Managing commute traffic : from parking provision with ride-sourcing to work hours arrangements considering household travels

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TECHNOLOGICAL
UNIVERSITY**

SINGAPORE

**Managing commute traffic: From parking
provision with ride-sourcing to work hours
arrangements considering household travels**

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School of Civil and Environmental Engineering

2019

**Managing commute traffic: From parking
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arrangements considering household travels**

Qida Su

School of Civil and Environmental Engineering

A thesis submitted to the Nanyang Technological University

in partial fulfilment of the requirements for the degree of

Doctor of Philosophy

2019

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

05 AUG, 2019

Date

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Su Qida

Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

06 AUG, 2019

Date

Assoc Prof. David Z.W. Wang

Authorship Attribution Statement

This thesis contains material from 1 paper published in the following peer-reviewed journal where I was the first author, and from 2 papers accepted at conferences in which I am listed as an author.

Chapter 3 is published as Su, Q., & Wang, D. Z. W. (2019). Morning commute problem with supply management considering parking and ride-sourcing. *Transportation Research Part C: Emerging Technologies*. doi:<https://doi.org/10.1016/j.trc.2018.12.015>.

The contributions of the co-authors are as follows:

- I proposed the research problem.
- Assoc Prof. David Z.W. Wang assisted in the study design.
- I prepared the manuscript drafts with all mathematical derivations.
- The manuscript was revised by Assoc Prof. David Z.W. Wang.

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- I proposed the research problem.
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The contributions of the co-authors are as follows:

- Assoc Prof. David Z.W. Wang proposed the research problem.
- I wrote the manuscript with all mathematical derivations.
- Assoc Prof. David Z.W. Wang revised the manuscript.

05 AUG, 2019

Date

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Su Qida

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Summary

The traffic mobility in peak hours has always been a challenge in city management. Due to the ubiquitous jobs-housing separation in most metropolitans, commuters inevitably need to travel between home and workplace with decision makings on the travel modes, departure time as well as the work schedule. This thesis then focuses on the commuting behavior and explores the management of morning commute problem from the supply of transportation facilities including parking spaces and ride-sourcing fleet size, to the travel demand of commuters with various work schedules and composite trip purposes.

In terms of supply, we notice that the emerging ride-sourcing service can have a substantial impact on road traffic and the parking demand at the CBD area. Thus, we investigate the morning commute problem with bottleneck congestion and parking space constraints in the presence of ride-sourcing service. We elaborate at length how ride-sourcing affects current parking provision management in peak hours. To best manage the commute traffic, the optimal supplies of parking spaces and ride-sourcing services in total travel cost minimization are explored in a bi-modal model, which indeed addresses a critical issue on how to regulate the supply of ride-sourcing service. This analysis has then been extended to the multi-modal case to incorporate transit as well as the parking reservation.

When it comes to the commute demand, with the hope to stagger the travel trips during peak hours so as to mitigate traffic congestion, we examine commuters' work start time/ work schedule choice considering both positive productivity externalities

and negative congestion externalities. Foremost, the compressed work schedule is compared to the normal work schedule and the optimal proportion of employers adopting compressed work schedule is derived. Multiple equilibria are found with the different number of compressed schedule users, depending on the agglomeration rate of productivity and the initial status. Furthermore, in view of the prevailing household travels with multiple trip purposes (e.g., daycare drop-off/pick-up) in peak hours, we incorporate such household travels into the work starting time choice model. The question on how household travelers differ from the conventional individual travelers in work schedule choice is being addressed. We further determine the optimal proportion of households and individuals adopting flextime in social benefit maximization. In some cases, the system can achieve the optimum naturally at long-run equilibrium without other policy intervention.

CHAPTER 1

INTRODUCTION

1.1 Background

Peak hour traffic congestion has always been a headache in metropolitans since the second industrial revolution. While the congestion is usually found in developed countries in the past, more and more developing countries start to suffer from gridlocks in recent decades. In 2017, it is reported that the five most congested countries are Thailand, Indonesia, Columbia, Venezuela, and Russia ("INRIX 2017 Global Traffic Scorecard Infographic," 2018). This is due to the imbalance between the tremendously increasing travel demand and the inadequate transportation facilities as well as the poor traffic management.

One vital composition leading to gridlock during peak hours is the commute trips. For instance, city dwellers in Los Angeles on average lost 102 hours on commuting in 2016 ("The hidden cost of congestion," 2018). Owing to the common jobs-housing separation, commuters need to travel daily between home and workplace and make trade-offs between the travel cost on commuting and the scheduling cost according to their work schedule.

Traditionally, regulators can manage the commute traffic from two directions. On the one hand, management emphasizes the supply of transportation infrastructure to spatially reallocate the commute trips, i.e., reallocate the travel trips from the congested facility to other unsaturated transportation infrastructure. For example, the reversible lane can temporally expand the highway capacity of one direction to improve the inbound/outbound flow in morning/evening peak hours. The ceiling of

the parking supply can restrain commuters from private autos driving and force them to switch to other travel modes.

On the other hand, management also focuses on the temporal redistribution of the commute demand. That is to redistribute the travel trips from peak hours to off-peak hours. For instance, congestion tolls (or pre-peak discount) internalize the negative externalities imposed on all commuters and encourage some of them altering their departure time. More directly, it is also suggested that flexible work schedules, which provide a degree of flexibility to employers and employees allowing them to change their work schedules to avoid rush hours, can alleviate the peak hour congestion.

However, there are limitations in some current proposed managing schemes. For one thing, in recent years due to technological advances, more and more e-transportation modes emerge. Regarding the form of service, there are e-dispatch for-hire vehicles (FHV), on-demand buses and dock-less shared bicycle, *etc.* For convenience, throughout this thesis, we hereinafter refer to those vehicles which provide FHV-like service as “eFHV” (if there are no special instructions), including the traditional FHV, the emerging e-dispatch ride-sourcing vehicles as well as the driverless FHV. In terms of engine, the era of hybrid vehicles and autonomous vehicles has begun. Such emerging travel modes offer commuters more travel options and therewith some conventional schemes may not be effective anymore. Take parking provision as an example, in the initial case with parking space constraints and without eFHV, the scale of parking space at central business district (CBD) can approximate the scale of traffic on the highway. Now with an increasing number of eFHV, some commuters no longer need a parking space at the destination and the demand for parking has changed. It is of great necessity to investigate the emerging modes’ effect on the commute traffic management.

And for another, though highly recognized to mitigate traffic congestion in literature, some managing schemes, such as road tolls and flexible work arrangement (FWA), are not widely applied in cities. Their obscure impact on equity and productivity impedes their popularization among commuters. Consider flextime/staggered hours, while commuters can choose their work start time so as to travel during off-peak hours, it is reported that only 22.8% of firms offer such options to employees in Singapore (*Conditions of employment*, 2018) as it casts doubt on the performance of employees when their work hours are staggered. Not to mention that commuters with multiple trip purposes, e.g., daycare pick up/drop off, exhibit different preferences towards work schedule. Whether the implementation of these management schemes can increase social welfare needs further examination.

Therefore, on account of these limitations, this thesis aims to revisit the commute traffic and address the limitations of its management mentioned above. Foremost, from the reallocation of supply of transportation infrastructure, we scrutinize the interaction between the emerging ride-sourcing service and parking provision as well as the railway transit on commute traffic during peak hours. From the temporal redistribution of travel demand, we model both employers and employees' preference towards different types of work schedule including compressed work week, flextime and staggered work hours. The impact of these work schedules on the peak hour congestion alleviation and social benefit's increase is explicitly examined. The research problem in this thesis is of great importance. It not only tackles the emerging challenges in commute traffic management but also excavates the potentials of some less popular managing schemes in travel mobility enhancement.

1.2 Key issues and objective

Figure 1-1 Schematic diagram of the thesis flow.

In this thesis, analysis and modeling will be divided into two parts with the first half focusing on the supply of transportation facility and the second half on the commute demand redistribution. More specifically, we will first investigate the supply management considering ride-sourcing, parking and transit to best manage the commute traffic. The optimal number of parking spaces and the optimal fleet size of ride-sourcing service will be explored. We will also examine different work schedules for their effects on flattening the commuting peak, considering commuters with multiple trip purposes. The system optimum on work schedule selection to maximize total net benefit will be discussed. Figure 1-1 shows the thesis flow with a schematic diagram.

The main objectives are then summarized as follows.

To formulate and analyze the bi-modal/multi-modal dynamic equilibrium problem to investigate commuters' tradeoffs between departure time choice and travel mode choice, in consideration of parking space constraint and the emerging ride-sourcing service.

To explore the optimal number of parking spaces and registered eFHV's and its control schemes to best manage the commute traffic.

To establish the bi-level economic model of two FWAs: compressed work week and flextime/staggered work hours, enclosing both positive production externalities and negative congestion externalities.

To scrutinize the optimal proportion of employees adopting FWAs in social benefit maximization, in consideration of commuters with multiple trip purposes.

1.3 Research Scope

The scopes of study in the following chapters in this thesis are summarized as below.

On one hand, a bi-modal analytical model of dynamic traffic congestion is developed in Chapter 3 in a 1-OD bottleneck network with homogeneous travelers, with adequate supply of parking spaces and ride-sourcing service, incorporating the travel behaviors of parking searching and ride-sourcing waiting.

In Chapter 4 , a multi-modal analytical model of dynamic traffic congestion is built up in a 1-OD bottleneck network with homogeneous travelers in consideration of parking spaces constraints, limited ride-sourcing service and a paralleled transit line with sufficiently large capacity.

On the other hand, a bi-level economic model of work schedule choice on compressed work schedule is developed in Chapter 5 with homogeneous employees considering agglomeration productivity, 1-OD pair day-long bottleneck congestion and home-activity utility.

Lastly, a bi-level economic model of work starting time choice is used in Chapter 6 of homogeneous employees with multiple trip purposes considering agglomeration productivity, and 1-OD pair bottleneck congestion with no late arrivals in the morning.

1.4 Outline

The thesis is organized as follows.

Chapter 2 briefly reviews the literature on commute traffic studies. For the supply management, we further revisit the studies related to parking management and ride-sourcing service. For demand control, we focus on the literature of those FWAs affecting commuters work start/end time.

Foremost, Chapter 3 and Chapter 4 explore the commute traffic in terms of supply management of parking spaces and ride-sourcing service.

Chapter 3 (Su & Wang, 2019b) develops a bi-modal morning commute problem with parking constraints considering both private auto driving and ride-sourcing service. The travel pattern of dynamic equilibrium is analyzed. In this chapter, only the traffic on the highway is considered.

Chapter 4 extends the work in the previous chapter to the multi-modal case to incorporate the railway transit. A railway transit line paralleled to the highway is added to the model so that the elasticity of highway commuters is considered. The optimal parking spaces provision and the optimal fleet size of ride-sourcing vehicles is determined.

Afterward, Chapter 5 and Chapter 6 examine the FWAs in commuting demand redistribution to best manage commute traffic.

Chapter 5 (Su & Wang, 2018) focuses on the compressed work week in travel demand management. A bi-level model considering economic agglomeration is developed to investigate employers' and employees' decision making on work schedule choice. The equilibrium commuting pattern is examined and the optimal proportions of firms in the compressed work week to maximize total net benefit are studied.

Similar to the previous chapter, Chapter 6 (Su & Wang, 2019a) formulates a bi-level model encapsulating both the positive productivity externalities and the negative congestion externalities considering the general flextime/staggered work hours. In addition, it deals with a significant but often overlooked group of commute trips, i.e., those with multiple trip purposes during commuting. The difference of those household and individual travelers in work start time preference is recognized and being compared. The optimal proportions of firms and optimal proportions of household commuters in flextime/staggered work hours are obtained in both short-run and long-run equilibrium.

Finally, Chapter 7 wraps up the thesis with some summaries and discusses the future research directions.

CHAPTER 2

LITERATURE REVIEW

2.1 Overview of Commute Traffic Studies

2.1.1 Single Bottleneck Model

The traffic congestion in commute traffic has always been a headache for traffic management authorities. Due to the rigid and intensive demand for travel during rush hour, the commute problem has aroused researchers' interest in recent decades. When commute traffic is considered, one important concern is the time-varying congestion. To understand such congestion dynamics, the bottleneck model is widely applied in transportation economics analysis. Vickrey (1969) was the first one to create the model for morning commute bringing the bottleneck congestion into consideration, and later Hendrickson and Kocur (1981) had studied the schedule delay in the time-dependent equilibrium with distributed arrivals at a single bottleneck. The proofs of existence and uniqueness of the single bottleneck equilibrium are shown by Daganzo (1985) and Smith (1984) respectively. Newell (1987) expand the model to study heterogeneous travelers with different values of time and schedule delay. He then created a morning congestion model where traffic is treated as a continuous fluid (1988). Kuwahara (1990) studied the morning commute problem in the case of two tandem bottleneck. Notably, Arnott et al. (1990b) revisited Vickrey (1969) bottleneck problem and made remarkable progress. He simplified the model with one route and one bottleneck connecting one single origin-destination pair which is abbreviated as the ADL model. It has provided an operational and fundamental framework for future research.

In fact, as summarized by De Palma and Fosgerau (2011), unlike the static congestion models, this ADL bottleneck model focuses on the intra-day timing. It features not only the departure time variation but also the scheduling cost difference among travelers. In this ADL model, the bottleneck represents any road segment or bridge with binding capacity constraint linking a single O-D pair (let us say residence and workplace). It allows travelers to pass at a fixed rate on a first-come-first-served basis. While commuters arrive at the bottleneck continuously during the morning peak, they have to seek the compromise between the congestion delay and scheduling delay. In other words, commuters may depart from home early or late enough in order to avoid long queuing delays in the bottleneck but arrive at work early or late correspondingly. On the contrary, those who preferred to arrive on time will suffer the longest queuing. In this way, a congested demand peak with a queue that first develops and then fades out is described in the ADL model. At equilibrium, no traveler can reduce their commute cost by changing their departure time choice unilaterally, and they all incur the same generalized travel cost.

2.1.2 **Bi-modal/multi-modal problems**

In the basic bottleneck model, attentions are usually paid to single mode traffic and some significant findings have been given. Nonetheless, two modes' or multi-modes' traffic could reflect the reality more thoroughly, especially when more and more new transportation modes emerge along with the development of technology. For this purpose, some researchers expanded the basic ADL model to the bi-modal or multi-modal problem. Tabuchi (1993) introduced a paralleled transit line to the bottleneck to study the modal split. Huang (2000) compared different schemes of transit fare and road toll in the case with two groups of commuters, and later Danielis and Marcucci (2002) examined the efficiency of road pricing regimes in the modal split problem. The dual-mode problem was also revisited as a second-best price problem (Kraus &

Yoshida, 2002). Gonzales and Daganzo (2012) studied the competing modes bottleneck with deterministic degraded capacity for vehicle users. Qian and Michael Zhang (2011a) modeled the multi-modal morning commute problem in consideration of carpooling in a traffic corridor. In their study, three modes, i.e., driving alone, transit and carpooling, are scrutinized in a nested logit model and sensitivity analysis on how various factors affect the mode shares and network performance is provided. Nie and Yin (2013) explored the management of travel choice in rush hours via tradable credit scheme. In recent years, the researcher also applied the bi-modal/multi-modal model to examine some other emerging travel modes, e.g., ride-sourcing service (will be elaborated in Section 2.3).

2.1.3 **Other studies in Bottleneck**

Bottleneck model is distinguished for its compact combination of the essential features of congestion dynamics. Besides multi-modal studies, it has been adopted to address various issues as well, including commuter heterogeneity (Newell, 1987; Arnott et al., 1994; Lindsey, 2004; Doan et al., 2011; Liu & Nie, 2011; van den Berg & Verhoef, 2011; Qian & Zhang, 2013; Xiao et al., 2014b), road pricing (Arnott et al., 1990a; Laih, 1994; Braid, 1996; Xiao et al., 2012), stochasticity study (Arnott et al., 1999a; Lindsey, 2009; Xiao et al., 2013c; Xiao et al., 2014a), Pareto improving strategy (Daganzo & Garcia, 2000; Xiao & Zhang, 2013b), elastic demand (Arnott et al., 1993; Yang & Huang, 1997), reliability analysis (Yin et al., 2004; Li et al., 2012; Siu & Lo, 2013), activity-based perspective study (Li et al., 2014), integration of day-long commute (Zhang et al., 2005; Daganzo, 2013; Gonzales & Daganzo, 2013), tradable credit scheme (Nie, 2013; Tian et al., 2013; Xiao et al., 2013a) , ridesharing studies (Yang & Huang, 1999; Dahlgren, 2002; Menendez & Daganzo, 2007) and parking management (will be further elaborated in Section 2.2).

2.1.3.1 Commuters with multiple trip purposes

Joint decision makings of travelers are very common and are examined in previous studies (Dubernet & Axhausen, 2013; Gupta & Vovsha, 2013; Ho & Mulley, 2015b; Ho & Mulley, 2015a; Fu et al., 2016). Recently, a branch of literature recognized a significant group of commuters who have multiple trip purposes and make joint travel decision during peak hours. One of the most typical examples is the household travels, which is used to describe the trips of families with school child so that parents need to drop-off/pick-up their child during commuting hours.

Jia et al. (2016) were the first to incorporate the household travels in the dynamic equilibrium to investigate commuters' travel behavior. Liu et al. (2016b) extended the study to the case with mixed travelers (i.e., household and individual travelers) and proposed the optimal schedule of school and work to reduce traffic congestion. Moreover, Zhang et al. (2017) revisited the household travels with a different school location choice. In these papers, it is found that the appropriate schedule coordination between school and work start time can significantly reduce the total travel cost. Naturally, the schedule preferences of household and individual commuters are different and should be examined in detail. However, the work schedule choices were scarcely discussed in these household studies. Only in Zhang et al. (2017) the differentiated school and work schedules were examined preliminarily without the consideration of productivity effect. Therefore, this thesis fills the research gap to examine how the commuters with multiple trip purposes behave in their work schedule choices. After all, it is the work schedule which affects commuters' within-day travel demand.

2.2   Parking Management

For decades, parking economics, albeit with recognized importance, have both theoretically and practically been slighted in comparison to other transportation issues. Directly transport economics treated parking as a fixed cost added on to the end of a trip (Arnott, 2011). Their significance as an essential element of urban transportation problem to city planners was rarely mentioned. Particularly, parking problems have often been tackled separately from urban congestion studies in the literature. While the conventional measures, e.g., congestion pricing, has been widely discussed, researchers recently start to explore the potentials of parking management as an alternative solution to traffic congestion. Generally, the main aspects covered in previous parking economics studies in relation to urban congestion includes parking cruising, parking pricing and parking availability (Inci, 2015).

2.2.1   Parking Cruising

Being a tragedy of commons, parking cruising represents the travel behavior that people cruise slowly around the destination for undersupplied parking spaces, regardless of his contribution to others' travel cost. Shoup (2006) pointed out the inefficiency of parking cruising with a comparison of the findings in 16 different studies in congested areas around the world. This search externality may result in substantial social welfare losses as it slows down traffic and imposes some time costs on all drivers. Internalization of this externality would be the key to efficient parking policy. Bifulco (1993) brought parking cruising times, types and fees into consideration in a static stochastic traffic assignment model. Arnott and Rowse (1999b) studied the expected driving time, parking time and parking search distance in a ring-road model with uniformly distributed parking lots. Arnott and Inci (2006)

then developed a bathtub model of curbside parking to investigate the effects of parking cruising on traffic and proposed some pricing solution schemes. This model was later extended to examine curbside parking policy with the integration of curbside parking, garage parking and traffic congestion (Arnott & Rowse, 2009). Besides, Qian and Rajagopal (2014a) investigated the efficiency of dynamic parking pricing in the reduction of welfare losses resulted from parking search. While some studies approximated the cruising time as the reciprocal of the expected vacancy rate (Axhausen et al., 1994; Anderson & de Palma, 2004; Horni et al., 2013; Qian et al., 2014a), Arnott and Williams (2017) disputed that such approximation could lead to underestimation of cruising time of curbside parking.

2.2.2 **Parking Pricing**

With more straightforward and cheaper investment compared to road pricing, parking pricing, however, is a somewhat neglected option of traffic management. Commuters' travel patterns can be shaped by not only road tolls but also parking fees. In a deterministic bottleneck model, Arnott et al. (1991) found that optimal parking fee system can be as efficient as optimal tolls and system optimal can be yielded by a joint scheme of time-varying toll and location-dependent parking fee. Glazer and Niskanen (1992) pointed out the limitation of parking pricing that high parking fees may vary commuters' parking duration and commuters may even forgo parking and use other transportation modes. A diagrammatic analysis was also carried out to compare the parking pricing and parking supply restriction (Verhoef et al., 1995). Anderson et al. (2004) pointed out that the benefits of parking pricing are significantly reduced if cruising for parking brings congestion externalities to both the parkers and through traffic. In addition, Qian et al. (2012) created a one OD pair two parking clusters model to explore the optimal parking fee and parking supply in morning commute management. Fosgerau and de Palma (2013) shown that a

combined morning and evening parking fee is superior to a fee only in the morning or evening. Given new information technologies, Qian and Rajagopal (2014b) proposed an occupancy-driven parking pricing strategy in a combination of information provision aiming for system cost minimization. Mackowski et al. (2015) developed a dynamic parking pricing scheme to maximize urban parking space utilization in a dynamic non-cooperative bi-level model.

2.2.3 **Parking Provision**

Using parking provision for morning commute traffic management has also received extensive research attention in the literature. Qian et al. (2011b) found that the availability of parking spaces can be a significant factor in commuters' travel decision making. Different parking permit distribution schemes were later compared by Zhang et al. (2011) in a many to one bottleneck model with the paralleled transit line. Qian et al. (2012) also examined the congestion mitigation via parking supply with choice behavior between two parking clusters. Yang et al. (2013) further proposed that an appropriate proportion of reserved parking spots can temporally alleviate road congestion and consequently increase the social benefit, and it is later extended to an expirable parking reservation scheme in (Liu et al., 2014).

Interestingly, most parking management studies have made a similar assumption that all road users require parking space at CBD area and ignored the non-parking road users constituted mostly by for-hire vehicles (taxis and limousines) or the emerging and rapidly growing ride-sourcing service vehicles. However, as will be discussed in Section 2.3 , given the rapidly increasing scale of e-dispatched for-hire vehicles, the common assumption without considering the FHV in previous parking management studies may not be reasonable anymore. Thus, to overcome this research inadequacy,

the effects of ride-sourcing service supply on commute traffic as well as parking demand are scrutinized in this thesis.

2.3 For Hire Vehicles and Ridesharing

2.3.1 For Hire Vehicles (FHVs)

In the past, working as the supplementary transportation mode to auto driving and transit, the traditional street hail taxis and the on-call limousines constituted the major part of FHVs. With relatively high market access threshold, the total number of FHVs was often controlled by the government and took up a small portion of the total road vehicles. *A priori* studies mainly focused on street hail taxicab service. In their model design, emphasis was more put on the characteristic of taxi including taxi availability (expected passengers' waiting time and market entry control), taxi utilization (expected fraction of time a taxi is occupied), drivers and passengers' mutual searching and multi-period taxi service (work shift of taxi drivers) (Yang & Wong, 1998; Wong et al., 2001; Yang et al., 2005; Yang et al., 2010). The implementation of taxi regulation policies such as fare structure and fleet size was investigated in these taxi network models. (Arnott, 1996; Yang et al., 2002; Salanova et al., 2011; Yang & Yang, 2011; Wong et al., 2014a; Wong et al., 2014b) These taxi regulation policies could also be applied to general FHVs. While better FHVs service can be provided with ripe policies, taxis themselves sharing the roads with daily traffic contributes to road congestions. Yet due to its limited quantity, the traffic congestion resulted from FHVs are marginal compared to private car driving and often ignored in traffic congestion research.

2.3.2 Ridesharing

Ridesharing, or carpooling, as the name implies, allows travelers to share their rides in a car, which is often considered as the origin of ride-sourcing service. Rather than being a new concept, its emergence should date back to World War II in U.S. Advantages of ridesharing include travel cost saving, travel time reduction, traffic congestion mitigation, fuel conservation, and air pollution abatement (Ferguson, 1997; Kelly, 2007; Morency, 2007; Chan & Shaheen, 2012). Notwithstanding the listed benefits, ridesharing disappointed the social expectation for a time of being a vital transportation mode in morning commute solution and was not widespread in the world because of the lack of efficient ride-matching system. Usually, with at least two participants sharing a vehicle for a trip and splitting travel costs like gas, toll, and parking fees, ridesharing requires coordination with respect to itineraries and schedules (Furuhata et al., 2013). Not until recently can the coordination obstacles be overcome with innovative technologies. Many literatures are talking about ridesharing regarding its matching system and management regulation (Dailey et al., 1999; Chan et al., 2012). Yet limited research on ridesharing and its effects on traffic congestion have been done (e.g., Qian et al., 2011a). Yang et al. (1999) discussed the ridesharing behavior and congestion toll in a multilane highway with or without high occupancy vehicle (HOV) lanes and showed that the optimal toll in carpool case is different from the original one. In another paper, the effects of HOV lanes were also examined with the aid of bottle diagnostics (Menendez et al., 2007). Dahlgren (2002) used the morning commute model to evaluate the potential implementation of high-occupancy toll (HOT) lanes. Xu et al. (2015) examined the congestion effects of ridesharing in a mixed complementarity model. Xiao et al. (2016) connected the ridesharing behaviors during peak hours with the parking availability. Liu and Li (2017) explored the pricing scheme of ridesharing in a dynamic context to best eliminate queuing delay.

2.3.3 **Ride-sourcing**

Recently, technological advances in mobile devices, global positioning systems (GPS) and social networks have strongly facilitated the mutual communication between travelers, which catalyzes the new type of transportation modes: e-dispatch FHVs. It is a kind of shared transport based on the app-based, on-demand ride-sourcing service (Rayle et al., 2014). Several app-based companies have emerged to seize this opportunity, e.g., Uber, Grab, Didi, Lyft. With this service, passengers and drivers can be matched in real time on mobile apps and thus waiting time for passengers and hailing time for drivers are sharply decreased. Not only conventional FHVs' professional drivers but also private car drivers can register at the e-dispatch ride-sourcing platform. Without fixed working hours or high entry threshold as the traditional FHVs, this service has attracted a large number of private car drivers who are willing to offer up their empty seats to other travelers and earn income.

In fact, it is reported that more than half of the drivers providing ride-sourcing service have other full-time or part-time jobs and half of them on average drive less than 10 hours per week ("New Survey: Drivers Choose Uber for its Flexibility and Convenience," 2015). That is to say, one key motivator for private car drivers to provide FHVs-like service is to earn extra income with considerable flexibility. Given this, there is perhaps no wonder that once-distinct regulatory categories of FHVs are now blurring in consequence of the e-dispatch technology (*For Hire Vehicle Transportation Study*, 2016).

Even though the recent e-dispatch ride-sourcing service has roots in ridesharing, i.e., car sharing, it shares traits of traditional FHVs. In ridesharing, drivers and passengers are required to have proximate destinations and similar time schedules. As for ride-sourcing, with the purpose of earnings, drivers usually do not share a destination with passengers. After a trip, they would like to cruise on the street until the next trip is

dispatched. From parking perspective, namely, while ridesharing drivers need to find parking space after arriving at the destination, ride-sourcing drivers do not need parking in most cases. Ride-sourcing differs from ridesharing, and hence the previous models on ridesharing should be designed *mutatis mutandis* to adapt themselves to ride-sourcing studies.

On this account, research on ride-sourcing service usually considered it as an extension of the traditional street hailing taxi service in recent years, to focus on its market equilibrium (He & Shen, 2015; Zha et al., 2016; Qian & Ukkusuri, 2017; Xu et al., 2017) and its pricing strategy (Wang et al., 2016; Zha et al., 2017). Moreover, comparisons are also made with the traditional street hailing taxi service (Nie, 2017) or ridesharing (Dias et al., 2017).

However, owing to the ever-increasing drivers using the e-dispatch ride-sourcing platform, their impacts upon road congestion inevitably become more and more noticeable, especially during peak hours. The traffic congestion studies in consideration of eFHV are now a necessity and are examined in this thesis.

2.4 Flexible Work Arrangement

As the alternative scheduling of work distinguished from traditional normal work hours, FWA allows employees to enjoy a degree of flexibility of their work schedule so as to better manage their work-life balance. The common FWAs include flextime, telecommuting, compressed work week, job sharing, part-time work hours, *etc* (Gap, 2003).

2.4.1 **Pros and cons of FWAs**

Since the 1970s, different types of FWAs have been introduced (Breugh, 1983; Dunham et al., 1987; Baltes et al., 1999). The pros and cons of these FWAs are discussed empirically at length in literature. Among several different formal FWAs, we reviewed the studies on flextime/staggered work hours (SWH) and compressed work week, as they are the two FWAs that pertain to time flexibility and hence affect the commute traffic temporally.

For flextime/ SWH, commuters can start working at several predetermined work start time or can even choose their work start time freely. Previous literature has shown both the advantages and disadvantages of flextime/SWH (Ralston, 1989; Christensen & Staines, 1990; Ezra & Deckman, 1996; Lucas & Heady, 2002; Byron, 2005; Gainey & Clenney, 2006; Allen et al., 2013).

As a great benefit with flexible work schedule, more and more employees are willing to adopt flextime/SWH. Several studies mentioned that employers could attract more qualified employees and can increase employees' retention by offering flextime. In terms of work-family balance, Shockley and Allen (2007) suggested that flextime is an effective policy in reducing work interference with family. In fact, flextime/SWH can not only ease work-family conflict but also promote work-family enrichment (Wayne et al., 2006; McNall et al., 2009). In terms of productivity, some literature argued that flextime could boost individual productivity as employees can schedule their work during their most productive times. In particular, evidence showed that there is positive wage differential associated with flextime for some types of employees (Gariety & Shaffer, 2001). Yet, Rau and Hyland (2002) pointed out that flextime/SWH is attractive to employees with high role conflict only.

However, while a large body of literature discussed the advantages of flextime from the perspective of employees, the impact of flextime to employers, particularly the

financial impact, remains much less well understood (Baltes et al., 1999; Kelly et al., 2008). After all, as shown in some case studies, though with increased revenue, the implementation of flextime inevitably increases the cost of employers (Lee & DeVOE, 2012). Not to mention that flextime/SWH is not for all employers depending on their industry and different organizations have their different optimal alternative working schedules.

In fact, the elusive financial impact on employers has also impeded the popularization of flextime/SWH. Highly recognized in the literature, yet the flextime/SWH are still not widely adopted by employers and employees, especially in the Asia Pacific. Take Singapore as an example, according to the report in Ministry of Manpower (*Singapore yearbook of manpower statistics*, 2019), only 25.4% of firms in both public and private sectors offer flextime/staggered hours in 2018.¹ Regarding economic agglomeration, such flextime/SWH staggers the working hours of employees, and hence the less agglomerated work hours lead to the lower instantaneous productivity of each employee. As for employees, whether to choose a firm offering flextime/SWH depends on numerous factors. A survey carried out by Karin and Danielle (2007) showed that in the Asia Pacific the most chosen reason to adopt flextime is to avoid congestion (89%), followed by household travels (daycare pick-up/drop off) (57%).

As for Compressed work week (CW), instead of working in standard workdays, CW allows employees to reallocate the same number of work hours per pay period to fewer days and to enjoy more days off for leisure. One of the most well-known types of CW is the 4/40 schedule, where the employee works four 10-hour days and have

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¹ 47% firms offer formal FWA include part-time work, flextime/staggered hours, formal tele-commuting, homeworking, job sharing, compressed work week. Besides, 82% firms offer ad-hoc FWA, including unplanned time-off and ad-hoc tele-commuting. However, the ad-hoc FWAs have no significant impacts on the long-run travel pattern.

one extra day off while the total number of weekly working hours remains unchanged. As an easy-to-implement alternative working schedule, CW can benefit both the employers and employees (Fottler, 1977; Ronen & Primps, 1981; Hung, 1994). Hung (1996) provides some demonstrations on how CW can reduce work commuting. CW has considerable potential to be a practical measure in travel demand management (TDM) to alleviate traffic congestion (Sundo & Fujii, 2005).

Nonetheless, regarding individual productivity, while some research showed that individual performance improves in CW due to increased job satisfaction (Foster et al., 1979; Golden, 2012), some argued that employee's efficiency decrease in the extended hours as a result of fatigue (Goodale & Aagaard, 1975; Cette et al., 2011). Similar to flextime/SWH, such alternative working schedule staggers the working hours of employees. Therefore, the less agglomerated work hours lead to the lower instantaneous productivity of each employee in terms of economic agglomeration. Work-family balance is another concern. CW can affect the utility of non-work activity in two ways. On the one hand, CW brings greater flexibility to employees (e.g., extended weekend) to manage work and household activity (Gannon, 1974). On the contrary, longer hours of work per day deprive employees of time for daily off-work recreation. However, to the best of our knowledge, there is no economic study on CW to indicate the employers' and employees' work schedule choices analytically, particularly given the attractive benefits of CW in reducing travel trips. Thus, this thesis also addresses this gap of research.

2.4.2 Analytical models

In fact, to better investigate people's work schedule choice and the corresponding travel behavior, a branch of literature has focused on developing analytical models on the work schedule (work start time) selection. Seminal work is carried out by

Henderson (1981). Incorporating the productivity and traffic congestion effect, he examined the equilibrium and optimal scheme of staggered work hours. He showed that work starting times are distributed continuously at equilibrium. Following Henderson's framework, Wilson (1992) scrutinized the choice of residential location, Arnott et al. (2005) generalized the model to include firms' heterogeneity and Arnott (2007) explored the potential of congestion tolling in congestion alleviation. Later Mun and Yonekawa (2006) focused on the flextime where employees can choose their work starting time freely. He modeled the traffic congestion via Vickrey (1969)'s bottleneck model and tried to figure out the equilibrium and an optimum number of firms using flextime. Yushimito et al. (2014) developed a two-stage model with upper level capturing the firm's decision via a variation of the generalized quadratic assignment problem and lower level encapsulating employees' behavior in a nonlinear complementarity problem. Takayama (2015) reformulated the discrete staggered work hours model with peak hour congestion as a potential game. Fosgerau and Small (2017) studied commuters' trip timing preference incorporating the productivity effects of work and leisure. Moreover, Zhu et al. (2018) proposed an activity-based bottleneck model to analyze the optimal work starting time.

It is noteworthy that one assumption usually made in previous studies is that all trips during peak hour share same single purpose only traveling from home to workplace and hence traffic congestion was caused solely by work trips in their models. Even for those encapsulating activity utilities (Li et al., 2014; Fosgerau et al., 2017; Zhu et al., 2018), they merely examined the respective utility at home and workplace, i.e., the utility at origin and destination. To the best of our knowledge, trips with other purposes (e.g., household travels) were seldom addressed. Their effects on traffic congestion and work schedule choices remain unknown and will be explored in this thesis.

CHAPTER 3

SUPPLY MANAGEMENT CONSIDERING PARKING AND RIDE-SOURCING SERVICE DURING PEAK HOURS

This chapter aims to examine how the consideration of the eFHV's would affect the management of morning commute traffic with parking space constraints. To the best of our knowledge, eFHV's effects on parking management and thus morning commute traffic management have not been explicitly addressed in previous studies. In this study, the equilibrium commute travel pattern taking into account the surging supply and demand of eFHV's is completely depicted. From eFHV's regulation perspective, we address the question of how we should control the number of registered eFHV's if the objective of best managing morning traffic is to be achieved.

The rest of this chapter is organized as follows. Section 3.1 formulates the bi-modal morning commute problem with parking constraints considering both private auto driving and ride-sourcing service. Section 3.2 studies the travel pattern in the dynamic equilibrium. We then analyze the system performance and investigate the optimal supply of parking spaces and eFHV's in Section 3.3 . Numerical examples are also presented in Section 3.4 . Finally, Section 3.5 wraps up the chapter with some summaries.

3.1 Model formulation

The following notations are used throughout the process of model formulation:

<i>Parameters</i>		
N		The total number of commuters during morning peak
M_C		The total number of parking spaces available during morning peak
M_F		The total number of eFHV available for service during morning peak
t^*		The common preferred arrival time at the workplace
α		Value of time
β		Unit time scheduling penalty of early arrival
s		Constant service capacity of the bottleneck
F_C		One-off parking fee at the workplace
F_F		Customer fare of ride-sourcing service from home to workplace
ε		A constant positive coefficient which links the effective occupancy of parking space to parking search time
ω		Sufficiently small positive number close to 0, indicating the proportion of private autos with a dramatic increase in parking search time
Δ		Extremely large coefficient approaching infinity
ρ		A constant positive coefficient in passenger waiting cost function
<i>Variables</i>		
N_C, N_F		The actual number of private auto and eFHV commuters, respectively
U_C, U_F		Actual (ultimate) utilization rate of private auto and eFHV, respectively
t_C^0, t_F^0		The earliest departure time from home of private auto and eFHV, respectively
$t_C^{1-\omega}$		The departure time of the $(1-\omega)M_C$ -th private auto
t_q^0		The queue ending time, if there is still departure at t_q^0
t_q^1		The queue ending time, if there is no departure at t_q^1
t_C^l, t_F^l		The last departure time from home of private auto and eFHV, respectively

$T(t)$	The travel time on the highway from home to workplace prior to parking for vehicle departing at t
$S(t)$	Parking searching time for private auto departing at t
$W(t)$	Ride-sourcing passenger waiting cost from booking time to the boarding time t
$D(t)$	Queue length at the bottleneck at t .
$r_C(t), r_F(t)$	The arrival rate at the bottleneck (the departure rate from home) of private auto and eFHV at t , respectively
$Cu_C(t), Cu_F(t)$	The cumulative arrivals at the bottleneck of private auto and eFHV at t , respectively
$P_C(t), P_F(t)$	Generalized travel cost of private auto and eFHV departing at t
P_C^*, P_F^*	Equilibrium travel cost of private auto and eFHV

3.1.1 Basic setting

Consider a traffic corridor connecting a CBD and residential area as shown in

Figure 3-1. A highway with a single bottleneck following FIFO principle is available to road commuters. Over the course of morning peak, homogenous commuters with same value of time α have two options commuting from home to workplace: drive alone, or take eFHV. Both private auto drivers and eFHV passengers share the highway usage, i.e., $N = N_C + N_F$. Commuters must arrive in the workplace before t^* , and no late arrival is allowed with $\alpha > \beta > 0$ based on the empirical evidence (Small, 1982). It is also assumed that the CBD is abstracted as a point and all parking spaces are located at this point so that walking time from parking spots to workplace is simply ignored. Furthermore, both private autos and eFHV are assumed to carry one commuter per vehicle per trip only and the consideration of ride sharing would be addressed in future studies. Thus, the actual vehicle quantities of private auto and eFHV precisely equal to the numbers of commuters in each mode, N_C and N_F .

Figure 3-1 Morning commute corridor network.

3.1.2 Private auto driving

We first revisit the morning commute bottleneck model in the absence of ride-sourcing service ($M_F = 0$), wherein all commuters travel to work via private auto driving, and each of them needs a parking spot at the destination ($M_C \geq N_C = N$). It is assumed that all private auto drivers have no designated spots for parking, and they need to search for a vacant parking spot with little information on the real-time occupancy. Once parked, they will not leave the CBD during morning peak hours. Every morning, commuters need to trade off between the travel time cost related to bottleneck queue length as well as parking searching, and the scheduling cost of being early at work. Thus, from the classical morning commute ADL model (Arnott et al., 1990b), the generalized private auto travel cost for commuters departing at time t , in an extended form including travel time cost, schedule delay cost, and parking fee, can be defined as follows:

$$P_C(t) = \alpha(T(t) + S(t)) + \beta(t^* - t - T(t) - S(t)) + F_C. \quad (3.1)$$

Note that the travel time in Eq. (3.1) includes the travel time on the highway with bottleneck and the parking search time. Here, the one-off parking fee F_C is assumed to be fixed. The free-flow travel time on the highway is assumed to be zero without loss of generality, and hence the highway only contains the single bottleneck, i.e., $T(t) = D(t)/s$, where $D(t)$ should satisfy

$$\frac{\partial D(t)}{\partial t} = \begin{cases} r_C(t) + r_F(t) - s, & r_C(t) + r_F(t) > s \text{ or } D(t) > 0; \\ 0, & r_C(t) + r_F(t) \leq s \text{ or } D(t) = 0. \end{cases} \quad (3.2)$$

For the case when ride-sourcing service is not considered, i.e., $M_F = 0$, we have

$r_F(t) \equiv 0$ and Eq.(3.2) will reduce to the same form as in classical ADL model with single-mode homogeneous travelers.

We further assume the parking spaces are assigned in the order of convenience by arrivals. While travelers with earlier departure can arrive at the parking facility earlier and find a parking space easier with smaller searching time cost, the later departures are getting more and more difficult to find a vacant space and bear the dramatically increasing searching time cost (Axhausen et al., 1994; Horni et al., 2013; Levy et al., 2013; Qian et al., 2014a; Inci & Lindsey, 2015). The parking search time for private auto commuters departing at time t is then given as² (Qian et al., 2014a),

$$S(t) = \begin{cases} \varepsilon \frac{Cu_C(t)}{M_C}, & \text{if } \frac{Cu_C(t)}{M_C} \leq 1 - \omega; \\ \varepsilon(1 - \omega) + \left(\frac{Cu_C(t)}{M_C} - 1 + \omega \right) \Delta, & \text{if } 1 - \omega < \frac{Cu_C(t)}{M_C} \leq 1; \end{cases} \quad (3.3)$$

where $Cu_C(t) = \int_{t_C^0}^t r_C(u) du$.

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² In the literature, it is found that the parking search time function is convex and is rather flat under low or medium occupancy, but it becomes much steeper when the occupancy is high. The searching time for the last few spots becomes dramatically large. Rather than using the traditional bi-nominal assumption-based searching time function, we adopt a convex continuous piecewise linear function to approximate the parking searching time. Besides, one may argue that when the real-time information of parking occupancy is available, the cruising for parking may have more to do with the vehicles circulating around rather than the vehicles already parked. In fact, different types of parking can induce different cruising behaviors. As we abstract the homogeneous parking spaces generally in a single point, we make a mostly used assumption that the search time is increasing with effective occupancy and hence later commuters would have a larger searching cost.

Henceforth, we term the period when $\frac{Cu_c(t)}{M_c} \leq 1 - \omega$ (i.e., $t \leq t_c^{1-\omega}$) as the *easy-to-*

park period and the period when $\frac{Cu_c(t)}{M_c} > 1 - \omega$ ($t > t_c^{1-\omega}$) as the *hard-to-park*

period. From Eq.(3.3), one can find that the first departure of private auto has zero searching cost for parking. When all parking spaces are occupied at the end of peak

hours, the last departure with $\frac{Cu_c(t)}{M_c} = 1$ has to spend $\varepsilon(1 - \omega) + \omega\Delta$ time in

parking search. With large enough Δ , there is rare departure of private autos during the *hard-to-park* period and hence we also term the scenarios with departures of private autos during *hard-to-park* period ($N_c > (1 - \omega)M_c$) as the *insufficiency*³ of

parking spaces. In fact, $\frac{Cu_c(t)}{M_c}$ indicates the effective occupancy of parking space

at the time when vehicles departing at time t leave the bottleneck, which includes the number of occupied spaces and the number of vehicles departing before t but still cruising for parking. The latter is expected to find the spaces earlier than those departing at t , when FIFO is enforced. It should be noted that, though in a simplified form, our parking searching time function is convex and monotonically increasing with the effective occupancy.

