled GLEs (1), under the conditions that the gain bandwidth is far broader than the spectral bandwidth of the formed solitons, and the steady-state laser operation, where the cavity losses is balanced by the saturated gain, the GLEs (1) are reduced to the incoherently coupled NLSEs shown in Ref. [19]. Therefore, it also explains why the antidark-bright vector solitons could be formed in our lasers.

V. CONCLUSION

In conclusion, we have experimentally confirmed the existence of dark-bright and antidark-bright vector solitons of the coupled NLSEs supported by third-order dispersion. By operating a dispersion-managed weakly birefringent cavity fiber laser near the net zero cavity dispersion point, we have experimentally observed both dark-bright vector solitons and antidark-bright vector solitons in the laser. The observed vector solitons exhibited typical features as those theoretically predicted for the vector solitons supported by third-order dispersion. Our experiment once again demonstrated that an appropriately designed fiber laser could be an ideal nonlinear test bed for the experimental study of the various forms of solitons of the KdV equation, NLSE, or their coupled equations.

ACKNOWLEDGMENTS

The research is partially supported by Minister of Education (MOE) Singapore, Grant No. 2018-T1-001-145, and the funds of Priority Academic Program Development of Jiangsu High Education Institutions (PAPD), National Natural Science Foundation of China (NSFC) (Grant No. 61575089).

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