Regarding within-day equilibrium travel pattern of the commuters, we have

$$P_c'(t) = 0, \quad Cu_c(t_c^0) = 0, \quad Cu_c(t_c^{1-\omega}) = (1 - \omega)M_c. \quad (3.4)$$

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³ Rather than the actual shortage of parking spaces, here the *insufficiency* represents the nominal insufficiency of parking spaces, where some private autos' drivers encounter difficulties in finding a parking spot during *hard-to-park* period.

3.1.3 Ride-sourcing service

Now we turn to the bi-modal case in which the ride-sourcing service is considered during morning peak hours ($M_F > 0$). Discouraged by the potential long parking search time with late departure, some commuters are willing to choose other alternative transportation modes. The fast-developing ride sourcing service provides them such an option. We postulate that all these eFHV do not need parking at the destination at CBD⁴ and only one home-to-work trip with one passenger can each eFHV complete during the morning commute. Additionally, the pricing mechanism for passengers of all eFHV is assumed to be identical without surge pricing and depends only on the travel distance⁵. Thus, in this model where all commuters travel along the same traffic corridor with constant distance, the fare F_F for eFHV commuters are identical and constant. It is also assumed that commuters have no preference in choosing a specific eFHV and the dispatch mechanism is the same for all ride-sourcing platforms. Every morning, eFHV commuters first make instant bookings of ride-sourcing service via mobile-app or phone call, then wait for the assigned eFHV to service. A commuter who make the booking earlier will board the eFHV and start the trip earlier as well. The time when a passenger boards his/her eFHV is assumed to be his/her departure time⁶. Based on the assumptions above, the

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⁴ Though some eFHV are in fact ridesharing vehicles and drivers need to park as well after arriving at the destination, we focus on ride-sourcing FHV in this study and thereby make this assumption. After all, drivers offer an eFHV ride in exchange for a fare income more often (Rayle et al., 2014).

⁵ We do not consider surge pricing in the thesis as it is closely relevant to the market equilibrium of the ride-sourcing platforms (Qian et al., 2017; Zha et al., 2017). Nevertheless, the modelling framework in the current form allows flexibility to incorporate surge pricing in future studies, e.g., to add additional term related to travel time in the generalized travel cost of eFHV so as to consider the travel time-based pricing.

⁶ The waiting time for the designated eFHV prior to boarding is not counted in the travel time. With the e-dispatch technology, passengers in ride-sourcing service are usually informed with the estimated waiting time for pick-up and they can use these waiting time freely rather than waiting (wasting) merely in traditional street-hailing taxi service.

generalized eFHV travel cost with departure time t , combining travel time cost, scheduling cost, waiting cost and ride-sourcing fare, is defined as

$$P_F(t) = \alpha T(t) + \beta(t^* - t - T(t)) + W(t) + F_F. \quad (3.5)$$

The passenger waiting cost $W(t)$ for ride-sourcing commuters captures their waiting time for pick-up after booking the ride-sourcing service successfully. It is assumed to be a convex function decreasing with the number of vacant eFHVs at his/her booking time, namely an increasing function of the number of occupied eFHVs, which is equal to $Cu_F(t)$ when FIFO is enforced⁷,

$$W(t) = \rho \frac{Cu_F(t)}{M_F - Cu_F(t)}, \quad (3.6)$$

where $Cu_F(t) = \int_{t_F^0}^t r_F(u) du$.

From Eq.(3.6), it can be found that the first departure of eFHV has negligible waiting cost, i.e., $W(t) = 0$. When the utilization of eFHV is 0.5, i.e., $Cu_F(t) = \frac{1}{2}M_F$, the corresponding waiting cost is ρ . When utilization rate of eFHV is getting higher and close to 1, i.e., $Cu_F(t) \rightarrow M_F$, the waiting cost for ride-sourcing customers will become significantly large, encouraging passengers to select other travel options or

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⁷ In fact, some previous studies applied a Cobb-Douglas matching function in an aggregate model to determine the customers' average waiting time in both traditional taxi and emerging ride-sourcing service analysis, depending on the customer demand and taxi/eFHV utilization rate (Yang et al., 2011; Zha et al., 2016). While the utilization rate is assumed to be time invariant in their models, we determine the dynamic waiting cost with the dynamic utilization rate at time t . One may verify that our waiting cost function in Eq.(3.6) is exactly in the same form as in Eq. (12) in (Yang et al., 2011) by taking the utilization rate $U = \frac{Cu_F(t)}{M_F}$ and setting the parameter

$\zeta = \rho, \alpha_1 / \alpha_2 = 1$.

make earlier booking. In terms of U_F , the waiting cost for last departure eFHV can also be derived, as

$$W(t_F^l) = \rho \frac{U_F}{1 - U_F}. \quad (3.7)$$

Hereinafter, the term “exclusive departure” is used to represent the scenarios when only private auto or ride-sourcing service has positive departure rate at this time point, while “mixed departure” describes the scenarios when there are positive departure rates for both travel modes at this time being. Indeed, Section 3.1.2 focuses on the case of exclusive departure with private autos only. Before we explore the possible mixed departure cases when $M_C, M_F > 0$, let us briefly investigate the extreme case when there is only ride-sourcing service on the highway ($M_C = 0$), i.e., $M_F \geq N_F = N$. Without private autos, there is exclusive departure of eFHVs only. The queue length $D(t)$ still follows Eq.(3.2) but with $r_F(t) \geq 0$ and $r_C(t) \equiv 0$. The within-day equilibrium should satisfy

$$P_F'(t) = 0, \quad Cu_F(t_F^0) = 0. \quad (3.8)$$

3.1.3.1 Bi-modal system considering both private auto and ride-sourcing service

Next, we return to the general bi-modal case with both private autos and eFHVs ($M_C, M_F > 0$). When the ride sourcing service is explicitly considered, the bottleneck services not only private auto commuters but also eFHV commuters at constant capacity, while $D(t)$ still follows Eq.(3.2) and both $r_C(t), r_F(t)$ are nonnegative. Every morning, commuters will choose the mode (either private auto or eFHV) with

the lowest travel cost at departure time t , if there is vacant parking space or vacant eFHV. Additionally, we assume $N \leq M_C + M_F$ so that the total number of parking spaces plus the total number of eFHVs can satisfy the total travel demand on the highway during peak hours. The actual number of private auto commuters N_C and eFHV commuters N_F are bounded by not only the total travel demand N , but also the respective available parking spaces M_C and eFHVs M_F . The equilibrium is achieved when no road commuter can lower his/her travel cost by unilaterally altering his/her mode of travel or the time of departure. This can be termed as a deterministic Wardropian bi-modal equilibrium. Consequently, the travel cost of private auto drivers and eFHV riders are identical at equilibrium, i.e.,

$$P_C^* = P_F^*. \quad (3.9)$$

Besides, the within-day equilibrium also needs to satisfy the conditions in Eqs.(3.4) and (3.8). By solving this system of ordinary differential equations, we can determine the departure rates of travelers from home (i.e., arrival rates at the bottleneck). It is possible to have both exclusive and mixed departure during peak hours, and different departure rates are generated depending on whether there is an existing queue or new queue forming. Specifically, in certain possible equilibrium travel departure pattern, some travelers depart after the queue dissipates, i.e., bottleneck queue forms at early stage of the peak hours and there is still some positive departure rate after the queue dissipates with no new queue being formed. We use the term “without queue” to describe such scenarios (see Figure 3-2(1c) or Figure 3-3(2b) for illustrative reference); meanwhile, the term “with queue” is used to depict the scenarios wherein travelers depart when there is an existing queue or new queue being formed. The underlying mechanism of queue formation is indeed the trade-off between the marginal cost and benefit from delaying departure (Arnott et al., 1990b).

The conditions of queue formation are discussed as follows. When there is exclusive departure of private autos, according to Eq.(3.2), when $M_C > \frac{\alpha - \beta}{\beta} \varepsilon s$, $\Rightarrow r_C^2 < s < r_C^1$. There is a queue being formed with increasing queue length in the bottleneck when $t \leq t_C^{1-\omega}$ and the queue length is shortening when $t > t_C^{1-\omega}$ till the queue ends at $t = t_q^0$. As Δ is large enough, we postulate $\Delta \gg \frac{\beta M_C}{(\alpha - \beta)s}$ and $\Delta \gg \varepsilon$, i.e., $\frac{\Delta}{M_C} \gg \frac{\beta}{(\alpha - \beta)s}$, and hence when $M_C \leq \frac{\alpha - \beta}{\beta} \varepsilon s$, $r_C^4 < r_C^3 \leq s$, there is no queue being formed in the bottleneck at any t .

When there is exclusive departure of eFHV's, from $Cu_F(t_F^0) = 0$, there's

$C_F^1 = \rho s - \alpha s t_F^0$ and $C_F^2 = \frac{\rho - \beta t_F^0}{\rho M_F}$. Thus, at $t = t_F^0$, the departure rates for the

earliest departure of eFHV's with and without queues are

$$r_F^1 \Big|_{t=t_F^0, M_C=0} = \frac{\alpha M_F s}{(\alpha - \beta) M_F + \rho s}; \quad (3.10)$$

$$r_F^2 \Big|_{t=t_F^0, M_C=0} = \frac{\beta M_F}{\rho}. \quad (3.11)$$

When $M_F > \frac{\rho}{\beta} s$, there is $r_F^1 \Big|_{t=t_F^0, M_C=0} > s$ and queue forms starting from $t = t_F^0$ till

queue ends at $t = t_q^0$. When $M_F \leq \frac{\rho}{\beta} s$, we have $r_F^2 \Big|_{t=t_F^0, M_C=0} \leq s$ and hence no queue

forms.

As for the bi-modal case with mixed departures, to ensure queue being formed, σ_1 and σ_2 in $r_C^{m1}, r_F^{m1}, r_C^{m2}, r_F^{m2}$ in Table 3-1 should satisfy,

$$(1+\sigma_1)\alpha \frac{\varepsilon}{M_c} s < 2\beta \left(1 + \frac{\varepsilon}{M_c} s\right) \quad \text{when } t \leq t_C^{1-\omega}; \quad (3.12)$$

$$(1+\sigma_2)\alpha \frac{\Delta}{M_c} s < 2\beta \left(1 + \frac{\Delta}{M_c} s\right) \quad \text{when } t > t_C^{1-\omega}. \quad (3.13)$$

When $M_c > \frac{\alpha-\beta}{\beta} \varepsilon s$, the inequality in Eq.(3.12) always holds, $r_C^{m1} + r_F^{m1} > s$ is

guaranteed. Since $\frac{\Delta}{M_c} \gg \frac{\beta}{(\alpha-\beta)s}$, it should also be stated that only if

$\sigma_2 \leq \frac{2\beta-\alpha}{\alpha}$, the inequality in Eq. (3.13) holds. Thus, when $M_c > \frac{\alpha-\beta}{\beta} \varepsilon s$, there

is a queue being formed at the early stage of morning peak (when $t \leq t_C^{1-\omega}$).

Table 3-1 Departure rates $r_C(t), r_F(t)$.

Departure Case	Queueing status	$r_C(t)$	$r_F(t)$
Exclusive departure	With queues	$r_C^1 = \frac{\alpha s}{(\alpha - \beta) \left(1 + \frac{\varepsilon}{M_C} s\right)}, \text{ when } t \leq t_C^{1-\omega}$ $r_C^2 = \frac{\alpha s}{(\alpha - \beta) \left(1 + \frac{\Delta}{M_C} s\right)}, \text{ when } t > t_C^{1-\omega}$	$r_F^1 = \frac{\alpha s}{2(\alpha - \beta)} \left(1 - \frac{A}{\sqrt{A^2 + 4(\alpha - \beta)\rho M_F s}}\right)$
	Without queues	$r_C^3 = \frac{\beta}{(\alpha - \beta)} \frac{M_C}{\varepsilon}, \text{ when } t \leq t_C^{1-\omega}$ $r_C^4 = \frac{\beta}{(\alpha - \beta)} \frac{M_C}{\Delta}, \text{ when } t > t_C^{1-\omega}$	$r_F^2 = \frac{\beta \rho M_F}{(\beta t + \rho M_F C_F^2)^2}$
Mixed departure	With queues	$r_C^{m1} = \frac{\alpha s}{2(\alpha - \beta) \left(1 + \frac{\varepsilon}{M_C} s\right)} (1 + \sigma_1), \text{ when } t \leq t_C^{1-\omega}$ $r_C^{m2} = \frac{\alpha s}{2(\alpha - \beta) \left(1 + \frac{\Delta}{M_C} s\right)} (1 + \sigma_2), \text{ when } t > t_C^{1-\omega}$	$r_F^{m1} = \frac{\alpha s}{2(\alpha - \beta)} (1 - \sigma_1), \text{ when } t \leq t_C^{1-\omega}$ $r_F^{m2} = \frac{\alpha s}{2(\alpha - \beta)} (1 - \sigma_2), \text{ when } t > t_C^{1-\omega}$
	Without queues	$r_C^3, \text{ when } t \leq t_C^{1-\omega}$ $r_C^4, \text{ when } t > t_C^{1-\omega}$	r_F^2

where $A = -(\alpha - \beta) M_F + \alpha s t + C_F^1$, $\sigma_1 = \frac{B_1}{\sqrt{B_1^2 + 4\rho(\alpha - \beta)\varepsilon M_F(M_C + \varepsilon s)}}$ and $\sigma_2 = \frac{B_2}{\sqrt{B_2^2 + 4\rho(\alpha - \beta)\Delta M_F(M_C + \Delta s)}}$ with

$B_1 = (\alpha s t - (\alpha - \beta) M_F) \varepsilon + C_1$ and $B_2 = (\alpha s t - (\alpha - \beta) M_F) \Delta + C_2$. And C_F^1, C_F^2, C_1 and C_2 are time-invariant constants

determined by the actual travel patterns of the two modes on the highway. Obviously, $\sigma_1, \sigma_2 \in (-1, 1)$.

The derivations of departure rates are similar to that in the classical ADL model (Arnott et al., 1990b; Jia et al., 2016). In Table 3-1, we present the solutions of the equilibrium departure rates in different situations. However, unlike most traditional bottleneck models, nonlinear departure rates are also generated, stemming from the

nonlinearity of ride-sourcing waiting cost. In fact, the departure rates in Table 3-1 are general results. They also include two extreme cases with only private autos or eFHV as special cases, which have exclusive departures solely as discussed in previous subsections. Obviously, $\frac{\partial r_C^1}{\partial t} = \frac{\partial r_C^2}{\partial t} = \frac{\partial r_C^3}{\partial t} = \frac{\partial r_C^4}{\partial t} = 0$. It can also be verified

$$\text{that } \frac{\partial r_F^1}{\partial t}, \frac{\partial r_F^2}{\partial t} < 0, \frac{\partial r_C^{m1}}{\partial t}, \frac{\partial r_C^{m2}}{\partial t} > 0 \text{ and } \frac{\partial r_F^{m1}}{\partial t}, \frac{\partial r_F^{m2}}{\partial t} < 0.$$

From Table 3-1, one may also notice that, in case of mixed departure, the without-queue departure rates of private autos and eFHV are identical to those with the exclusive departure. Indeed, without queuing, there is no longer any mutual interaction of the two groups in travel cost and only the exclusive costs count, i.e., parking search cost for private autos and waiting cost for ride-sourcing. Apparently,

$$\frac{\partial(r_C^3 + r_F^2)}{\partial t} = \frac{\partial(r_C^4 + r_F^2)}{\partial t} < 0. \text{ Summarizing Table 3-1, we have the following Lemma}$$

3.1.

Lemma 3.1 *When there is mixed departure,*

1. *the total departure rates from home for both private autos and eFHV*
 $r_C(t) + r_F(t)$ *decrease as the departure time* t *advances;*
2. *the departure rate of private autos* $r_C(t)$ *within the same period (easy-to-park or hard-to-park) either increases as the departure time* t *advances or it does not vary with* t , *despite that it drops dramatically at* $t = t_C^{1-\omega}$;
3. *the departure rate of eFHV* $r_F(t)$ *decreases as the departure time* t *advances.*

Intuitively, at the late stage of peak hours, the difficulties in finding a vacant parking space or a vacant eFHV restrain the departures of commuters and they are encouraged to depart earlier. Furthermore, the decreasing departure rates indicate that, if N is sufficiently large, there may exist a certain period starting from queue formation till queue dissipation (at t_q^0) with continuous positive departures in between, after which the queue is no longer formed even there are positive departures. We call such period the *longest possible queuing period* and will explore it in following sections.

In a word, we formulate the model in this section incorporating both the parking search in private auto driving and the order-to-pick-up waiting cost in ride-sourcing service. We also examine the properties of the equilibrium travel departure rate in different cases. Though different departure rates of both private autos and eFHVs are derived, whether they will occur during morning peak hours depends on the actual travel pattern at equilibrium, which will be discussed in the next section.

At the end of this chapter, more discussions on the trend of departure rates are presented in what follows, and the reasoning is similar to that in the classical ADL model as in (Arnott et al., 1990b), except for the consideration of parking search and ride-sourcing's waiting cost. With regard to eFHVs, since $Cu_F'(t) \geq 0$ and

$Cu_F(t) < M_F$ from definition, and $\frac{\partial W(t)}{\partial Cu_F(t)} = \rho \frac{M_F}{(M_F - Cu_F(t))^2} > 0$, there is

$$W'(t) = \rho \frac{M_F Cu_F'(t)}{(M_F - Cu_F(t))^2} \geq 0, \quad (1)$$

$$W''(t) = \rho M_F \frac{2(Cu_F'(t))^2 + (M_F - Cu_F(t))Cu_F''(t)}{(M_F - Cu_F(t))^3}. \quad (2)$$

The marginal cost in waiting for eFHV is affected not only by $Cu_F'(t)$, (i.e., $r_F(t)$), but also by $Cu_F(t)$, the cumulative arrivals. In the mixed departure case, we also have

$T'(t) = \frac{Cu_C'(t) + Cu_F'(t)}{s} - 1$ when there is a queue, and $T'(t) = 0$ when there is

no queue. To achieve $P_F'(t) = 0$, it should satisfy $\alpha T'(t) + W'(t) \equiv \beta(1 + T'(t))$ at

the time t with departure, i.e., $T'(t) + \frac{W'(t)}{\alpha - \beta} \equiv \beta$. The marginal cost of both

increases in travel cost and parking search time should equal the marginal benefit

from postponing departure. When $Cu_F''(t) \geq 0$,

$\partial \left(T'(t) + \frac{W'(t)}{\alpha - \beta} \right) / \partial t = T''(t) + \frac{W''(t)}{\alpha - \beta} > 0$. Only when the arrival rate of eFHV

decreases with time, i.e., $Cu_F''(t) < 0$, it is possible to reach the congruence above.

The decreasing arrival rate balances the fast-increasing waiting cost and the travel cost equilibrium.

In addition, one may notice that the departure rates of private autos during mixed departure period increase or does not vary with departure time in our model. Since

$Cu_C'(t) \geq 0$ and $Cu_C(t) < M_C$, and $S'(t) = \frac{\kappa}{M_C} Cu_C'(t) \geq 0, (\kappa = \varepsilon, \Delta)$,

$S''(t) = \frac{\kappa}{M_C} Cu_C''(t)$. The marginal cost of searching time for parking increase only

with $Cu_C'(t)$, namely the arrival rate of private auto. To achieve the bi-modal

equilibrium with $P_C'(t) = P_F'(t) = 0$, it should satisfy not only

$\alpha T'(t) + W'(t) \equiv \beta(1 + T'(t))$ but also $\alpha(T'(t) + S'(t)) \equiv \beta(1 + T'(t) + S'(t))$ at

the time t with departure. Since $Cu_F''(t) < 0$, there is $Cu_C''(t) \geq 0$ to maintain the

congruence. Nevertheless, the departure rates of private auto drop dramatically at

$t = t_C^{1-\omega}$.

3.2 Within-day commuting equilibrium

In this section, we investigate the within-day morning commute traffic equilibrium. The two extreme cases with private autos only ($M_F = 0$) and eFHV only ($M_C = 0$) will be briefly discussed first in Section 3.2.1, followed by the general bi-modal equilibrium with given $M_C, M_F > 0$ in Section 3.2.2. We would identify the occurrence condition for each equilibrium travel pattern. Before we start, the following Proposition 3.1 is noteworthy to be presented based on the model formulation in Section 3.1 .

Proposition 3.1 *Neither the parking spaces M_C nor the eFHV M_F will be used up over the course of morning commute, when $M_C, M_F > 0$ and $M_C + M_F > N$.*

Proof. This proposition can be proofed by contradiction. From Eq.(3.3), if the parking spaces are used up, the searching time for parking for last departure private auto is $\varepsilon(1-\omega) + \omega\Delta$. Since Δ is large enough, the searching time for last parking space can be long enough to induce fairly high travel cost. When there are other travel options, e.g., ride-sourcing, last departure private auto would without doubt switch his/her travel mode. Likewise, from Eq.(3.6), if the eFHV are used up, the waiting cost of last departure eFHV becomes infinity. When there are still parking spaces left, he/she will drive to work instead. Therefore, it is impossible to use up either of the supply (parking spaces or eFHV) when the total supply exceeds the demand of commuters. $\square$

In analyzing the equilibrium travel pattern, Proposition 3.1 excludes the possibility when either private autos or eFHV no longer depart simply because there is no parking space or no more eFHV to take. Basically, the decisions on which commute mode to choose and when to depart are determined only by the travel cost, rather than

the capacity of parking space or eFHV provision. The actual number of commuters N_C, N_F is endogenously determined as an outcome of the equilibrium. To reiterate, the scenarios with private auto departures after $t_C^{1-\omega}$ are considered as *parking spaces insufficiencies*.

Several assumptions are also made. Foremost, we let $F_C < F_F$. Namely, we assume the monetary fare for a ride-sourcing trip is generally higher than parking fee. From Eqs. (3.1) and (3.5), this assumption guarantees $P_C(t)|_{Cu_C(t)+Cu_F(t)=0} < P_F(t)|_{Cu_C(t)+Cu_F(t)=0}$. In other words, the earliest departures on the highway are private autos, $t_C^0 < t_F^0$. Besides, it is assumed that $M_C > \frac{\alpha - \beta}{\beta} \varepsilon s$ and

$M_F > \frac{\rho s}{\beta}$, which are the critical conditions of queue formation for both exclusive and mixed departure in the early stage of morning peak. The without-queue period ($r_C(t) + r_F(t) \leq s$) only exists in the late stage of the peak hour.

Now we attempt to determine the equilibrium commute travel departure pattern and the analysis results are delineated in Figure 3-2, Figure 3-3 and Figure 3-4, wherein solid blue and red curves represent the departure rates from home of private autos and eFHVs respectively and dashed black curves represent the departures from the bottleneck (i.e., the arrivals at workplace of eFHVs). Also, dashed blue curves represent the arrivals at workplace of private autos with extra parking search time.

The conditions for the occurrence of each pattern and the corresponding equilibrium commute cost P_C^*, P_F^* are summarized in Table 3-2 and Table 3-3. Generally, the number of total commuters N determines the occurrence of different equilibrium travel patterns.

Figure 3-2 The travel patterns of private autos at equilibrium in the absence of ride-sourcing.

Figure 3-3 The travel patterns of eFHV's at equilibrium in the absence of private autos.

3.2.1 Equilibrium in the two extreme cases

In this subsection, for illustration purpose, we only focus on the extreme case with the absence of ride-sourcing.

In the absence of ride sourcing service ($M_F = 0$), there are three possible travel patterns of private auto commuters at equilibrium following the analysis in Section 3.1.2. As shown in Figure 3-2, Case (1a) depicts the situation when the supply of parking spaces is much more than the total number of commuters and the last private autos can depart within the *easy-to-park* period. Case (1b) describes the situation when the supply of parking spaces is smaller but still much larger than the total number of commuters so that even though the last private auto departs within the

hard-to-park period, the queue ends at t_q^1 after the last departure. Lastly, Case (1c) depicts the situation when the supply of parking spaces is even smaller, and there are still departures after the queue ends t_q^0 until the last departure arrives at workplace at $t = t^*$. Evidently, Case(1b) and (1c) are the scenarios with parking spaces *insufficiencies*.

Table 3-2 Conditions and equilibrium commute cost for cases depicted in Figure 3-2 and Figure 3-3.

<i>Equilibrium</i>	<i>Conditions</i>	P_C^*, P_F^*
Case (1a)	$N \leq (1-\omega)M_C$	$F_C + \beta \left(\frac{N}{s} + \frac{N}{M_C} \varepsilon \right)$
Case (1b)	$N > (1-\omega)M_C$ and $N \leq \frac{(\alpha-\beta)s(\Delta-\varepsilon)(1-\omega)M_C}{(\alpha-\beta)s\Delta-\beta M_C}$	$F_C + \beta \left(\frac{N}{s} + \left(\frac{N}{M_C} - 1 + \omega \right) \Delta + (1-\omega)\varepsilon \right)$
Case (1c)	$N > \frac{(\alpha-\beta)s(\Delta-\varepsilon)(1-\omega)M_C}{(\alpha-\beta)s\Delta-\beta M_C}$	$F_C + \alpha \left(\frac{N}{M_C} - 1 + \omega \right) \Delta + \alpha(1-\omega)\varepsilon$
Case (2a)	$N \leq M_F - \frac{\rho}{\beta} s$	$F_F + \beta \frac{N}{s}$
Case (2b)	$N > M_F - \frac{\rho}{\beta} s$	$F_F + \rho \frac{N}{M_F - N}$

Proposition 3.2 When $M_F = 0$ and $N \geq Cu_C(t_q^0)$, there exists a longest possible queuing period $[t_C^0, t_q^0]$, where the cumulative number of arrivals of private autos at t_q^0 is

$$Cu_C(t_q^0|_{M_F=0}) = \frac{(\alpha-\beta)s(\Delta-\varepsilon)(1-\omega)M_C}{(\alpha-\beta)s\Delta-\beta M_C}. \quad (3.14)$$

Proof. Since there is a queue forming at $t = t_C^0$, and $r_C^2 < s < r_C^1$ the bottleneck services at full capacity from t_C^0 . The departure time of the $(1-\omega)M_C$ -th private auto $t_C^{1-\omega}$ can be determined by solving the equation $Cu_C(t_C^{1-\omega}) = (1-\omega)M_C$,

$$t_C^{1-\omega} \Big|_{M_F=0} = t_C^0 + \frac{(M_C + \varepsilon s)(\alpha - \beta)(1-\omega)}{s\alpha}. \quad (3.15)$$

Likewise, we can determine t_q^0 by solving the equation $Cu_C(t_q^0) = (t_q^0 - t_C^0)s$,

$$t_q^0 \Big|_{M_F=0} = t_C^0 + \frac{(\alpha - \beta)(\Delta - \varepsilon)(1-\omega)M_C}{(\alpha - \beta)s\Delta - \beta M_C}. \quad (3.16)$$

It can be verified that since $\Delta \gg \frac{\beta M_C}{(\alpha - \beta)s}$, there is $t_q^0 \Big|_{M_F=0} > t_C^{1-\omega} \Big|_{M_F=0}$. And the cumulative arrivals at $t_q^0 \Big|_{M_F=0}$ can be derived as $(t_q^0 \Big|_{M_F=0} - t_C^0)s$. $\square$

In Proposition 3.2, the cumulative number of private auto arrivals at the bottleneck at t_q^0 is equal to the critical value of N distinguishing Case (1b) and (1c). This is indeed the maximum number of private autos drivers suffering from bottleneck congestion with given M_C . When $N < Cu_C(t_q^0)$, all private auto drivers need to queue in the bottleneck; when $N > Cu_C(t_q^0)$, some late private autos are free from bottleneck congestion with departure rates smaller than s , however, at the price of bearing higher parking search cost.

Lemma 3.2 *The latest departure time of private autos t_C^l is independent of the travel pattern and determined by t_C^0 ,*

$$t_C^l = \frac{\beta}{\alpha} t_C^0 + \left(1 - \frac{\beta}{\alpha}\right) t^* . \quad (3.17)$$

Proof. This lemma can be proved by the equalized travel cost. Since the travel cost of the first private auto equals to that of the last private auto, who arrives at workplace at t^* , we have $P_C(t_C^0) = P_C(t_C^l)$ and $t_C^l + T(t_C^l) + S(t_C^l) = t^*$. After mathematical manipulations, we can derive t_C^l . $\square$

Lemma 3.2 indicates that, with consideration of parking search time, the last departure departs at a definite time point only depending on the length of morning peak $t^* - t_C^0$. In fact, this lemma also applies to the bi-modal case when $M_C, M_F > 0$ as long as private autos depart first. No matter when and how the eFHV depart, the last private autos will depart at the same time for a given t_C^0 . Moreover, after t_C^l , only eFHVs will depart from home.

As shown in Table 3-2, the equilibrium travel cost P_C^* , which determines t_C^0 , varies with the number of commuters $\varepsilon, \Delta N$ and the supply of parking spaces M_C . It is worth noting that, as parking search time is considered in our model, t_C^0 should be early enough not only to allow all travelers to leave the bottleneck before t^* , but also to find the parking space before t^* . The more commuters on the highway or the less parking spaces provided, the higher travel cost will be imposed at equilibrium. Note that in Case (1a) and (1b), as the last departure arrives at the bottleneck before the queue ends, the equilibrium travel cost will be lower for larger bottleneck capacity. However, for Case (1c), the queuing in bottleneck no longer plays a substantial role in determining the travel cost. Rather, the parking search factor (i.e.,) determines the equilibrium travel cost. Specifically, in an extreme case when $\Delta \rightarrow \infty$, it is extraordinarily costly for private autos to depart within the *hard-to-park* period. One

can observe that, in such situations, the equilibrium travel cost P_C^* is prohibitively large and the actual parking spaces occupied $N_C = (1 - \omega)M_C$ in both Case (1b) and (1c), which is indeed parking *insufficiency*. In those circumstances, commuters may consider give up driving and switch to other possible travel options even when there are vacant parking spaces ($N < M_C$). When the total number of commuters is assumed to be fixed, the introduction of alternative transportation modes, e.g., ride-sourcing service, becomes particularly imperative.

In the extreme case when $M_C = 0$, there are two possible travel patterns of eFHV commuters at equilibrium. As shown in Figure 3-3, Case (2a) depicts the situation when the supply of ride-sourcing service during morning peak hours is relatively large so that the queue ends after the last departure of eFHVs. And Case (2b) depicts the situation when the supply of ride-sourcing is smaller, and there are still departures after the queue ends at t_q^0 until $t = t^*$.

From Section 3.1.3, there is a queue forming at $t = t_F^0$, and the departure rates of eFHVs decrease with time. Obviously, the *longest possible queuing period* is $[t_F^0, t_q^0]$.

When $M_C = 0$ and $N \geq Cu_F(t_q^0)$, by solving the equation $Cu_F(t_q^0) = (t_q^0 - t_F^0)s$, the queue ending time t_q^0 and its corresponding cumulative arrivals can be determined, where

$$t_q^0|_{M_C=0} = t_F^0 + \frac{M_F}{s} - \frac{\rho}{\beta}. \quad (3.18)$$

The cumulative number of arrivals of eFHVs at t_q^0 is

$$Cu_F(t_q^0) = M_F - \frac{\rho}{\beta}s. \quad (3.19)$$

Eq.(3.19) indicates the critical N between Case (2a) and Case (2b), which is independent of the travel demand. When $N < Cu_F(t_q^0)$, all eFHV queue in the bottleneck and conversely when $N > Cu_F(t_q^0)$, some late eFHV bearing high waiting cost are free from bottleneck congestion.

Comparing to the travel pattern in the single-mode equilibrium with private autos only, the travel pattern in the equilibrium with ride-sourcing only is similar but with different departure rates and later departures from the bottleneck due to the exempt from parking searching. As for the equilibrium travel cost P_F^* in Table 3-2, which determines the earliest departure time of private auto t_F^0 , varies with the number of commuters. The more commuters on the highway, the higher travel cost is at equilibrium as expected. Note that in Case (2a), since last departure arrives at the bottleneck before the queue ends, with larger the bottleneck capacity, the equilibrium travel cost is lower. The equilibrium travel cost does not change with the supply of ride-sourcing service. As for Case (2b), the queuing in bottleneck no longer plays a necessary role. Instead, the waiting cost factor (i.e., ρ) and the supply level of eFHV determine the equilibrium travel cost. When the number of eFHV available for service during peak hours is barely enough, the last eFHV has infinite waiting cost, and he/she can hardly find a vacant eFHV even when he/she departs earlier (still the last) on the next day. As proofed in Proposition 3.1, the last eFHV needs to switch to other travel modes, e.g., private autos driving.

3.2.2 Bi-modal commuting equilibrium

Next, we would examine the bi-modal commuting equilibrium, when $M_C, M_F > 0$.

There are nine possible equilibrium travel patterns for the commuters with both private auto and eFHV as alternative choices (as depicted in Figure 3-2 and Figure 3-4).

Case (3a) and (3f) show the cases when the cost of eFHV is prohibitively high and no one uses ride-sourcing service at equilibrium, which is indeed the case with private autos only as depicted in Figure 3-2. The travel patterns of other cases are shown in Figure 3-4. Case (3b) and (3g) illustrate the cases when the first eFHV departs after the last private auto leaving the bottleneck, with the last private auto departing in *easy-to-park* period and *hard-to-park* period, respectively. Case (3c) and (3h) delineate the cases when the first eFHV departs after last private auto but prior to its leaving from the bottleneck, with the last private auto departing in *easy-to-park* period and *hard-to-park* period, respectively. Further, Case (3d), (3e) display the cases when the first eFHV departs prior to last private auto in *easy-to-park* period, whereas last private auto departs in *easy-to-park* period in Case (3d), and *hard-to-park* period in Case (3e). Lastly, Case (3i) shows the case when the first eFHV departs prior to last private autos in *hard-to-park* period. One can easily observe that Case (3e)-(3i) are the cases with *insufficient* parking spaces as discussed in Section 3.1.2.

Figure 3-4 The travel patterns of private autos and eFHV at equilibrium when $M_C, M_F > 0$.

Table 3-3 summarizes the occurrence conditions and the corresponding time sequence, as well as the equilibrium travel cost. Some new notations are also defined in Table 3-3 with superscripts to indicate the boundary cases. For instance, $N = N^{(a,f)}$ represents the boundary conditions between Case (3a) and (3f), i.e., when last private autos depart right at the end of *easy-to-park* period without eFHV usage.

Table 3-3 Occurrence conditions, time sequence and equilibrium travel cost in the bi-modal equilibrium.

Case	Conditions	Time sequence	$P_C^*, P_F^* \#$
(3a)	$\phi \geq 0$ or ($\phi < 0$ and $N \leq N^{(a,f)}$), $N \leq N^{(a,b)}$	$t_C^0 \rightarrow t_C^l (< t_C^{1-\omega}) \rightarrow t_q^1 \rightarrow t^*$	$F_C + \beta \left(\frac{N}{s} + \frac{N}{M_C} \varepsilon \right)$
(3b)	$\phi \geq 0$ or ($\phi < 0$ and $N \leq N^{(b,g)}$), $N \in (N^{(a,b)}, N^{(b,c)}]$	$t_C^0 \rightarrow t_C^l (< t_C^{1-\omega}) \rightarrow t_q^1 \rightarrow t_F^0 \rightarrow t_F^l \rightarrow t^*$	$F_C + \frac{M_C + s\varepsilon}{2M_C + s\varepsilon} \left(F_F - F_C + \beta \frac{N}{s} \right)$
(3c)	$\phi \geq 0$, $N \in (N^{(b,c)}, N^{(c,d)}]$; or $\phi < 0$, $N \leq N^{(c,h)}$ and $N \in (N^{(b,c)}, N^{(c,h,i)}]$	$t_C^0 \rightarrow t_C^l (< t_C^{1-\omega}) \rightarrow t_F^0 (< t_q^0) \rightarrow t_F^l \rightarrow t^*$	$F_C + \beta \frac{N}{s}$
(3d)	$\phi \geq 0$ and $N \in (N^{(c,d)}, N^{(d,e)}]$	$t_C^0 \rightarrow t_F^0 \rightarrow t_C^l (< t_C^{1-\omega}) \rightarrow t_F^l \rightarrow t^*$	$F_C + \beta \frac{N}{s}$
(3e)	$\phi \geq 0$ and $N > N^{(d,e)}$	$t_C^0 \rightarrow t_F^0 \rightarrow t_C^{1-\omega} \rightarrow t_C^l \rightarrow t_F^l \rightarrow t^*$	$F_C + \beta \frac{N}{s}$
(3f)	$\phi < 0$, $N > N^{(a,f)}$ and $N \leq N^{(f,g,h)}$	$t_C^0 \rightarrow t_C^{1-\omega} \rightarrow t_C^l \rightarrow t^*$	$F_C + \beta \left(\frac{N}{s} + \left(\frac{N}{M_C} - 1 + \omega \right) \Delta + (1-\omega)\varepsilon \right)$
(3g)	$\phi < 0$, $N > N^{(b,g)}$ If $M_C \in (M_C^1, M_C^2)$, $N \in (N^{(f,g,h)}, N^{(g,h)}]$; else if $M_C < M_C^1$, $N > N^{(f,g,h)}$	$t_C^0 \rightarrow t_C^{1-\omega} \rightarrow t_C^l \rightarrow t_q^0 \left(\frac{1}{t_q} \right) \rightarrow t_F^0 \rightarrow t_F^l \rightarrow t^*$	$F_C - \frac{M_C \beta (\Delta - \varepsilon) (1-\omega)}{2M_C + s\Delta}$ $+ \frac{M_C + s\Delta}{2M_C + s\Delta} \left(F_F - F_C + \beta \frac{N}{s} \right)$
(3h)	$\phi < 0$ and $N > N^{(c,h)}$ If $M_C \in (M_C^1, M_C^2)$, $N \in (N^{(g,h)}, N^{(c,h,i)}]$; else if $M_C > M_C^2$, $N \in (N^{(f,g,h)}, N^{(c,h,i)}]$	$t_C^0 \rightarrow t_C^{1-\omega} \rightarrow t_C^l \rightarrow t_F^0 (< t_q^0) \rightarrow t_F^l \rightarrow t^*$	$F_C + \beta \frac{N}{s}$
(3i)	$\phi < 0$, $M_C > M_C^1$ and $N > N^{(c,h,i)}$	$t_C^0 \rightarrow t_C^{1-\omega} \rightarrow t_F^0 \rightarrow t_C^l \rightarrow t_F^l \rightarrow t^*$	$F_C + \beta \frac{N}{s}$

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where $N^{(a,f)} = (1-\omega)M_C$, $N^{(a,b)} = \frac{F_F-F_C}{\beta} \frac{M_C}{M_C+\varepsilon s} s$, $N^{(b,g)} = (2M_C+\varepsilon s)(1-\omega) - \frac{F_F-F_C}{\beta} s$, $N^{(b,c)} = \frac{F_F-F_C}{\beta} \frac{M_C+\varepsilon s}{M_C} s$,

$$N^{(c,d)} = \frac{F_F-F_C}{\alpha-\beta} \frac{M_C+\varepsilon s}{\varepsilon} , \quad N^{(c,h)} = (1-\omega)(M_C+\varepsilon s) , \quad N^{(c,h,i)} = \frac{(F_F-F_C)s\Delta-\phi M_C}{(\alpha-\beta)\Delta} , \quad N^{(d,e)} = \frac{\phi}{\phi+\rho} M_F + (1-\omega)(M_C+s\varepsilon) ,$$

$$N^{(f,g,h)} = \frac{F_F-F_C+\beta(\Delta-\varepsilon)(1-\omega)}{\beta} \frac{M_C s}{M_C+s\Delta} , \quad N^{(g,h)} = \frac{F_F-F_C-\beta(\Delta-\varepsilon)(1-\omega)}{\beta} s + \frac{F_F-F_C}{\beta M_C} s^2 \Delta , \quad M_C^1 = \frac{-(F_F-F_C)(\alpha-\beta)s\Delta}{\beta\phi} ,$$

$$M_C^2 = \frac{(F_F-F_C)s\Delta}{\beta(\Delta-\varepsilon)(1-\omega)} . \quad \# : \text{Only the travel costs of the scenarios with the last departure before } t_q^0 \text{ are presented.}$$

It is important to note that, in Figure 3-4, only the scenarios with departure before t_q^0 are sketched for illustration purpose. It is possible for the last vehicle to depart after the queue end at t_q^0 , i.e., after the *longest possible queuing period*. To be specific, for private autos in Case (3g), the last private auto can depart after the queue ends, with travel pattern similar to Case (1c) in Figure 3-2. As for eFHV, the last eFHV can depart after queue ends in all cases depicted in Figure 3-4, with similar travel pattern in Case (2b) in Figure 3-3. These scenarios, though not sketched, would be further discussed in the next section.

In addition, the cumulative arrivals at the bottleneck in the bi-modal equilibrium are calculated at critical time points and the results are shown in Table 3-4. One may verify the results in Table 3-4 by solving $r_C(t) = Cu_C'(t)$ and $r_F(t) = Cu_F'(t)$. Besides, time points for each bi-modal equilibrium cases are also computed and presented in Table 3-5 and their derivations are presented below.

When $\phi \geq 0$, ride-sourcing service is already more cost attractive to commuters at $t_C^{1-\omega}$. Thus, $t_F^0 \leq t_C^{1-\omega}$ at equilibrium. On the other hand, when $\phi < 0$, driving to work with private autos is still cost attractive at $t_C^{1-\omega}$, and commuters are willing to drive to work after $t_C^{1-\omega}$, thus $t_F^0 > t_C^{1-\omega}$.

Thus, with $F_C < F_F$ and $t_F^0 \leq t_C^l$, the first eFHV departs within the *easy-to-park* period of private autos if $\phi \geq 0$; and the first eFHV departs within the *hard-to-park* period of private autos if $\phi < 0$, where t_F^0 is derived from $P_C(t_F^0) = P_F(t_F^0) \Big|_{Cu_F(t_F^0)=0}$.

$$t_F^0 = \begin{cases} t_C^0 + \frac{(F_F - F_C)(M_C + \varepsilon s)}{\alpha}, & \text{when } \phi \geq 0 \\ & \text{(Case (3d),(3e));} \\ t_C^0 + \frac{F_F - F_C}{\alpha s} \frac{M_C}{\varepsilon} - \frac{\phi}{\alpha} \frac{\Delta}{\varepsilon} + \frac{\alpha - \beta}{\alpha} (1 - \omega) \varepsilon, & \text{when } \phi < 0 \\ & \text{(Case (3i)).} \end{cases} \quad (3.20)$$

As for the cases when eFHVs departing after t_C^l in Case (3b) (3c) (3g) and (3h), the corresponding t_F^0 are recalculated considering the departure gap between private autos and eFHVs, and there is

$$t_F^0 = \begin{cases} t_C^0 + \frac{F_F - F_C}{\beta} & \text{Case (3b) (3g);} \\ t_C^0 + \frac{F_F - F_C}{\alpha} + \frac{\alpha - \beta}{\alpha s} Cu_C(t_C^l) & \text{Case (3c) (3h).} \end{cases} \quad (3.21)$$

where t_C^l is determined in Lemma 3.2.

When $\phi \geq 0$ and the last private auto departs in the *hard-to-park* period ($t_C^{1-\omega} < t_C^l$), from $Cu_C(t_C^{1-\omega}) = (1 - \omega)M_C$, the departure time of the $(1 - \omega)M_C$ -th private auto is

$$t_C^{1-\omega} \Big|_{\phi \geq 0} = t_C^0 + \frac{\phi - F_C + F_F}{\alpha} \left(\frac{M_C}{\varepsilon s} + 1 \right) + \frac{\alpha - \beta}{\alpha s} \frac{\phi}{\phi + \rho} M_F. \quad (3.22)$$

Note that when $\phi < 0$, $\Rightarrow t_C^{1-\omega} < t_F^0$, the departure time of the $(1 - \omega)M_C$ -th private auto $t_C^{1-\omega}$ is equivalent to $t_C^{1-\omega} \Big|_{M_F=0}$ in Eq.(3.15).

We further define

$$\phi = F_C + (1 - \omega)(\alpha - \beta)\varepsilon - F_F; \quad (3.23)$$

$$\varphi = F_C - (1 - \omega)(\alpha - \beta)(\Delta - \varepsilon) - F_F. \quad (3.24)$$

where ϕ is the instantaneous travel cost difference of private autos and eFHV at $t_C^{1-\omega}$, when there is no eFHV departure within the *easy-to-park* period. One can verify that $P_C(t_C^{1-\omega}) - P_F(t_C^{1-\omega})|_{Cu_F(t_C^{1-\omega})=0} = F_C + (\alpha - \beta)\varepsilon(1 - \omega) - F_F = \phi$. Moreover, ϕ is the instantaneous cost difference of the time-invariant term between private autos and eFHV in the *hard-to-park* period, i.e., $\phi + (\alpha - \beta)\frac{Cu_C(t)}{M_C}\Delta = \rho\frac{Cu_F(t)}{M_F - Cu_F(t)}$. With a negative coefficient of Δ in Eq.(3.24), one can derive that $\phi < 0$ and $\phi < \phi$.

Table 3-4 Cumulative arrivals $Cu_C(t), Cu_F(t)$ at the bottleneck when $M_C, M_F > 0$.

Time	$Cu_C(t)$	$Cu_F(t)$	Occurrence Case
$t \in [t_C^0, \text{Min}[t_C^{1-\omega}, t_F^0]]$	$\frac{\alpha s(t - t_C^0)}{(\alpha - \beta)(M_C + s\varepsilon)} M_C$	0	(3a)-(3i)
$t \in [t_C^{1-\omega}, t_F^0]$, when $t_C^{1-\omega} \leq t_F^0$	$\frac{\alpha s(t - t_C^0) - (\alpha - \beta)(\Delta - \varepsilon)(-1 + \omega)s}{(\alpha - \beta)(M_C + s\Delta)} M_C$	0	(3f)-(3i)
$t \in [t_F^0, t_C^{1-\omega}]$, when $t_C^{1-\omega} > t_F^0$	$\frac{B_1 - 2(M_C + s\varepsilon)(F_C - F_F + \rho) + B_1/\sigma_1}{2(\alpha - \beta)\varepsilon(M_C + s\varepsilon)} M_C$	$\frac{B_1 + 2M_F(\alpha - \beta)\varepsilon - B_1/\sigma_1}{2(\alpha - \beta)\varepsilon}$	(3d) and (3e)
$t \leq t_C^l$ $t \in [\text{Max}[t_F^0, t_C^{1-\omega}], t_q^0]$	$\frac{B_2 - 2(M_C + s\Delta)(\varphi + \rho) + B_2/\sigma_2}{2(\alpha - \beta)\Delta(M_C + s\Delta)} M_C$	$\frac{B_2 + 2M_F(\alpha - \beta)\Delta - B_2/\sigma_2}{2(\alpha - \beta)\Delta}$	(3e) and (3i)
$t \in [t_q^0, t^*]$	$\frac{M_C \beta}{(\alpha - \beta)\Delta} t + C_3$	$M_F - \frac{\rho M_F}{\beta t + \rho M_F C_F^2}$	private autos:(3e)-(3i); eFHV:(3b)-(3e) (3g)-(3i)
$t > t_C^l$ $t \in [\text{Max}[t_F^0, t_C^l], t_q^0]$	$Cu_C(t_C^l)$	$\frac{A + 2(\alpha - \beta)M_F - \sqrt{A^2 + 4(\alpha - \beta)\rho M_F s}}{2(\alpha - \beta)}$	(3b)-(3i)
$t \in [t_q^0, t^*]$	$Cu_C(t_C^l)$	$M_F - \frac{\rho M_F}{\beta t + \rho M_F C_F^2}$	(3b)-(3i)

where $A, B_1, B_2, \sigma_1, \sigma_2$ are defined in Table 3-1 with $C_1 = -\alpha \varepsilon s t_C^0 + (F_C - F_F + \rho)(M_C + \varepsilon s)$ and

$C_2 = -\alpha \Delta s t_C^0 - (1 - \omega) M_C (\alpha - \beta)(\Delta - \varepsilon) + (F_C - F_F + \rho)(M_C + \Delta s)$. And C_3, C_F^1, C_F^2 are determined by the continuity

of the actual travel pattern.

3.2.2.1 Travel departure evolution

We now try to explore the departure evolution in the bi-modal equilibrium. Unlike private autos driving, ride-sourcing riders do not need parking and thereby are free from parking search during peak hours. Private auto drivers need to depart earlier to reserve some time for parking search. With the assumption of $F_C < F_F$, the private autos will depart first at t_C^0 . Thus, bounded by not only bottleneck congestion but also the parking search time, the last departure time of private autos t_C^l can also be determined in a similar way as in Lemma 3.2, which is independent of the equilibrium travel pattern. As for eFHV, though waiting cost is time-variant, it is not counted in the travel time as defined in Section 3.1.3 and therefore the departure time of the last eFHV t_F^l is determined by the bottleneck congestion only. Apparently, $t_C^l < t_F^l$.

Commuters choose their mode of travel by comparing the instantaneous travel cost at their time of departure. The mode with lower travel cost is chosen. As private auto departs first, there are private autos only on the highway for a certain time period, until $t = t_F^0$. As shown in Table 3-5, t_F^0 is independent from the ride-sourcing supply in all cases. This is because the ride-sourcing supply only affects the waiting cost in the travel cost composition and the first eFHV has zero waiting cost. Ride-sourcing will not be chosen until the travel cost of the first eFHV equals the travel cost of private autos.

Whether the first eFHV will depart within the *easy-to park* period or *hard-to-park* period depends on the sequence of t_F^0 and $t_C^{1-\omega}$, which is determined by the sign of ϕ . When $\phi = 0$, t_F^0 coincides with $t_C^{1-\omega}$. Since $F_C < F_F$, only if the extra cost induced by parking search for the $(1-\omega)M_C$ -th private autos, $(\alpha - \beta)\varepsilon(1-\omega)$, is

sufficiently large, will it be achieved that $\phi = F_C + (\alpha - \beta)\varepsilon(1 - \omega) - F_F \geq 0$, and

$t_F^0 \leq t_C^{1-\omega}$. Otherwise, $\phi < 0$ and $t_F^0 > t_C^{1-\omega}$.

Table 3-5 Time points for each bi-modal case depicted in Figure 3-2 and Figure 3-4.

Case	$t_C^0 \#$	t_F^0	$t_C^{1-\omega}$
(3a)	$t^* - \frac{N}{s} - \varepsilon \frac{N}{M_C}$	N/A	N/A
(3b)	$t^* - \frac{1}{\beta} \frac{M_C + s\varepsilon}{2M_C + s\varepsilon} \left(F_F - F_C + \beta \frac{N}{s} \right)$	$t_C^0 + \frac{F_F - F_C}{\beta}$	N/A
(3c)	$t^* - \frac{N}{s}$	$t_C^0 + \frac{F_F - F_C}{\alpha} + \frac{\alpha - \beta}{\alpha s} Cu_C(t_C^l)$	N/A
(3d)			N/A
(3e)		$t_C^0 + \frac{(F_F - F_C)(M_C + \varepsilon s)}{\alpha \varepsilon s}$	$t_C^0 + \frac{\phi - F_C + F_F}{\alpha} \left(\frac{M_C + 1}{\varepsilon s} \right) + \frac{\alpha - \beta}{\alpha s} \frac{\phi}{\phi + \rho} M_F$
(3f)	$t^* - \left(\frac{N}{s} + \left(\frac{N}{M_C} - 1 + \omega \right) \Delta - (1 - \omega) \varepsilon \right)$	N/A	$t_C^0 + \frac{(M_C + \varepsilon s)(\alpha - \beta)(1 - \omega)}{s\alpha}$
(3g)	$t^* + \frac{M_C(\Delta - \varepsilon)(1 - \omega)}{2M_C + s\Delta} - \frac{1}{\beta} \frac{M_C + s\Delta}{2M_C + s\Delta} \left(F_F - F_C + \beta \frac{N}{s} \right)$	$t_C^0 + \frac{F_F - F_C}{\beta}$	
(3h)	$t^* - \frac{N}{s}$	$t_C^0 + \frac{F_F - F_C}{\alpha} + \frac{\alpha - \beta}{\alpha s} Cu_C(t_C^l)$	
(3i)		$t_C^0 + \frac{F_F - F_C}{\alpha s} \frac{M_C}{\varepsilon} - \frac{\phi}{\alpha} \frac{\Delta}{\varepsilon} + \frac{\alpha - \beta}{\alpha} (1 - \omega) \varepsilon$	

: Only the t_C^0 of the scenarios with last departure before t_q^0 are presented.

Though we derive t_F^0 in Table 3-5 by solving $P_C(t_F^0) = P_F(t_F^0)$, whether there will be positive eFHV departure at t_F^0 is still unknown and we have Lemma 3.3 on departures evolution as follows.

Lemma 3.3 *When $t_F^0 < t_C^l$, both the private autos and eFHVs depart simultaneously (mixed departure) after t_F^0 until last departure at t_C^l . After t_C^l , only the eFHVs depart on the highway.*

Proof. We prove this lemma by comparing the departure rates of private autos and eFHVs, which is split into two parts by the queue status.

When there is an existing queue or a new queue being formed at the bottleneck: If the exclusive departure rate of either type of commuters is larger than the total departure rates of mixed departure, that type of commuters will depart more following the exclusive departure rates rather than the mixed departure rate. The other type of commuters will not depart as the queue length in the bottleneck increase more than expected. On the other hand, if the exclusive departure rate is smaller than the total departure rates of mixed departure, both types of commuters will depart simultaneously following the mixed departure rates. In our case, it suffices to find $\max\{r_C^1, r_F^1, r_C^{m1} + r_F^{m1}\}$ when $t \leq t_C^{1-\omega}$, and find $\max\{r_C^2, r_F^1, r_C^{m2} + r_F^{m2}\}$ when $t_C^{1-\omega} < t \leq t_q^0$. When $t \leq t_C^{1-\omega}$ and $\phi \geq 0$, it can be verified that $r_C^{m1} + r_F^{m1} > r_C^1$. We also

have $r_C^{m1} + r_F^{m1} \Big|_{t=t_F^0} > r_F^1 \Big|_{t=t_F^0}$ and $\left| \frac{\partial(r_C^{m1} + r_F^{m1})}{\partial t} \right| \leq \left| \frac{\partial r_F^1}{\partial t} \right|$. Since $\frac{\partial r_F^1}{\partial t} < 0$, $\frac{\partial(r_C^{m1} + r_F^{m1})}{\partial t} < 0$,

there is $r_C^{m1} + r_F^{m1} > r_F^1$ when $t \geq t_F^0$. Thus, $\max\{r_C^1, r_F^1, r_C^{m1} + r_F^{m1}\}$ in this case is $r_C^{m1} + r_F^{m1}$. Likewise, it can be verified that $\max\{r_C^2, r_F^1, r_C^{m2} + r_F^{m2}\}$ is $r_C^{m2} + r_F^{m2}$. Thus, both private autos and eFHVs depart simultaneously till last departure or queue ends.

When there is no queue at the bottleneck: As mentioned earlier in Section 3.1.3.1, the instantaneous travel costs of both types are mutually independent. When $t \geq t_q^0$, it can be verified that $r_C^4 + r_F^2 < s$, both private autos and eFHV's will depart simultaneously even after queue ends. It should also be noted that it is not necessary to compare the without-queue departure rate when $t \leq t_C^{1-\omega}$ since the queue length is increasing. $\square$

In the literature, many previous studies investigated bottleneck problems with heterogeneous travelers. In these studies, the heterogeneity is usually defined by variation in the parameters of value of time, unit scheduling cost and the preferred time of arrival between different groups of travelers. However, in this study, we are investigating two competing modes; more specifically, how private autos driving and ride-sourcing service interact in the bottleneck congestion. While previous studies in the literature determined the equilibrium travel pattern by finding the upper, nonnegative envelope of all groups' isocost queueing curves (Hendrickson et al., 1981; Newell, 1987; Lindsey, 2004), Lemma 3.3 motivates us to examine the instantaneous maximum total departure rates, which ensure the longest possible queues at that moment (i.e., the upper envelope). so that we can determine the evolution of equilibrium travel pattern with mixed departure. In addition, one interesting finding on the cumulative arrivals at $t_C^{1-\omega}$ is presented in Lemma 3.4.

Lemma 3.4 *When $\phi \geq 0$ and $t_C^{1-\omega} < t_C^l$, both the cumulative departures of private autos and the cumulative departures of eFHV's at $t_C^{1-\omega}$, are independent of the total number of commuters, wherein*

$$Cu_C(t_C^{1-\omega}) = M_C(1-\omega), \quad Cu_F(t_C^{1-\omega}) = M_F \left(1 - \frac{\rho}{\phi + \rho} \right) \quad \text{when } \phi \geq 0. \quad (3.25)$$

Proof. One can verify that at $t = t_C^{1-\omega} \Big|_{\phi \geq 0}$, $Cu_F(t_C^{1-\omega})$ is time-invariant and determined by the ratio of ϕ and ρ . $\square$

Lemma 3.4 applies to Case (3e) only. The underlying reasoning of the time-invariant cumulative eFHV at $t = t_C^{1-\omega} \Big|_{\phi \geq 0}$ is the equilibrium travel cost. Specifically, at the time when the $(1-\omega)M_C$ -th private auto departs, his/her travel cost should be equal to the travel cost of the eFHV with simultaneous departure. As they share the same queuing time, the parking search time for the $(1-\omega)M_C$ -th private auto is given and independent of M_C , the only term changed due to the departure of eFHV prior to $t_C^{1-\omega}$ is the waiting cost of eFHV. To ensure $P_C(t_C^{1-\omega}) = P_F(t_C^{1-\omega})$, it has to be fulfilled

that $\rho \frac{Cu_F(t_C^{1-\omega})}{M_F - Cu_F(t_C^{1-\omega})} = \phi$. After mathematical manipulations, one can easily derive $Cu_F(t_C^{1-\omega})$.

In this subsection, we determine and discuss some critical points and the travel departure evolution in the within-day dynamic equilibrium. At equilibrium, the actual number of occupied parking spaces is the cumulative arrivals of private autos at the bottleneck at t_C^l , i.e., $N_C = Cu_C(t_C^l)$ (Presented in Table 3-6). Thus, $N_F = Cu_F(t_C^l) = N - Cu_C(t_C^l)$, and the actual utilization rate of parking spaces and eFHV can also be derived, where

$$U_C = \frac{N_C}{M_C} = \frac{Cu_C(t_C^l)}{M_C}, \quad (3.26)$$

$$U_F = \frac{N_F}{M_F} = \frac{N - Cu_C(t_C^l)}{M_F}. \quad (3.27)$$

In fact, with given M_C, M_F , we can determine the resultant equilibrium travel pattern. In the next section, we will continue to analyze the bi-modal equilibrium focusing on the *longest possible queuing period* and the system performance.

Table 3-6 Actual number of occupied parking spaces N_C .

Case	$N_C = Cu_C(t_C^l)$ #	Case	$N_C = Cu_C(t_C^l)$ #
(3a)	N	(3f)	N
(3b)	$\frac{M_C}{2M_C+s\varepsilon} \left(\frac{F_F-F_C}{\beta} s+N \right)$	(3g)	$\frac{M_C}{2M_C+s\Delta} \left(\frac{F_F-F_C+\beta(\Delta-\varepsilon)(1-\omega)}{\beta} s+N \right)$
(3c)	$\frac{M_C}{M_C+s\varepsilon} N$	(3h)	$\frac{M_C}{M_C+s\Delta} (N+s(\Delta-\varepsilon)(1-\omega))$
(3d)	$\frac{M_C}{2(M_C+s\varepsilon)} (N-M_F) + \frac{(F_F-F_C-\rho)}{2(\alpha-\beta)\varepsilon} M_C$ $+ \frac{M_C}{2(M_C+s\varepsilon)(\alpha-\beta)\varepsilon} \sqrt{4(M_C+s\varepsilon)M_F(\alpha-\beta)\varepsilon\rho + \left((N-M_F)(\alpha-\beta)\varepsilon - (M_C+s\varepsilon)(F_F-F_C-\rho) \right)^2}$	(3i)	$\frac{M_C}{2(M_C+s\Delta)} (N-M_F+s(\Delta-\varepsilon)(1-\omega)) - \frac{M_C}{2(\alpha-\beta)\Delta} (\varphi+\rho)$ $+ \frac{M_C}{2(\alpha-\beta)\Delta(M_C+s\Delta)} \sqrt{4(M_C+s\Delta)M_F(\alpha-\beta)\Delta\rho + \left((\alpha-\beta)\Delta(N-M_F+s(\Delta-\varepsilon)(1-\omega)) - (M_C+s\Delta)(\varphi+\rho) \right)^2}$
(3e)	$\frac{M_C}{2(M_C+s\Delta)} (N-M_F+s(\Delta-\varepsilon)(1-\omega)) - \frac{M_C}{2(\alpha-\beta)\Delta} (\varphi+\rho)$ $+ \frac{M_C}{2(\alpha-\beta)\Delta(M_C+s\Delta)} \sqrt{4(M_C+s\Delta)M_F(\alpha-\beta)\Delta\rho + \left((\alpha-\beta)\Delta(N-M_F+s(\Delta-\varepsilon)(1-\omega)) - (M_C+s\Delta)(\varphi+\rho) \right)^2}$		

: Only N_C in the scenarios with last departure before t_q^0 are presented.

3.3 Optimal supply of eFHV with best system performance

As indicated in Table 3-1 and Lemma 3.1, we know that the term *longest possible queueing period* plays an important role in analyzing the bi-modal equilibrium travel pattern. In fact, the *longest possible queueing period* in the extreme case with the

absence of ride-sourcing has been examined in Proposition 3.2. In this section, we would first introduce the *longest possible queuing period* in the bi-modal equilibrium, and then investigate its effects on equilibrium travel cost. Besides, in order to best manage the morning commute traffic, we also seek to address the problem on how to determine the optimal supply of parking space and eFHV.

3.3.1 Longest possible queuing period in bi-modal equilibrium

As mentioned earlier in Section 3.2.2, the scenarios with continuous departures after t_q^0 are not explicitly depicted in Figure 3-4. However, the existence of these scenarios is still possible. When there is positive demand for eFHVs during the morning peak and the first eFHV departs before the last private autos leaving the bottleneck, i.e., Case (3c)-(3e) and (3h)-(3i), where only one bottleneck queue exists, we have Lemma 3.5⁸.

Lemma 3.5 *Given $M_C, M_F > 0$, when the first eFHV departs before the last private autos leaving the bottleneck, there exists a longest possible queuing period $[t_C^0, t_q^0]$, within which the travel departures of both private autos and eFHVs suffer from bottleneck queueing.*

The existence of the *longest possible queuing period* is in line with our findings in Proposition 3.2 in the single-mode equilibrium. Such queuing period results from the consideration of parking search time and waiting cost for ride-sourcing service. In fact, when only the travel time on the highway with bottleneck congestion is

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⁸ . For Case (3a) (3b) (3f) and (3g), wherein either there is no demand for eFHVs or there is two separate queues in peak hours, the *longest possible queuing periods* of private autos and eFHVs are determined separately and are independent of the travel demand and travel pattern.

considered in the travel cost composition as in the standard bottleneck model (Arnott et al., 1990b), the queuing period is determined by the demand only. Without scheduling cost, last departure reaching workplace at t^* must bear the longest queue time at the bottleneck, when no late arrival is allowed. However, in our model, due to the consideration of the parking search time and eFHV waiting cost, it is possible that the queue ends at t_q^0 before t^* even though the scheduling cost is small for late departures.

However, even though there is the *longest possible queuing period* in the bi-modal equilibrium, the maximum possible number of private autos and eFHVs suffering from bottleneck congestion can hardly be solved analytically due to the intractability in algebra as shown in Table 3-6.

In fact, we also attempt to discuss the lengths of the *longest possible queuing period* in terms of cumulative departures. To determine the *longest possible queuing period* without information of the utilization of eFHVs, we need to consider three scenarios: 1. When there is no demand for eFHVs in Case (3a)(3f); 2. When there are two separate queues in peak hours in Case (3b)(3g); 3. When there is demand for eFHVs and only one queue in peak hours in other cases.

Obviously, as discussed in Section 3.2.1, the longest queueing period when there is no demand for eFHVs is determined in Proposition 3.2.

When there is two separate queues, to ensure the cumulative departures from bottleneck within the longest queueing period, there are extra conditions in addition to those listed in Table 3-3. For Case (3b), it is impossible to end the queue during the departure of private autos, and hence the travel demand on the highway should

satisfy $N \leq Cu_C(t_C^l) + \left(M_F - \frac{\rho}{\beta} s \right)$ to allow all departures of eFHVs from

bottleneck within the longest queueing period. The latter term $M_F - \frac{\rho}{\beta}s$ is

determined in Eq.(3.19). Similarly, for Case (3g), the travel demand should satisfy

$$N \leq \frac{(\alpha - \beta)s(\Delta - \varepsilon)(1 - \omega)M_C}{(\alpha - \beta)s\Delta - \beta M_C} + \left(M_F - \frac{\rho}{\beta}s \right) \text{ to allow all departures of private}$$

autos and eFHV's from bottleneck within the longest queueing period.

Lastly, when there is demand for eFHV's and only one queue in peak hours with

$t_C^l < t_q^0$, to determine the *longest possible queueing period* and its corresponding

cumulative departures, it suffices to solve the equations,

$$\begin{cases} Cu_C(t_C^l) + Cu_F(t_q^0) = (t_q^0 - t_C^0)s \\ t_q^0 = t^* \end{cases}, \quad (3.28)$$

so that the last eFHV's leaving the bottleneck at t^* happen to be the first vehicles

without queuing. Note that $t_C^l, Cu_C(t), Cu_F(t)$ are provided in Lemma 3.2 and Table

3-4.

When private autos stop to depart and eFHV's depart on the highway only, the queues

in that period affect the length of *longest possible queueing period*. With mathematical

manipulations, there is

$$N \leq (t_q^0 - t_C^0)s = M_F + Cu_C(t_C^l) - \frac{C_q - \sqrt{(C_q)^2 - 4M_F s \beta \rho}}{2\beta}, \quad (3.29)$$

where
$$C_q = \alpha Cu_C(t_C^l) + (\alpha - \beta)(Cu_F(t_C^l) - s(t^* - t_C^0)) + M_F \beta + \frac{M_F s \rho}{M_F - Cu_F(t_C^l)}$$

when $t_F^0 \leq t_C^l$ and $C_q = \beta(M_F + Cu_C(t_C^l)) + (F_C - F_F + \rho)s$ when $t_F^0 > t_C^l$.

Evidently, when $t_F^0 \leq t_C^l, \Rightarrow Cu_F(t_C^l) > 0$. However, it is difficult to solve Eq.(3.29)

with complex and tedious algebra in particular when $t_F^0 \leq t_C^l$, as the right-hand side

is a function of t_C^0 , which is also determined by N . The length of *longest possible queuing period* $(t_q^0 - t_C^0)$ is a function of N . Nevertheless, for Case (3c), when $t_F^0 > t_C^l$, and we can derive the cumulative departures within the *longest possible queuing period*,

$$N|_{\text{Case (3c)}} \leq \frac{M_F(M_C + s\varepsilon)}{2s\varepsilon} + \frac{s(F_F - F_C - \rho)}{2\beta} + \frac{\sqrt{\left((M_C + s\varepsilon)M_F\beta + s^2\varepsilon(F_F - F_C - \rho)\right)^2 - 4M_Fs^2(F_F - F_C)\beta\varepsilon(M_C + s\varepsilon)}}{2s\beta\varepsilon}. \quad (3.30)$$

As for Case (3h), since Δ is sufficiently large, the departures after $t_C^{1-\omega}$ can be very small and infinitely close to 0 when $\Delta \rightarrow \infty$. Likewise, for Case (3e) and (3i), when last private auto departs after $t_C^{1-\omega}$, we have the following with $\Delta \rightarrow \infty$.

$$N|_{\substack{\text{Case (3e) (3i)} \\ \text{Case (3h)} \\ \Delta \rightarrow \infty}} \leq \frac{(1-\omega)M_C + M_F}{2} + \frac{(F_F - F_C - \rho)s}{2\beta} + \frac{\sqrt{\left(\beta((1-\omega)M_C + M_F) - s(F_F - F_C - \rho)\right)^2 - 4M_Fs\beta\rho}}{2\beta}. \quad (3.31)$$

In addition, when last private autos depart after queue ends, i.e., $t_C^l \geq t_q^0$, the *longest possible queuing period* is independent of the travel demand. One may refer to Case (3e) and (3i) in Figure 3-4 but leave out the eFHV-only departure periods. As the private autos depart first, again there is a queue forming at $t = t_C^0$, and $r_C^2 < s < r_C^1$ the bottleneck services in full capacity from t_C^0 . We also have $r_C^{m1} + r_F^{m1} > s$, when

$\frac{\varepsilon}{M_C} < \frac{\beta}{(\alpha - \beta)s}$. Therefore, when $t \leq t_C^{1-\omega}$, the queue length is ever increasing if

$N \geq Cu_C(t_C^{1-\omega}) + Cu_F(t_C^{1-\omega})$, and $t_q^0 > t_C^{1-\omega}$. Now we can derive the t_q^0 by solving the equations $Cu_C(t_q^0) + Cu_F(t_q^0) = (t_q^0 - t_C^0)s$ in both cases when $\phi \geq 0$ and $\phi < 0$. It can be verified that $t_q^0|_{\phi \geq 0} = t_q^0|_{\phi < 0}$ and $t_q^0|_{M_C, M_F > 0}$ is derived.

$$\begin{aligned}
t_q^0|_{M_C, M_F > 0} = & t_C^0 + \frac{F_F - F_C - \rho}{2\beta} \\
& + \frac{\beta(-(\alpha - \beta)\Delta M_F + (F_F - F_C + \phi)M_C)}{2\beta(M_C\beta - s(\alpha - \beta)\Delta)} \\
& - \frac{\sqrt{4\rho(M_C\beta - s(\alpha - \beta)\Delta)(\alpha - \beta)\beta\Delta M_F + \left((\alpha - \beta)\beta\Delta M_F - s(\alpha - \beta)\Delta(F_F - F_C - \rho) - M_C\beta(\rho + \phi)\right)^2}}{2\beta(M_C\beta - s(\alpha - \beta)\Delta)}
\end{aligned} \tag{3.32}$$

Though the expression in Eq.(3.32) is still too complex in algebra after simplification, interestingly we can find that the queuing period when $N \geq Cu_C(t_q^0) + Cu_F(t_q^0)$ does not change with the given travel demand.

Nevertheless, we have the following proposition on the actual utilization rate of eFHV's U_F .

Proposition 3.3 *Given N , there exists a critical actual utilization rate of eFHV's U_F^* , such that the last commuter departs right at the end of the longest possible queuing period (t_q^0), where*

$$U_F^* = \begin{cases} 1 - \frac{s\rho}{-(F_F - F_C)s + \beta N + s\rho}, & \text{Case(3c)-(3e), (3h)-(3i);} \\ 1 - \frac{(2M_C + s\varepsilon)s\rho}{\left(- (F_F - F_C)M_C s + \beta N (M_C + s\varepsilon)\right) + (2M_C + s\varepsilon)s\rho}, & \text{Case(3b);} \\ 1 - \frac{(2M_C + s\Delta)s\rho}{\left(- (F_F - F_C)M_C s + \beta N (M_C + s\Delta) + \beta M_C s (\Delta - \varepsilon)(1 - \omega) + (2M_C + s\Delta)s\rho\right)}, & \text{Case(3g).} \end{cases} \tag{3.33}$$

Proof. As $t_F^l > t_C^l$, the last departure departing at $t_F^l = t_q^0$ must be an eFHV. Based on the equilibrium principle, the travel cost of the first private autos without queuing cost equals to that of the last eFHVs departing at t_q^0 . We have

$$P_C^* = \beta(t^* - t_C^0) + F_C = \rho \frac{N_F}{M_F - N_F} + F_F = P_F^*.$$

In the case when first eFHV departing

before last private auto leaving the bottleneck, i.e., Case (3c)-(3e) and (3h)-(3i), the critical U_F can be derived through Eq.(3.27) as follows. When

$$\beta(t^* - t_C^0) + F_C = \beta \frac{N}{s} + F_C > \rho \frac{N_F}{M_F - N_F} + F_F,$$

last departures bear queueing cost

departing before t_q^0 , with $U_F = \frac{N_F}{M_F} < U_F^*$. Conversely, when

$$\beta \frac{N}{s} + F_C < \rho \frac{N_F}{M_F - N_F} + F_F, \rightarrow t^* - t_C^0 > \frac{N}{s},$$

there will exist without-queue

departure after t_q^0 . Thus, $U_F > U_F^*$. Only when $U_F = U_F^*$, last departure departs at

t_q^0 . As for Case (3b) and (3g), wherein first eFHV departs after last private auto

leaving the bottleneck, we can derive critical U_F in a similar way but with different

$$P_C^*.$$

□

In fact, one can easily observe that the U_F^* in Proposition 3.3 are determined by the N, M_C and some other parameters, regardless of the equilibrium travel pattern.

3.3.2 Equilibrium travel cost

Now, we seek to understand how the *longest possible queuing period* affects the equilibrium travel cost. In Table 3-3, we only present the equilibrium travel costs for

different scenarios with last departures before t_q^0 . In the following proposition, we discuss the properties of equilibrium travel cost for the scenarios when the last commuter departs after t_q^0 .

Proposition 3.4 *With a given N , the equilibrium travel cost for the scenarios with the last departure after t_q^0 is always higher than that of the scenarios with the last departure before t_q^0 .*

Proof. We can calculate equilibrium travel cost by $P_F^* = P_C^* = P_C(t_C^0) = \beta(t^* - t_C^0) + F_C$. When the last vehicle departs before t_q^0 , and there is only one queue during morning peak, i.e., all cases except Case (3b) and (3g), t_C^0 is directly determined by the capacity of bottleneck, where $t_C^0 = t^* - \frac{N}{s}$ when there is eFHV demand or $t_C^0 = t^* - \frac{N}{s} - S(t_C^l) \Big|_{Cu_C(t_C^l)=N}$ when only private auto is used at equilibrium. As for Case (3b) and (3g), t_C^0 is indirectly determined by the actual number of private autos, where $t_C^0 = t^* - \frac{N - Cu_C(t_C^l)}{s} - \frac{F_F - F_C}{\beta}$. However, when last vehicle departs after t_q^0 , some vehicles free from bottleneck congestion leave the bottleneck at a rate smaller than s . The time needed for all vehicles to pass through the bottleneck increases, and hence t_C^0 is smaller. Thus, the equilibrium travel cost increases and is larger than that of the cases with the last departure before t_q^0 . $\square$

Proposition 3.4 implies that, to ensure a low equilibrium travel cost, it is desirable to avoid the without-queue departures after t_q^0 . That is to say, all commuters should depart within the *longest possible queuing period*. By ensuring all departures depart

before t_q^0 , the capacity of the highway can be fully utilized. In fact, as shown in Table 3-3, the equilibrium travel costs in most cases remain unchanged when last vehicles depart before t_q^0 . It is also worth noting that, if there is still private auto departures at t_q^0 , there must be without-queue departures after t_q^0 as $t_F^l > t_C^l$.

Moreover, if the value of U_F is known and given, the equilibrium travel cost with the last departure after t_q^0 can be expressed as,

$$P_C^* = P_F^* = F_F + \rho \frac{U_F}{1 - U_F} . \quad (3.34)$$

Eq.(3.34) is derived with $P_C^* = P_F^*$ at equilibrium. Since $t_C^l < t_F^l$, when there is departure after t_q^0 , the last eFHV has neither queueing cost nor scheduling cost, regardless of the sequence of t_C^l, t_q^0 . In other words, no matter last private autos depart before or after t_q^0 , the individual travel cost remains unchanged.

To reiterate, as discussed in the proof of Proposition 3.3 when the last vehicle departs after t_q^0 , there is $U_F > U_F^*$. Conversely, when the last vehicle departs before t_q^0 , $U_F < U_F^*$. Therefore, one can verify easily that the P_C^* in Eq.(3.34) is larger than

those listed in Table 3-3 when $U_F \leq U_F^*$, which further supports Proposition 3.4. One

can also observe that $\frac{\partial P_C^*}{\partial U_F} = \frac{\rho}{(1 - U_F)^2} > 0$ when $U_F > U_F^*$, i.e., the equilibrium

travel cost reduces when U_F becomes smaller. The elasticities of P_C^* with respect to U_F (denoted as $E_{P_C^*(U_F)}$) can be determined:

$$E_{P_C^*(U_F)} = \frac{U_F}{P_C^*} \frac{\partial P_C^*}{\partial U_F} = \frac{1}{1-U_F} \frac{\frac{U_F}{1-U_F} \rho}{F_F + \frac{U_F}{1-U_F} \rho} . \quad (3.35)$$

When $U_F > \frac{1}{1+\sqrt{\rho/F_F}}$, $E_{P_C^*(U_F)} > 1$, the slight decrease in U_F can reduce the equilibrium travel cost P_C^* substantially.

3.3.3 Optimal supply of parking spaces and eFHV

To best manage the overall system, it is desired to minimize the total system travel costs. As all the commuters share the same individual travel cost at equilibrium, to achieve the best system performance with minimum total travel costs is equivalent to minimizing the individual equilibrium travel cost. Therefore, proposition 4.2 is also very indicative if the best system performance is sought after. More specifically, to minimize the total travel costs, it is imperative to ensure the equilibrium travel pattern is achieved with all departures within the *longest possible queuing period*. Thus, combining Eq.(3.27), Proposition 3.3 and Proposition 3.4, we have the following proposition on the supply of parking spaces and eFHV.

Proposition 3.5 *To minimize system total travel costs, the optimal supply of parking spaces and eFHV (M_C, M_F) should satisfy,*

$$M_C \geq M_C^*, M_F \geq M_F^*, \quad (3.36)$$

where (M_C^*, M_F^*) is the solution of

$$\frac{N - Cu_C(t_C^l)}{M_F} = U_F^*. \quad (3.37)$$

The left-hand side of Eq.(3.37) is exactly the definition of U_F as in Eq.(3.27) and the right-hand side U_F^* is defined in Proposition 3.3. It amounts to ensure $U_F = U_F^*$ in Eq.(3.37). That is to say, with the parking spaces and eFHV's supplied at the amount of (M_C^*, M_F^*) , the last vehicle departs at t_q^0 . Intuitively, when there are abundant parking spaces and eFHV's, U_C, U_F are mainly dependent on the travel demand N . With smaller parking spaces or eFHV's supply, the critical constraints determining U_C, U_F become the travel supply, i.e., (M_C, M_F) . When the supply is even lower, there is without-queue departure after t_q^0 . In that case, the total system travel time becomes larger. Thus, to achieve the minimum total travel costs in the system, we have to ensure $U_F \leq U_F^*$ so as to avoid without-queue departure, i.e., the supply of parking spaces and eFHV's should be greater than or equal to (M_C^*, M_F^*) .

Figure 3-5 Optimal supply of parking space and eFHVs.

With given N , the $(M_C^*, M_F^*)^9$ are delineated in solid blue lines in Figure 3-5, where the other parameters will be provided in Section 3.4 in the numerical example. Naturally, the ride-sourcing service is a substitute for private autos driving, if given constant total highway users N is fixed. The curve in Figure 3-5 describing (M_C^*, M_F^*) can also be considered as the indifference curve of the supply of parking

⁹ For large M_C^* , the corresponding M_F^* may be smaller than $\frac{\rho^s}{\beta}$, which contravenes our assumption $M_F < \frac{\rho^s}{\beta}$, which is postulated to focus on the travel pattern with bottleneck queuing. Nevertheless, as shown in Figure 3-4, only in Case (3b) & (3g), it is possible to have departures of eFHVs without queue forming when $M_F < \frac{\rho^s}{\beta}$, as the first eFHVs depart after queue dissipates. For all other bi-modal cases, the first eFHVs depart with queues.

spaces and eFHV ensuring $U_F = U_F^*$. Indeed, any combination of the supplies of parking spaces and eFHV (M_C, M_F) above this curve would lead to the minimum system total costs.

Indeed, this interesting finding that the supply of parking spaces and eFHV combine together to affect the system performance is indicative for the supply management of eFHV. For some urban contexts with very limited parking spaces provision, the lowest supply required to achieve the minimum system total costs, i.e., M_F^* , is relatively high. Thus, it is desirable to promote eFHV to offer more supplies, even with the assistance of certain subsidy policies,. On the other hand, if the supply of parking spaces are sufficient, the lower bound for the required eFHV supply M_F^* is easy to be fulfilled. In this case, one may consider to control the eFHV registration to avoid oversupply, as numerical evidence (Referred to Section 3.4.2 and Figure 3-7) shows that the total queueing cost spent in the bottleneck, another important system performance indicator, may increase with over supplied eFHV. That is to say, in this situation, it is recommended for the eFHV management to control the supply of eFHV to a close value of the derived M_F^* , through setting the limit for eFHV registration, so that the total system travel costs could be minimized while the bottleneck queueing time is well managed.

Figure 3-6 Domains of occurrence of different cases in Table 3-3.

3.4 Numerical examples

In this section, we would illustrate the bi-modal equilibrium and system performance discussed in Section 3.2 and 3.3 in numerical examples. Following Jia et al. (2016), we set parameters of value of time, schedule delay penalties and the highway bottleneck capacity as follows: $\alpha = 0.3$ (SGD\$/min), $\beta = 0.1$ (SGD\$/min), $s = 200$ (veh/min). In addition, we normalize the preferred arrival time at workplace is $t^* = 0$ such that early arrivals arrive at workplace at a negative t . We further assume

$\omega = 0.2$, $\varepsilon = 50$ (min), $\Delta = 100,000$ (min) and $F_C = 4.5$ (SGD\$/commuter) for private autos. As for eFHV, we let $\rho = 5$ (SGD\$), and consider two cases with $F_F = 10$ and 17 (SGD\$/commuter), so that both cases with $\phi \geq 0$ and $\phi < 0$ can be investigated.

3.4.1 The occurrence domains of equilibrium travel patterns

We would firstly exemplify the occurrence of the equilibrium travel pattern in different cases as detailed in Table 3-3. With $M_C > \frac{\alpha - \beta}{\beta} \varepsilon s = 20,000$, Figure 3-6 clearly illustrates the domains of occurrence for all different cases shown in Table 3 with respect to different values of M_C and N , when $M_F = 25,000$. The labeled curves in different colors represent the domain boundaries among different cases. The solid black line indicates an upper bound of highway travelers which is restrained by the supply of both parking spaces and eFHV. In particular, the dashed squares A and B in Figure 3-6 II with $\phi < 0$ are further magnified and plotted on the right-hand side of the figure.

In Figure 3-6, one interesting finding is, with different values of M_F , the domains of occurrence remain the same in most cases with unchanged curves of boundaries, except for the curves $N = M_C + M_F$ (in solid black) and $N = N^{(d,e)}$ (in yellow). It can be deduced that the domains of Case (3d) and (3e) in Figure 3-5 I and Case (3g) and (3i) in Figure 3-5 II (the vertical distances along with same M_C) become larger when M_F increases.

Besides, with larger M_C , the domains of Case (3a) and (3c) expand while the domain of Case (3b) shrinks. The possible interpretation is that, if more parking spaces are

provided (i.e., with larger value of M_C), more commuters choose private auto for commuting and ride-sourcing service only becomes more attractive at the late stage of the morning peak. Private auto drivers can now park within the *easy-to-park* period, and hence the domain of Case (3a) is expanded. With larger M_C , the relative delay (compared to the length of morning peak) of the last private auto leaving the bottleneck squeezes the domain of Case (3b), when the first eFHV departs after last private auto leaving bottleneck but before t^* . The larger M_C with higher private auto usage also contributes to the longer time for the bottleneck to clear the existing queue of private autos, which enlarges the domain of Case (3c).

Meanwhile, when $\phi < 0$, we also notice that the domains of Case (3f) and (3h) are very small. The rapidly increasing parking searching time in the potential *hard-to-park* period induces commuters to choose eFHVs earlier. Thus, in Figure 3-5 II one can observe that, when N increases, the equilibrium switches from Case (3a) to (3g) swiftly if M_C is small, and the equilibrium switches from Case (3c) to (3i) quickly if M_C is large. Similar to the single-mode equilibrium in Section 3.2.1, in the extreme case when $\Delta \rightarrow \infty$, private auto commuters will depart only in the *easy-to-park* period with the maximum private auto utilization $(1-\omega)M_C$. In this situation, cases (3e)-(3i) no longer exist. While Case (3e) and (3f) are replaced by Case (3d) and (3a) respectively, Case (3g)-(3i) are all taken over by Case (3c). One may verify that all N_C in Case (3e)-(3i) in Table 3-4 equal to $(1-\omega)M_C$ when $\Delta \rightarrow \infty$.

3.4.2 Optimal supply of eFHVs

Next, we illustrate the optimal supply of parking spaces and eFHVs. As introduced earlier in Section 3.3.3, Figure 3-5 depicts the indifference curve at $U_F = U_F^*$ with

respect to M_C, M_F at $N = 40,000$. From Eq.(3.33), $U_F^* = 0.74 \& 0.6$ respectively at $F_F = 10 \& 17$. It can be verified that, as M_C increases, the equilibrium travel patterns switch from Case (3e) to (3d) and finally to (3c) in Figure 3-5 I and from Case (3g) to (3i), then to (3h) and finally to (3c) in Figure 3-5 II, which are also shown along the horizontal dashed black lines in Figure 3-6.

Since M_C is usually predetermined, we further focus on the system performance in two examples with given M_C (black nodes in Figure 3-6). To note, the two nodes are selected to focus on the bi-modal case with parking space *insufficiency*. The total travel cost and its composition with respect to U_F are depicted in Figure 3-7. While the equilibrium switches from Case (3d) to (3e) with increasing U_F in Figure 3-7I, the equilibrium is located at Case (3h) only regardless of the value of U_F in Figure 3-7II. The change of equilibrium case in Figure 3-7I even with given N, M_C is consistent with our observation in Figure 3-5, where the boundary $N^{(d,e)}$ between Case (3d)&(3e) (i.e., $t_C^l = t_C^{1-\omega}$) increase with M_F . In fact, rather than focusing on the identification of equilibrium cases, Figure 3-7 seeks to reveal the trends of total travel cost and the interior cost variation. Apparently, the total travel costs remain unchanged at SGD\$980,000 when U_F is small, until $U_F = U_F^*$. The critical (M_C^*, M_F^*) is also highlighted in yellow nodes in Figure 3-5. With rapidly increasing total travel cost when $U_F > U_F^*$, Proposition 3.4 are validated in Figure 3-7. Meanwhile, the total waiting cost of ride-sourcing increases with U_F . Furthermore, it is worth noting that the total queueing cost decreases gradually with U_F . The total queueing cost decreases from 217,152 when the actual utilization of eFHV's $U_F \rightarrow 0$

to 126,976 at $U_F = 0.74$ ($M_C^* = 25,000$ and $M_F^* = 26,895$) with 41.5% reduction in Figure 3-7 I, and decreases from 137,698 when $U_F \rightarrow 0$ to 106,371 at $U_F = 0.6$ ($M_C^* = 35,000$ and $M_F^* = 19,988$) with 22.8% reduction in Figure 3-7 II. Based on this result, rather than providing more parking spaces or eFHV, it is more desirable to maintain the supply $M_C = M_C^*$ and $M_F = M_F^*$ in morning commute management, so that the total travel cost is minimized with lowest total queuing cost.

Figure 3-7 Total travel cost and its composition in bi-modal equilibrium with respect to U_F .

3.5 Concluding remarks

In this chapter, the equilibrium morning commute traffic is explored in an extended single OD bottleneck model with parking and ride-sourcing service. The through traffic characteristic without parking demand of eFHV is explicitly examined in our work. In addition, the increasing parking difficulties for later departures are also captured in a convex piecewise parking searching function. The within-day equilibrium travel pattern with given private auto and eFHV supply M_C, M_F is derived.

Moreover, due to the explicit consideration of the parking search time and ride-sourcing waiting time, we briefly introduce the definition of *longest possible queuing period*. The travel cost is lower when all commuters depart within the *longest possible queuing period*. Further, we notice that there is an optimal range of ride-sourcing service utilization rate so as to minimize the total travel cost. Based on this optimal range, the lower bound of the supply of parking spaces and eFHV can be determined. From regulators' perspective, we notice that the supply of parking spaces and eFHV should be coordinated to fulfill the requirement if minimum system travel cost is to be achieved.

CHAPTER 4

PARKING PROVISION MANAGEMENT IN MORNING COMMUTE PROBLEM CONSIDERING RIDE-SOURCING AND TRANSIT

In this chapter, we extend our work from bi-modal case with highway commuters only to multi-modal case together with transit riders, so that the elasticity of highway commuters can be well embedded. In addition, the no-late-arrival assumption is relaxed. The parking reservation is also incorporated in this extension. From parking management perspective, we address the question more fully on how to use the parking provision for best managing morning traffic by obtaining the optimal number of parking space provision, in particular when both ride-sourcing service and railway transit are present as alternative travel modes for commuting.

The rest of the chapter is organized as follows. Section 4.1 revisits the classical bi-modal morning commute problem with parking constraints firstly, then further expand it to a multi-modal problem. Section 4.2 studies the equilibrium commuting pattern. We then analyze the network performance and introduce the optimal number of parking spaces and eFHV to improve traffic efficiency in Section 4.3 . Finally, Section 4.4 summarizes this chapter.

4.1 Model formulation

The following notations are used throughout the process of model formulation:

<i>Parameters</i>		
N		The total number of commuters during morning peak
M_c		The total number of parking spaces available during morning peak
t^*		The common preferred arrival time at the workplace
α		Value of time
β, γ		Unit time scheduling penalty of early and late arrival, respectively
s		Constant service capacity of the bottleneck
F_c		One-off parking fee at the workplace
F_F		The aggregated generalized cost of eFHV, excluding the travel time cost and schedule cost
ε		A constant coefficient that satisfies for private autos without parking reservation and $\varepsilon = 0$ for reserved private autos.
F_T		Transit fare
θ		A positive constant indicating the crowdedness effects in railway transit
S_0		Initial parking searching time
<i>Variables</i>		
N_c, N_F, N_T		The actual number of private auto, eFHV, and transit commuters, respectively
t_1		The earliest departure time from home for all road users
t_c^l, t_F^l		The last departure time from home of private auto and eFHV, respectively
$T(t)$		The travel time on the highway from home to workplace prior to parking for vehicle departing at t
$D(t)$		Queue length at the bottleneck at t .
$r_c(t), r_F(t)$		The arrival rate at the bottleneck (the departure rate from home) of private auto and eFHV at t , respectively
$P_c(t), P_F(t)$		Generalized travel cost of private auto and eFHV departing at t

P_F^*, P_T^*	Equilibrium travel cost of eFHV and transit railway, respectively
P_C^{r*}, P_C^{u*}	Equilibrium travel cost of private autos, with and without parking reservation, respectively
N_C^0, N_F^0	The virtual demand for parking spaces in the absence of ride-sourcing, and the virtual demand for eFHVs in the absence of parking spaces

4.1.1 Preliminary

As shown in Figure 4-1, a paralleling transit line is added to the one-to-one network used in Chapter 3 . Therefore, the homogenous travelers with value of time α now have three options commuting from home to workplace: drive alone, take eFHVs or catch a train. While only transit commuters use railway, both private auto drivers and eFHV passengers share the highway as in the previous chapter. Let N_C, N_F, N_T denote the number of private auto, eFHV and transit commuters respectively. The total number of commuters, $N = N_C + N_F + N_T$, is assumed to be given. Commuters have a common preferred arrival time t^* at the destination. To note, we relax the assumption of no-late-arrival in this chapter. That is to say that early or late arrival is penalized (schedule penalty for a unit time of early or late arrival: β, γ) with $\gamma > \alpha > \beta > 0$ (Small, 1982). Besides, the settings of parking spaces and ride-sourcing service follow those in Chapter 3 .

4.1.2 Bi-modal case with private autos and transit

We first look at the bi-modal morning commute bottleneck model in the absence of ride-sourcing ($N_F = 0$), wherein all commuters using private auto need a parking spot at the destination. Every morning, road users need to trade off between the travel time cost related to bottleneck queue length and the scheduling cost of being early or late at work. Furthermore, the parking reservation is considered in this chapter. If there is a binding parking capacity, drivers who do not have reserved parking spot

would have to compete with each other for parking with earlier departure, and thus bear an extra time cost resulted from parking space searching while those who have parking reservation can be free from such worry and enjoy a lower commute cost (Yang et al., 2013). The parking searching time for private auto commuters departing at time t is assumed to be a decreasing function of the number of non-occupied parking spaces at destination at the time when he left the bottleneck, namely an increasing function of the number of non-reserved vehicles departing before t (Axhausen et al., 1994). Thus from the classical morning commute ADL model (Arnott et al., 1990b), private auto travel cost for commuters departing at time t , in a modified form including travel time cost, schedule delay cost and parking fee, can be defined as follows:

$$P_c(t) = \alpha(T(t) + S(t)) + \beta \max\{0, t^* - t - T(t) - S(t)\} + \gamma \max\{0, t + T(t) + S(t) - t^*\} + F_c, \quad (4.1)$$

where $T(t)$ is the travel time for departure time t on the highway prior to parking, $S(t)$ is the parking searching time for vehicle departing at time t , F_c is the parking fee and it is assumed to be a one-off fee.

Figure 4-1 Morning commute corridor network with parallel transit line

Here, free-flow travel time is assumed to be zero without loss of generality, and hence the highway only contains a bottleneck with a constant service capacity s ($s > 0$),

i.e. $T(t) = D(t)/s$, where $D(t)$ is the queue length at the bottleneck at t . In addition, unlike the previous chapter, we linearize the parking searching time function, which is given as,

$$S(t) = S_0 + \varepsilon \cdot \frac{\int_{t_1}^t r_c(u) du}{N_c}, \quad (4.2)$$

where S_0 is the initial searching time, t_1 is the earliest departure time from home for all road users, $r_c(t)$ is the flow rate of private auto entering the bottleneck at time t , N_c is total number of private autos, or total number of parking spaces if there is binding parking capacity, and ε is a constant coefficient that satisfies for private autos without parking reservation and $\varepsilon = 0$ for reserved private autos.

Thus, similar to standard bottleneck analysis (Arnott et al., 1990b), from within-day equilibrium, the arrival rates at the bottleneck for private auto commuters arriving at destination before and after t^* respectively are given by

$$r_{c1} = \frac{\alpha}{(\alpha - \beta) \left(1 + \frac{s\varepsilon}{N_c}\right)} s, \quad r_{c2} = \frac{\alpha}{(\alpha + \gamma) \left(1 + \frac{s\varepsilon}{N_c}\right)} s. \quad (4.3)$$

The equilibrium commute cost from home to workplace for private auto drivers without parking restraint then become

$$P_c^{u*}(N_c) = \frac{\beta\gamma}{\beta + \gamma} \frac{N_c}{s} + \alpha S_0 + \frac{\beta(\alpha + \gamma)}{\beta + \gamma} \varepsilon + F_c. \quad (4.4)$$

As for the transit commuters, we assume that the operating capacity of the railway is sufficiently large to the extent that the unpunctuality cost is not considered for transit commuters while the crowdedness in the transit still needs to be considered. Therefore, including the crowding effects, the commute cost of transit travelers P_T^*

can be given as an increasing function of the number of commuters using transit. As was done in Huang (2000), we let

$$P_T^*(N_T) = F_T + \theta N_T, \quad (4.5)$$

where F_T and θ represent the transit fare and unit cost of crowding or discomfort from home to work on transit respectively.

From $P_C^{u*}(N_C) = P_T^*(N_T)$ and $N = N_C + N_T$, we can obtain the virtual demand for parking space in the absence of ride-sourcing N_C^0 ,

$$N_C^0 = \frac{s(\beta + \gamma)}{\mathbb{C}} \left[\theta N + F_T - \alpha S_0 - F_C + \frac{\beta(\alpha + \gamma)}{\beta + \gamma} \varepsilon \right], \quad (4.6)$$

where $\mathbb{C} = (\beta + \gamma)\theta s + \beta\gamma$, $N_C^0 \geq 0$. Obviously, $\mathbb{C} > 0$.

When there is binding parking capacity, i.e. $M_C < N_C^0$, where M_C is the number of parking spaces available at the destination ($M_C > 0$), the number of private auto commuters is restrained by M_C , and hence the number of transit commuters become $N - M_C$. In the absence of ride sourcing service, the equilibrium travel patterns of private auto commuters when there's parking constraint are similar to those shown in Figure 1 and 2 in (Yang et al., 2013), but with different departure rates and later arrival at the workplace due to the consideration of parking searching time in our setting. For simplicity and illustration purpose, we omit the sketches of these results here. Note that when all parking spaces are reserved, the arrival rates at the bottleneck for commuters arriving at destination before and after t^* are given by $r_1 = \frac{\alpha}{(\alpha - \beta)} s$

and $r_2 = \frac{\alpha}{(\alpha + \gamma)} s$, where r_1 and r_2 are the special forms of r_{C1} and r_{C2}

correspondingly when $\varepsilon = 0$ based on our setting for reserved parking vehicles. According to Eq.(4.4), the commute cost of private auto drivers in all-reserved case P_C^{r0} is

$$P_C^{r0} = P_C^*(M_c) = \frac{\beta\gamma}{\beta + \gamma} \frac{M_c}{s} + \alpha S_0 + F_C \quad . \quad (4.7)$$

4.1.3 Multi-modal case

Now we turn to the case in which the ride-sourcing service is considered ($N_F > 0$). Similar to Chapter 3 , we postulate that all these eFHV's do not need parking at CBD and only one home to work trip with one passenger can each eFHV driver completes during morning commute. In this chapter, it is assumed that commuters have no preference in choosing a specific eFHV and there are enough eFHV's so that the waiting time for eFHV after booking is an equal constant for everyone independent of customers' booking time¹⁰. The generalized travel cost of eFHV's, departing at time t , is defined:

$$P_F(t) = \alpha T(t) + \beta \max\{0, t^* - t - T(t)\} + \gamma \max\{0, t + T(t) - t^*\} + F_F \quad , \quad (4.8)$$

where in this chapter F_F aggregates the generalized cost of eFHV other than travel time cost and schedule cost, which is principally composed of the travel fare and the waiting cost. Since travel fare and waiting time are assumed to be constant, F_F is a constant in our model. In comparison with the private auto travel cost, eFHV travel

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¹⁰ We make this assumption for convenience of analysis. Some previous studies in the literature applied a matching function to determine the customers' average waiting time in traditional taxi analysis, depending on the customer demand and taxi utilization rate(Yang et al., 2011). As for ride sourcing service, when the number of eFHV's is sufficient, it is possible that the waiting time for ride-sourcing service becomes very short, with the aid of current advanced technology. In fact, the average waiting time of eFHV customers can be obtained in an aggregate model using the matching function given the current eFHV supply.

cost cuts out the parking-related cost, i.e., parking fee and parking searching time, and add the exclusive cost of eFHV instead.

When considering the ride sourcing service, the bottleneck services not only private auto commuters but also eFHV commuters at constant capacity and the bottleneck queue length $D(t)$ should satisfy

$$\frac{\partial D(t)}{\partial t} = \begin{cases} r_C(t) + r_F(t) - s, & r_C(t) + r_F(t) > s \text{ or } D(t) > 0; \\ 0, & r_C(t) + r_F(t) \leq s \text{ or } D(t) = 0. \end{cases} \quad (4.9)$$

where $r_F(t)$ is the flow rate of eFHVs entering the bottleneck at time t . At departure time t , if $\min\{P_F(t), P_C(t)\} > P_T^*(N_T)$, commuters without parking reservation will choose transit to work; conversely if $\min\{P_F(t), P_C(t)\} < P_T^*(N_T)$, they will use the road transport to workplace and will choose the mode (either private auto or eFHV) with the lowest travel cost at time t , if available. As for reserved parking commuters, they will drive to work only if their commute cost is equal to or lower than any other road users' travel cost. Furthermore, the number of available parking spaces is still M_C , and one can verify easily that $N_C \leq M_C$. The actual number of private auto commuters N_C is now not only bounded by the parking spaces M_C but also by the eFHVs N_F . The performance of ride-sourcing service can alter commuters' mode choice. This can be termed as a deterministic Wardropian multi-modal equilibrium. Consequently, the commute cost of non-reserved private auto drivers (denoted by P_C^u), eFHV riders (denoted by P_F^*) and transit passengers, are identical at equilibrium, and higher than that of reserved commuters (denoted by P_C^r), i.e.,

$$P_C^{u*} = P_F^* = P_T^* (N - N_C - N_F) \geq P_C^{r*} . \quad ^{11} \quad (4.10)$$

Regarding within-day equilibrium, it should satisfy

$$\frac{\partial P_C(t)}{\partial t} = 0 \text{ and } \frac{\partial P_F(t)}{\partial t} = 0 . \quad (4.11)$$

4.2 Multi-modal equilibrium considering ride-sourcing

In this section, we investigate the multi-modal morning traffic equilibrium on the condition that the supply of ride-sourcing service is not limited. Namely, we assume that there is sufficiently large fleet size of eFHV to meet the commute demand.¹² Meanwhile, parking supply M_C falls short of demand, and all parking spaces are not reserved.

Since the private auto drivers have to reserve some time for parking other than passing through the bottleneck, for the same preferred arrival time, the private auto driver arriving on time will always depart earlier than the on-time arrival eFHV passengers. Let t_{3C} and t_{3F} denote the departure time of on-time arrival private autos and eFHV, and we have $t_{3C} < t_{3F}$. Thus, the morning rush hours can be divided

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¹¹ Here, it is reasonable to assume $P_C^{u*} \geq P_C^{r*}$. If $P_C^{u*} < P_C^{r*}$, it means that those who have parking reservation in advance have to pay even higher cost during morning commute compared to those who have not parking reservation. In the end, no one would like to make parking reservation and all private auto commuters will compete for parking.

¹² This seems to be impractical at first glance, however if the market entry barrier of being an eFHV drivers become low enough and the extra income from driving eFHV is attractive, or if the technology of autonomous car is well developed and driverless cars can be widely employed in FHV industry, the supply of eFHV can possibly satisfy its demand.

into three time windows: Both private autos and eFHV arrive early; private autos arrive late while eFHV arrive early; both private autos and eFHV arrive late.

Lemma 4.1 *There is no simultaneous departures of private autos and eFHV at equilibrium at any t .*

From Eq.(4.11), if there is a time point when both private autos and eFHV have departures simultaneously, the departure rates of private autos and eFHV should satisfy

$$\begin{cases} \left(1 + \frac{s\mathcal{E}}{N_C}\right) r_C + r_F = \frac{s\alpha}{(\alpha - \beta)} = r_C + r_F, & \text{when } t \leq t_{3C}; \\ \left(1 + \frac{s\mathcal{E}}{N_C}\right) r_C + r_F = \frac{s\alpha}{(\alpha + \gamma)}, r_C + r_F = \frac{s\alpha}{(\alpha - \beta)}, & \text{when } t_{3C} < t \leq t_{3F}; \\ \left(1 + \frac{s\mathcal{E}}{N_C}\right) r_C + r_F = \frac{s\alpha}{(\alpha + \gamma)} = r_C + r_F, & \text{when } t > t_{3F}; \end{cases} \quad (4.12)$$

where $r_C, r_F \geq 0$.

when both private autos and eFHV have departures, $r_C, r_F > 0$. Obviously, for each time window, we cannot find such positive r_C and r_F respectively to satisfy the equilibrium conditions as shown above. In other words, it is impossible to have simultaneous positive departures of the two modes at equilibrium. Therefore, for private autos, the early and late arrival departure rates are r_{C1} and r_{C2} , while the early and late arrival departure rates of eFHV are r_1 and r_2 as long as there is positive departure. This is also consistent with the findings in Lindsey (2004).

Let $F_F^0 = (\alpha - \beta) \cdot S_0 + F_C$. When $F_F \leq F_F^0$, from Eqs.(4.1) and (4.8), there always exists $P_F(t) \leq P_C(t)$ for any t , which means that riding eFHV is better than driving during morning peak. When eFHV's supply is sufficient, all commuters will not drive to work, and only eFHV use the highway.

When $F_F > F_F^0$, private auto driving is used for commuting. Since $P_F(t_1) > P_C(t_1)$, there are only private autos departures at t_1 . However, the latter travel pattern of bottleneck becomes different depending on the parking spaces as well as other parameters. At first private autos arrive at the bottleneck at a constant rate r_{C1} and no eFHV is on the road, the commute cost of eFHV riders gradually decrease and eventually equals to the cost of private auto drivers. The time point with equal commute cost may occur before or after the full occupancy of all parking spaces.

Lemma 4.2 *Once the commute costs of the two modes equal, i.e., at $t = t_2$, there is no longer any private autos departure when $t > t_2$ and the road will be filled with eFHVs only, if the supply of eFHV is sufficient.*

This lemma can be proved by contradiction. Suppose there exists a time point t_2 , where $P_F(t_2) = P^*$, the equilibrium commute cost. If there is still positive private auto departure from t_2 to $t_2 + \Delta t$ ($\Delta t > 0$), we have $P_C(t_2) = P_C(t_2 + \Delta t)$ and no eFHVs departures. When $t_2 \leq t_{3C}$, according to Eq.(4.1),

$$P_C(t_2 + \Delta t) - P_C(t_2) = (\alpha - \beta) \left[\frac{(r_{C1} - s)}{s} \Delta t + \frac{\varepsilon r_{C1}}{M_C} \Delta t \right] - \beta \Delta t = 0. \quad (4.13)$$

The difference between the commute cost of eFHVs at $t_2 + \Delta t$ and t_2 is

$$\begin{aligned} P_F(t_2 + \Delta t) - P_F(t_2) &= (\alpha - \beta) \left[\frac{(r_{C1} - s)}{s} \Delta t \right] - \beta \Delta t < 0, \\ \rightarrow P_F(t_2 + \Delta t) &< P^*. \end{aligned} \quad (4.14)$$

Likewise, it can be proved that when $t_{3C} < t_2 \leq t_{3F}$ or $t_2 > t_{3F}$, $\rightarrow P_F(t_2 + \Delta t) < P^*$.

We find that the commute cost of eFHV at $t_2 + \Delta t$ will always be lower than the equilibrium commute cost and this contradicts the principle of user equilibrium. Thus, at equilibrium, no one will choose private autos anymore after t_2 . If bottleneck queues have not been cleared at t_2 , there must be eFHV departures at later times, and the last road user takes eFHV to work. Otherwise, it is possible that no one chooses eFHV when the cost of eFHV is pretty high or when eFHV departing at t_2 arrives already late. On this account, we notice that the number of parking spaces may not always be binding in the case when $F_F > F_F^0$ since the private auto driving becomes no longer cost competitive after a specific time point.

Lemma 4.3 *When both private autos and eFHV are used for commuting, the total number of road users $N_F + N_C$ is a constant independent of N_C or M_C ,*

$$N^0 = \frac{(\beta + \gamma)s}{\mathbb{C}} (F_T + \theta N - K),$$

$$\text{where } K = \frac{(\alpha - \beta)\gamma}{\beta + \gamma} S_0 + \frac{\beta}{\beta + \gamma} F_F + \frac{\gamma}{\beta + \gamma} F_C, \quad (4.15)$$

if the bottleneck queue has never been cleared before the last arrival of road users, i.e., when bottleneck services at full capacity without a break during morning commute.

If there are both private autos' and eFHV's departures, we have

$$N_F + M_C \geq N_F + N_C = (t_4 - t_1)s, \quad (4.16)$$

$$P_C(t_1) = P_F(t_4). \quad (4.17)$$

where t_4 denotes the last departure time of road users during morning peak. Let

$$M_2^u = \frac{[F_F - (F_C + (\alpha - \beta)S_0)]s}{\beta}, \text{ then after some substitutions, we have}$$

$$t_4 = \frac{(\beta + \gamma)t^* - \beta t_1 - \beta M_2^u / s}{\gamma}, \quad (4.18)$$

and hence
$$N_F + M_C \geq N_F + N_C = \frac{(\beta + \gamma)}{\gamma}(t^* - t_1)s - \frac{\beta}{\gamma}M_2^u,$$

$$\rightarrow t^* - t_1 = \frac{\gamma(N_F + N_C) + \beta M_2^u}{(\beta + \gamma)s},$$

$$\rightarrow P^* = P_C(t_1) = \frac{\beta\gamma}{(\beta + \gamma)s}(N_F + N_C) + K = P_T^*(N - N_F - N_C),$$

$$\rightarrow N_F + N_C = \frac{(\beta + \gamma)s}{\mathbb{C}}(F_T + \theta N - K) = N^0.$$

Therefore, the individual equilibrium commute cost does not change after N_C and thereby does not change after M_C when there does not ever exist an empty bottleneck before the end of morning peak.

Now we attempt to determine the travel pattern of road users. Let

$$\begin{aligned} F_F^1 &= (\alpha - \beta)(S_0 + \varepsilon) + F_C, \quad M_1^u = \frac{P_T^*(N - N^0) - F_C - \alpha S_0 - \beta \varepsilon}{\beta} s, \\ F_F^2 &= (\alpha + \gamma)(S_0 + \varepsilon) + F_C, \quad M_3^u = \frac{P_T^*(N - N^0) - F_F^0}{\beta} s, \\ M_4^u &= \frac{(\alpha + \gamma)\varepsilon}{F_F - F_F^2} M_u^3, \\ M_5^u &= \frac{(\beta + \gamma)[P_T^*(N - N^0) - F_C - \alpha S_0] - \beta(F_F - F_F^0) - (\alpha + \gamma)\beta \varepsilon}{\beta(\beta + \gamma)} s. \end{aligned}$$

Apparently, $F_F^0 < F_F^1 < F_F^2$, $M_1^u < M_3^u$, $M_2^u < M_3^u$. Then the domains of different commuting travel patterns with respect to total number of parking spaces M_C and the generalized cost of eFHV's excluding the traveling and scheduling cost, F_F , is

shown in Figure 4-2. Note that we only consider the morning commute problem with parking spaces restraint, i.e., $M_C < N_C^0$.

Figure 4-3 (a) - (j) then depict the possible equilibrium commuting patterns, where blue curve AB and H(F)E represent the cumulative arrival at the bottleneck of private autos and eFHV's respectively and red curve DC and GE represent their cumulative arrival at the destination respectively.¹³ t_C^l represents the departure time of the last private auto during morning peak.

Note that in case (a) (c) (e) (f) (g) (h), both private autos and eFHV's are used, and the bottleneck queue has never been cleared before the last arrival of road users. From Lemma 4.3, the total number of road users in these cases equals to N^0 and the

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¹³ Figure 4-3(a) sketches the case when the commute cost of eFHV's and private autos become equal before the full occupancy of all parking spaces and both last private car and first eFHV arrive at CBD early. Figure 4-3 (b) and (c) sketch the cases when the commute cost of eFHV's and private autos become equal after the full occupancy of all parking spaces and both last private car and first eFHV arrive at CBD early. Figure 4-3 (d) (e) sketch the cases when the commute cost of eFHV's and private autos become equal after the full occupancy of all parking spaces, last private car arrives at CBD late while first eFHV arrives early. Figure 4-3 (f) sketch the case when the commute cost of eFHV's and private autos become equal after the full occupancy of all parking spaces and both last private car and first eFHV arrive at CBD late. Figure 4-3 (g) sketches the case when the commute cost of eFHV's and private autos become equal before the full occupancy of all parking spaces and last private car arrives at CBD late while first eFHV arrives early. Figure 4-3 (h) sketches the case when the commute cost of eFHV's and private autos become equal before the full occupancy of all parking spaces and both last private car and first eFHV arrive at CBD late. Figure 4-3 (i) sketches an extreme case when no one uses eFHV and commuters drive to work only during morning peak. Figure 4-3 (j) sketches another extreme case when no one uses car driving and commuters take eFHV's to work only during morning peak.

equilibrium commute cost $P_{(a)(c)(e)(f)(g)(h)}^* = P_T^*(N - N^0)$, which is independent of the number of parking spaces M_C .

Though the equilibrium cost does not change after M_C , all parking spaces are taken up in cases (c) (e) (f). Yet in cases (a) (g) (h), the actual number of private auto drivers is even less than the number of parking spaces, which indicates the oversupply of parking spaces.

Now we compare and contrast the equilibrium with or without ride-sourcing service. According to Eq.(4.10), given the parking spaces M_C , the system commute cost difference (denoted by Δ , the system commute cost with ride sourcing minus that without ride sourcing) $\Delta = [P_T^*(N - N^0) - P_T^*(N - M_C)] \cdot N$. When $F_F \leq F_F^2$, there is $N_C^0 \leq N^0$. Since $M_C < N_C^0$, $M_C < N^0 \rightarrow N - M_C > N - N^0 \rightarrow \Delta < 0$. When $F_F > F_F^2$, $N_C^0 > N^0$, whether $\Delta \leq 0$ depends on the value of M_C . It cannot be determined directly whether the addition of ride-sourcing service into bottleneck problem will make the bottleneck performance at equilibrium better with lower system commute cost. Yet in most cases when there is under-supply of parking spaces, M_C can be considered smaller than N^0 , $\Delta < 0$, i.e., the introduction of ride-sourcing improves system performance.

As for case (b) and (d), both private autos and eFHV are used and the first eFHV departs after the bottleneck queue is cleared. All parking spaces are used up during morning peak. The equilibrium commute cost becomes $P_{(b)(d)}^* = \frac{\beta\gamma}{\mathbb{C}} [F_T + \theta(N - M_C) - F_F] + F_F = P_T^*(N - M_C - N_F)$. It can be verified that the commute cost of road users in case (b) and (d) is a strictly decreasing function of M_C . As shown in Figure 4-2, the number of parking spaces should satisfy $M_C \leq M_2^u$.

At $M_C = M_2^u$, the equilibrium commute cost reaches the minimum

$$P_{(b),(d)\min}^* = \frac{\beta\gamma}{(\beta + \gamma)s} N^0 + K = P_T^*(N - N^0), \text{ namely } P_{(b),(d)}^* \geq P_T^*(N - N^0). \text{ The}$$

system commute cost difference is computed as

$$\Delta = P_T^*(N - M_C - N_F) - P_T^*(N - M_C). \text{ Since } N_F > 0, \text{ we always have } \Delta < 0 \text{ in case (b) and (d).}$$

In case (i), the number of road users equals the number of parking spaces M_C . The

equilibrium commute cost $P_{(i)}^* = P_T^*(N - M_C)$ is a strictly decreasing function of

M_C . In comparison with the case without ride-sourcing, the system commute cost

$$\text{difference } \Delta = P_T^*(N - M_C) - P_T^*(N - M_C) = 0.$$

Lastly in case (j), all commuters will not drive to work, and apparently the number of

road users is independent of the number of parking spaces M_C . The equilibrium

$$\text{commute cost becomes } P_{(j)}^* = P_F^*(N_F^0) = \frac{\beta\gamma}{(\beta + \gamma)s} \frac{N_F^0}{s} + F_F = P_T^*(N - N_F^0), \text{ where}$$

$$N_F^0 = \frac{s(\beta + \gamma)(F_T - F_F + \theta N)}{\mathbb{C}}, \text{ representing the virtual demand of eFHV's.}$$

Without loss of generality, we have $N_F^0 > 0$; otherwise, no one uses the highway,

and hence we can assume that $F_F < F_T + \theta N$ always exists. The system commute

$$\text{cost difference } \Delta = [P_T^*(N - N_F^0) - P_T^*(N - M_C)] \cdot N. \text{ When } N_C^0 < N_F^0 \text{ or } N_C^0 \geq N_F^0$$

and $M_C < N_F^0$, one can derive that $\Delta < 0$; when $N_C^0 \geq N_F^0$ and $M_C \geq N_F^0$, we have

$$\Delta \geq 0. \text{ In most cases when the parking spaces are not sufficient, we have } M_C < N_F^0,$$

i.e., $\Delta < 0$.

From the above analysis, one can find, among all the cases shown in Figure 4-5, only in case (b) (d) and (i), increasing the parking spaces supply can reduce the equilibrium commute cost. In other cases, the equilibrium commute cost is independent of the number of parking spaces when eFHV's are sufficient. However, the reasons behind lower commute cost in (b) (d) and (i) are different. In case (i), the ride-sourcing service is no longer attractive, and it is indeed the bi-modal equilibrium. While in (b) (d), it is because the shortage of parking spaces is extremely acute, and the marginal benefit of parking spaces increment to system cost reduction remains high. Once the scale of parking spaces is expanded to a threshold, i.e. M_2'' , a further increase in parking facilities provision brings no benefit to the system performance. Therefore, the morning commute management via parking spaces control becomes ineffectual in many instances. The total number of road users can no longer be constricted by parking spaces provision as in Zhang et al. (2011), neither can the total queuing delay at bottleneck (a proxy of bottleneck performance). Yet the existence of ride-sourcing service can lower the system commute cost of commuters when there are insufficient parking spaces, as it fills the unsaturated road capacity resulted from parking space paucity. Such system commute cost reduction induced by ride-sourcing also implies that the commute cost derived from the traditional bi-modal equilibrium is usually overestimated.

Figure 4-3 (a)-(j): Possible Commuting patterns with parking space restraint in the presence of sufficient ride-sourcing service

4.3 The optimal number of parking space and eFHV

Having solved the equilibrium morning traffic pattern considering ride-sourcing, we seek to address the problem that how we can obtain the optimal number of parking spaces from the perspective of best managing the morning commute traffic.

Likewise, parking spaces provided M_C are assumed to be insufficient in this section.

The number of eFHV which can service morning commute, N_F , is no longer assumed to be sufficient and is smaller than the virtual demand N_F^0 .¹⁴ Additionally, we assume $F_F \leq F_F^0$. Prior researches (Zhang et al., 2011; Yang et al., 2013) have suggested that system commute cost will always be reduced when all parking permits are distributed in advance, i.e., all parking spaces are reserved, compared to the case in which all parking spaces are not reserved. To this end, all private auto drivers in this part are assumed to have parking reservation¹⁵, namely $\varepsilon = 0$ for all private autos.

Besides, several variables $\mathbb{D}$, $\mathbb{E}$, $\mathbb{A}$, $\mathbb{B}$, N_F^1 to N_F^6 and M^1 to M^7 , are hereinafter defined for reference and comparison purpose. Let

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¹⁴ Various factors can affect the supply of eFHV in practice, for instance, the income of the ride-sourcing drivers, the profit of ride-sourcing firms, the average demand of ride-sourcing service and the government policy. Zha et al. (2016) discussed the equilibrium supply of ride-sourcing vehicles in an aggregate model, which is not detailed in this study as it is beyond the focus of this paper. Once N_F is given, the major issue addressed hereinafter is whether parking space control is still feasible in morning commute management, when eFHV is presented.

¹⁵ When the parking spaces at CBD are all employer-provided or with monthly/yearly booking and no public parking are set aside, i.e. drivers are not allowed to park at CBD during morning peak without parking permit, we can state that all parking spaces have been reserved.

$$\begin{aligned}
\mathbb{D} &= F_F^0 - F_F, & N_F^1 &= \frac{\mathbb{D} + \mathbb{E} - \theta N}{2\beta + \theta s} s, & M^1 &= \frac{\beta + \gamma}{2} \frac{\mathbb{A}}{\mathbb{C}}, \\
\mathbb{E} &= \theta N + F_T - F_F - \beta S_0, & N_F^2 &= \frac{\beta(\mathbb{D} + \mathbb{E} - \theta N) + \gamma \mathbb{E}}{2\beta^2 + \mathbb{C}}, & M^2 &= (\beta + \gamma) \frac{\mathbb{B}}{\mathbb{C}}, \\
\mathbb{A} &= [\mathbb{E} - \mathbb{D} + \theta(N - N_F)] s, & N_F^3 &= \frac{\mathbb{D}}{\beta} s, & M^3 &= \frac{\beta \mathbb{A} + \gamma \mathbb{B}}{2\mathbb{C}}, \\
\mathbb{B} &= (\mathbb{E} - \theta N_F) s - \beta N_F, & N_F^4 &= \frac{\mathbb{E} - \mathbb{D}}{\theta}, & M^4 &= (\beta + \gamma) \frac{\mathbb{A} - \theta N s}{\mathbb{C}}, \\
&& N_F^5 &= \frac{\beta(\mathbb{E} - \mathbb{D}) + \gamma \mathbb{E}}{\mathbb{C}} s, & M^5 &= N_F^5 - N_F, \\
&& N_F^6 &= N_F^5 - M^6, & M^6 &= \frac{\beta \theta N s}{\mathbb{C}}, \\
&& N_F^7 &= \frac{\mathbb{E} s - \frac{\mathbb{C}}{\beta + \gamma} M_C}{\beta + \theta s}, & M^7 &= N_F^5 - N_F^3. \\
&& N_F^8 &= N_F^4 - \frac{\mathbb{C}}{(\beta + \gamma) \theta s} M_C, \\
&& N_F^9 &= N_F^5 - M_C,
\end{aligned}$$

Obviously, $\mathbb{D} \geq 0$ and $\mathbb{C} = (\beta + \gamma) \theta s + \beta \gamma$. From Eq.(4.6), the equilibrium private auto drivers without parking restraint $N_C^0 \geq 0$. When all drivers have parking reservation ($\varepsilon = 0$), $\theta N + F_T - \alpha S_0 - F_C \geq 0$.

Then we have $\mathbb{E} = \theta N + F_T - F_F - \beta S_0 = \theta N + F_T - \alpha S_0 - F_C + \mathbb{D} \geq 0$. Besides, one may verify that the equations $M_C = M^2$ and $N_F = N_F^7$ are equivalent. Such equivalence also applies to: 1. $M_C = M^4$ and $N_F = N_F^8$, and 2. $M_C = M^5$ and $N_F = N_F^9$.

Following the setting above, the number of eFHV commuters and private auto commuters are limited to N_F and M_C , respectively. Since ride-sourcing is more attractive than driving at any time point ($P_F(t) \leq P_C(t)$ for any t) in our setting, road users will compete for the insufficient eFHVs first with earlier trip booking and earlier departure. However, as shown in Eq.(4.10), the equilibrium commute cost for eFHV passengers is less than that of private auto drivers with parking reservation.

This is because private auto mode will not be chosen until full occupancy of all eFHV and eFHV commuters have to spend more on scheduling due to such eFHV competition. As a result, all private auto drivers in this case depart later than eFHV passengers with a departure gap. The larger scheduling cost of eFHV passengers has finally turned eFHV's superiority over driving into inferiority.

Possible scenarios of cumulative departures from home and arrivals at destination at equilibrium depending on different N_F and M_C are presented below.¹⁶ The commuting cost for each scenario including schedule delay and queuing delay costs are then calculated. Time points for each scenario are presented in Table 4-1.

Scenario 1): All eFHV passengers arrives at destination early (before t^*). The first private auto departs after the bottleneck queue is cleared and hence the bottleneck services without flow interaction between eFHV and private autos.

Scenario 2): All eFHV passengers arrives at destination early. Some private autos enter the bottleneck before the queue is cleared and wait for pass till all eFHV leaves the bottleneck.

Figure 4-4 depicts the critical case between Scenario 1) and Scenario 2) in which the first private auto happens to enter the bottleneck right after the last eFHV leaves the bottleneck. Curve AB and GF represent the cumulative arrival at the bottleneck of private autos and eFHV respectively and curve DC and GE represent their cumulative arrival at the destination correspondingly.

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¹⁶ There is no such scenario when some eFHV passengers arrive at destination late and some private autos arrive early. Note $t_{3F} > t_{3C}$ as discussed in Section 3.2. In this case, we should have the last eFHV departure time $t_F^l > t_{3F}$, the first private auto departure time $t_2 < t_{3F}$ and hence $t_2 < t_F^l$, contradicting the fact that all private autos depart later than eFHV.

Scenario 3): Some eFHV passengers arrive at destination late (after t^*). All private autos arrive late. From Eq.(4.11), it is impossible to have late-arrival departure after the queue is cleared. So the first private auto can only depart before the bottleneck queue is cleared.

The individual travel cost and the total travel cost of all commuters in the previous scenarios will be discussed in the following subsections: first to determine the optimal number of parking space M_C^O with a given N_F in Section 4.3.1, and second to determine the optimal number of vehicles providing ride sourcing service N_F^O with a given M_C in Section 4.3.2.

Table 4-1 Time points for each scenario.

Time	Scenario 1)	Scenario 2)	Scenario 3)
t_1	$t^* - \frac{P_F^* - F_F}{\beta}$		
t_F^l	$t^* - \frac{P_F^* - F_F}{\beta} + \frac{N_F}{s}$	$t^* - \frac{P_F^* - F_F}{\beta} + \frac{\alpha - \beta}{\alpha} \frac{N_F}{s}$	$t^* - \frac{\alpha + \beta + \gamma}{\alpha} \frac{P_F^* - F_F}{\beta} + \frac{\alpha + \gamma}{\alpha} \frac{N_F}{s}$
t_2	$t^* + \frac{P_C^* - (\alpha + \gamma)S_0 - F_C}{\gamma} - \frac{N_C}{s}$	$t^* + \frac{\alpha}{\alpha + \beta + \gamma} \frac{P_C^* - (\alpha + \gamma)S_0 - F_C}{\gamma} - \frac{\alpha + \gamma}{\alpha + \beta + \gamma} \frac{N_C}{s} - S_0$	$t^* + \frac{P_C^* - (\alpha + \gamma)S_0 - F_C}{\gamma} - \frac{\alpha + \gamma}{\alpha} \frac{N_C}{s}$
t_4	$t^* + \frac{P_C^* - (\alpha + \gamma)S_0 - F_C}{\gamma}$		

4.3.1 Optimal number of parking space

We first assume the number of eFHV N_F is given during morning commute.

Though there are several scenarios upon different departure and arrival times, the

individual commute cost for eFHV's riders and transit passengers are equal, i.e.

$P_F^* = P_T^* (N - N_F - N_C)$, according to Eq.(4.10). As for private auto drivers, their

individual commute cost can be summarized in two cases:

$$P_C^{r*} = \begin{cases} \frac{\beta\gamma}{(\beta+\gamma)s} N_C + \alpha S_0 + F_C, & \text{if } N_F \leq N_F^3 \\ \text{or} \\ N_F > N_F^3 \text{ and } M_C \leq M^2 \text{ (i);} \\ \left(\frac{\gamma [F_F s + \beta (N_C + N_F) - P_F^* s]}{\beta s} \right) / \left(+(\alpha + \gamma) S_0 + F_C \right), & \text{if } N_F > N_F^3 \\ \text{and } M_C > M^2 \text{ (ii);} \end{cases} \quad (4.19)$$

where $N_C \leq M_C$ and if $N_F \leq N_F^3$, $N_C = \min\{M_C, M^4\}$; if $N_F > N_F^3$,

$N_C = \min\{M_C, M^5\}$.

Figure 4-4 Critical commuting equilibrium between Scenario 1) and Scenario 2), with parking

space restraint in the presence of insufficient ride-sourcing service, when $F_F \leq F_F^0$.

Figure 4-5 Domain of optimal number of parking spaces N_F^o in the presence of insufficient ride-sourcing service with respect to N and N_F , when $F_F \leq F_F^0$.

Figure 4-6 Domain of optimal number of eFHVs N_F^o with parking space restraint, with respect to N and M_C , when $F_F \leq F_F^0$.

Figure 4-7 Efficiency contours η with respect to N and M_C , when $F_F \leq F_F^0$.

Therefore, the parking spaces provided M_C is binding only when M_C is smaller than a specific value (M^4 or M^5 in this study); otherwise, the number of private auto commuters is less than M_C and the parking space provision is redundant.

More intuitively, case (i) in Eq.(4.19) indicates that there exists one period with zero flow at the bottleneck, i.e., Scenario 1); case (ii) represents that there never exists a period with zero flow at the bottleneck, i.e., Scenario 2) and 3).

Now let's move on to the system commute cost (denoted by TC). TC is the total cost for all commuters, including private auto cost, transit cost, and eFHV cost and is given as follows,

$$\begin{aligned} TC &= P_C^{r*} \cdot N_C + P_F^* \cdot N_F + P_T^* (N - N_F - N_C) \cdot (N - N_F - N_C) \\ &= P_C^{r*} \cdot N_C + P_F^* \cdot (N - N_C), \end{aligned} \quad (4.20)$$

where P_C^{r*} is determined in Eq.(4.19) and $P_F^* = P_T^* (N - N_F - N_C)$.

Given the number of eFHV during morning peak N_F , as shown in Figure 4-5, we have the following proposition.

Proposition 4.1 *For given N_F , there is always an optimal number of parking spaces*

$M_C = M_C^O(N_F)$, which leads the system commute cost to a minimum.

$$M_C^O(N_F) = \begin{cases} \min\{M^1, M^4\}, & \text{if } N_F \leq N_F^4 \text{ and } N_F \leq N_F^1; \\ M^4, & \text{if } N_F \leq N_F^4 \text{ and } N_F > N_F^1 \\ & \text{and } N_F \leq N_F^3; \\ M^2, & \text{if } N_F \leq N_F^4 \text{ and } N_F > N_F^3 \\ & \text{and } N_F \leq N_F^2; \\ M^3, & \text{if } N_F \leq N_F^6 \text{ and } N_F > N_F^3 \\ & \text{and } N_F > N_F^2; \\ M^5, & \text{if } N_F \leq N_F^5 \text{ and } N_F > N_F^6 \\ & \text{and } N_F > N_F^3; \\ 0, & \text{otherwise.} \end{cases} \quad (4.21)$$

Proof. We will derive the optimal number of parking spaces regarding different quantities of eFHV in this appendix. As all parking spaces are now reserved, and private auto drivers will not drive to work without parking reservation.

According to Eqs.(4.19) and (4.20), when there once exists a period with zero flow at the bottleneck, namely when $N_C < M^2$ as shown in Figure 4-4, the system commute cost (denoted by TC^i) becomes

$$TC^i = \left[\frac{\beta\gamma}{(\beta+\gamma)s} N_C + \alpha S_0 + F_C \right] \cdot N_C + \left[F_T + \theta(N - N_F - N_C) \right] \cdot (N - N_C), \quad (4.22)$$

where $N_F + N_C < N$. The first-order derivative of TC^i with respect to N_C is then given by

$$\frac{dTC^i}{dN_C} = 2 \left(\frac{\beta\gamma}{(\beta+\gamma)s} + \theta \right) \cdot N_C + F_C - F_T + \alpha S_0 + \theta N_F - 2\theta N, \quad (4.23)$$

When $N_C > M^1$, $\frac{dTC^i}{dN_C} > 0$; when $N_C \leq M^1$, $\frac{dTC^i}{dN_C} \leq 0$.

Moreover, since $P_C^{r*} \leq P_F^*$ from Eq.(4.10),

$$\rightarrow \frac{\beta\gamma}{(\beta+\gamma)s} N_C + \alpha S_0 + F_C \leq F_T + \theta(N - N_F - N_C),$$

$$\rightarrow N_C \leq -\frac{s(\beta+\gamma)(F_C - F_T + \alpha S_0 + \theta N_F - \theta N)}{\mathbb{C}} = M^4$$

From $N_C \geq 0$, there is $M^4 \geq 0$, $\rightarrow N_F \leq \frac{-F_C + F_T - \alpha S_0 + \theta N}{\theta} = N_F^4$.

Besides, we have

$$\frac{d^2TC^i}{d(N_C)^2} = 2 \left(\frac{\beta\gamma}{(\beta+\gamma)s} + \theta \right) > 0. \quad (4.24)$$

As for the case when $N_C \geq M^2$, i.e., there never exists a time window with zero flow

at the bottleneck, the system commute cost (denoted by TC^{ii}) is

$$TC^{ii} = \left\{ \begin{aligned} &F_C + (\alpha + \gamma)S_0 \\ &+ \frac{\gamma[F_F s + \beta(N_F + N_C) - (\theta(N - N_F - N_C) + F_T)s]}{\beta s} \end{aligned} \right\} \cdot N_C. \quad (4.25)$$

$$+ [F_T + \theta(N - N_F - N_C)] \cdot (N - N_C)$$

The first-order derivative of TC^{ii} with respect to N_C is then given by

$$\begin{aligned} \frac{dTC^{ii}}{dN_C} = & \frac{2\mathbb{C}}{\beta s} N_C + F_C - F_T + \frac{\gamma N_F}{s} + \frac{\gamma(F_F - F_T)}{\beta} \\ & + (\alpha + \gamma)S_0 + \theta N_F - 2\theta N - \frac{\gamma\theta}{\beta}(N - N_F). \end{aligned} \quad (4.26)$$

It can be verified that when $N_C > M^3$, $\frac{dTC^{ii}}{dN_C} > 0$; when $N_C \leq M^3$, $\frac{dTC^{ii}}{dN_C} \leq 0$.

Similarly, we have $P_C^{r*} \leq P_F^*$,

$$\begin{aligned}
& \rightarrow F_C + (\alpha + \gamma)S_0 + \frac{\gamma[F_F s + \beta(N_F + N_C) - (\theta(N - N_F - N_C) + F_T)s]}{\beta s} \\
& \leq F_T + \theta(N - N_F - N_C) \\
& \rightarrow N_C \leq \frac{\theta N(\beta + \gamma)s - [\beta F_C + \gamma F_F + \beta(\alpha + \gamma)S_0 - F_T(\beta + \gamma)]s}{\mathbb{C}} - N_F = M^5
\end{aligned}$$

From $N_C \geq 0$, there is $M^5 \geq 0$,

$$\rightarrow N_F \leq \frac{(\beta + \gamma)(\theta N + F_T) - \beta F_C - \beta(\alpha + \gamma)S_0 - \gamma F_F}{\mathbb{C}} = N_F^5. \quad \text{The second-order}$$

derivative of TC^{ii} is

$$\frac{d^2 TC^{ii}}{d(N_C)^2} = \frac{2\mathbb{C}}{\beta s} > 0. \quad (4.27)$$

Though Eqs.(4.23),(4.24),(4.26) and (4.27) show that TC^i and TC^{ii} are strictly convex in N_C respectively, the global total cost function TC is a piecewise function and is not always convex in N_C .

Comparing M^1 , M^2 and M^3 , we have

$$\begin{aligned}
M^1 & \geq M^2, \text{ iff } N_F \geq N_F^1; \\
M^3 & \geq M^2, \text{ iff } N_F \geq N_F^2.
\end{aligned} \quad (4.28)$$

Without considering the non-negativity of N_C and $P_C^* \leq P_F^*$, the minimum of TC corresponding to each segment is

$$\begin{aligned}
\text{If } N_C \leq M^2: & \quad \text{if } N_F \geq N_F^1, \quad TC_{min} = TC(N_C = M^2); \\
& \quad \text{else,} \quad TC_{min} = TC(N_C = M^1); \\
\text{else:} & \quad \text{if } N_F \geq N_F^2, \quad TC_{min} = TC(N_C = M^3); \\
& \quad \text{else,} \quad TC_{min} = TC(N_C = M^2).
\end{aligned} \quad (4.29)$$

Evidently, the result in Eq.(4.29) is not completed and should be further combined with the other constraints including M^4 , M^5 . Then comparing M^2 , M^4 and M^5 , it can be verified that

$$\frac{M^4 - M^2}{M^5 - M^2} = \frac{\beta + \gamma}{\beta}; \quad (4.30)$$

and

$$M^4 > M^5 \geq M^2, \text{ if } N_F \geq N_F^3; M^4 < M^5 \leq M^2, \text{ if } N_F \leq N_F^3. \quad (4.31)$$

Furthermore, the relationship between N_F^1 , N_F^2 and N_F^3 can also be found. Let

$$\bar{N} = \frac{\theta \mathbb{D}s + \beta(\mathbb{D} - \mathbb{E} + \theta N)}{2\beta\theta + \theta^2 s}, \text{ we have}$$

$$N_F^2 \geq N_F^1, \quad \text{iff } N \geq \bar{N}; \quad (4.32)$$

When $N_F^3 \leq N_F^1$, $\bar{N} \leq 0$, thus $N \geq \bar{N}$ is always satisfied, $\rightarrow N_F^2 \geq N_F^1$. Thus, it is impossible to have the cases when

$$N_F^3 \leq N_F^2 \leq N_F^1 \text{ or } N_F^2 \leq N_F^3 \leq N_F^1. \quad (4.33)$$

In addition, when $N < 2\bar{N}$, $N_F^4 < N_F^1$; when $N < \frac{(2\beta + \gamma)(2\beta + \theta s)}{\beta(2\beta + \theta s + \gamma)}\bar{N}$, $N_F^4 < N_F^2$;

when $N < \left(2 + \frac{\theta s}{\beta}\right)\bar{N}$, $N_F^5 < N_F^3$; thus when $N < \bar{N}$, namely when $N_F^2 < N_F^1$,

there is $N_F^4 < N_F^2 < N_F^1$ and $N_F^5 < N_F^3$.

Now from Eq.(4.33), the optimal parking spaces problem can be separated into four different cases depending on the relationship of N_F^1 , N_F^2 and N_F^3 , i.e.,

①;②;③;④. For different N_F , the optimal number of parking spaces $M_C^O(N_F)$

can be calculated below in Table 4-2.

Table 4-2 Optimal Number of Parking Spaces $M_C^O(N_F)$.

N_F	① $\mathbb{S} = N_F^3,$ $\mathbb{M} = N_F^1,$ $\mathbb{L} = N_F^2;$	② $\mathbb{S} = N_F^1,$ $\mathbb{M} = N_F^3,$ $\mathbb{L} = N_F^2;$	③ $\mathbb{S} = N_F^1,$ $\mathbb{M} = N_F^2,$ $\mathbb{L} = N_F^3;$	④ $\mathbb{S} = N_F^2,$ $\mathbb{M} = N_F^1,$ $\mathbb{L} = N_F^3;$
$(-\infty, \mathbb{S}]$	$\min\{M^1, M^4\}$	$\min\{M^1, M^4\}$	$\min\{M^1, M^4\}$	$\min\{M^1, M^4\}$
$(\mathbb{S}, \mathbb{M}]$	M^1	M^4	M^4	0
$(\mathbb{M}, \mathbb{L}]$	M^2	M^2	M^4	0
$(\mathbb{L}, +\infty)$	$\min\{M^3, M^5\}$	$\min\{M^3, M^5\}$	$\min\{M^3, M^5\}$	0

Note:

1. For each case, $\mathbb{S}$, $\mathbb{M}$ and $\mathbb{L}$ represent the relationship of N_F^1, N_F^2 and N_F^3 , where $\mathbb{S} \leq \mathbb{M} \leq \mathbb{L}$, e.g., in case

① we have $N_F^3 \leq N_F^1 \leq N_F^2$.

2. It is assumed that in this table N_F satisfies that $N_F \leq N_F^4$ when $N_C < M^2$, or $N_F \leq N_F^5$ when $N_C \geq M^2$;

otherwise $M^4 \leq 0$, $N_C \geq M^4$ when $N_C < M^2$, and $M^5 \leq 0$, $N_C \geq M^5$ when $N_C \geq M^2$, $P_C^{r*} \geq P_F^*$, no one prefers private autos and the optimal number of parking spaces should be zero.

3. In case ④, $N_F^2 \leq N_F^1 \rightarrow N_F^4 \leq N_F^2$ as mentioned earlier. When $N_C < M^2$, if $N_F \geq N_F^2 \rightarrow N_F \geq N_F^4$, from Note 2.,

the optimal number of parking spaces should be zero. When $N_C \geq M^2$, as it can be verified that $M^3 \leq 0$ is satisfied

when $N_F^2 \leq N_F^1 \rightarrow \min\{M^3, M^5\} \leq 0$. Due to the non-negativity of N_C , the optimal number of parking spaces is zero.

After summarizing the results in Table 4-2, the optimal number of parking spaces

$M_C^O(N_F)$ is given as shown in Eq.(4.21). $\square$

From Proposition 4.1, one can easily find that, with different levels of ride-sourcing service supply, the corresponding optimal parking spaces supply can be different. A mismatch parking supply results in social loss. When the current supply of parking spaces falls below the optimum at current N_F , system commute cost increases;

conversely when the current supply of parking spaces outstrips this optimum, the system commute cost either increases or remains the same with unnecessary operating expense on redundant parking spaces. In terms of morning commute management, when the ride-sourcing service generally performs better than the private auto driving during morning peak ($F_F \leq F_F^0$), the increase of parking spaces supply does not surely reduce system commute cost, considering ride-sourcing service. Instead, an optimal number of parking permits coordinated with the current number of eFHV's can improve travel efficiency. When necessary, it is cost effective to temporally block up the superfluous parking spaces (e.g., curbside parking) and limit the distributed parking permits to optimum during morning peak to achieve minimum system commute costs.

4.3.2 **Optimal number of eFHV's**

In the previous subsection, the optimal parking provision is obtained. However, one may argue that the number of parking space is difficult to be manipulated. Particularly for those office buildings, once the construction is finished, the provided parking spaces can be hardly changed. In this subsection, we then focus on the question from the eFHV's regulation perspective, i.e., what is the optimal number of eFHV's, when the number of parking space M is given, so that the system travel cost can be minimized. Indeed, it is imperative for the regulators to address the question that: should the number of registered eFHV's be controlled? If yes, how the fleet size of eFHV's should be determined? Granted that many different factors and considerations stipulate the answer to such a question, we can at least attempt to address this problem in this study from the perspective of managing morning commute traffic.

The individual commute cost of private auto users is summarized below:

$$P_C^* = \begin{cases} \frac{\beta\gamma}{(\beta+\gamma)s} N_C + \alpha S_0 + F_C, & \text{if } N_F \leq N_F^7, \\ & \text{or } N_F > N_F^7 \\ & \text{and } M_C \geq M^7 \text{ (i);} \\ \frac{\gamma[F_F s + \beta(N_C + N_F) - P_F^* s]}{\beta s + (\alpha + \gamma)S_0 + F_C}, & \text{if } N_F > N_F^7 \\ & \text{and } M_C < M^7 \text{ (ii);} \end{cases} \quad (4.34)$$

where $N_C \leq M_C$ and if $M_C \geq M^7$, $N_C = \min\{M_C, M^4\}$; if $M_C < M^7$, $N_C = \min\{M_C, M^5\}$

The individual cost of private auto users is almost the same as that in Section 4.3.1, except the slightly different domain definition. Note that $N_F = N_F^7$ is equivalent to $M_C = M^2$. Such domain difference arises from the switch of reference frame. Likewise, case (i) in Eq.(4.34) represents Scenario 1) and case (ii) represents Scenario 2) and 3).

We then directly apply Eqs.(4.20) and (4.34) to analyze the system commute cost. Given the number of parking space during morning peak M_C , as shown in Figure 4-6, we have the following proposition.

Proposition 4.2 *For given $M_C \leq N_C^0$, there is always an optimal number of eFHV's $N_F = N_F^O(M_C)$, which leads the system commute cost to a minimum without any unused parking space.*

$$N_F^O(M_C) = \begin{cases} N_F^8, & \text{if } M_C > M^7; \\ N_F^7, & \text{if } M_C \leq M^7 \text{ and } M_C > M^6; \\ N_F^9, & \text{if } M_C \leq M^7 \text{ and } M_C \leq M^6. \end{cases} \quad (4.35)$$

Proof. When $N_C < M^2$, i.e., $N_F < N_F^7$, it is Scenario 1). The system commute cost is shown in (4.22). Moreover, the first-order derivative of TC^i with respect to N_F is then given by

$$\frac{dTC^i}{dN_F} = -\theta(N - N_C) < 0 . \quad (4.36)$$

Moreover, since $P_C^{r*} \leq P_F^*$, $\rightarrow \frac{\beta\gamma}{(\beta+\gamma)s} N_C + \alpha S_0 + F_C \leq F_T + \theta(N - N_F - N_C)$,
 $\rightarrow N_F \leq N_F^8$.

As for the case when $N_F \geq N_F^7$, i.e., Scenario 2) and 3), the system commute cost is shown in Eq.(4.25). The first-order derivative of TC^{ii} with respect to N_F is then given by

$$\frac{dTC^{ii}}{dN_F} = \frac{\gamma(\beta + s\theta)}{s\beta} N_C - \theta(N - N_C). \quad (4.37)$$

It can be verified that when $N_C > M^6$, $\frac{dTC^{ii}}{dN_F} > 0$; when $N_C \leq M^6$, $\frac{dTC^{ii}}{dN_F} \leq 0$.

Similarly, we have $P_C^{r*} \leq P_F^*$,

$$\rightarrow F_C + (\alpha + \gamma)S_0 + \frac{\gamma[F_F s + \beta(N_F + N_C) - (\theta(N - N_F - N_C) + F_T)s]}{\beta s}, \rightarrow N_F \leq N_F^9.$$

$$\leq F_T + \theta(N - N_F - N_C)$$

One can further verify that

$$TC^i(N_F = N_F^7) = TC^{ii}(N_F = N_F^7) . \quad (4.38)$$

Therefore, given Eqs.(4.36), (4.37) and (4.38), the global total cost function TC is a continuous and convex function of N_F . When $N_C > M^6$, TC reaches its minimum at $N_F = N_F^7$, i.e., at the case when the first private auto happens to enter the bottleneck right after the last eFHV leaves the bottleneck; when $N_C < M^6$, TC

reaches its minimum at $N_F = N_F^0$, i.e., at the case when all road users are eFHV only.

However, the minimum is not stable by setting N_F to N_F^7 or N_F^0 directly. The actual number of private auto users may differ when ride sourcing is more cost competitive than private auto driving, i.e., $P_C^{r*} > P_F^*$. Besides, when $N_F = N_F^0$, all parking spaces become idle.

Comparing N_F^7, N_F^8 and N_F^9 , we have

$$\frac{N_F^9 - N_F^7}{N_F^8 - N_F^7} = \frac{\beta \theta s}{\mathbb{C}} < 1; \quad (4.39)$$

and

$$\begin{aligned} N_F^8 &\geq N_F^9 \geq N_F^7 && \text{if } M_C \leq M^7; \\ N_F^8 &\leq N_F^9 \leq N_F^7 && \text{if } M_C \geq M^7. \end{aligned} \quad (4.40)$$

One can verify that when $M_C \leq N_C^0$, $N_F^7, N_F^9 > 0$, $N_F^8 \geq 0$.

When $M_C \geq M^7$, if it is in Scenario 2) or 3) at equilibrium, i.e., $N_F > N_F^7$, we have

$P_C^{r*} > P_F^*$, the actual number of private auto users decreases until $M_C = M^5$, i.e.,

$N_F = N_F^9$. Since $N_F = N_F^9 \leq N_F^7$ when $P_C^{r*} = P_F^*$, the equilibrium will go to Scenario

1). Once it is in Scenario 1), N_F^8 bounds and finally $N_C = M^4$. Therefore, it is

impossible for the equilibrium to stay at Scenario 2) or 3), when $M_C \geq M^7$. Namely

no matter what N_F is, when $M_C \geq M^7$, we have Scenario 1) at equilibrium. By

setting $N_F = N_F^8$, TC reaches its minimum without unused parking space.

When $M_C < M^7$, it is possible for the equilibrium to stay in any scenarios. If $N_C > M^6$, as discussed, TC reaches its minimum at $N_F = N_F^7 \leq N_F^9 \leq N_F^8$. If $N_C \leq M^6$, TC reaches its minimum at $N_F = N_F^9 \leq N_F^0$. After summarizing the preceding results, the optimal number of eFHV's $N_F^O(M_C)$ is given as shown in Eq.(4.35). $\square$

By switching the frame of reference, the results to maximize total commute become different compared to Section 4.3.1. When the current supply of eFHV's is less than the optimum at current M_C , the system commute cost increases. The insufficient eFHV supply directly results in the individual cost increase of transit riders along with ride sourcing passengers and can lower the individual cost of private auto riders. On the contrary, when the current supply of eFHV's exceed the optimum, the oversupplied eFHV's attract private auto users to switch their commute mode and hence lower the parking demand, given $F_F \leq F_F^0$. In fact, the supply of eFHV's can be regarded as the aggregated number of eFHV's on road, independent of their types or owners. Inspired by traditional taxi industry, the number of eFHV's on road can be managed via various regulations, e.g., entry restrictions, fleet size control. Our work provides regulators a macroscopic tool of quantity management of ride sourcing vehicles during morning commute. Based on our results, regulators can further set an optimal number of eFHV's licenses and distribute them to various ride sourcing service providers, particularly when there's a sparse number of parking spaces.

4.4 Numerical examples

4.4.1.1 Number of parking space

We now use a numerical example to illustrate the results of optimal number of parking space. Following Jia et al. (2016), the value of time, schedule delay penalties are taken as follows: $\alpha = 0.3$ (SGD\$/min), $\beta = 0.1$ (SGD\$/min), $\gamma = 0.4$ (SGD\$/min). The highway bottleneck capacity is $s = 200$ (veh/min). We further suppose $\theta = 0.001$, $F_T = 1$ (SGD\$/commuter), $F_C = 4$ (SGD\$/commuter), $F_F = 3$ (SGD\$/commuter), $S_0 = 3$ (min). Then for instance, when the total number of commuters is 100,000, if parking provision meets the virtual parking demand 68,643, we have system commute cost $TC = 3.24 \times 10^6$ (SGD\$) without ride-sourcing service. Now with 10,000 eFHV, the total cost becomes $TC = 3.14 \times 10^6$ (SGD\$), and the actual number of occupied parking space is 59,557 with 9,086 parking space idle. By reducing the available parking space to 59,071 for morning commute, we can further lower the system commute cost to $TC = 2.99 \times 10^6$ (SGD\$).

4.4.1.2 Number of eFHV

To illustrate the optimal number of eFHV, we take the same values of parameters as in Section 4.4.1.1. The efficiency of the optimal eFHV supply is also evaluated. Comparison and contrast are carried out between the benchmark case when $N_F = 0$ and the optimal case when $N_F = N_F^O(M_C)$. We then define the percentage of cost reduction as

$$\eta = \frac{TC(N_F = 0) - TC(N_F = N_F^O(M_C))}{TC(N_F = 0)} . \quad (4.41)$$

The contour of relative efficiency η is then shown in Figure 4-7. When the total number of commuters is 100,000, and the number of parking space provided is

50,000, by providing 18,466 eFHV for morning commute, we can then achieve the optimum with $\eta = 34.4\%$ reduction of system commute cost from 3.79×10^6 (SGD\$) to 2.49×10^6 (SGD\$).

4.5 Concluding remarks

This chapter extends the bi-modal model in the previous chapter to multi-modal case with additional paralleled transit line. In the presence of railway transit, we notice that the morning commute traffic management via parking spaces control is unproductive except in times of severe scarcity of parking space or public's disinclination towards ride-sourcing service. When ride-sourcing service is in general cost-competitive in contrast with private auto driving, there is an optimal number of parking spaces corresponding to different eFHV quantities, leading to system commute cost minimum, on the condition that all parking spaces are reserved. As a general policy, it is worth noting that, with consideration of ride-sourcing, the supply of parking spaces larger than optimum can bring about the increase of system commute cost as well as unnecessary spending on operational cost of parking space. In this case, the parking space provision should be reduced. One of the most manageable schemes to reduce parking supply with flexibility is to block some curbside parking in peak hour. This is consistent with the findings in some previous studies, e.g., Arnott et al. (2015). While Arnott et al. (2015) suggested the curbside parking elimination to alleviate congestion with less parking cruising, we propose the curbside parking control from the perspective of system commute cost minimum considering ride-sourcing. In addition to quantity control of parking space, we also examine the optimal number of eFHV in morning commute with a given number of parking spaces so as to minimize the system commute cost. This result indeed

provides the regulators a reference of the aggregated number of eFHV licenses to be released when they are regulating the emerging ride sourcing industry.

CHAPTER 5

WHETHER COMPRESSED WORK SCHEDULE IS ATTRACTIVE? AN ECONOMIC STUDY CONSIDERING PERIODIC COMMUTE PATTERN AND PRODUCTIVITY

In Chapter 3 and Chapter 4 , we explore the commute traffic management from the perspective of supply management. Focuses are laid on the supplies of different travel modes and their effects on commuters' travel behaviors. One important assumption made in these two chapters is that the homogeneous commuters share the same working schedule with fixed work start time, which is mostly in line with the reality. Each commuter shows a strong inclination towards on-time arrival and their departures are inevitably centralized due to the identical work start time. Though the travel trips can be restrained via supply management, the rigid work schedule results in the rather high scheduling cost of some commuters. Thus, to better manage the commute traffic, it is far from enough to emphasize on the supply side only, and the temporal redistribution of travel demand is worthy of research attention. In the next two chapters, we will examine the commute traffic management in terms of travel demand. More specifically, we aim to flatten the peak hour traffic via voluntary departure time choice of commuters, i.e., flexible work arrangements.

Foremost, we focus on the commute pattern in the presence of compressed work schedule (CW) and its effects on productivity, congestion, and home activity. While the individual work-related productivity is determined under agglomeration economy, the day-long individual non-work utility combining travel disutility and home activity utility is formulated via a modified activity-based bottleneck model. We aim at

examining how CW influences employees' commuting behavior and work performance and in turn, how these factors influence employers' choice of CW and its schedule design. To this end, a bi-level economic model is developed to capture this series of decision makings of both the employees and employers. By investigating the intrinsic interrelationship between CW and its resulting commute pattern, the potential of CW in being a useful TDM measure can be better understood. In addition, our study also provides revealing insights to the government agencies on when to introduce CW and how to optimize the benefits of CW. More practical guidelines of CW are given to employers in future implementation as well.

The rest of this chapter is organized as follows. First, Section 5.1 introduces the model framework in a bi-level economic model. The individual activity-based utilities in both normal schedule and CW schedule are presented. Then Section 5.2 and 5.3 examine the two levels' equilibrium respectively and determines commute pattern, the equilibrium number of workers in CW and its corresponding equilibrium schedule design. Finally, we wrap up the chapter in Section 5.4 with some concluding remarks and future research directions.

5.1 Model formulation

The following notations are used throughout the process of model formulation.

Commuters composition	
N	The total number of commuters
N_I	The number of commuters in work schedule $I(=R, C)$, where R, C represent normal (rigid) schedule and compressed work schedule(CW), respectively
Work schedule design	
D_I	The number of workdays of work schedule $I(=R, C)$ per pay period
H	Total hours of work per day in normal schedule
ΔH	The daily extended work hours of CW
$t_m^*, t_{I,e}^*$	The uniform work start time; The work end time of workers in schedule $I(=R, C)$
Daily commute related	
t_h^d, t_w^d	The departure time from home in the morning and from work in the evening, respectively
α, β, λ	Value of time; Unit time scheduling penalty of early arrival for work; Unit time scheduling penalty of later departure from work
u_h, u_h	The marginal home-activity utility in the morning before work and in the evening after work
s	Constant service capacity of the bottleneck
$T_m(t), T_e(t)$	The travel time from home to the workplace in the morning and from workplace to home in the evening for vehicle departing at t

$q_I(t_h^d, t_w^d)$	The number of commuters in schedule $I(=R, C)$ with the departure time choice of $\{t_h^d, t_w^d\}$.
$r_{I,m}(t), r_{I,e}(t)$	The flow rate of commuters in work schedule $I(=R, C)$ entering the bottleneck at time t in the morning and evening
Productivity related	
u_w	The marginal production utility
$N_I^d(t)$	The number of workers on duty at time t in schedule $I(=R, C)$
κ	A constant parameter in u_w indicating the rate of agglomeration
System performance indicators	
π_I	The periodic net profit per worker of firms in schedule $I(=R, C)$
w_I	The periodic wage expenses per worker of firms in schedule $I(=R, C)$
P_I	The periodic productivity per worker in schedule $I(=R, C)$
U_I^J	The daily individual type $J(=A, T)$ utility of workers in schedule $I(=R, C)$, where A, T stand for home-activity utility and travel disutility, respectively
U_I, U_I	The daily individual non-work utility of workers in schedule $I(=R, C)$ during CW workdays and CW off-work days
NU_I^{A-T}	The periodic non-work utility of workers in schedule $I(=R, C)$, where A, T stand for home-activity utility and travel disutility, respectively
NU_I	The periodic net utility of workers in schedule $I(=R, C)$
TNU	The total periodic net utility of all workers

Figure 5-1 A monocentric city with daily home-bottleneck-workplace network.

5.1.1 Basic Setting

Consider a city with a CBD and a residential area connected by a highway with a single bottleneck (Figure 5-1). During workdays, all employees need to commute from home to the workplace in CBD in the morning and from workplace back home in the evening, following their work schedules. The total number of commuters N is assumed to be given.

In terms of work schedule, we make the following assumptions.

Assumption 1: There are only two schedule options available: normal (rigid) schedule and compressed work schedule(CW), i.e., $N = N_R + N_C$, and each firm can adopt one work schedule only. In addition, we assume a fixed normal schedule with invariant $D_R \in \mathbb{Z}^+$ and $H \geq 2\frac{N}{s}$.

Assumption 2: The schedule designs are the same for all those firms adopting the same work schedule, and all workers (normal and CW) share the uniform work start time t_m^* in the morning.

Assumption 3: For each pay period (e.g., one week/ one month), the number of total work hours for each worker is fixed and given, i.e., $D_R H = D_C (H + \Delta H)$.

Assumption 4: The number of workdays in CW is no larger than that in normal schedule, i.e., $D_C \leq D_R$, and $D_C \in \mathbb{Q}^+$ ¹⁷. The daily extended hours of CW is assumed to be no larger than the normal work hours, i.e., $\Delta H \leq H$.

Admittedly, there exist some other flexible work schedules (e.g., staggered hours, flextime), and different firms can have different schedule designs. However, as a preliminary analysis, we make Assumption 1 and 2 for simplification purpose. The lower bound of H is assumed as well such that the length of normal work hours is longer than the durations of the commuting hours¹⁸. In addition, it is reasonable to set the uniform t_m^* as the time schedule is usually tight in the morning and employees work over-time mostly in the evening. At the same time, Assumption 3 and 4 guarantee the nonnegativity and boundedness of ΔH and D_C . Visibly, the following conditions with respect to ΔH and D_C hold based on our assumptions.

$$\Delta H = \frac{D_R - D_C}{D_C} H, \quad (5.1)$$

$$\frac{1}{2} D_R \leq D_C \leq D_R. \quad (5.2)$$

It should be noted that in the extreme case $D_R = D_C$, we have $\Delta H = 0$ and there is no difference between the two schedules in schedule design, or we can say CW does not exist.

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¹⁷ Instead of an integer, D_C is relaxed to be a rational number so that the properties of continuous functions can be applied in later calculations. After all, for every $D_C \in \mathbb{Q}^+ (\notin \mathbb{Z}^+)$, we can always find a $m \in \mathbb{Z}^+$ such that $mD_C \in \mathbb{Z}^+$ is the total number of CW workdays in m per pay periods.

¹⁸ Here the duration of commuting hours is approximated by the total duration for all commuters N to travel roundtrips between home and workplace through the bottleneck at full capacity s .

Conventionally, employees with normal working hours start work at t_m^* and work for H hours per day till $t_{R,e}^*$, D_R days per pay period. Now with CW, despite the same work start time t_m^* in the morning, some employees can enjoy work flexibility via working ΔH extended hours per workday in the afternoon till $t_{C,e}^*$ for the benefit of $D_R - D_C$ fewer workdays. The schedule design of both normal and CW work schedule per pay period is shown in Figure 5-2.

Figure 5-2 Diagram of normal schedule and CW per pay period.

5.1.2 Bi-level decision makings

Now we try to formulate both the employers' and employees' behavior in a bi-level model.

In the lower level, the employees can change neither their job nor work schedule and they seek to maximize their daily utility by changing their commute trip-timing. In the morning, they need to decide the departure time from home as a tradeoff between longer home staying and on-time arrival at work. After work in the evening, they

have to choose the departure time from work, particularly for those with extended hours, where earlier arrival at home means fewer hours' deprivation from after-work recreation. Namely, they need to make their choices of $\{t_h^d, t_w^d\}$ till no employees in the same work schedule can increase their daily non-work utility U_I , U_I ($I = R, C$) (See Section 5.1.3) unilaterally at short-run equilibrium.

In the upper level, CW users can maximize their benefit with optimal schedule design $\{\Delta H, D_C\}$. Further, all employers and employees can decide whether to adopt or work in CW. That is to say, $\{N_R, N_C\}$ are endogenously determined by the long-run choice behavior, with the optimal CW schedule design at that proportion of CW users.

In fact, from employers' perspective, whether to adopt CW in the long run depends on the profit per worker, which is the difference between workers' productivity (See Section 5.1.4) and the corresponding periodic wage expenses, and is defined as,

$$\pi_I = P_I - w_I, \quad (I = R, C). \quad (5.3)$$

From employees' perspective, whether to work in a firm adopting flextime depends on the periodic individual net utility, which combines the wage utility (work-related utility) and the non-work utility, and is defined as,

$$NU_I = w_I + NU_I^{A-T}, \quad (I = R, C), \quad (5.4)$$

where

$$NU_I^{A-T} = D_C U_I + (D_R - D_C) U_I. \quad (5.5)$$

We further made the following assumption regarding employers' behavior.

Assumption 5: All firms produce homogeneous goods under perfect competition and employees with identical skills can switch firms freely.

This assumption suggests that all firms are price takers at long-run equilibrium. Under perfect competition, all of them earn zero profit and thus have no incentive to alter their work schedule choice, where

$$\pi_R = \pi_C = 0, \rightarrow P_R = w_R, P_C = w_C. \quad (5.6)$$

From Eq.(5.4),(5.6), the following equations hold.

$$NU_R = P_R + NU_R^{A-T}, NU_C = P_C + NU_C^{A-T}. \quad (5.7)$$

Eq.(5.7) shows that the individual net utility of employees in the long-run is the sum of his/her productivity and non-work utility. For every given number of $\{N_R, N_C\}$, CW users maximize their individual net utilities with a best CW schedule design, i.e., $\{\Delta H, D_C\} = \arg \max NU_C|_{\{N_R, N_C\}}$, while the normal schedule remains unchanged. And at equilibrium, each employee cannot increase his/her individual net utility by changing the type of firms at which he/she is employed unilaterally. Therefore, in the case when both schedules are adopted at equilibrium, the individual net utilities are equal, regardless of their work schedule, i.e.,

$$NU_R = NU_C. \quad (5.8)$$

At equilibrium, it is also possible to have only one schedule $I^* (= R, C)$ adopted by all employers and employees, wherein

$$NU_{I=I^*} > NU_{I \neq I^*}, \quad (I = R, C). \quad (5.9)$$

It is also noteworthy that the aforementioned model enclosing both employees' and employers' behaviors is similar to those in (Mun et al., 2006; Takayama, 2015).

Before we scrutinize the properties of the bi-level equilibrium in Section 5.2 and 5.3, we first formulate the relevant utilities perceived by employees in Section 5.1.3 and 5.1.4.

5.1.3 Day-long non-work utility

To encapsulate the effects of congestion and home-activity on employees' short-run travel behavior, a day-long activity-travel scheduling model is developed, where the daily individual non-work utility is

$$U_I \text{ (or } U_I) = U_I^A - U_I^T, \quad (I = R, C), \quad (5.10)$$

and it depends on the type of day whether it is U_I for CW workdays or U_I for CW off-work days.

Different from Li et al. (2014)'s activity-based model, we distinguish the work activity from the non-work activity and only consider congestion and home-activity in this section. In fact, we will show in Section 5.1.4 that the daily work-related utility is not affected by the short-run trip timing decisions under mild assumptions.

5.1.3.1 Travel disutility

The total daily disutility of a commuter due to travel $U_I^T, (I = R, C)$, is the sum of the home-to-work and work-to-home travel time costs and the scheduling costs of these round-trip commutes. The following assumptions regarding travel disutility are made in our analysis.

Assumption 6: The free flow travel times from-home-to-work and from-work-to-home are zero without loss of generality. The bottleneck with constant inbound/outbound capacity s follows FIFO. The CBD area is abstracted as a point with all firms located at that point.

Assumption 7: Neither late arrival at work nor early leave from work is allowed so that all employees in the same schedule start and end work at the same time.

Assumption 8: The value of α, β, λ are the same among all homogeneous employees with $\alpha > \beta > 0$, $\alpha > \lambda > 0$ (Small, 1982; de Palma & Lindsey, 2002).

Assumption 6 is made following the standard bottleneck model and its extensions. Assumption 7 is made for the completeness of the daily productivity and the morning and evening commute trips with simplification. In addition, Assumption 8 provides the linkage between VOT and the individual productivity. As the value of time is the monetary amount one employer would be willing to pay to reduce travel time, it can be considered as unit time income, which should be no larger than the maximum unit time productivity.

Therefore, the daily commute disutility for employees departing from home at t_h^d in the morning and from work at t_w^d in the evening, in firms adopting normal and CW schedule is defined as,

$$U_I^T(t_h^d, t_w^d) = \alpha T_m(t_h^d) + \beta \cdot \max\{0, t_m^* - t_h^d - T_m(t_h^d)\} + \alpha T_e(t_w^d) + \lambda \cdot \max\{0, t_w^d - t_{I,e}^*\}, \quad (I = R, C), \quad (5.11)$$

where $t_{R,e}^* = t_m^* + H$ and $t_{C,e}^* = t_m^* + H + \Delta H$.

To note, $T_k(t) = \frac{Q_k(t)}{s}$ ($k = m, e$) is the travel time for departure time t on the highway and $Q_k(t)$ ($k = m, e$) is the queue length at the bottleneck at t in the morning and evening, and should satisfy

$$\frac{\partial Q_k(t)}{\partial t} = \begin{cases} \sum_{I=R,C} r_{I,k}(u) - s, & \sum_{I=R,C} r_{I,k}(u) > s \text{ or } Q_k(t) > 0; \\ 0, & \sum_{I=R,C} r_{I,k}(u) \leq s \text{ or } Q_k(t) = 0, \end{cases} \quad (5.12)$$

where $r_{I,k}(t)$ ($k = m, e$) is the flow rate of commuters in work schedule I entering the bottleneck at time t in the morning or evening.

5.1.3.2 Home-activity utility

The home activities consist of the home activities before work in the morning and after work in the evening. Following Li et al. (2014), we define the marginal home-activity utilities in the morning $u_h(t)$ and in the evening $u_h(t)$, respectively and we fix two times $t=0$ and $t=24$ that represent the start and end of a day. The daily individual home-activity utility can then be expressed as

$$U_I^A(t_h^d, t_w^d) = \int_0^{t_h^d} u_h(t) dt + \int_{t_w^d + T_e(t_w^d)}^{24} u_h(t) dt, \quad (I = R, C), \quad (5.13)$$

where $t_w^d + T_e(t_w^d)$ is the actual home arrival time in the evening after work.

5.1.4 Work-related productivity utility

In Li et al. (2014)'s model, the work utility is modelled as a function of time of a day and work duration without considering the production effect. In fact, the productivity of employees determines not only their work utility but also employers' work schedule choice as discussed in Section 5.1.2. Thus, when it comes to the work schedule analysis, it is necessary to incorporate the productivity effect as in several previous works (Henderson, 1981; Takayama, 2015). The following assumptions regarding productivity are made.

Assumption 9: The individual marginal work utility equals to his/her marginal productivity.

Assumption 10: The agglomeration economy is assumed so that employers prefer to agglomerate the work hours of employees (Henderson, 1981; Takayama, 2015).

Assumption 9 incorporates the productivity effects into employees' work-related utility. After all, as shown in Eq.(5.6), the productivity per worker equals to his/her periodic work-related utility indeed and it is reasonable to make this assumption. Assumption 10 further suggests that the more employees work simultaneously, the higher the productivity they perform.

On this account, we define the marginal work utility as a function of the time of a day t , the number of workers on duty at that moment in schedule $I^* (= R, C)$, $N_{I=I^*}^d(t)$, and the number of workers on duty in the other schedule, $N_{I \neq I^*}^d(t)$, i.e., $u_w(t, N_R^d(t), N_C^d(t))$, with $\frac{\partial u_w(\cdot)}{\partial N_R^d} > 0$ and $\frac{\partial u_w(\cdot)}{\partial N_C^d} > 0$ ¹⁹. Thus, the daily individual work utility (productivity) is $\int_{t_m^*}^{t_{I,e}^*} u_w(t, N_R^d(t), N_C^d(t)) dt$, ($I = R, C$). The periodic individual productivity of worker in schedule I can be further represented as,

$$P_I = D_I \int_{t_m^*}^{t_{I,e}^*} u_w(t, N_R^d(t), N_C^d(t)) dt, I = R, C, \quad (5.14)$$

where $N_R^d(t)$, $N_C^d(t)$ are piecewise step functions based on our assumptions, which are given by,

$$N_R^d(t) = \begin{cases} N_R, & t \in [t_m^*, t_{R,e}^*] \text{ on normal workdays,} \\ 0, & \text{otherwise,} \end{cases} \quad (5.15)$$

$$N_C^d(t) = \begin{cases} N_C, & t \in [t_m^*, t_{C,e}^*] \text{ on CW workdays,} \\ 0, & \text{otherwise.} \end{cases} \quad (5.16)$$

Lemma 5.1: *Both the daily and periodic individual productivity of workers in both schedules P_I ($I = R, C$) are independent of his/her daily trip timing $\{t_h^d, t_w^d\}$.*

Proof. As the work start and end time of both schedules $t_m^*, t_{R,e}^*, t_{C,e}^*$ only depend on the schedule design, i.e., the value of ΔH , which is prescribed in the short-run, the integral of marginal productivity over a prescribed period is independent to the short-run variables evidently. Besides, periodic productivity is the sum of daily productivity

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¹⁹ It should be noted that the workers in one schedule can affect the marginal work utility of those in the other schedule as there is intra-firm productivity effect.

on all workdays D_R, D_C , which are also prescribed and independent of one's $\{t_h^d, t_w^d\}$. Thus, the lemma is proofed. $\square$

To summarize, a bi-level economic model is set up in this section. The daily trip timing $\{t_h^d, t_w^d\}$ of employees in the short-run is determined by their day-long non-work utility U_I or U_I ($I = R, C$). With a given number of CW employees, the individual net utility is maximized with optimal schedule design $\{\Delta H, D_C\}$. The proportion of CW workers, i.e., $\{N_R, N_C\}$, is further determined in the long run equilibrium. In the following two sections, we will elaborate on the equilibrium properties of this bi-level model in both short-run and long-run, respectively.

5.2 Short-run equilibrium

As we focus on the interrelationship between employees' commuting behavior and work performance and employers' choice of work schedule as well as its schedule design, we made the following simplifications hereinafter to make the algebra in the paper much less tedious (but indeed still sufficiently complex) with more revealing insight.

Assumption 11: In terms of travel disutility, we assume $\lambda = \beta$.

Assumption 12: In terms of home-activity utility, the constant marginal utility is assumed with $u_h(t) = u_h(t) = u_h$ ²⁰.

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²⁰ As was stated in (Li et al., 2014), when the non-constant marginal utility is assumed, the closed-form solutions cannot be obtained and a numerical approach, e.g., method of successive averages, is needed. We leave the non-constant marginal utility assumption for future study.

Assumption 13: In terms of productivity, the marginal productivity function is specified as follows (Takayama, 2015),

$$u_w(t, N_R^d(t), N_C^d(t)) = \kappa \cdot (N_R^d(t) + N_C^d(t)), \quad (5.17)$$

where $\kappa > 0$ is a constant parameter indicating the rate of agglomeration.

In other words, $u_w(\cdot)$ in both schedules is linearly increasing with the instantaneous total number of employees on duty, i.e., $N_R^d(t) + N_C^d(t)$. For a same number of employees on duty, the marginal productivity is higher with larger κ . Note that when all employees are on duty during the overlapped hours of both schedules, i.e., $N_R^d(t) + N_C^d(t) = N$, employees are most productive with maximum marginal productivity at κN . On the contrary, when there is only one schedule at work, i.e., during the CW extended hours or on the CW off-work days, employees become less productive due to less agglomeration.

We now investigate the short-run equilibrium on commuters' daily trip timing. Following (Li et al., 2014), we let $q_I(t_h^d, t_w^d)$ denote the number of commuters in schedule $I (= R, C)$ with the departure time choice of $\{t_h^d, t_w^d\}$. As proofed in Lemma 5.1, $q_I(t_h^d, t_w^d)$ does not affect one's daily productivity and hence the short-run daily equilibrium conditions discussed in Section 5.1 can be formulated as a complementarity problem with respect to non-work activity only, as follows.

$$\begin{cases} q_I(t_h^d, t_w^d) [U_I^A(t_h^d, t_w^d) - U_I^T(t_h^d, t_w^d) - U_I(\text{or } U_I)] = 0, & \forall t_h^d, t_w^d, \\ q_I(t_h^d, t_w^d) \geq 0, U_I^A(t_h^d, t_w^d) - U_I^T(t_h^d, t_w^d) - U_I(\text{or } U_I) \geq 0, & I = R, C, \end{cases} \quad (5.18)$$

where $U_I^T(t_h^d, t_w^d)$ and $U_I^A(t_h^d, t_w^d)$ are defined in Eq.(5.11) and (5.13), and $U_I(\text{or } U_I)$ is the daily non-work related utility as defined in Eq.(5.10).

Given the departure pattern $q_I(t_h^d, t_w^d)$ of workers in schedule $I (= R, C)$, $r_{I,m}(t_h^d)$ and $r_{I,e}(t_w^d)$ can further be calculated by

$$r_{I,m}(t_h^d) = \int_{t_{I,e}^*}^{t_{I,e}^1} q_I(t_h^d, t) dt, \forall t_h^d, \text{ and} \quad (5.19)$$

$$r_{I,e}(t_w^d) = \int_{t_{I,m}^0}^{t_{I,m}^*} q_I(t, t_w^d) dt, \forall t_w^d, \quad I (= R, C), \quad (5.20)$$

where $t_{I,m}^0$ and $t_{I,e}^1$ are the times at which the first commuters in schedule $I (= R, C)$ depart in the morning, and the last commuters in that schedule depart in the evening, respectively.

Lemma 5.2 *In both schedules, the employees' departure time choices in the morning and evening, $\{t_h^d, t_w^d\}$, are independent.*

Proof. The independence of morning and evening departure time choices means that there is no interaction between the marginal contributions of t_h^d, t_w^d to the daily utility. Thus, from Eq.(5.11)(5.13), the first-order partial derivatives of the daily non-work utility U_I can be derived with respect to $\{t_h^d, t_w^d\}$ as,

$$\begin{aligned} \frac{\partial U_I}{\partial t_h^d} &= \frac{\partial (U_I^A(\cdot) - U_I^T(\cdot))}{\partial t_h^d} \\ &= u_h + \beta - (\alpha - \beta) \frac{\partial T_m(t_h^d)}{\partial t_h^d}, \quad \forall t_h^d \in [t_{I,m}^0, t_{I,m}^*], \end{aligned} \quad (5.21)$$

$$\begin{aligned} \frac{\partial U_I}{\partial t_w^d} &= \frac{\partial (U_I^A(\cdot) - U_I^T(\cdot))}{\partial t_w^d} \\ &= -u_h - \beta - (\alpha + u_h) \frac{\partial T_e(t_w^d)}{\partial t_w^d}, \quad \forall t_w^d \in [t_{I,e}^*, t_{I,e}^1], \quad I = R, C. \end{aligned} \quad (5.22)$$

With Eq.(5.21)(5.22), we obtain $\frac{\partial U_I}{\partial t_h^d \partial t_w^d} = 0$ readily and it completes the proof. $\square$

It is worth mentioning that Lemma 5.2 still holds even though we relax the Assumption 7, Assumption 10 and Assumption 11 with a time-varying home-activity utility function and different scheduling penalties, as was proofed in Li et al. (2014).

Therefore, at short-run equilibrium, it should satisfy,

$$\frac{\partial U_I}{\partial t_h^d} = 0, \frac{\partial U_I}{\partial t_w^d} = 0. \quad (5.23)$$

In view of Eq.(5.12),(5.21),(5.22) and (5.23), the departure rate of both schedules in the morning and evening can be obtained readily as,

$$r_{R,m}(t_h^d) + r_{C,m}(t_h^d) = \frac{\alpha + u_h}{\alpha - \beta} s, \quad (5.24)$$

$$r_{R,e}(t_w^d) = r_{C,e}(t_w^d) = \frac{\alpha - \beta}{\alpha + u_h} s. \quad (5.25)$$

Obviously, $\frac{\alpha + u_h}{\alpha - \beta} s$ and $\frac{\alpha - \beta}{\alpha + u_h} s$ are constants with $\frac{\alpha + u_h}{\alpha - \beta} s > s > \frac{\alpha - \beta}{\alpha + u_h} s > 0$.

Again, commuters in both schedules share the same VOT and scheduling penalties. In the morning, the departure orders from home are indeterminate as commuters also share the same work start time t_m^* and hence only the total departure rates in all schedules is determinate as in Eq.(5.24). As for the evening peak, commuters in different work schedules have different work end times, and there is no overlap between these two groups of departures whereas both groups have the same value of evening departure rates as in Eq.(5.25).

5.2.1 Equilibrium commute pattern

Similar to standard bottleneck analysis, the possible commute patterns in the morning and evening peak are derived and shown in Figure 5-3 and Figure 5-4.

On CW off-work days, only employees in normal schedule need to commute, and hence, there is only one departure pattern in both morning and evening peak as shown in Figure 5-3 and Figure 5-4 (d). As for the CW workers, these days are the compensated rest time for their over-time working during workdays and they can perceive the full-day home-activity utility without commuting.

On CW workdays, there is only one departure pattern in the morning (Figure 5-3) whereas there are three possible departure patterns (Figure 5-4 (a),(b),(d)) in the evening depending on ΔH , and further on D_R, D_C via substitutions with Eq.(5.1).

The conditions for the occurrence of each pattern are summarized in Table 5-1 and the occurrence domain of stable equilibrium with respect to N_R and D_C is shown in Figure 5-5.

On the morning of CW workdays in
Case (1)-(3)
and CW off-work days

Figure 5-3 Equilibrium commute pattern in the morning peak.

a) On the evening of CW workdays in
Case (1)

b) On the evening of CW workdays in
Case (2)

c) Critical case between in Case (1)&
(2) on the evening of CW workdays

d) On the evening of CW workdays in
Case (3)
and CW off-work days

Figure 5-4 Equilibrium commute pattern in the evening peak.

Table 5-1 Conditions for the occurrence of each pattern in the evening peak depicted in Figure 5-4.

Commute pattern	Conditions with respect to ΔH	Conditions with respect to D_R, D_C
Case 1)	$\Delta H \in \left[\frac{N_R}{s}, H \right]$	$D_C \in \left[\frac{1}{2}D_R, \frac{H}{N_R/s+H}D_R \right]$
Case 2)	$\Delta H \in \left[\Delta H^1, \frac{N_R}{s} \right)$ and $N_R \in (N_R^0, N]$,	$D_C \in \left(\frac{H}{N_R/s+H}D_R, \frac{H}{\Delta H^1+H}D_R \right]$ and $N_R \in (N_R^0, N]$
	$\Delta H \in \left(0, \frac{N_R}{s} \right)$ and $N_R \in [0, N_R^0]$	$D_C \in \left(\frac{H}{N_R/s+H}D_R, D_R \right)$ and $N_R \in [0, N_R^0]$
Case 3)	$\Delta H \in (0, \Delta H^1)$ and $N_R \in (N_R^0, N]$,	$D_C \in \left(\frac{H}{\Delta H^1+H}D_R, D_R \right)$ and $N_R \in (N_R^0, N]$,
	$\Delta H = 0$	$D_C = D_R$

where $\Delta H^1 = \frac{\alpha+u_h}{\alpha-\beta} \frac{N_R - N_R^0}{s}$ with $N_R^0 = \frac{\beta+u_h}{\alpha+u_h} N$ denoting the critical ΔH between Case (2) and (3) for any

$$N_R > N_R^0.$$

Figure 5-5 The occurrence domain of stable equilibrium in the evening peak depicted in Figure 5-4,

with respect to N_R and D_C , where $N_R^0 = \frac{\beta+u_h}{\alpha+u_h} N$ and $\Delta H^1 = \frac{\alpha+u_h}{\alpha-\beta} \frac{N_R - N_R^0}{s}$.

We further distinguish the following three cases of daily equilibrium commute patterns on CW workdays according to the corresponding evening patterns.

Case (1): As shown in Figure 5-3, in the morning all commuters arrive at the workplace no later than t_m^* . As mentioned earlier, during the morning peak period, the exact departure rates of travelers in different work schedule remain unknown.

Nonetheless, as long as the total departure rate from home equals to $\frac{\alpha + u_h}{\alpha - \beta} s$

(Eq.(5.24)), the overall travel pattern is the same and such departure order indeterminacy does not alter the equilibrium utility of commuters in any schedules.

In the evening, since no early departure is allowed, there are a number of commuters in both schedules leaving workplace simultaneously right at the moment of $t_{R,e}^*$ or

$t_{C,e}^*$. After that, commuters in each schedule depart at a constant rate $\frac{\alpha - \beta}{\alpha + u_h} s$. Also,

the extended hours $\Delta H \in \left[\frac{N_R}{s}, H \right]$ of CW is long enough so that the first commuters in CW depart from workplace to enter the bottleneck after all commuters in normal schedule have left the bottleneck and arrived at home already. The departures of the two groups are completely separated with two separate queues.

Case (2): In the morning, the travel pattern is the same as that in Case (1). While in the evening, some commuters still depart from workplace simultaneously at $t_{R,e}^*$ or

$t_{C,e}^*$ in both schedules. However, the extended hours ΔH are shorter and hence the

last commuters in normal schedule arrive at home after the first commuters in CW depart from the workplace. There is only one queue in the evening peak but with double peaks. The critical evening departure pattern between Case (1) and (2) is also depicted in Figure 5-4(c), where the last commuters of normal schedule arrive at home right at the time when the first commuters of CW depart from the workplace,

$$\text{i.e., } \Delta H = \frac{N_R}{s} \left(\text{or } D_C = \frac{H}{N_R / s + H} D_R \right).$$

Case (3): Again, the morning travel pattern is the same as that in Case (1). While in the evening, some commuters in normal schedule still depart from workplace simultaneously at $t_{R,e}^*$, whereas all commuters in CW depart late after $t_{C,e}^*$. This is because of the even shorter extended hours ΔH and the large proportion of normal schedule workers N_R . In fact, from Figure 5-5, it can be observed that Case (3) occurs mostly when N_R and D_C is large enough. Only one queue with single peak exists in the evening peak in this case. The critical evening departure pattern between Case (2) and (3) is similar to that in Figure 5-4(d), with the last normal workers departing right at $t_{C,e}^*$, where $\Delta H = \Delta H^1$, $\left(\text{or } D_C = \frac{H}{\Delta H^1 + H} D_R \right)$.

In Figure 5-5, it is worth noting that in the extreme case when CW does not exist with $\Delta H = 0$ ($D_C = D_R$), the daily equilibrium individual utility should be equal for any $N_R \in [0, N]$. We classify this extreme case into Case (3) as it can be verified that the equilibrium non-work utilities in this case equal to those in Case (3) when $\Delta H = 0$.²¹ Following Eqs.(5.11)-(5.13), (5.23)-(5.25) and the discussions above, we can derive the daily equilibrium non-work utilities readily for each case including travel disutility and the home-activity utility in both CW workdays and off-work days, which are summarized in Table 5-2 and Table 5-3. With some manipulations, here the daily utilities are presented with respect to $\Delta H, N_R$.

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²¹ In fact, Case (2) can not apply to this extreme case for the reason that there is one more additional equilibrium case if early leaving from work is allowed. That is the case when all normal and some CW workers depart early from work but with only one queue and single peak. With the increase of scheduling penalty for early leaving from work, the domain of this case is squeezing. And lastly when no early leaving is allowed with infinitely high penalty, this case no longer exists.

From Table 5-2 and Table 5-3, it is observed that the daily non-work utility is independent of ΔH or N_R and only depends on the number of total commuters N in some cases. Yet it should be pointed out that though ΔH may not affect the short-run daily non-work utility, it can affect the periodic utility since ΔH is determined by D_R, D_C in the end. Besides, from these two tables, it can also be verified that $U_R < U_C$ on CW off-work days, and $U_R < U_C$ in Case (3) on CW workdays. However, for the other cases on CW workdays, it remains unclear which work schedule brings employees higher non-work utility by considering one-day travel pattern only. Indeed, it is the periodic utility that affects an employee's work schedule preference in long-run, as will be discussed later.

In this section, the daily commute patterns of commuters in both schedules on CW workdays and off-work days are examined, and the daily individual non-work utilities are obtained correspondingly. In the next section, we will explore the non-work utility in a longer dimension, i.e., per pay period, and further examine the long-run equilibrium considering the effect of productivity.

Table 5-2 Daily equilibrium non-work utility in patterns depicted in Figure 5-3 and Figure 5-4 on CW off-work days.

Case		Morning	Evening	Daily=Morning + Evening (U_R, U_C)
1)-	R	$u_h^* t_m^* - (\beta + u_h) \frac{N_R}{s}$	$u_h (24 - t_{R,e}^*) - (\beta + u_h) \frac{N_R}{s}$	$u_h (24 - H) - (\beta + u_h) \frac{2N_R}{s}$
3)	C	-		$24u_h$

Note: R and C stand for Normal schedule and CW, respectively.

Table 5-3 Daily equilibrium non-work utility in patterns depicted in Figure 5-3 and Figure 5-4 on CW workdays.

Case		Morning	Evening	Daily=Morning + Evening $(U_R \cdot U_C)$
1)	R	$u_h t_m^* - (\beta + u_h) \frac{N}{s}$	$u_h \left(24 - t_{R,e}^* \right) - (\beta + u_h) \frac{N_R}{s}$	$u_h (24 - H) - (\beta + u_h) \frac{N + N_R}{s}$
	C		$u_h \left(24 - t_{R,e}^* - \Delta H \right) - (\beta + u_h) \frac{N - N_R}{s}$	$u_h (24 - H - \Delta H) - (\beta + u_h) \frac{2N - N_R}{s}$
2)	R		$u_h \left(24 - t_{R,e}^* \right) - (\alpha + u_h) \frac{N_R}{s} + (\alpha - \beta) \Delta H$	$u_h (24 - H) - (\beta + u_h) \frac{N}{s} - (\alpha + u_h) \frac{N_R}{s} + (\alpha - \beta) \Delta H$
	C		$u_h \left(24 - t_{R,e}^* \right) - (\beta + u_h) \frac{N}{s} + \beta \Delta H$	$u_h (24 - H) - (\beta + u_h) \frac{2N}{s} + \beta \Delta H$
3)	R		$u_h \left(24 - t_{R,e}^* \right) - (\beta + u_h) \frac{N}{s}$	$u_h (24 - H) - (\beta + u_h) \frac{2N}{s}$
	C		$u_h \left(24 - t_{R,e}^* \right) - (\beta + u_h) \frac{N}{s} + \beta \Delta H$	$u_h (24 - H) - (\beta + u_h) \frac{2N}{s} + \beta \Delta H$

Note: R and C stand for Normal schedule and CW, respectively.

5.3 Long-run equilibrium

As mentioned in Section 5.1.2, at long-run equilibrium, CW users optimize their work schedule so as to maximize their individual net utilities. Meanwhile, both employers and employees can decide whether to adopt or work in CW as well by evaluating their perceived benefit. To model this upper-level equilibrium, we first summarize the periodic individual net utilities in both schedules combining productivity utility and non-work utility in Section 5.3.1, then determine the equilibrium CW schedule design with given number of $\{N_R, N_C\}$ in Section 5.3.2, and lastly figure out the equilibrium numbers of CW and normal schedule workers in long-run in Section 5.3.3.

5.3.1 Periodic individual net utility

From Section 5.2 , it is observed that there is only one equilibrium case in the morning peaks on CW workdays and on the whole day of CW off-work days. Besides, as will be shown in Section 5.3.1.1, we have only one case of periodic productivity as well. Thus, the same categorization as in the daily commute pattern analysis in Section 5.2 is used to classify the different cases in periodic non-work utility and later the periodic net utility, with the only difference lying in the length of extended hours in the evening peak on CW workdays.

5.3.1.1 Periodic individual productivity

Foremost, following Eq.(5.1), (5.14)-(5.16) and Assumption 12, the periodic individual productivity can be readily calculated by,

$$\begin{aligned} P_R &= (D_R - D_C) H \kappa N_R + D_C H \kappa N \\ &= \kappa H (D_R N_R + D_C (N - N_R)), \end{aligned} \quad (5.26)$$

$$\begin{aligned} P_C &= D_C \Delta H \kappa (N - N_R) + D_C H \kappa N \\ &= \kappa H (D_R (N - N_R) + D_C N_R), \end{aligned} \quad (5.27)$$

where the two terms in Eq.(5.26) represent the normal schedule productivity on CW off-work days and CW workdays respectively, and the two terms in Eq.(5.27) denote the CW productivity during the extended hours and normal hours, respectively. With regard to P_R and P_C , we have the following lemmas and propositions.

Lemma 5.3 *When CW exists, i.e., $D_R > D_C$, the periodic individual productivity of each schedule increases with the number of workers in that schedule, and decreases with the number of workers in the other schedule. The individual productivity reaches the highest when all workers use the same schedule (either CW or normal schedule).*

Lemma 5.4 *When both work schedules are in use, i.e., $N_R, N_C > 0$, the periodic individual productivity of both schedules increases with CW workdays, D_C .*

Proposition 5.1 *When more employees adopt CW than normal schedule, i.e., $N_R < N_C$, the individual productivity of CW is higher than that of normal schedule, and vice versa. The individual productivities in both schedules are equal if and only if $N_R = N_C$.*

In fact, Lemma 5.3 , Lemma 5.4 and Proposition 5.1 can be attributed to the agglomeration assumption in Assumption 10. More workers in a schedule lead to higher individual productivity of that schedule. Due to the symmetry of the productivity functions as in Eqs.(5.26)(5.27), they become equal in the middle. At the same time, more CW workdays indicate more agglomerated work hours of all workers, which also results in higher individual productivity.

Proposition 5.2 *There is positive externality of individual productivity in either schedule regarding the employees in that schedule.*

Proof. The productivity externality in either schedule can be calculated as the difference between the marginal total productivity and the marginal individual productivity of that schedule, as,

$$N_R \frac{\partial P_R}{\partial N_R} = \kappa H N_R (D_R - D_C) \geq 0, \quad (5.28)$$

$$N_C \frac{\partial P_C}{\partial N_C} = \kappa H (N - N_R) (D_R - D_C) \geq 0. \quad (5.29)$$

Evidently, the productivity externalities are positive in both schedules. $\square$

Table 5-4 Periodic individual non-work utility.

Case		NU_R^{A-T}, NU_C^{A-T}
1)	R	$u_h (24-H) D_R - 2(u_h + \beta) \frac{N_R}{s} D_R - (u_h + \beta) \frac{N-N_R}{s} D_C$
	C	$u_h (24-H) D_R - (u_h + \beta) \frac{2N-N_R}{s} D_C$
2)	R	$u_h (24-H) D_R + (\alpha - \beta) H (D_R - D_C) - 2(u_h + \beta) \frac{N_R}{s} D_R - \left((\alpha - \beta) \frac{N_R}{s} + (u_h + \beta) \frac{N-N_R}{s} \right) D_C$
	C	$(24u_h + \beta H) D_R - (\beta + u_h) \left(H + \frac{2N}{s} \right) D_C$
3)	R	$u_h (24-H) D_R - 2(u_h + \beta) \frac{N_R}{s} D_R - 2(u_h + \beta) \frac{N-N_R}{s} D_C$
	C	$(24u_h + \beta H) D_R - (\beta + u_h) \left(H + \frac{2N}{s} \right) D_C$

Table 5-5 First-order partial derivatives of periodic individual non-work utility and periodic

individual net utility in both schedules, with respect to N_R and D_C .

Case		$\frac{\partial NU_R^{A-T}}{\partial N_R}, \frac{\partial NU_C^{A-T}}{\partial N_R}$	$\frac{\partial NU_R^{A-T}}{\partial D_C}, \frac{\partial NU_C^{A-T}}{\partial D_C}$	$\frac{\partial NU_R}{\partial N_R}, \frac{\partial NU_C}{\partial N_R}$	$\frac{\partial NU_R}{\partial D_C}, \frac{\partial NU_C}{\partial D_C}$
1)	R	$-(u_h + \beta) \frac{2D_R - D_C}{s} < 0$	$-(u_h + \beta) \frac{N - N_R}{s} < 0$	$\left(\kappa H - 2 \frac{u_h + \beta}{s} \right) D_R - \left(\kappa H - \frac{u_h + \beta}{s} \right) D_C$	$\left(\kappa H - \frac{u_h + \beta}{s} \right) (N - N_R)$
	C	$(u_h + \beta) \frac{D_C}{s} > 0$	$-(u_h + \beta) \frac{2N - N_R}{s} < 0$	$-\kappa H D_R + (u_h + \beta + \kappa H s) \frac{D_C}{s}$	$-2 \frac{u_h + \beta}{s} N + \left(\kappa H + \frac{u_h + \beta}{s} \right) N_R$
2)	R	$\left((u_h + \beta) - (\alpha - \beta) \right) \frac{D_C}{s}$ $-2(u_h + \beta) \frac{D_R}{s} < 0$	$-(\alpha - \beta) \left(\frac{N_R}{s} + H \right)$ $-(u_h + \beta) \frac{N - N_R}{s} < 0$	$\left(\kappa H - 2 \frac{u_h + \beta}{s} \right) D_R$ $-\left(\kappa H - \frac{u_h + \beta}{s} + \frac{\alpha - \beta}{s} \right) D_C$	$-(\alpha - \beta) H - \left(\frac{u_h + \beta}{s} - \kappa H \right) N$ $+ \left(\frac{u_h + \beta}{s} - \frac{\alpha - \beta}{s} - \kappa H \right) N_R$
	C	0	$-(\beta + u_h) \left(H + \frac{2N}{s} \right) < 0$	$-\kappa H D_R + \kappa H D_C$	$-(u_h + \beta) H - 2 \frac{u_h + \beta}{s} N + \kappa H N_R$
3)	R	$-2(u_h + \beta) \frac{D_R - D_C}{s} \leq 0$	$-2(u_h + \beta) \frac{N - N_R}{s} < 0$	$\left(\kappa H - 2 \frac{u_h + \beta}{s} \right) D_R - \left(\kappa H - 2 \frac{u_h + \beta}{s} \right) D_C$	$\left(\kappa H - 2 \frac{u_h + \beta}{s} \right) (N - N_R)$
	C	0	$-(\beta + u_h) \left(H + \frac{2N}{s} \right) < 0$	$-\kappa H D_R + \kappa H D_C$	$-(u_h + \beta) H - 2 \frac{u_h + \beta}{s} N + \kappa H N_R$

In other words, an additional employee in one schedule makes all employees in that schedule better off with improved productivity. Nonetheless, it does not suffice to say that the workers in the other schedule are also better off in terms of productivity as there is one less employee in one schedule with one additional employee in the other. In fact, in addition to the positive productivity externality, workers also value the non-work utility in their schedule choice, which will be discussed next.

5.3.1.2 Periodic individual non-work utility

In Table 5-4, we next summarize the periodic individual non-work utility as defined in Eq.(5.5), with the results in Table 5-2 and Table 5-3. The first-order partial derivatives with respect to N_R, D_C are also summarized in Table 5-5. With regard to NU_R^{A-T} and NU_C^{A-T} , the following lemmas and propositions hold.

Lemma 5.5 *For normal schedule, the periodic non-work utility decreases with N_R . As for CW, the periodic individual non-work utility either decreases or is invariant with N_C .*

Lemma 5.6 *For both schedules, the periodic non-work utility decreases with D_C .*

Proposition 5.3 *The periodic individual non-work utility of CW is higher than that of normal schedule, when $D_C \in \left(\frac{H}{\Delta H^1 + H} D_R, D_R \right]$ and $N_R \in \left(\frac{\beta + u_h}{\alpha + u_h} N, +\infty \right)$.*

In other words, with regard to the number of users, the more users in one schedule cannot lower the travel disutility in that schedule. With regard to the CW schedule, the reduced workdays can benefit not only the CW users, but also the normal schedule users. In addition, Proposition 5.3 suggests that when CW users are the minority with

short enough extended hours (Case (3)), they enjoy higher non-work utility compared to normal workers, yet bearing lower productivity accordingly.

Proposition 5.4 *For normal schedule, there is negative externality of individual non-work utility for all cases regarding the normal schedule employees. As for CW, there is negative externality of individual non-work utility regarding CW employees when*

$$D_C \in \left[\frac{1}{2} D_R, \frac{H}{N_R / s + H} D_R \right].$$

Proof. The non-work utility externality in either schedule can be calculated as the difference between the marginal total non-work utility and the marginal individual non-work utility of that schedule, as,

$$N_R \frac{\partial NU_R^{A-T}}{\partial N_R} = \begin{cases} -(2D_R - D_C) \frac{(u_h + \beta) N_R}{s} < 0, & \text{Case (1),} \\ -\left(\frac{(2D_R - D_C)(u_h + \beta)}{+D_C(\alpha + \beta)} \right) \frac{N_R}{s} < 0, & \text{Case (2),} \\ -(D_R - D_C) \frac{(u_h + \beta) N_R}{s} \leq 0, & \text{Case (3);} \end{cases} \quad (5.30)$$

$$N_C \frac{\partial NU_C^{A-T}}{\partial N_C} = \begin{cases} -(N - N_R) \frac{D_C (u_h + \beta)}{s} < 0, & \text{Case (1),} \\ 0, & \text{Case (2) \& (3).} \end{cases} \quad (5.31)$$

Apparently, the non-work utility externalities are negative for all cases in normal schedule. For CW, there is negative non-work utility externalities in Case (1), i.e.,

$$\text{when } D_C \in \left[\frac{1}{2} D_R, \frac{H}{N_R / s + H} D_R \right]. \quad \square$$

Therefore, when there are more employees in normal schedule, every normal schedule employee gets worse off in non-work utility as a result of more severe congestion and thereby longer deprivation from home activity. Meanwhile, when there are more CW employees, every CW workers get worse off only if the two departure clusters are totally separated as in Case (1) in Figure 5-4. For other cases,

the change in CW employees does not affect the non-work utility in CW as shown in Table 5-4.

So far, both the periodic productivity and non-work utility are formulated, and at equilibrium it is all about their balance. Simply by summing up the productivity P_R, P_C stated in Eq.(5.26), (5.27) and the non-work utilities NU_R^{A-T}, NU_C^{A-T} listed in Table 5-4, we can obtain the periodic individual net utility NU_R, NU_C for each equilibrium cases right away. We omit the summary of periodic individual net utility to save place, but still summarize their first order derivatives with respect to N_R, D_C in Table 5-5 for reference. With NU_R, NU_C , the equilibrium schedule design of CW and employees' work schedule choices in long-run equilibrium can be determined.

5.3.2 Equilibrium schedule design

Now we turn to address the question on how the CW users optimize their work schedule with the purpose of benefit maximization. That is to find the optimal value of D_C with a prescribed N_R .

From Eq.(5.6) and (5.7), we learn that the firms have zero profit under perfect competition in the long-run and hence only NU_C , which CW users aim to maximize, is considered in the determination of optimal work schedule. The equilibrium number of CW workdays $D_C^*(N_R)$ in long-run is the D_C leading to the maximum of NU_C at that given N_R . Based on the results of NU_C in Section 5.3.1, there are three $D_C^*(N_R)$ in different zones depending on the value of N_R and κ , and we have the following lemma.

Figure 5-6 Equilibrium number of CW workdays $D_C^*(N_R)$, with respect to N_R and κ , where

$$N_R^{ab} = \frac{2(u_h + \beta)}{\kappa H s + (u_h + \beta)} N \quad \text{and} \quad N_R^{bc} = \frac{(u_h + \beta)(2N + Hs)}{\kappa H s}.$$

Lemma 5.7 Given $N_R \in [0, N]$, the equilibrium number of workdays in CW is dependent of the given N_R , and can be determined as follows,

$$D_C^*(N_R) = \begin{cases} \frac{1}{2} D_R, & \text{when } N_R \in [0, \min\{N_R^{ab}, N\}], \\ \frac{H}{N_R / s + H} D_R, & \text{when } N_R \in \left(\min\{N_R^{ab}, N\}, \min\{N_R^{bc}, N\} \right), \\ D_R & \text{when } N_R \in \left(\min\{N_R^{bc}, N\}, N \right], \end{cases} \quad (5.32)$$

where $N_R^{ab} = \frac{2(u_h + \beta)}{\kappa H s + (u_h + \beta)} N$ and $N_R^{bc} = \frac{(u_h + \beta)(2N + Hs)}{\kappa H s}$. Besides, when

$N_R = N_R^{ab}$ or $N_R = N_R^{bc}$, CW users are indifferent to their schedule design if N_R is given.

In fact, it can be verified that N_R^{ab} and N_R^{bc} are the critical N_R leading to

$$\frac{\partial N U_C}{\partial D_C} = 0 \quad \text{of different equilibrium cases and there is } N_R^{ab} < N_R^{bc} \quad \text{and,}$$

$$\min \{N_R^{ab}, N\} = \begin{cases} N_R^{ab}, & \text{when } \kappa > \kappa_{ab}, \\ N, & \text{when } \kappa \leq \kappa_{ab}, \text{ and} \end{cases} \quad (5.33)$$

$$\min \{N_R^{bc}, N\} = \begin{cases} N_R^{bc}, & \text{when } \kappa > \kappa_{bc}, \\ N, & \text{when } \kappa \leq \kappa_{bc}, \end{cases} \quad (5.34)$$

where $\kappa_{ab} = \frac{u_h + \beta}{Hs}$ and $\kappa_{bc} = \frac{(2 + Hs/N)(u_h + \beta)}{Hs}$ with $\kappa_{ab} < \kappa_{bc}$.

More intuitively, Figure 5-6 depicts $D_C^*(N_R)$ with respect to N_R and κ . Apparently, all the equilibrium $D_C^*(N_R)$ are the boundaries of the different equilibrium cases discussed in Section 5.3.1. When N_R is smaller than N_R^{ab} in Zone (a), CW users minimize D_C at equilibrium. This is the scenario when the non-work utility decrease surpasses the productivity utility increase, for one additional CW workday. More staggered work hours between the two groups (larger ΔH , i.e., smaller D_C) lead to higher NU_C . When N_R is between N_R^{ab} and N_R^{bc} in Zone (b), CW users reach equilibrium at the critical case between Case (1) and (2), in which the evening commute pattern is depicted in Figure 5-4(c). This is the scenario when the dominant type of utility is unclear. At equilibrium, the productivity utility is maximized without commuting intervention in the evening peak. Lastly, when N_R is larger than N_R^{bc} in Zone (c), CW users maximize D_C till $D_C = D_R$ at equilibrium and hence CW no longer exists. That is the scenario when the non-work utility decrease falls behind the productivity utility increase, for one additional CW workday. More agglomerated work hours between the two groups (smaller ΔH , i.e., larger D_C) result in higher NU_C . It should be noted that with increasing κ , N_R^{ab} and N_R^{bc} are getting smaller and smaller, and it becomes more often when CW users

prefer not to stagger their work schedule as the productivity utility becomes dominant. At the extreme case when $\kappa \rightarrow +\infty$, there are $N_R^{ab} \rightarrow 0$ and $N_R^{bc} \rightarrow 0$, and all commuters use the same normal schedule at equilibrium. Moreover, it is necessary to clarify the equilibrium cases depicted in Figure 5-5 and the equilibrium CW workdays' zones depicted in Figure 5-6, where the former is the result of the difference in short-run commuting pattern, and the latter is the consequence of long-run schedule optimization of CW users.

5.3.3 Equilibrium schedule choice

Now that the number of CW workdays at equilibrium is determined, we next examine the one last question in equilibrium, i.e., the equilibrium number of CW and normal schedule users. As mentioned earlier, whether to adopt or work in a schedule in the long run depends on the individual net utilities. More and more employers and employees will choose the schedule with higher individual net utilities, until either the net utilities of the two schedules become equal or no one chooses the other schedule.

Therefore, we compare the individual net utilities of the two groups, wherein CW users have optimized their schedule design. Namely, we compare $NU_R|_{D_C=D_C^*(N_R)}$ and $NU_C|_{D_C=D_C^*(N_R)}$, and have the following proposition.

Proposition 5.5 *At long-run equilibrium when CW users have optimized their work schedule, there can be multiple equilibria depending on the value of κ and the initial N_R , and the equilibrium number of normal schedule workers N_R^* are given as follows,*

$$N_R^* = (\text{or } \in) \begin{cases} N_R^a, & \text{when } \kappa \in (0, \kappa_{ab}), \\ 0, & \text{when } \kappa \in [\kappa_{ab}, +\infty), \\ N_R^b, & \text{when } \kappa \in [\kappa_b, +\infty), \\ [N_R^{bc}, N], & \text{when } \kappa \in [\kappa_{bc}, +\infty), \end{cases} \quad (5.35)$$

To visualize Proposition 5.5, Figure 5-7 delineates the trend of long-run equilibrium with arrows and the equilibrium number of employees in normal schedule with solid blue highlights. In Figure 5-7, the gray arrows show the equilibrium trends inside that area and the black arrows specify the trends on the boundaries. For instance, the gray down-arrow in Zone (a) indicates that for any N_R inside the area ABCFED, the net utility of CW is higher and N_R will decrease until the equilibrium is reached along the curves ABC. In addition, even though the equilibrium CW schedules in Zone (a) and (b) are different as discussed in Section 5.3.2, the two black down-arrows on the two sides of curve DE imply that employers tend to switch to CW around this curve. That is to say, for any $\kappa \in (\kappa_{ab}, \kappa_b)$, the N_R originally located in Zone (b) will continue to decrease to Zone (a) and finally reach equilibrium with $N_R = 0$. Particularly, the double-headed arrows in Zone (c) indicate that the net utilities in both schedules are equal regardless of N_R and hence the equilibrium is achieved for any N_R in the area HIJ (shaded in blue).

Therefore, it can be observed from Figure 5-7 that different values of κ lead to different schedule choices at equilibrium. When $\kappa < \kappa_{ab}$, the long-run equilibrium is unique and both schedules are in use with less than a quarter of workers in normal schedule. This is the scenario when the rate of agglomeration is too low, and the non-work utility dominates in the net utility of both schedules. Thus, following Lemma 5.5, the net utilities in both schedules decrease with their number of users and intersect at $N_R = N_R^a$. When κ is getting larger with $\kappa_{ab} \leq \kappa \leq \min\{\kappa_{bc}, \kappa_b\}$, all workers use CW at long-run equilibrium. This is the scenario when the rate of agglomeration is getting large and the productivity starts to dominate in the net utility of CW, while the productivity in normal schedule cannot cover the larger commute

disutility yet due to additional workdays. The net utility of normal schedule is smaller than that of CW for all $N_R \in [0, N]$ and thus $N_R = 0$ at equilibrium. Note that $\min\{\kappa_{bc}, \kappa_b\}$ depends on the value of H and N . When κ is getting even larger with $\kappa > \min\{\kappa_{bc}, \kappa_b\}$, there are multiple equilibria with different N_R . For very small N_R , the agglomeration effect in productivity is prominent in CW and no one chooses normal schedule at equilibrium. For larger N_R , there is no dominant work schedule and the equilibrium is obtained at $N_R = N_R^b$ (when $\kappa > \kappa_b$). If N_R is large enough, CW no longer exists at long-run equilibrium, and it amounts to that all workers use normal schedule

To sum up, we scrutinize the long-run work schedule equilibrium in this section. By formulating the periodic individual net utility of both schedules, the positive productivity externality and negative congestion externality are explicitly determined. For CW users, there are three possible CW schedule designs at equilibrium such that their individual net utility is maximized. Based on these optimal CW schedule designs, the equilibrium schedule choices are further determined. It is found that there can be multiple equilibria depending on the agglomeration rate and the initial schedule choices.

5.4 Concluding remarks

This chapter presents a preliminary yet complete economic analysis of the compressed work schedule (CW). With the expected benefit to reduce commute trips per pay period significantly, CW, however, is less common compared to other flexible work arrangements. Our work then attempts to investigate commuters' work schedule choice and the corresponding travel behavior in the presence of CW. The

employees' productivity, commuting behavior, as well as work-family balance are fully embedded in a modified activity-based model. A bi-level model encapsulating employers' and employees' choices on their work schedule is also developed, with explicit consideration of externalities resulted from productivity, travel disutility, and home-activity utility.

In the short run, the equilibrium commute patterns are different with different non-work utility depending on the schedule design of the two schedules. At long-run equilibrium, while CW users can maximize their utility with three different optimal schedule design, the normal schedule remains unchanged. With the optimal CW schedule, the equilibrium schedule choices of employers and employees are further determined, and there exist multiple equilibria depending on the agglomeration rate and the initial proportions of users in different schedules. Interestingly, when the initial case with all normal schedule workers is considered, it is socially beneficial to introduce CW if and only if the agglomeration rate is low. Otherwise, the CW becomes trashy in the case of high agglomeration rate as all workers would prefer normal schedule at long-run equilibrium. This can partially explain the less popularity of CW among flexible work arrangements.

CHAPTER 6

BOTTLENECK CONGESTION AND WORK START TIME DISTRIBUTION CONSIDERING HOUSEHOLD TRAVELS

While the previous chapter investigates the CW in commute traffic management, we now turn to look at another more popular flexible work arrangement, the flextime/staggered work hours. More importantly, we incorporate the commuters with multiple trip purposes. This chapter aims to examine the interrelationship between bottleneck congestion and work starting time choice behavior considering household travels. Following Mun et al. (2006), we assume that employees can choose their work starting time freely within a specified period (flex-interval) if flextime/SWH are offered. A bi-level economic model is developed enclosing positive production externalities and negative congestion externalities with commuters of multiple trip purposes. Both the short-term equilibrium with within-day travel behavior, and the long-term equilibrium with work schedule choices are analyzed. The question on how household travelers differ from the conventional individual travelers in work schedule choice is being addressed. We further inspect the effects of different flextime design on different travelers' work schedule choice and determine the system optimum in both total commute cost minimization and total net benefit maximization.

The rest of this chapter is organized as follows. Section 6.1 presents the model framework incorporating the production and traffic congestion effect of mixed travelers. The short-term equilibrium is examined in Section 6.2 , followed by the long-term equilibrium discussed in Section 6.3 . Next, the optimal group

compositions of firms adopting flextime are studied in Section 6.4 . Finally, Section 6.5 wraps up the paper with concluding remarks.

6.1 Model formulation

The following notations are used throughout the process of model formulation.

<i>Parameters</i>	
N	The total number of commuters
N_H, N_I	The equilibrium number of household and individual commuters, respectively
μ	The proportion of household commuters, $\mu \in [0,1]$.
t^s	The uniform school start time
t^w	The work start time of normal(rigid) schedule workers
α	Value of time
β	Unit time scheduling penalty of early arrival
s	Constant service capacity of the bottleneck
H	Total hours of work per day
θ	The length ratio of the actual flex-interval to the longest flex-interval, $\theta \in (0,1]$
Δ	Length of the flex-interval for flextime commuters
<i>Variables</i>	
N^r, N^f	The equilibrium number of commuters in normal work schedule and flextime, respectively
$N^i(t)$	The number of workers on duty in work schedule $i(=r, f)$ at time t
v_H, v_I	The proportions of workers adopting normal work schedule among household and individual travelers respectively, $v_H, v_I \in [0,1]$
N_J^i	The equilibrium number of group $J(=H, I)$ commuters in work schedule $i(=r, f)$
π^i	The profit per worker per day of firms in work schedule $i(=r, f)$
F^i	The output per worker per day of firms in work schedule $i(=r, f)$
ω^i	The wage per worker per day of firms in work schedule $i(=r, f)$

u_J^i	The individual net income of group $J(=H,I)$ commuters in work schedule $i(=r,f)$
P_J^i	The aggregated travel cost of group $J(=H,I)$ commuters in work schedule $i(=r,f)$
δ_J^i	The net benefit per worker of group $J(=H,I)$ commuters in work schedule $i(=r,f)$
$y(t)$	The instantaneous output at time t for normal schedule worker
κ	A constant parameter in $y(t)$
N_f^e	The number of flextime commuters who arrive at workplace early before the start of flex-interval $t^w - \Delta$
$T(t)$	The travel time on the highway from home to workplace prior to parking for vehicle departing at t
$r_J^i(t)$	The flow rate of group $J(=H,I)$ commuters in work schedule $i(=r,f)$ entering the bottleneck at time t
$p_J^i(t)$	Generalized travel cost of group $J(=H,I)$ commuters in work schedule $i(=r,f)$ departing at t

6.1.1 Basic setting

Consider a city with a CBD and a residential area connected by a highway with a single bottleneck, and there is a school in between after the bottleneck (Figure 6-1). Commuters travel from home to workplace in CBD in the morning and from workplace back home in the evening. The total number of commuters N is assumed to be given.

In terms of trip purpose, two groups of commuters are considered: individual and household commuters. While individual travelers N_I need to drive to workplace only with a desired arrival time/interval depending on one's work schedule, there is another portion of commuters N_H , the household travelers. They need to first send their children to school before the fixed school starting time t^s and then reach workplace in the morning, and pick them up from school before home in the evening. Each vehicle is assumed to take only one worker and one child at most. Namely, the number of vehicles on road equals the total number of commuters, and we have $N = N_H + N_I$.

Figure 6-1 A monocentric city with home-bottleneck-school-workplace network.

In terms of work schedule, it is assumed that there are only two schedule options available: normal (rigid) work hours and flextime. Conventionally, employees with normal working hours start work at a fixed time point t^w ($t^w > t^s$) and work for H $\left(H > 2 \frac{N}{s}\right)$ hours per day²², where s is the service capacity of the single bottleneck.

Now with flextime, some employees can enjoy flexibility in work starting time selection. Assume the flextime schedule is the same for all firms adopting it. Flextime commuters choose to start work freely before the latest work starting time within the

²² The daily work hour H is assumed to be longer than the total time of both morning and evening commuting periods.

flex-interval, where its length is defined by Δ ($\Delta < H$) and the maximum length of Δ is $\frac{N}{s}$. Let $\theta = \frac{\Delta}{N} s$, where $\theta \in (0, 1]$. Obviously, $\Delta = \frac{\theta N}{s}$ reaches a maximum when $\theta = 1$. It is also assumed that once the flextime commuters reach the workplace within the flex-interval, they will start work immediately. Early arrivals need to wait till the flex-interval. The length of work hours per day H remains unchanged and is the same as that of normal schedule workers. Therefore, the core hours where all employees must be at work is the difference between the possible earliest work quitting time and the possible latest work starting time, i.e., $H - \Delta$. We further assume no late arrival is allowed for any types of workers and the latest work starting time for flextime workers coincides with the work starting time of normal workers. In other words, all employees must be at work after t^w . Thus, the flex-interval starts at $t^w - \Delta$. And the number of workers adopting normal schedule and flextime are denoted by N^r and N^f respectively. Evidently, $N = N^r + N^f$.

Let $\mu \in [0, 1]$ denote the proportion of household travelers, i.e., $N_H = \mu N$, thus $N_I = (1 - \mu)N$. Further, we let $\nu_H, \nu_I \in [0, 1]$ denote the proportion of workers adopting normal work schedule among household and individual travelers respectively. Hence, the number of workers of each group in each type of work schedule can be derived readily, where

$$N_H^r = \nu_H N_H = \nu_H \mu N, \quad (6.1)$$

$$N_H^f = (1 - \nu_H) N_H = (1 - \nu_H) \mu N, \quad (6.2)$$

$$N_I^r = \nu_I N_I = \nu_I (1 - \mu) N, \quad (6.3)$$

$$N_I^f = (1 - \nu_I) N_I = (1 - \nu_I) (1 - \mu) N. \quad (6.4)$$

6.1.1.1 Employers and employees' behavior

Now we try to formulate the work starting time choice in a bi-level model. From employers' perspective, whether to adopt flextime depends on the workers' productivity and the corresponding wage expenses. All firms aim to maximize their profit per worker, which is defined as

$$\pi^i = F^i - w^i, \quad (6.5)$$

where F and w is the output and wage per worker per day respectively and $i(=r, f)$ indicates the adopted work schedule (r represents normal schedule, f represents flextime).

On the other hand, from employees' perspective, whether to work in a firm adopting flextime depends on the net income, and each worker seeks to maximize it, which is defined as

$$u_J^i = w^i - P_J^i, \quad (6.6)$$

where $J(=H, I)$ indicates the traveler group (household or individual travelers) and P_J^i represents the travel cost of group J commuters working in firms adopting schedule i at short-run equilibrium.

Further, it is assumed that firms produce homogeneous goods under perfect competition and employees with identical skills can switch firms freely. Therefore, at long-run equilibrium, each employee cannot increase his or her net income by changing the type of firms at which he/she is employed unilaterally. The daily commute patterns remain consistent. In addition, all firms are price takers with zero profit at equilibrium, and thus they have no incentive to alter their work schedule arrangement. That is

$$\pi^r = \pi^f = 0, \rightarrow F^r = w^r, F^f = w^f, \quad (6.7)$$

$$u_H^r = u_H^f, u_I^r = u_I^f. \quad (6.8)$$

Therefore, for group J commuters, in the case when both schedules are adopted at equilibrium, the individual net benefits of that type of employees are equal, regardless of the employer type (normal schedule or flextime), i.e.,

$$\delta_J^r = F^r - P_J^r = F^f - P_J^f = \delta_J^f. \quad (6.9)$$

Note that at equilibrium it is possible to have only one schedule i^* adopted for group J commuters. In that case, there is

$$\delta_J^{i=i^*} = F^{i=i^*} - P_J^{i=i^*} > F^{i \neq i^*} - P_J^{i \neq i^*} = \delta_J^{i \neq i^*}. \quad (6.10)$$

The long-run equilibrium will be elaborated in more details in Section 6.3 .

6.1.2 Productivity Effect

Firms agglomerate spatially and temporally in the CBD areas to boost the agglomeration economies. One typical assumption in the literature on temporal agglomeration is that the unit time output per worker is independent of daily working hours and increases with the number of workers on duty at that time unit (Henderson, 1981; Takayama, 2015). Namely, in terms of productivity, it is assumed that there is positive externality to agglomerate work hours of employees. Following this, the individual instantaneous output at time t for normal schedule worker is defined as an increasing function of the total number of workers on duty at time t , $N^r(t) + N^f(t)$, and is given as

$$y(t) = \kappa \cdot f\left(N^r(t) + N^f(t)\right), \quad (6.11)$$

where $\kappa > 0$ is a constant parameter and $f(\cdot)$ is a shift factor representing agglomeration economy with $f'(\cdot) > 0$. Note that when all commuters are on duty,

i.e., $N^r(t) + N^f(t) = N$, the instantaneous output $y(t)$ reaches maximum equal to $\kappa f(N)$. The individual outputs per day for workers with normal schedule are then given as follows.

$$F^r = \int_{t^w}^{t^w+H} y(t) dt = \kappa \int_{t^w}^{t^w+H} f(N^r(t) + N^f(t)) dt. \quad (6.12)$$

As for flextime workers, there is intra-firm productivity effect, where the output of firms adopting flextime may be lower when some workers are not present during flex-intervals. For simplicity, we assume that the intra-firm productivity effect roots in the total labor input only, i.e., the number of total workers on duty. Therefore, the individual instantaneous output at time t for flextime workers is defined in the same form as in Eq.(6.11). Admittedly, the daily individual outputs of different flextime users differ depending on one's work starting time. However, from employer's perspective, it is the total productivity of all workers in flextime that matters, and undifferentiated wages are paid to all homogeneous workers. Given the undifferentiated wages, the net benefit perceived by the same group users in flextime remains unchanged, regardless of one's actual output. After all, it is also difficult to evaluate individual productivity only by one's work starting time in real life. On this account, we take the average output as the individual output of workers in flextime, as follows.

$$F^f = \frac{1}{N^f} \left(N^e \cdot \int_{t^w-\Delta}^{t^w-\Delta+H} y(t) dt + \sum_{t'}^{t''} \left(\partial(N^f(t^a)) \int_{t^a}^{t^a+H} y(t) dt \right) dt^a \right), \quad (6.13)$$

where N^e represents the number of flextime commuters who arrive at workplace early before the start of flex-interval $t^w - \Delta$, and t', t'' denote the first and last work starting time of a continuous period within the flex-interval, in which flextime

workers start work continuously. Moreover, $\partial(N^f(t))$ represents the number of flextime workers who start to work at time t , and thus $\partial(N^f(t^a)) \int_{t^a}^{t^a+H} y(t) dt$ is the total daily output of commuters starting work at t^a . To note, $N^f(t'') - N^f(t')$ is the number of flextime commuters starting work from t' to t'' . Multiple t' and t'' may exist depending on the actual arrival pattern at workplace.

6.1.3 Commute Cost

The daily departure pattern is determined by work schedules as well as the school schedule. To better investigate the dynamics of congestion effect resulting from the two work schedules on the two groups of commuters, the bottleneck model (Vickrey, 1969) is applied. The bottleneck with capacity s follows FIFO. The CBD area is abstracted as a point with all firms located at that point, and the school is abstracted as a point right after the bottleneck. Without loss of generality, free-flow travel time between home and school, and between school and workplace are assumed to be zero (Liu et al., 2016b).

The value of time α is assumed to be same among all homogeneous employees, and it is the monetary amount one employer would be willing to pay to reduce travel time, which can be considered as unit time income. In our setting, as the individual wage equals to individual output at equilibrium and the maximum unit time output is $\kappa f(N)$, there is $\alpha \leq \kappa f(N)$, where the equality sign holds if and only if no employer adopts flextime. The schedule penalty for a unit time of early arrival β is then assumed to be less than α (Small, 1982), i.e., $\kappa f(N) \geq \alpha > \beta > 0$. We further

denote $\rho = \frac{\beta}{\kappa f(N)}$ to represent the relative schedule penalty compared to the

instantaneous individual output ($0 < \rho < 1$). Over the course of morning peak, individual commuters travel from home to workplace and determine his/her departure time depending on the queuing cost and the scheduling cost of early arrival at workplace. As for household travelers, they make a composite decision choice on their departure time considering both work and school trips. They need to first arrive at school before t^s and then arrive at workplace no later than t^w . Following (Jia et al., 2016; Liu et al., 2016b), the value of time and the unit time scheduling penalty for the child in household travels are simply assumed to be the same as those for employees, i.e., α, β . Furthermore, the commute costs for household employees combine the travel costs of both the child's school trip and parent's work trip.

Therefore, the commute costs for individual and household employees in firms adopting normal work schedule departing at time t are

$$p_I^r(t) = \alpha \cdot (T(t)) + \beta \cdot \max\{0, t^w - t - T(t)\}, \quad (6.14)$$

$$p_H^r(t) = \alpha \cdot (T(t)) + \beta \cdot \max\{0, t^s - t - T(t)\} + \alpha \cdot (T(t)) + \beta \cdot \max\{0, t^w - t - T(t)\}, \quad (6.15)$$

where $p_H^r(t)$ encapsulates both the school trip cost on the first line and the work trip cost on the second line. Similarly, the commute costs for individual and household employees in firms adopting flextime departing at time t are

$$p_I^f(t) = \alpha \cdot (T(t)) + \beta \cdot \max\{0, (t^w - \Delta) - t - T(t)\}, \quad (6.16)$$

$$p_H^f(t) = \alpha \cdot (T(t)) + \beta \cdot \max\{0, t^s - t - T(t)\} + \alpha \cdot (T(t)) + \beta \cdot \max\{0, (t^w - \Delta) - t - T(t)\}. \quad (6.17)$$

To note, $T(t) = \frac{D(t)}{s}$ is the travel time for departure time t on the highway and

$D(t)$ is the queue length at the bottleneck at t and should satisfy

$$\frac{\partial D(t)}{\partial t} = \begin{cases} \sum_{\substack{i=r,f \\ J=H,I}} r_j^i(u) - s, & \sum_{\substack{i=r,f \\ J=H,I}} r_j^i(u) > s \text{ or } D(t) > 0; \\ 0, & \sum_{\substack{i=r,f \\ J=H,I}} r_j^i(u) \leq s \text{ or } D(t) = 0, \end{cases} \quad (6.18)$$

where $r_j^i(t)$ is the flow rate of group J commuters in work schedule i entering the bottleneck at time t .

One may notice that even for flextime, when employees arrive at workplace before $t^w - \Delta$, there is work related scheduling cost. When previous study assumes that flextime users have zero scheduling cost (Mun et al., 2006), this study relaxes this assumption and tries to investigate the effect of length of flex-interval on commuters' travel behavior. After all, commuters with other trip purposes (e.g., household travels), may not be able to arrive at workplace within the flex-interval, and hence bear certain scheduling penalty, particularly when the school starting time is too early, or the flex-interval is too short.

When considering household travels, schedule coordination has been discussed in the literature, and it is found that proper coordination between work and school schedule can achieve minimization of total commute cost (Liu et al., 2016b; Zhang et al., 2017). Since our focus is on the attractiveness of flexible work schedules to household and individual travelers, it is reasonable to assume that the gap between school starting time and work starting time of normal schedules workers, i.e., $t^w - t^s$, is fixed and given as,

$$t^w - t^s = \frac{N_I}{s}. \quad (6.19)$$

This gap value is the optimal gap to minimize total travel cost in the case when all commuters adopt normal schedule only, and it is determined following (Liu et al., 2016b). For comparisons in following chapters, we consider the case when no commuter adopts flextime as the benchmark case, in which the rigid schedule

difference is already optimized, further reduction in total travel cost mainly results from flextime only. Since no late arrivals for school and work are allowed in our setting, Eq.(6.19) can be easily obtained from Eq.(13) in (Liu et al., 2016b) by setting the unit time late arrival penalty $\gamma \rightarrow +\infty$.

The preceding discussion focused on travel cost during morning peak hours. As for evening peak, following previous studies (Vickrey, 1973; Hurdle, 1981; Hurdle et al., 1983; de Palma et al., 2002), it is assumed that one's travel cost in evening peak equals to his/her travel cost in the morning. In fact, similar to morning peak, household travelers who will pick up their children before going back home in the evening, also need to bear additional cost related to their children (e.g., travel time cost, late pick-up penalties), while the individual travelers still don't need to. Besides, it applies to both morning and evening peak hours that the travel cost of flextime workers is generally smaller than the travel cost of normal schedule workers, as the former can choose their work starting/leaving time freely to avoid the peak hours. Therefore, only the morning peak is examined in what follows.

6.2 Short-run equilibrium

In this section, the questions on how the work start time distribution affects congestion travel time are addressed with the analysis of the equilibrium commute pattern. In the short run, employees cannot change their firms so that the numbers of individual and household workers in normal work schedule, i.e., ν_H, ν_I are given.

Based on our definitions, $N_H^r, N_H^f, N_I^r, N_I^f$ can be derived. Commuters in different groups seek to minimize their travel cost only and at equilibrium travel cost of the same group in same work schedule equals.

Since no late arrival is allowed for school and work, derived from within-day travel equilibrium and Eqs.(5.11)-(5.12), the equilibrium arrival rates at the bottleneck for individual and household commuters with normal work schedule are equal and given by

$$r_I^r = r_H^r = \frac{\alpha}{\alpha - \beta} s > s. \quad (6.20)$$

As for flextime, the equilibrium arrival rates at the bottleneck for individual commuters who arrive workplace before and within the flex-interval, respectively, are given by

$$r_I^f = r_I^r; \quad r_I^p = s. \quad (6.21)$$

Similarly, the equilibrium arrival rates at the bottleneck for household commuters who arrive workplace before and within the flex-interval are

$$r_H^f = r_H^r; \quad r_H^p = \frac{2\alpha}{2\alpha - \beta} s. \quad (6.22)$$

One may refer to (Arnott et al., 1990b; Liu et al., 2016b) for details derivation of Eqs.(6.20)-(6.22). Apparently, $r_I^r = r_H^r = r_I^e = r_H^e > r_H^p > r_I^p = s$. With the above formulations, the possible dynamic commute patterns at user equilibrium are derived and shown in Figure 6-2 depending on ν_H, ν_I and θ . Note that while Figure 6-2 delineates the arrivals at the bottleneck (solid black lines) and the departures from the bottleneck (dashed black lines), it also depicts the equilibrium work starting time before t^w (solid red lines) of travelers in both normal and flextime schedules. As zero free-flow travel time between school and workplace is assumed²³, the departure time from bottleneck coincides with the arrival time at school and workplace. This is

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²³ The case with positive free flow travel time has been briefly discussed in (Liu et al., 2016b)

also the work start time for flextime workers if they arrive at workplace within the flex-interval. We will discuss the work starting time and its corresponding productivity in Section 6.3 with more details.

As it is assumed that the school and work starting time difference has been optimized for no flextime case ($\mu=0$), we also note that the two clusters of travels (individual and household) are separated entirely. The time gap $t^w - t^s$ is large enough for the bottleneck to digest all the individual flows and individual travelers will depart only after t^s . Thus, there are four commute patterns at equilibrium. The conditions for the occurrence of each pattern are summarized in Table 6-1.

Case a): In this case, the flex-interval $\Delta \in \left(0, \frac{N_I^r}{s}\right]$. All household travelers N_H

arrive at school and work early, and all individual travelers with flextime N_I^f arrive at workplace before the flex-interval. From definitions, it should satisfy $\theta + \mu < 1$

and $v_I \geq \frac{\theta}{1-\mu}$. As for individual travelers with normal schedule, some of them arrive

at workplace before the flex-interval. It is worth mentioning that the arrival orders earlier than flex-interval of household and individual travelers with different work schedule are indeterminate. Namely, during the period from t^0 to t^s and from t^s to $t^w - \Delta$, the exact departure rates of different types of travelers remain unknown.

However, as long as the total arrival rate at bottleneck equals to

$r_I^r = r_H^r = r_I^f = r_H^f = \frac{\alpha}{\alpha - \beta} s$, the overall traffic pattern will be the same, and hence the

equilibrium travel cost is determinate. In fact, the commute pattern in Case a) is identical to the case when all travelers adopt normal work schedule, i.e.,

$v_H = 1, v_I = 1$.

Case b): In this case, the flex-interval $\Delta \in \left(\frac{N_I^r}{s}, \frac{N_I}{s} \right]$. All household travelers N_H

arrive at school and work early while some individual travelers with flextime arrive at workplace in the flex-interval. From definitions, it should satisfy $\theta + \mu < 1$ and

$\nu_I < \frac{\theta}{1-\mu}$. All individual travelers with normal schedule arrive at workplace later

than the ones with flextime. In other words, the arrival order is determinate. The

critical case between Case a) and b) occurs when $\nu_I = \frac{\theta}{1-\mu}$, where last individual

travelers with flextime arrive at workplace right at the start of flex-interval.

Case c): In this case, the flex-interval $\Delta \in \left(\frac{N_I}{s}, \frac{N - N_H^f}{s} \right]$. All individual travelers

N_I arrive at bottleneck within the flex-interval, and all household travelers with

flextime N_H^f arrive at workplace before the flex-interval. From definitions, it should

satisfy $\theta + \mu > 1$ and $\nu_H \geq \frac{\theta + \mu - 1}{\mu}$. Likewise, though the arrival order before the

flex-interval of household travelers is indeterminate, the overall commute pattern will

still be the same as $r_I^e = r_H^e = \frac{\alpha}{\alpha - \beta} s$.

Case d): In this case, the flex-interval $\Delta \in \left(\frac{N - N_H^f}{s}, \frac{N}{s} \right]$. All individual travelers

N_I as well as the household travelers with normal schedule N_H^r arrive at bottleneck

within the flex-interval while some household travelers with flextime N_H^f arrive at

workplace before the flex-interval. From definitions, it should satisfy $\theta + \mu > 1$ and

$v_H < \frac{\theta + \mu - 1}{\mu}$. The arrival order is determinate and all household travelers with

normal schedule arrive at school and workplace later than those with flextime. When

$v_H = \frac{\theta + \mu - 1}{\mu}$, it is the critical case between Case c) and d), where last household

traveler with flextime departs from the bottleneck right at the start of flex-interval.

One should also notice that the queue has dissipated in case c) and d) during the

departures of individuals with flextime. As $t^w - t^s$ is predetermined by the total

number of individual travelers, the individuals adopting flextime can arrive at

workplace within the flex-interval without incurring any queuing cost and their

minimum arrival rate is s ²⁴.

Note that from Eq. (6.19), when $\theta + \mu = 1$, $t^w - t^s = \frac{N_I}{s} = \Delta$. The flex-interval start

right at the school starting time and the commute pattern is the same as the one in

Case c).

Following Eqs. (5.11)-(6.17) and the discussion above, we can derive the equilibrium

commute cost for each case including schedule penalty and queuing delay costs

readily and it is summarized in Table 6-2. With some manipulations, here the

commute cost is presented with respect to $v_H, v_I, \mu, \theta, \beta, N$ and s .

Detailed derivations of Table 6-2 is presented as follows. For Case a), the commute

cost in the morning $P_H^f|_{\text{Morning}} = \beta \frac{N_H}{s} + \beta \left(\frac{N_H}{s} + t^w - \Delta - t^s \right)$,

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²⁴ If the arrival rate is lower than s , the bottleneck cannot service all the individual commuters N_I within the time gap between t^s and t^w , the individual commuters left have to depart earlier before t^s traveling together with household commuters which increase their travel cost and will choose to travel after t^s at equilibrium, until the arrival rate become s .

$$P_H^r|_{\text{Morning}} = \beta \frac{N_H}{s} + \beta \left(\frac{N_H}{s} + t^w - t^s \right) \quad , \quad P_I^f|_{\text{Morning}} = \beta (t^w - \Delta - t^s) \quad \text{and}$$

$P_I^r|_{\text{Morning}} = \beta \frac{N_I}{s}$. To note, the first terms in $P_H^f|_{\text{Morning}}, P_H^r|_{\text{Morning}}$ are the commute costs of the school trip and the second terms are the costs of work trip. For Case b),

the commute costs of household commuters $P_H^f|_{\text{Morning}}, P_H^r|_{\text{Morning}}$ are the same as

those in Case a). As for individual commuters, $P_I^f|_{\text{Morning}}$ is the same as that in Case

a), $P_I^r|_{\text{Morning}} = \beta \left(\frac{N_I^r}{s} + t^w - \Delta - t^s \right)$. For Case c), $P_H^f|_{\text{Morning}}, P_H^r|_{\text{Morning}}$ are the same as

those in Case a) as well, and $P_I^f|_{\text{Morning}} = 0, P_I^r|_{\text{Morning}} = \beta \frac{N_I^r}{s}$. Lastly for Case d),

$P_H^f|_{\text{Morning}}$ is the same as that in Case a), and

$$P_H^r|_{\text{Morning}} = \beta \left(\frac{N_H + N_H^r}{s} + t^w - \Delta - t^s \right) + \beta \left(t^w - t^s + \left(\frac{N_H + N_H^r}{s} + t^w - \Delta - t^s \right) \right), \text{ while}$$

$P_I^f|_{\text{Morning}}, P_I^r|_{\text{Morning}}$ are the same as those in Case c). As the evening commute cost is

assumed to be the same as that in the morning, Table 6-2 is derived after mathematical manipulations.

As shown in Table 6-2, it is obvious that in Case a), b) and c), there is $P_H^r > P_H^f$, and

in Case a), c) and d), there is $P_I^r > P_I^f$ (with $\mu < 1$ and $\theta > 0$). Since $0 \leq v_I < \frac{\theta}{1-\mu}$

in Case b) and $0 \leq v_H < \frac{\theta + \mu - 1}{\mu}$ in Case d), one can also easily verify that $P_I^r > P_I^f$

in Case b) and $P_H^r > P_H^f$ in Case d). The equilibrium commute cost of travelers with

normal schedule is generally higher than that with flextime as intuited for any groups of travelers.

Figure 6-2 Possible equilibrium commute patterns and work starting time patterns.

Table 6-1 Conditions for the occurrence of each pattern depicted in Figure 6-2.

Equilibrium	Conditions		N^f
Case a)	$\theta + \mu < 1$	$v_I \geq \frac{\theta}{1-\mu}$	$(1-v_I + (v_I - v_H)\mu)N$
Case b)		$v_I < \frac{\theta}{1-\mu}$	$(1-\theta - v_H\mu)N$
Case c)	$\theta + \mu = 1$		$\mu(1-v_H)N$
	$\theta + \mu > 1$	$v_H \geq \frac{\theta + \mu - 1}{\mu}$	
Case d)		$v_H < \frac{\theta + \mu - 1}{\mu}$	$(1-\theta)N$

Table 6-2 Equilibrium commute cost of different types of travelers in patterns depicted in Figure 6-2.

Traveler type	Schedule Case / Work	a)	b)	c)	d)
Household	Normal	$P_H^r = 2(1 + \mu) \frac{\beta N}{s}$			$P_H^r = 2(3 - \mu - 2\theta + 2\mu v_H) \frac{\beta N}{s}$
	Flextime	$P_H^f = 2(1 + \mu - \theta) \frac{\beta N}{s}$			
Individual	Normal	$P_I^r = 2(1 - \mu) \frac{\beta N}{s}$	$P_I^r = 2((1 - \mu)(1 + v_I) - \theta) \frac{\beta N}{s}$	$P_I^r = 2(1 - \mu) v_I \frac{\beta N}{s}$	
	Flextime	$P_I^f = 2(1 - \mu - \theta) \frac{\beta N}{s}$			$P_I^f = 0$

6.3 Long-run equilibrium

Now we turn to the long-run equilibrium, where the employers can alter their offered work schedule and employees can select the corresponding employers freely to maximize their net benefit. We will first elaborate on the equilibrium conditions in Section 6.3.1 and focus on several corner cases in Section 6.3.2.

6.3.1 Equilibrium conditions

As discussed in Section 6.1.1.1, each commuter in the group J (household or individual) cannot increase his/her net benefit $\delta_J^i = F^i - P_J^i$ by changing the type of firms with work schedule i (normal schedule or flextime) unilaterally at equilibrium. Since there are four different commute patterns at short-run equilibrium depending on v_H, v_I, θ and μ , it is necessary to first investigate the productivity of each commute pattern case by case separately.

Referring to Figure 6-2, we focus on the red lines which show the work starting time distribution and therewith derive the corresponding individual output. It is worth noting that different commute patterns only affect the work starting time of flextime workers and all the normal schedule workers will still start work at t^w . Moreover, the employers value the total output only and the wage is identical across the commuters with the same work schedule and is independent of the commuter's group. As delineated in Figure 6-2, the work starting time distributions vary across the four cases.

For Case a), though the arrival order is found indeterminate indeed in some periods as discussed in Section 6.2, such indeterminacy will not affect the distribution of work starting time because in this case all flextime workers arrive workplace before the flex-interval and will start work at $t^w - \Delta$.

For Case b), all household workers and some individual workers with flextime $N_H^f + N_I^f$ start work at $t^w - \Delta$ while some individual flextime workers N_I^f arrive workplace within the flex-interval and start work at a constant rate s .

For Case c), similarly all household workers with flextime N_H^f start work at $t^w - \Delta$. In this case, all individual flextime workers N_I^f start work at the constant rate s .

Lastly for Case d), the arrival order of different groups in different schedule is determinate. As can be seen in Figure 6-2 (d), a part of household workers with flextime start work at $t^w - \Delta$, followed by the other part of household workers with flextime who start work at the constant rate s . After a break interval without any employees starting work (during which the household workers with normal schedule arrive at workplace), the individual workers with flextime will start work with the constant rate s .

Hereinafter, we also specify the forms of function in productivity effects in Eq.(6.11) as $f(x) = x$. Based on Eqs (6.11)-(6.13) and the work starting time distribution depicted in Figure 6-2, the individual daily output can be derived and is summarized in Table 6-3. From Table 6-3, it can be verified that the individual output of worker with the normal schedule is not always larger than that with flextime depending on the value of ν_H, ν_I .

The derivation of Table 6-3 with details is shown below. While the daily individual output of commuters in normal work schedule is easy to derive following Eq.(6.12), we focus on the individual output of flextime commuters. For commuters in flextime, as shown in Eq.(6.13), the average individual output combines the output of both the arrivals earlier than the start of flex-interval N^e_f , and the continuous arrivals within the flex-interval $N^f(t'') - N^f(t')$. Again, the commuters N^e_f start (and leave) work at the same time and the derivation of their output is similar to that in normal work schedule. Attention is then paid to the continuous arrivals $N^f(t'') - N^f(t')$ and in what follows, the average output of arrivals during time interval $[t', t'']$ is derived.

As shown in Figure 6-3, there are continuous arrivals at workplace (work start) between t' and t'' with arrival rate s , and hence their leaving from work is between $t' + H$ and $t'' + H$ with identical leaving rate s . In a trial when the interval $[t', t'']$ is discretized into three unit sub-intervals, the individual output of arrivals at the first sub-interval is the sum output of zone A1, B1, C1, D and A2, where D is the identical output during $[t'', t' + H]$ shared by all arrivals within $[t', t'']$. Similarly, the output of the second interval is the sum output of zone B1, C1, D, A2 and B2 and lastly the output of the third interval is the sum output of zone C1, D, A2, B2 and C2. The total output is $(A1 + 3 \times A2) + (2 \times B1 + 2 \times B2) + (3 \times C1 + C2) + 3 \times D$. In addition,

it should be noted that the cumulative work-start commuters at any t equal to the cumulative work-end commuters at $t + H$. Thus, after mathematical manipulations,

we can derive the average output as $\frac{2(t''-t')(9N+2(t''-t')s)}{27}$. Likewise, when

$[t', t'']$ is discretized into n sub-intervals, the average output of these commuters

$\frac{(1+n)(t''-t')(3nN+(n-1)(t''-t')s)}{6n^2}$. When $n \rightarrow +\infty$, there is

$$\begin{aligned} & \frac{1}{N^f(t'') - N^f(t')} \int_{t'}^{t''} \left(\partial \left(N^f(t^a) \right) \int_{t^a}^{t^a+H} y(t) dt \right) dt^a \\ &= \frac{(t''-t')(3N+(t''-t')s)}{6} \end{aligned} \quad (6.23)$$

With Eq.(6.23), we can derive the average output of commuters in flextime.

Figure 6-3 The average output of continuous arrivals at workplace during time interval $[t', t'']$.

Table 6-3 Daily individual output of different types of travelers in patterns depicted in Figure 6-2.

Case	Normal Schedule (F^r)	Flextime (F^f)
a)	$\kappa HN + (\varepsilon \theta) \frac{\kappa N^2}{s}$	$\kappa HN + (-(\varepsilon+1)\theta) \frac{\kappa N^2}{s}$
b)	$\kappa HN + \frac{1}{2} \left(\theta(\theta+2\mu v_H-2)+(1-\mu)^2 v_I^2 \right) \frac{\kappa N^2}{s}$	$\kappa HN + \frac{1}{6\varepsilon} \left(\begin{aligned} & -4\theta^3+(1-\mu)^2 v_I^2(6-9\mu v_H-5v_I+5\mu v_I) \\ & -6\theta(\mu v_H-1)(\mu(v_H+v_I)-v_I) \\ & +3\theta^2(2+v_I-\mu(3v_H+v_I)) \end{aligned} \right) \frac{\kappa N^2}{s}$

c)	$\kappa HN + \frac{1}{2} \left(\frac{\mu(2\theta(v_H-1) + (\mu-2)(v_I^2-1))}{+v_I^2-1} \right) \frac{\kappa N^2}{s}$	$\kappa HN + \frac{1}{6\varepsilon} \left(\frac{\mu(-2(6+\mu(4\mu-9))-6\theta(2+\mu(v_H-2))(v_H-1)+9(\mu-1)^2 v_H)}{+2-3v_I+3\mu(3+(\mu-3)\mu+2\theta(-1+\mu+v_H-\mu v_H))v_I} \right) \frac{\kappa N^2}{s}$
d)	$\kappa HN + \frac{1}{2} \left(\frac{(\theta-2)\theta+(\mu-1)^2 v_I^2}{+\mu v_H(2+\mu(v_H-2))} \right) \frac{\kappa N^2}{s}$	$\kappa HN + \left(\frac{-1}{6\varepsilon} \right) \left(\frac{17+2\varepsilon^3+\theta^3-18\mu+3\mu^3+3\theta^2(\mu-1)(v_I-1)+3\theta(-2+(\mu+v_I-\mu v_I)^2)}{+3(\mu-1)(12+(\mu-10)\mu)v_I-15(\mu-2)(\mu-1)^2 v_I^2} \right) \frac{\kappa N^2}{s}$

where $\varepsilon = -1 + \mu v_H + (1 - \mu)v_I$. One can verify that when $0 \leq v_H, v_I \leq 1$, there is $-1 \leq \varepsilon \leq 0$.

So far, we have already established work schedule choice model and derived the equilibrium commute cost and individual output as well. The net benefit $\delta_J^i = F^i - P_J^i$ of each group of commuters with each work schedule in different cases can be derived readily. Again, commuters are assumed to choose their work schedule deterministically by comparing the net benefit (utility) of each work schedule. The higher the net benefit of a work schedule is, the more commuters switch to that schedule.

To better investigate travelers' decision-making process between the two work schedules, we now define a function $D_J(v_H, v_I)$ with respect to $v_H, v_I \in [0, 1]$, to represent the difference of net benefit in the group $J (= H, I)$ between normal schedule and flextime, where

$$D_J(v_H, v_I) = \delta_J^r - \delta_J^f. \quad (6.24)$$

Evidently, when $D_J(v_H, v_I) > 0$, adopting normal schedule can enjoy higher net benefit and commuters in the group J will stick to normal schedule, i.e., $v_J \nearrow$. On the contrary, when $D_J(v_H, v_I) < 0$, it is more beneficial to adopt flextime and

commuters in the group J will switch to flextime, i.e., $v_J \searrow$. From Table 5-1, multiple cases and therewith multiple forms of $D_J(v_H, v_I)$ exist depending on the value of θ and μ and in fact $D_J(v_H, v_I)$ is a piecewise function with respect to v_H or v_I . For instance, when $\theta + \mu < 1$, only Case a) and b) occur. Thus, the derivations of $D_J(v_H, v_I) = \delta_J^r - \delta_J^f = (F^r - P_J^r) - (F^f - P_J^f)$ first follow Case b) when $0 \leq v_I < \frac{\theta}{1-\mu}$ and later follow Case a) when $\frac{\theta}{1-\mu} \leq v_I \leq 1$. Similarly, $D_J(v_H, v_I)$ when $\theta + \mu > 1$ can also be derived.

To find the equilibrium number of commuters adopting flextime, it suffices to find the stable points within a virtual 1×1 unit square, where v_H and v_I are the x and y axis. However, the stability conditions vary depending on the locations of the equilibrium points: inside the square, at the four corners, and on the four boundaries. Foremost, we examined the case when the equilibrium falls inside the square, i.e., both work schedules are adopted in both groups of commuters at equilibrium. From Eq.(5.8), it should be fulfilled that $v_A, v_B \quad D_H(v_A, v_B) = 0$, which are the necessary conditions to attain the equilibrium at point $(D_J(v_A, v_B) = 0)$. Nevertheless, such conditions are not sufficient to guarantee stability.

Lemma 6.1 *Assume $v_A, v_B \in (0, 1)$. Point $(v_H, v_I) = (v_A, v_B)$ is a stable equilibrium point, if and only if, it satisfies that, $D_J(v_A, v_B) = 0$ and*

$$\frac{\partial D_J(v_A, v_B)}{\partial v_J} < 0, \text{ where } J = H, I.$$

Proof. This lemma can be proved by contradiction. As mentioned above, when $D_J(v_H, v_I) > 0$, v_J increases; when $D_J(v_H, v_I) < 0$, v_J decreases. Suppose at point (v_A, v_B) with $D_H(v_A, v_B) = 0$, there is $\left. \frac{\partial D_H(v_H, v_I)}{\partial v_H} \right|_{v_H=v_A, v_I=v_B} \geq 0$. Obviously, we have $\lim_{x \rightarrow 0} D_H(v_A + x, v_B) \geq 0$ and $\lim_{x \rightarrow 0} D_H(v_A - x, v_B) \leq 0$ ($x \in \mathbb{R}^+$). In other words, increasing v_H from v_A to $v_A + x$ leads to the relative net benefit increase of normal work schedule compared to flextime, and hence v_H increases. Contrarily, decreasing v_H from v_A to $v_A - x$ leads to the relative net benefit increase of flextime, and hence v_H decreases. The equilibrium is not stable. Similarly, we have the same contradiction for v_I . Therefore, for every equilibrium inside the unit square, Lemma 6.1 guarantees its stability.

Proposition 6.1 *When $\theta \in (0, 1 - \mu]$, there is no equilibrium solution when both schedules are adopted by both household and individual commuters, i.e., $N_H^r, N_H^f, N_I^r, N_I^f > 0$.*

Proof. When $\theta \in (0, 1 - \mu]$, only Case a) or b) may occur. For Case a), by solving

$$D_J(v_H, v_I) = 0 \quad (J = H, I), \rightarrow v_I = \frac{1 - 2v_H\mu + \rho}{2 - 2\mu}. \text{ Substituting the } (v_H, v_I) \text{ into}$$

$$\frac{\partial D_J(v_H, v_I)}{\partial v_J}, \quad \text{there are} \quad \frac{\partial D_H(v_H, v_I)}{\partial v_H} = \frac{2\theta\mu\kappa N^2}{s} > 0 \quad \text{and}$$

$$\frac{\partial D_I(v_H, v_I)}{\partial v_I} = \frac{2\theta(1 - \mu)\kappa N^2}{s} > 0, \text{ which contradict with Lemma 6.1.}$$

Likewise, for Case b), from $D_J(v_H, v_I) = 0$, there is $v_H = \frac{1-2\theta+\rho}{2\mu}$, $v_I = \frac{\theta}{1-\mu}$ and

hence $\frac{\partial D_H(v_H, v_I)}{\partial v_H} = \frac{2\theta\mu\kappa N^2}{s} > 0$, which also contradicts with Lemma 6.1.

Thus, there is no internal equilibrium $v_H, v_I \in (0,1)$ when $\theta \in (0, 1-\mu]$. $\square$

Later we will further show in Corollary 6-2 that when $\theta \in (0, 1-\mu]$, the equilibrium can only be obtained at the two corners of the unit square. At least one group of commuters all adopt the same type of work schedule.

6.3.2 Corner and boundary solutions

Next, let's move on looking at the four corners as well as the boundary cases in details. While the stable equilibrium inside the unit square should satisfy the conditions listed in Lemma 6.1, the equilibrium points on the boundaries are not necessary to follow those conditions. Instead, according to Eq.(5.9), we can have $D_J(v_H, v_I) > 0 (< 0)$ when $v_J = 1 (= 0)$, where $J = H$ or I . It suffices to the cases when one work schedule is still superior to another even though all workers within the same group adopted it, for at least one group of commuters.

For the four corners cases, we have the following Lemma 6.2.

Lemma 6.2 Assume binary variables $v_A, v_B \in \{0,1\}$. Point $(v_H, v_I) = (v_A, v_B)$ is a stable point at user equilibrium, if and only if, it satisfies that

$$\left\{ \begin{array}{l} (v_J - 0.5)D_J(v_A, v_B) > 0 \quad (J = H, I); \\ \text{or } D_J(v_A, v_B) = 0 \text{ and } \frac{\partial D_J(v_A, v_B)}{\partial v_J} < 0 \quad (J = H, I), \quad \forall v_A, v_B \in \{0,1\}. \end{array} \right. \quad (6.25)$$

Proof. The first row of the condition above is equivalent to $D_J(v_A, v_B) > 0$ if $v_J = 1$; and $D_J(v_A, v_B) < 0$ if $v_J = 0$ ($J = H, I$). From Eq.(5.9), $D_J(v_H, v_I) > 0 (< 0)$ has immediately guaranteed the stability of the equilibrium since v_J has reached the maximum(minimum). The second row covers the occasions when the net benefits of both work schedules are equal coincidently at the corner, and its stability should be guaranteed by similar conditions listed in Lemma 6.1.

As for the boundary equilibrium, i.e., either v_H or v_I reaches the boundary, it is, in fact, a relaxation of Lemma 6.2 altering the domain of one of the independent variables from two discrete points $\{0,1\}$ to a continuous interval $[0,1]$. Analogously, we have the following lemma.

Lemma 6.3 Assume v_A is a binary variable $\{0,1\}$ and $v_B \in [0,1]$. Point $(v_H, v_I) = (v_A, v_B)$ (or (v_B, v_A)) is a stable point at user equilibrium, if and only if, it satisfies both the two conditions presented below.

1.

$$\left\{ \begin{array}{l} (v_A - 0.5)D_H(v_A, v_B) > 0 \text{ (or } (v_A - 0.5)D_I(v_B, v_A) > 0); \\ \text{or } D_H(v_A, v_B) = 0 \text{ and } \frac{\partial D_H(v_A, v_B)}{\partial v_H} < 0 \left(\text{or } D_I(v_B, v_A) = 0 \text{ and } \frac{\partial D_I(v_B, v_A)}{\partial v_I} < 0 \right), \forall v_A \in \{0,1\}; \end{array} \right.$$

2.

$$D_I(v_A, v_B) = 0 \text{ and } \frac{\partial D_I(v_A, v_B)}{\partial v_I} < 0 \left(\text{or } D_H(v_B, v_A) = 0 \text{ and } \frac{\partial D_H(v_B, v_A)}{\partial v_H} < 0 \right), \forall v_A \in \{0,1\}.$$

Proof. Condition one is in the similar form of Eq.(6.25) to assure the equilibrium points fall on the boundaries and condition two is in the same form of Lemma 6.1 to determine the exact location of the equilibrium point. One can refer to Lemma 6.1

and Lemma 6.2 for the reasoning. Note that when $v_B = 0, 1$ at extreme cases, it is equivalent to the equilibrium at the four corners.

Following Lemma 6.2 and Lemma 6.3, it is now possible to examine the stability of boundary equilibrium more specifically.

6.3.2.1 Corner solutions

We first look at the case when both household and individual workers adopt normal work schedule only, i.e., $N_H^r = N_H$ and $N_I^r = N_I$ ($v_H = 1, v_I = 1$). As mentioned in Section 6.2, in this case, the commute pattern follows Case a) in Figure 6-2, but the work starting time pattern does not so.

Proposition 6.2 *The situation in which all commuters adopt the normal work schedule, i.e., $N_H^r = N_H$ and $N_I^r = N_I$, can be realized at equilibrium, if and only if,*

$$\rho < 0.5 \left(\beta < \frac{1}{2} \kappa N \right).$$

For the case when both household and individual workers adopt flextime only, i.e., $N_H^f = N_H$ and $N_I^f = N_I$ ($v_H = 0, v_I = 0$), we have the following proposition.

Proposition 6.3 *The situation in which all commuters adopt flextime, i.e., $N_H^f = N_H$ and $N_I^f = N_I$ can be realized at equilibrium, if and only if,*

$$\theta \in [0, 2(1-\mu)) \quad \text{or} \quad \theta \in [2(1-\mu), 1] \quad \text{and} \quad \rho < \rho^{d*}, \quad \text{where}$$

$$\rho^{d*} = \frac{\theta^3 - 3\theta^2\mu + 3\theta(\mu^2 + 2\mu - 1) + 3(1-\mu)^2(1+\mu)}{12(\theta + 2\mu - 2)}.$$

The occurrence domain of stable equilibrium at $N_H^f = N_H$ and $N_I^f = N_I$ with respect to μ and θ is shown in Figure 6-4. From Figure 6-4, there is Corollary 6-1.

Corollary 6-1 When $\rho \leq \frac{1}{3}$, the situation in which all commuters adopt flextime can always be realized at equilibrium.

As for the other two cases when $N_H^f = N_H$ and $N_I^r = N_I$ ($\nu_H = 0, \nu_I = 1$) or $N_H^r = N_H$ and $N_I^f = N_I$ ($\nu_H = 1, \nu_I = 0$), we have the following proposition.

Proposition 6.4 It can never be realized at equilibrium in the situation that all commuters in the same group (household or individual) adopt the same work schedule, and meanwhile, the adopted work schedules of the two groups are different, i.e., $N_H^f = N_H$ and $N_I^r = N_I$ or $N_H^r = N_H$ and $N_I^f = N_I$.

Proof of Proposition 6.2-4. For $(\nu_H, \nu_I) = (1, 1)$, only Case a) and c) may occur. For

$$\text{Case a), } D_H(1, 1) = D_I(1, 1) = \frac{\theta(0.5 - \rho)\kappa N^2}{s} . \quad \text{When } \rho < 0.5 ,$$

$$D_H(1, 1), D_I(1, 1) > 0 . \quad \text{For Case c), } D_H(1, 1) = \frac{\theta(0.5 - \rho)\kappa N^2}{s} ,$$

$$D_I(1, 1) = \frac{(1 - \mu)(0.5 - \rho)\kappa N^2}{s} . \quad \text{Again when } \rho < 0.5 , D_H(1, 1), D_I(1, 1) > 0 \text{ and}$$

thus $(\nu_H, \nu_I) = (1, 1)$ is stable. Proposition 6.2 is proved.

For $(\nu_H, \nu_I) = (0, 0)$, only Case b) and d) may occur. For Case b),

$$D_H(0, 0) = -\frac{(6 - 9\theta + 4\theta^2 + 12\rho)\theta\kappa N^2}{6s} , \quad D_I(0, 0) = -\frac{(6 - 9\theta + 4\theta^2)\theta\kappa N^2}{6s} . \quad \text{One}$$

may verify that $D_H(0, 0), D_I(0, 0) < 0$ for all $\theta \in (0, 1 - \mu)$. As for Case d),

$$D_H(0, 0) = \frac{2(\theta + 2\mu - 2)(\rho - \rho^{d*})\kappa N^2}{s} , \quad \text{where } \rho^{d*} \text{ is defined as in Proposition}$$

6.3. When $\theta < 2 - 2\mu$, $\rho^{d*} < 0$ and hence $D_H(0, 0) < 0$. When $\theta \geq 2 - 2\mu$, if

$\rho < \rho^{d*}$, $\rightarrow D_H(0,0) < 0$. In addition,

$$D_I(0,0) = -\frac{(\theta^3 - 3\theta^2\mu + 3(1-\mu)^2(1+\mu) + 3\theta(\mu(2+\mu)-1))\kappa N^2}{6s} . \text{ It can be}$$

verified that $D_I(0,0) < 0$ for all $\theta \in (1-\mu, 1)$. Proposition 6.3 is proved.

For $(\nu_H, \nu_I) = (1, 0)$, Case b) and c) may occur. For Case b),

$$D_H(1,0) = -\frac{\theta(4\theta^2 + 3\theta(4\mu-3) + 6(1-\mu)(1-2\mu+2\rho))\kappa N^2}{6s(1-\mu)} ,$$

$$D_I(1,0) = -\frac{\theta(6-9\theta+4\theta^2-18\mu+12\theta\mu+12\mu^2)\kappa N^2}{6s(1-\mu)} . \text{ It can be verified that the}$$

conditions when $D_H(1,0) > 0$ and $D_I(1,0) < 0$ cannot hold simultaneously. For

$$\text{Case c), } D_H(1,0) = -\frac{(1-5\mu+4\mu^2+12\theta\rho)\kappa N^2}{6s} , D_I(1,0) = -\frac{(1-\mu)(1-4\mu)\kappa N^2}{6s} .$$

Again $D_H(1,0) > 0$ and $D_I(1,0) < 0$ cannot hold simultaneously. Thus, equilibrium cannot be attained at $(\nu_H, \nu_I) = (1, 0)$.

Lastly for $(\nu_H, \nu_I) = (0, 1)$, Case a) and d) may occur. In Case a)

$$D_H(0,1) = D_I(0,1) = \frac{\theta(1-2\mu-2\rho)\kappa N^2}{s} . \text{ It is impossible to have}$$

$$D_H(0,1)D_I(0,1) < 0 . \text{ For Case d)}$$

$$D_H(0,1) = -\frac{\left(\theta^3 - 3\theta^2\mu + 3\theta(-1+\mu^2+\mu(2-4\rho)) + (1-\mu)(2+5\mu^2+\mu(-7+24\rho))\right)\kappa N^2}{6\mu s} , \text{ and}$$

$$D_I(0,1) = -\frac{\left(\theta^3 - 3\theta^2\mu + 3\theta(-1+2\mu+\mu^2) + (1-\mu)(2+5\mu^2+\mu(-7+12\rho))\right)\kappa N^2}{6\mu s} . \text{ It can also be verified that}$$

$D_H(0,1) < 0$ and $D_I(0,1) > 0$ cannot hold simultaneously. Thus, equilibrium cannot be attained at $(\nu_H, \nu_I) = (0,1)$. Proposition 6.4 is proofed. $\square$

In Proposition 6.2-6.4, the stability of the equilibrium at four corner cases is examined. Foremost, Proposition 6.2 implies that when all firms adopt normal work schedule initially, in the cases with relatively small unit time scheduling penalty of early arrival, firms no longer have incentives to switching their offered work schedule to flextime, regardless of the proportions of household workers and the length of the flex-interval. In Proposition 6.3, it is also found that the equilibrium can be attained in most cases when all firms adopt flextime unless the flex-interval is long and the proportion of household workers is high. Lastly, Proposition 6.4 indicates that it is impossible to reach equilibrium at the other two corners without other policy intervention. The agglomeration of work schedule of any group of commuters (household or individual) has attracted the commuters in the other group to select that work schedule so as to maximize their individual net benefit.

Figure 6-4 The occurrence domain of stable equilibrium with respect to μ and θ .

6.3.2.2 Boundary solutions

Now we explore the cases when the equilibrium solutions fall on the boundaries. Different commute and work starting time pattern may occur in one boundary depending on the values of $\mu, \theta, \nu_H, \nu_I$. For example, when the equilibrium is obtained with all individual workers adopting flextime, i.e., $N_I^f = N_I$ ($\nu_I = 0$), Case b), c) and d) can occur. Following the stability conditions stated in Lemma 6.3, we have Proposition 6.5.

Proposition 6.5 *When $\theta \in (0, 1 - \mu]$, it cannot be realized at equilibrium when all travelers in the same group (households or individuals) adopt one uniform work schedule unless all the travelers in the other group adopt the same schedule.*

Proof. To prove Proposition 6.5, it suffices to prove the equilibrium cannot be attained at the boundaries when $\theta \in (0, 1 - \mu]$. For the boundary at $\nu_I = 0$, only Case b) may occur. According to Lemma 6.3, if there is one equilibrium $(\nu_H, 0)$ on the boundary (except the corner point), it should satisfy that $D_H(\nu_H, 0) = 0$,

$$\frac{\partial D_H(\nu_H, 0)}{\nu_H} < 0 \text{ and } \begin{cases} D_I(\nu_H, 0) < 0 \\ D_I(\nu_H, 0) = 0, \frac{\partial D_I(\nu_H, 0)}{\nu_I} < 0 \end{cases}. \text{ However, it can be verified}$$

that there is no such $\nu_H \in (0, 1)$ and hence there is no equilibrium on the boundary at $\nu_I = 0$. For the boundary at $\nu_H = 0$, both Case a) and Case b) may occur. , if there is one equilibrium $(0, \nu_I)$ on the boundary (except the corner point), it should satisfy

$$\text{that } D_I(0, \nu_I) = 0, \frac{\partial D_I(0, \nu_I)}{\nu_I} < 0 \text{ and } \begin{cases} D_H(0, \nu_I) < 0 \\ D_H(0, \nu_I) = 0, \frac{\partial D_H(0, \nu_I)}{\nu_H} < 0 \end{cases}. \text{ Again,}$$

there is no such $\nu_I \in (0, 1)$. For similar reasoning, it also can be found that at the

other boundaries at $v_I = 1$ (Case a)) and $v_H = 1$ (both Case a) and b)), no equilibrium is attained. $\square$

Proposition 6.5 shows that under certain conditions when the length of flex-interval is too short to cover the arrival period of all individual workers, the equilibrium cannot be obtained at all boundaries, except for the corner points. In fact, combining Proposition 6.1-4.5, the following corollary holds.

Corollary 6-2 *When $\theta \in (0, 1 - \mu]$, it can only be realized at equilibrium when all travelers adopt the same work schedule (either normal schedule or flextime).*

Corollary 6-2 indicates that when the scheme of flextime is not flexible enough, there is no distinct advantage of one schedule over the other. Therefore, various groups of commuters show no preference in selecting one specific schedule and they only have the inclinations towards the same work schedule in order to maximize their individual net benefit.

As for the boundary equilibrium at large θ as well as other interior equilibrium, a numerical example is presented below in light of the intractability in algebra.

6.4 System optimum

We have examined both the short-run and long-run equilibrium and the corresponding stability conditions in Section 6.2 and 6.3 . Following the equilibrium analysis, now we turn to evaluate the economic efficiency of different proportions of flextime users of different groups of commuters.

6.4.1 Total commute cost minimization

Regarding short-run equilibrium, the total commute cost is calculated as the sum of individual commute cost, as

$$TC = \sum_J \sum_i N_J^i P_J^i \quad (6.26)$$

where $J = H, I$ and $i = r, f$. To achieve system optimum in short-run, we need to minimize the total commute cost.

Given μ , Table 6-4 summarizes the total commute cost TC , the derivative of TC with respect to v_H, v_I, θ and the minimum TC in all the four patterns depicted in

Figure 6-2. It can be seen that $\frac{dTC}{dv_I} \geq 0, \frac{dTC}{d\theta} \leq 0$ for all patterns and $\frac{dTC}{dv_H} \geq 0$ in

Case a), b), c). In Case d), whether $\frac{dTC}{dv_H} \geq 0$ depends on the values of μ, θ . To

minimize the total commute cost, v_I should reach its minimum and θ should reach its maximum. As for v_H , for Case a), b), c), it should reach its minimum and for Case d) either minimum or maximum should be reached depending on the values of μ, θ .

Namely, increasing the length of flex-interval Δ or increasing the number of individual flextime workers N_I^f can always lower the total commute cost TC and thereby reduce the negative externalities in congestion as expected. Increasing the number of household flextime workers N_H^f can also lower TC in most cases.

However, in the case when μ, θ are relatively large and N_H^f is small, i.e., when

$\mu > \frac{1}{2}, 2(1-\mu) \leq \theta \leq 1$ and $v_H < \frac{\theta + \mu - 1}{\mu}$, increasing N_H^f (reducing v_H) can lead

to decrease initially and then increase of TC , and it reaches a minimum at

$v_H = \frac{-2 + \theta + 2\mu}{4\mu}$. In such case, when there are already too many household travelers

adopting flextime ($v_H < \frac{-2 + \theta + 2\mu}{4\mu}$), further increase the household flextime

workers can lead to larger negative externalities. This can be ascribed to the rigidity of school start time. While in this paper we focus on the distribution of work start time and its corresponding effect in congestion alleviation, the congestion due to school trips cannot be reduced without the aid of other measures, e.g., road tolls. Such inevitable school trips' congestion prevents the total commute cost from decreasing continuously by encouraging more household workers to adopt flextime.

Table 6-4 Total commute cost and the related derivatives in each pattern depicted in Figure 6-2.

Case	a)	b)	c)	d)
TC	$\left(\frac{1-\mu+2\mu^2}{+\varepsilon\theta} \right) \frac{2\beta N^2}{s}$	$\left(\frac{1-\mu+2\mu^2}{-\theta(1-\mu)v_H} \right) \frac{2\beta N^2}{s}$	$\left(\frac{\mu+\mu^2}{-\theta\mu(1-v_H)} \right) \frac{2\beta N^2}{s}$	$\left(\frac{2\mu^2 v_H^2}{+(2\mu-\theta\mu-2\mu^2)v_H} \right) \frac{2\beta N^2}{s}$ $\left(\frac{\mu-\theta\mu+\mu^2}{+(1-\mu)^2 v_I^2} \right) \frac{2\beta N^2}{s}$
$\frac{dTC}{dv_H}$	$\mu\theta \frac{2\beta N^2}{s}$			$\mu(2-\theta-2\mu+4\mu v_H) \frac{2\beta N^2}{s}$
$\frac{dTC}{dv_I}$	$2(1-\mu)^2 v_I \frac{2\beta N^2}{s}$	$\theta(1-\mu) \frac{2\beta N^2}{s}$	$2(1-\mu)^2 v_I \frac{2\beta N^2}{s}$	
$\frac{dTC}{d\theta}$	$-(1-\mu v_H) \frac{2\beta N^2}{s}$	$\varepsilon \frac{2\beta N^2}{s}$	$-(1-v_H) \mu \frac{2\beta N^2}{s}$	$-\mu(1+v_H) \frac{2\beta N^2}{s}$
Min TC	$(1-2\mu+3\mu^2) \frac{2\beta N^2}{s}$	$2\mu^2 \frac{2\beta N^2}{s}$	$\mu(1+\mu) \frac{2\beta N^2}{s}$	$\left\{ \begin{array}{l} \frac{(-1+4\mu+4\mu^2) 2\beta N^2}{s} \\ \frac{8\mu^2 2\beta N^2}{s} \end{array} \right.$

Note: $\varepsilon = -1 + \mu v_H + (1-\mu)v_I$ and the Min TC in Case d) depends on the values of μ :

$$\left\{ \begin{array}{ll} \frac{(-1+4\mu+4\mu^2) 2\beta N^2}{s} & \text{when } \mu > \frac{1}{2}, \\ \frac{8\mu^2 2\beta N^2}{s} & \text{otherwise.} \end{array} \right.$$

By comparing the minimum of TC among these four cases, it can also be verified that the minimum of TC in Case d) is the smallest and the following proposition holds.

Proposition 6.6 *The global minimum of the total commute cost TC is obtained when*

$$\theta=1, v_I=0 \text{ and } v_H = \begin{cases} \frac{-2+\theta+2\mu}{4\mu} & \text{when } \mu \in \left(\frac{1}{2}, 1\right]; \\ 0 & \text{otherwise.} \end{cases}$$

6.4.2 Total net benefit maximization

In terms of long-run equilibrium, we define the total net benefit as the sum of the net benefit of all travelers, as

$$TB = \sum_J \sum_i N_J^i \delta_J^i \quad (6.27)$$

where $J = H, I$ and $i = r, f$. To achieve long-run system optimum, the total net benefit TB (summarized in Table 6-5) should be maximized.

One may notice that, for all TB in these four cases, they share an identical term

$H\kappa N^2$, and $+\frac{\kappa N^3}{s}$ is the identical coefficient of a composite term with respect to

v_H, v_I, θ, μ and ρ only. We can further generalize the total net benefit as

$$TB = H\kappa N^2 + \frac{\kappa N^3}{s} g(v_H, v_I, \theta, \mu, \rho) \quad , \quad \text{where } g(\cdot) \text{ is a function of}$$

v_H, v_I, θ, μ and ρ as shown in Table 6-5. Therefore, when the values of κ, N, H

and s are given, to maximize TB amounts to maximize $g(\cdot)$.

Proposition 6.7 *The total net benefit decreases with the unit time scheduling penalties $\beta(\rho)$, ceteris paribus.*

Proof. When $\theta + \mu < 1$, only Case a) and b) may occur. In Case a),

$$\frac{\partial g}{\partial \rho} = -1 + \mu - 2\mu^2 - \varepsilon\theta \quad . \quad \text{Again } \varepsilon = -1 + \mu v_H + (1 - \mu)v_I \quad . \quad \text{Since } -1 \leq \varepsilon \leq 0 \quad ,$$

$$\frac{\partial g}{\partial \rho \partial \theta} \geq 0, \text{ when } \theta = 1 - \mu, \text{ we have } \text{Max} \frac{\partial g}{\partial \rho} = -(1 + \varepsilon)(1 - \mu) - 2\mu^2 \leq 0,$$

$\mu, \theta \in (0, 1]$. Additionally, we have $\frac{\partial \rho}{\partial \beta} > 0$, from $\frac{\partial g}{\partial \rho} \leq 0$ we can get $\frac{\partial g}{\partial \beta} \leq 0$. In

Case b), $\frac{\partial g}{\partial \rho} = -(1 + \varepsilon)(1 - \mu) - 2\mu^2 + (1 - \mu)^2 (1 - v_I) v_I$. Since

$$\frac{\partial g}{\partial \rho \partial v_I} = -2(1 - \mu)^2 \leq 0, \text{ when } v_I = 0, \text{ we have}$$

$$\text{Max} \frac{\partial g}{\partial \rho} = -(1 + \varepsilon)(1 - \mu) - 2\mu^2 \leq 0, \mu, \theta \in (0, 1] \text{ and thus } \frac{\partial g}{\partial \beta} \leq 0. \square$$

Table 6-5 Total net benefits in each pattern depicted in Figure 6-2.

Case	Total Net Benefit: $TB = H\kappa N^2 + \frac{\kappa N^3}{s} g(v_H, v_I, \theta, \mu, \rho)$
a)	$H\kappa N^2 + \frac{\kappa N^3}{s} \left(\theta(\mu(v_H - v_I) + v_I - 1)(2\mu(v_H - v_I) + 2v_I - \rho) - (1 - \mu + 2\mu^2)\rho \right)$
b)	$H\kappa N^2 + \frac{\kappa N^3}{s} \left(\frac{2}{3}\theta^3 + \theta^2(2\mu v_H - 1) + \frac{4}{3}(1 - \mu)^3 v_I^3 + (1 - \mu)^2 v_I^2(2\mu v_H - \rho - 1) - \theta(1 - \mu v_H)(2\mu v_H - \rho) - (1 - \mu + 2\mu^2)\rho \right)$
c)	$H\kappa N^2 + \frac{\kappa N^3}{s} \left(\frac{-1}{3} + \frac{4}{3}v_I^3 + 2\mu((-1 + \theta)(-1 + v_H) + (1 + v_H)v_I^2 - 2v_I^3) + \frac{1}{3}\mu^3(4 - 4v_I^3 + 6v_H(-1 + v_I^2)) \right. \\ \left. + \mu^2(-3 + 2\theta(-1 + v_H)^2 + 4v_H(1 - v_I^2) + v_I^2(4v_I - \rho - 1) - \rho) + \mu(2v_I^2 - 1 + \theta - \theta v_I)\rho - v_I^2(1 + \rho) \right)$
d)	$H\kappa N^2 + \frac{\kappa N^3}{s} \left(\frac{1}{2} + \frac{1}{6}\theta^3 + \frac{1}{2}\theta^2(1 + \mu(v_H - 1)) + \frac{1}{6}\mu^3(3 + v_H(3 + 5(-3 + v_H)v_H) + 12v_H v_I^2 - 8v_I^3) \right. \\ \left. + \frac{1}{2}\theta(-3 + \mu^2(-1 + v_H)^2 + 2\mu(1 + v_H(-1 + \rho) + \rho)) \right. \\ \left. + \frac{1}{3}v_I^2(4v_I - 3(1 + \rho)) + \frac{1}{2}\mu(v_H(1 + 4v_I^2 - 4\rho) - 1 - 2\rho + 4v_I^2(1 - 2v_I + \rho)) \right. \\ \left. - \frac{1}{2}\mu^2(1 + v_H(2 + 8v_I^2 - 4\rho) + 2\rho + 2v_I^2(1 - 4v_I + \rho) + v_H^2(-5 + 4\rho)) \right)$

Apparently, larger unit time scheduling penalty leads to higher cost in commuting to both workplace and school, and further contributes to the lower total net benefit. Only in the extreme case when all travelers are individuals adopting a flextime scheme

with long enough flex-interval (i.e., $\mu=0, \theta=1, v_I=0$), the total net benefit is irrelevant to the value of ρ . Additionally, Proposition 6.7 also implies that $g(\cdot)$ is a decreasing function of ρ .

Besides, with given θ, μ, ρ , the Hessian of $g(\cdot)$ is

$$H(v_H, v_I) = \begin{bmatrix} \frac{\partial^2 g}{\partial v_H^2} & \frac{\partial^2 g}{\partial v_H \partial v_I} \\ \frac{\partial^2 g}{\partial v_I \partial v_H} & \frac{\partial^2 g}{\partial v_I^2} \end{bmatrix} \quad (6.28)$$

For each case, if there is a local maximum within the domains at (v_A, v_B) , from the second partial derivative test, it should satisfy the following three conditions.

$$\left. \frac{\partial g}{\partial v_H} \right|_{(v_H, v_I)=(v_A, v_B)} = 0, \quad \left. \frac{\partial g}{\partial v_I} \right|_{(v_H, v_I)=(v_A, v_B)} = 0; \quad (6.29)$$

$$\left. \frac{\partial^2 g}{\partial v_H^2} \right|_{(v_H, v_I)=(v_A, v_B)} < 0; \quad (6.30)$$

$$\det(H(v_A, v_B)) > 0, \quad (6.31)$$

where Eq.(6.29) determines the location of the critical points (v_A, v_B) and Eq.(6.30)

and (6.31) guarantee that the Hessian at (v_A, v_B) is negative definite.

By verifying the conditions above and comparing the $g(\cdot)$ at every boundary, there is Proposition 6.8 and Proposition 6.9.

Proposition 6.8 *With given θ, μ, ρ , when $\theta \in (0, 1-\mu]$, the system optimum is attained at*

$$(v_H^*, v_I^*) = \begin{cases} (1,1), & \rho \in \left(0, \frac{1}{2}\theta - \frac{1}{3}\theta^2\right); \\ (0,0), & \rho \in \left(\frac{1}{2}\theta - \frac{1}{3}\theta^2, 1\right). \end{cases} \quad (6.32)$$

Proof. Foremost, we determine if there is any local maximum for each case. When

$\theta \in (0, 1-\mu]$, it is Case a) or b) and there is always $\frac{\partial g}{\partial v_H^2} = 4\theta\mu^2 \geq 0$ for all

$v_H, v_I \in [0, 1]$. Thus, from Eq.(6.30), there is no local maximum within the domains

of those cases. Next, we compare the values of $g(\cdot)$ at the boundaries. That is to

compare $g(\cdot)|_{v_H=0, v_I=\frac{\theta}{1-\mu}}$, $g(\cdot)|_{v_H=1, v_I=\frac{\theta}{1-\mu}}$, $g(\cdot)|_{v_H=\frac{\mu+\theta-1}{\mu}, v_I=0}$, $g(\cdot)|_{v_H=\frac{\mu+\theta-1}{\mu}, v_I=1}$ and the

other four corners in each case. It can be found that when $\theta \in (0, 1-\mu]$, the

maximum of $g(\cdot)$ can only be attained at the corners and Eq.(6.32) is derived. $\square$

It should be noted that when $\rho = \frac{1}{2}\theta - \frac{1}{3}\theta^2$, the total net benefits at $(0,0)$ and $(1,1)$

are equal and $\left(\frac{1}{2}\theta - \frac{1}{3}\theta^2\right)$ reaches its minimum 0 at $\theta = 0$ and its maximum $\frac{3}{16}$ at

$\theta = 0.75$. That is to say that when $\rho > \frac{3}{16}$, the optimum is attained at

$(v_H^*, v_I^*) = (0,0)$ only for all $\theta \in (0, 1-\mu]$.

Proposition 6.9 *With given θ, μ, ρ , when $\theta \in (1-\mu, 1]$, if it further satisfies that*

$$\mu \in \left(2(\sqrt{2}-1), 1\right], \quad \theta \in \left(2\sqrt{2(1-\mu)} + \mu - 1, 1\right] \quad \text{and}$$

$$\rho \in \left[\frac{2\theta(1+\mu) - \theta^2 - (1-\mu)^2}{4(\theta + 2\mu - 2)}, 1\right), \text{ the system optimum can only be attained when all}$$

individuals adopt flextime ($v_I^* = 0$), and the corresponding v_H^* is,

$$v_H^* = 1 - \frac{5 + \theta - 8\rho + 2\sqrt{5(1-\mu)^2 - \theta^2 + \theta(5-9\rho)}}{5\mu}; \quad (6.33)$$

and else, the system optimum is attained at $(v_H^*, v_I^*) = (0, 0)$ or $(1, 1)$.

Proof. Similar to the proof of Proposition 6.8, when $\theta \in (1-\mu, 1]$, it can be verified

that $\frac{\partial g}{\partial v_H^2} = 4\theta\mu^2 \geq 0$ in the domain of Case c). In other words, there is no local

maximum within Case c). As for Case d), it can be found that there is one local maximum within the domain on the boundary of $v_I = 0$ for some values of θ, μ, ρ .

Thus, we compare the values of $g(\cdot)$ at the boundaries as well as the point at a local

maximum. That is to compare $g(\cdot)|_{v_H=v_H^*, v_I=0}$, $g(\cdot)|_{v_H=0, v_I=\frac{\theta}{1-\mu}}$, $g(\cdot)|_{v_H=1, v_I=\frac{\theta}{1-\mu}}$, $g(\cdot)|_{v_H=\frac{\mu+\theta-1}{\mu}, v_I=0}$, $g(\cdot)|_{v_H=\frac{\mu+\theta-1}{\mu}, v_I=1}$ and the other four corners. It can be verified that

when there exists a local maximum, it also reaches the maximum of $g(\cdot)$ for all

$v_H, v_I \in [0, 1]$ and hence Proposition 6.9 is proofed. $\square$

To note, in the constraints of Proposition 6.9, the lower bound of θ $(2\sqrt{2(1-\mu)} + \mu - 1)$ reaches its minimum 0 at $\mu = 1$ and its maximum 1 at

$\mu = 2(\sqrt{2} - 1)$ and the lower bound of ρ $\left(\frac{2\theta(1+\mu) - \theta^2 - (1-\mu)^2}{4(\theta + 2\mu - 2)}\right)$ reaches its

minimum 0.75 at $\mu = 1, \theta = 1$ and its maximum 1 at

$\mu = 2(\sqrt{2} - 1), \theta = 2\sqrt{2-2\mu} + \mu - 1$. In addition, though there is optimum with

$v_H^* > 0$, it can be verified that the v_H^* in Eq.(6.33) reaches its maximum 0.07 at

$\rho = 1, \mu = 1$ and $\theta = 1$. It suffices to say that almost all commuters adopt flextime at

system optimum in this case. In fact, during the proofs of Proposition 6.8 and

Proposition 6.9 when comparing the values of $g(\cdot)$ at the four corners, we also notice Corollary 6-3 as follows.

Corollary 6-3 *The system optimum cannot be obtained when all commuters in the same group (household or individual) adopt the same work schedule, and meanwhile, the adopted work schedules of the two groups are different, i.e., $N_H^f = N_H$ and $N_I^r = N_I$ or $N_H^r = N_H$ and $N_I^f = N_I$.*

Proof. If $(v_H, v_I) = (0, 1)$ is the system optimum, apparently it should satisfy that

$$g(\cdot)|_{v_H=1, v_I=1} < g(\cdot)|_{v_H=0, v_I=1} \quad \text{and} \quad g(\cdot)|_{v_H=0, v_I=0} < g(\cdot)|_{v_H=0, v_I=1}, \exists \theta, \mu, \rho \in (0, 1). \quad \text{When}$$

$$\theta + \mu < 1, \quad \text{with} \quad g(\cdot)|_{v_H=1, v_I=1} - g(\cdot)|_{v_H=0, v_I=1} = 2\mu\theta(1 - \mu - \rho) \quad \text{and}$$

$$g(\cdot)|_{v_H=0, v_I=0} - g(\cdot)|_{v_H=0, v_I=1} = \frac{(\theta(2\theta - 3) + 6(1 - \mu)(\mu + \rho))\theta}{3}, \quad \text{there is no satisfied}$$

θ, μ, ρ . When $\theta + \mu \geq 1$, we cannot find such θ, μ, ρ either with

$$g(\cdot)|_{v_H=1, v_I=1} - g(\cdot)|_{v_H=0, v_I=1} = -\frac{1}{6} \left(\frac{\theta^3 + 3\theta^2(1 - \mu) + 5(1 - \mu)^3}{+3\theta(-3 + \mu(2 + \mu + 4\rho))} \right) \quad \text{and}$$

$$g(\cdot)|_{v_H=0, v_I=0} - g(\cdot)|_{v_H=0, v_I=1} = \frac{(1 - \mu)^2(-1 + 4\mu + 6\rho)}{3}.$$

If $(v_H, v_I) = (1, 0)$ is the system optimum, obviously there are

$$g(\cdot)|_{v_H=1, v_I=1} < g(\cdot)|_{v_H=1, v_I=0} \quad \text{and} \quad g(\cdot)|_{v_H=0, v_I=0} < g(\cdot)|_{v_H=1, v_I=0}, \quad \exists \theta, \mu, \rho \in (0, 1).$$

Similarly, there is no such θ, μ, ρ to satisfy the conditions above. Thus,

$(v_H, v_I) = (0, 1)$ or $(1, 0)$ cannot be the system optimum. $\square$

In order to maximize total net benefit, Proposition 6.8 and Proposition 6.9 indicate the optimal proportions of flextime users in household and individual travelers. For the flextime scheme with low flexibility, it is implied in Proposition 6.8 that all travelers should agglomerate in their work schedule choice to reach system optimum.

Whether to agglomerate towards normal schedule or flextime depends on the scheduling penalties ρ . When ρ is small, the gains in reduced negative congestion externalities is smaller than the loss in reduced positive production externalities when more commuters adopt flextime. Namely, normal schedule is more superior than flextime at that ρ . Likewise, for large ρ , it shows a strong tendency to adopt flextime at system optimum due to the superiority of flextime. Even with increased flexibility in flextime, the optimum is usually attained with work schedule agglomeration for all commuters. Only when the household proportion is large, the scheme of flextime is flexible enough and the unit time scheduling penalty is rather high (i.e., the conditions listed in Proposition 6.9), it reaches the optimum with a small number of household commuters following normal schedule and the majority (including individuals) following flextime. For household commuters, the higher output in adopting normal schedule and the lower commute cost in adopting flextime under these circumstances have led to such optimum. In addition, Corollary 6-3 also shows that the system optimum cannot be reached when the household and individual commuters agglomerate at different work schedules, respectively.

From discussions above, one may also find that the points at system optimum do not always coincide with the points at user equilibrium. For instance, when $\theta \in (0, 1 - \mu]$, while the equilibrium can be attained at $(\nu_H, \nu_I) = (1, 1) \& (0, 0)$, the long-run system optimum can be attained at either $(0, 0)$ or $(1, 1)$ depending on the value of ρ . To shed some light on the difference between equilibrium and system optimum, we look at the scenarios when $\theta \in (0, 1 - \mu]$ and $\rho \in (0, 0.5)$. In Proposition 6.2, it has stated that $(\nu_H, \nu_I) = (1, 1)$ is always stable at equilibrium when $\rho < 0.5$. In other words, when we postulate only normal work schedule is provided to all commuters initially,

it achieves equilibrium already. Thus, we consider $(v_H, v_I) = (1, 1)$ as a benchmark case to compare with the total net benefit at system optimum.

In fact, for any ρ , the total net benefit at $(1, 1)$ and first increases and later decreases with μ , and it is irrelevant to θ ,

$$TB|_{v_H=1, v_I=1} = H\kappa N^2 + \frac{\kappa N^3}{s}(-1 + \mu - 2\mu^2)\rho, \quad (6.34)$$

where $TB|_{v_H=1, v_I=1}$ reaches its maximum at $\mu = 0.25$. From Proposition 6.8, when $\theta \in (0, 1 - \mu]$ and $\rho \in (0, 0.5)$, the percentage difference of total net benefit η is defined as

$$\eta = \frac{TB|_{SO} - TB|_{v_H=1, v_I=1}}{TB|_{v_H=1, v_I=1} - H\kappa N^2} = \begin{cases} 0, & \text{if } \rho \in \left(0, \frac{1}{2}\theta - \frac{1}{3}\theta^2\right); \\ \frac{((2\theta - 3)\theta + 6\rho)\theta}{6(2\mu^2 - \mu + 1)\rho}, & \text{if } \rho \in \left(\frac{1}{2}\theta - \frac{1}{3}\theta^2, \frac{1}{2}\right); \end{cases} \quad (6.35)$$

where $H\kappa N^2$ is the identical term in TB in Table 6-5. It can be readily verified that $\text{Min}(\eta) = 0$ when $\rho < \frac{1}{2}\theta - \frac{1}{3}\theta^2$ or $\theta = 0$ and $\text{Max}(\eta) = \frac{2}{3}$ when $\rho = 0.5$, $\theta = 1$ and $\mu = 0$. In other words, by driving the system from user equilibrium to long-run system optimum, the total net benefit can increase up to 66.7%.

6.4.2.1 Pigouvian policy

To attain the system optimum, the most straightforward approach is the Pigouvian tax/subsidy policy. In Pigouvian policy, the amounts of tax/subsidy σ_H, σ_I are imposed/provided to different groups (either household or individual) of commuters differentially in any work schedules (let's say normal schedule). The updated net benefit difference between normal schedule and flextime become

$D_H^p(v_H, v_I) = D_H(v_H, v_I) + \sigma_H$ and $D_I^p(v_H, v_I) = D_I(v_H, v_I) + \sigma_I$. Thus, the stability of system optimum implies that there exists σ_H^*, σ_I^* allowing $D_J^d(v_H^*, v_I^*)$ ($J = H, I$) to satisfy Lemma 6.1-4.3, which suffices to translate the surfaces of net benefit difference (e.g., Figure 6-5) upward/downward in the 3D plot to attain stable equilibrium at the given system optimum (v_H^*, v_I^*) . In Section 6.5.2, the feasibility of Pigouvian policy is illustrated numerically. Nevertheless, whether it is always feasible in Pigouvian policy to have such satisfied σ_H^*, σ_I^* remains unsolved and further examinations are required.

6.5 Numerical example

6.5.1 Long-run equilibrium

We now illustrate the long-run equilibrium conditions numerically. Suppose $s = 100$, $H = 400$, $N = 10000$ and $\kappa = 0.001$. Different values of β, θ and μ can lead to different equilibrium. In

Figure 6-5, the 3D plots of the net benefit difference $D_J(v_H, v_I)$ ($J = H, I$) and their contour plots at $D_J(v_H, v_I) = 0$ are shown with given ρ, θ and μ . Note that in the 3D plot, the surfaces in blue and orange represent the net benefit difference of household and individual travelers, respectively. The plane in red is the reference plane at 0. Thus, as discussed earlier, when $D_J(v_H, v_I) > 0$, normal work schedule is more attractive than flextime to group J commuters, and vice versa. Furthermore, the solid lines in blue and orange in the contour plot represent the contours of household and individual travelers at $D_J(v_H, v_I) = 0$, respectively. The thick dashed lines in blue and orange represent the unilateral equilibrium points of household and

individual travelers. Their intersections, which are shown in red dots, are the actual equilibrium points.

In Figure 6-5, for $\mu = 0.2$ and $\theta = 0.6$, it is Case a) and b) with $\theta + \mu < 1$. We can find out that when $\rho = 0.25 < 0.5$ (i.e., $\beta = 2.5$), there are two possible equilibrium points and their coordinates (v_H, v_I) are $(1,1)$ and $(0,0)$. When $\rho = 0.8 > 0.5$ ($\beta = 8$), $(1,1)$ is no longer a stable equilibrium and there is only one equilibrium point at $(0,0)$. As for $\mu = 0.4$ and $\theta = 0.9$, it is Case c) and d) with $\theta + \mu > 1$. Likewise, there are two stable equilibrium points $(1,1)$ and $(0,0)$ when $\rho = 0.25$. Only one stable equilibrium point at $(0,0)$ when $\rho = 0.8$.

We further enumerate the user equilibrium points with different $\theta, \mu \in [0,1]$ in step size 0.1 at two different $\rho = 0.25 \& 0.8$. At $\rho = 0.25$, it can be found that the possible equilibrium points locate at $(v_H, v_I) = (1,1) \& (0,0)$ for all $\theta, \mu \in [0,1]$ only. As for $\rho = 0.8$, the equilibrium point locates at $(v_H, v_I) = (0,0)$ in most cases as shown in Table 6-6. Only when the values of μ, θ are large, the equilibrium is attained at the boundary of $v_I = 0$, when all individuals adopt flextime and some households choose normal work schedule. For these two cases, it is worth noting that there is interior equilibrium neither in the case with $\theta + \mu < 1$ as proofed in Proposition 6.1, nor in the case with $\theta + \mu > 1$ from observations. Based on the results, Proposition 6.1-6.5 are verified.

Figure 6-5 The 3D plots and contour plots at $D_J(v_H, v_I) = 0$ ($J = H, I$) of the net benefit

difference between normal schedule and flextime.

Table 6-6 The equilibrium points (v_H, v_I) at given (μ, θ) when $\rho = 0.8$

$\mu \backslash \theta$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.0	(0,0)						(0.04,0)	(0.21,0)	(0.33,0)	(0.42,0)	(0.47,0)
0.9	(0,0)							(0.11,0)	(0.24,0)	(0.33,0)	(0.39,0)
0.8	(0,0)							(0.02,0)	(0.16,0)	(0.26,0)	(0.32,0)
0.7	(0,0)								(0.08,0)	(0.19,0)	(0.26,0)
0.6	(0,0)								(0.01,0)	(0.12,0)	(0.2,0)
0.5	(0,0)									(0.07,0)	(0.15,0)
0.4	(0,0)									(0.02,0)	(0.11,0)
0.3	(0,0)										(0.08,0)
0.2	(0,0)										(0.05,0)
0.1	(0,0)										(0.02,0)
0.0	(0,0)										

6.5.2 System optimum

To illustrate the system optimum in total net benefit maximization, we postulate the values of s, H, N, κ are given as before. At different $\theta, \mu \in [0, 1]$ with step size 0.1, the values of v_H, v_I at system optimum are enumerated. Comparing with Section 6.5.1, the system optimum at $\rho = 0.25$ for all $\theta, \mu \in [0, 1]$ locates at $(0, 0)$ only, which verifies Proposition 6.3 for $\theta \in (0, 1 - \mu]$ as $\rho = 0.25 > \frac{3}{16}$. As for the system optimum at $\rho = 0.8$ (presented in Table 6-7), one can see that only when μ is large enough, the system optimum is reached with $v_H > 0$. For $\theta \in (0, 1 - \mu]$, Proposition 6.3 is also verified with $\rho = 0.8 > \frac{3}{16}$. For $\theta \in (1 - \mu, 1]$, it can be derived from Proposition 6.4, that only when $\mu > 0.95$ it is possible to have optimum with $v_H^* > 0$

at $\rho = 0.8$, even though the $\text{Max } \nu_H^* < 0.02$. Evidently, with step size 0.1, we can only notice the optimum with $\nu_H > 0$ at $\mu = 1$ in this case.

Table 6-7 The corresponding (ν_H, ν_I) at system optimum with given μ, θ at $\rho = 0.8$.

$\mu \backslash \theta$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.0	(N/A,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0.02,N/A)
0.9	(N/A,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0.01,N/A)
0.8	(N/A,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,N/A)
0.7	(N/A,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,N/A)
0.6	(N/A,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,N/A)
0.5	(N/A,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,N/A)
0.4	(N/A,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,N/A)
0.3	(N/A,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,N/A)
0.2	(N/A,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,N/A)
0.1	(N/A,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,N/A)
0.0	(N/A,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,N/A)

To further verify Proposition 6.3 and Proposition 6.4 and better understand commuters' work schedule choice at system optimum, we discretize θ, μ with step size 0.01 at $\rho = 0.25 \& 0.8$ and calculate the optimal ν_H, ν_I respectively. The density plot²⁵ of the distribution of optimal ν_H, ν_I is shown in Figure 6-6. It can be observed from Figure 6-6 that for all $\theta, \mu \in [0, 1]$ at $\rho = 0.25 \& 0.8$, the optimums mostly (97%~99%) locate at the corner when all commuters adopt flextime.

²⁵ The overflow of fills outside the unit square in Figure 6-6 represents the optimums at the boundaries, which is resulted from the counts in bins with different the initial points. Since we focus on the distribution of optimums, the overflow sketched in Figure 6-6 cannot alter its distribution.

Lastly, to illustrate the Pigouvian policy, we take a look at Figure 6-5a when $\rho = 0.25$, $\mu = 0.2$ and $\theta = 0.6$. It can be derived that with

$$\begin{cases} \sigma_H^* \in (-\infty, 345) \& \sigma_I^* \in (-\infty, -300) \\ \sigma_H^* \in (-\infty, -300) \& \sigma_I^* \in [-300, -60) \end{cases} , \text{ the system optimum } (v_H, v_I) = (0, 0)$$

becomes the only equilibrium point with $\eta = 19.09\%$ increase of total net benefit.

The differentiated Pigouvian policy can obtain the system optimum in this case.

Figure 6-6 Density plot of the distribution of optimal v_H, v_I , when $\rho = 0.25$ & 0.8 .

6.6 Concluding remarks

To sum up, in this chapter a bi-level economic model is developed to examine the interrelationship between bottleneck congestion and commuters' work schedule choice behavior (either flextime or normal(rigid) work schedule). In addition to encapsulating negative congestion externalities and positive production externalities, we extend the model to incorporate the commuters with both home-to-school and school-to-work trip purposes, i.e., the household travels. The tradeoffs of commuters with different trip purposes between commute cost and productivity are explicitly examined. Furthermore, when the previous study assumes that flextime users have zero scheduling cost (Mun et al., 2006), this study relaxes this assumption so that the effect of length of flex-interval on commuters' travel behavior can also be investigated.

Unlike the conventional individual travelers, household travelers need to finish the school trip first followed by the work trip. In the short-run, when commuters cannot alter their work as well as the work schedule, the commute costs of both household and individual travelers with normal schedule are generally higher than those with flextime at equilibrium. In the long-run, when employers can alter the offered work schedule and employees can reselect their work, the properties of equilibrium are explored. In the case with relatively small unit time scheduling penalty ($\rho < 0.5$), firms have no incentives to switch to flextime from the initial normal schedule, regardless of the proportions of household workers and the length of the flex-interval. In addition, when the flex-interval is short ($\theta \in (0, 1 - \mu]$), all commuters agglomerate in work schedule choice at equilibrium.

We further scrutinize the system optimum at both short-run to minimize total commute cost and at long-run to maximize the total net benefit. For short-run, the system optimum is attained with long enough flex-interval ($\theta = 1$), all flextime

individual travelers and a specific proportion of flextime household travelers. For long-run, it is found that in order to reach system optimum, all commuters should agglomerate in one work schedule mostly and the schedule to adopt depends on the specific parameters. In some cases, achieving system optimum from the original equilibrium via effective measures (e.g., differentiated Pigouvian policy) can lead to up to 66.7% increase in total net benefit. Nonetheless, it should be noted that the long-run system optimum can never be reached when the work schedule choices of each group of commuters cluster within the same group but differ from that of the other group.

CHAPTER 7

CONCLUSION

7.1 Major findings

In this thesis, we explored the commute traffic management in terms of supply and demand during peak hours. In the first half, the focus is laid on the supply of transportation facilities such as parking spaces' provision and the fleet size of ride-sourcing; and in the second half, the emphasis is placed on the dynamic commuting demand of travelers with different work schedules and trip purposes.

Foremost, from the perspective of supply, the impact of emerging ride-sourcing service on commute traffic is explicitly investigated in an extended bottleneck model. We first look at the bi-modal case considering highway users only and later extend it to the multi-modal case to incorporate transit railway. In particular, the interaction between the supply of ride-sourcing service and the supply of parking spaces at CBD during peak hours is fully captured.

While previous literature on parking provision (Zhang et al., 2011; Yang et al., 2013) managed to improve system performance with appropriate parking allocation and reservation, our analysis implied that these approaches could be less effective with the popularization of ride-sourcing service. It is found that the demand for parking spaces has decreased in the presence of ride-sourcing service. This is due to the nature of ride-sourcing vehicles as a type of for-hire vehicles, which needs no parking at the destination. In fact, in order to minimize total travel cost, there exists an optimal quantity combination (range) of the parking spaces provided and ride-sourcing vehicles in service during peak hours.

However, management on the supply of transportation facilities does not alter the intense demand for on-time arrival at work in consequence of the usual rigid work schedule. For instance, even though commuters arrive at workplace earlier to secure a vacant parking spot or an available ride-sourcing vehicle, extra scheduling cost has been imposed on them. Therefore, to improve commuting efficiency, we also look at the commute traffic management from the perspective of dynamic commuting demand, i.e., the work-start-time redistribution with the aid of FWAs.

Focusing on the two FWAs pertained to time flexibility (CW and flextime/SWH), bi-level economic models are developed to investigate how the system strike a balance between the positive production externalities and the negative congestion externalities. In the analysis of CWs, it is found that popularizing CW can reduce total commute cost but not necessarily increase social welfare and there exist an optimal number of CW commuters to maximize the total net benefit. Furthermore, the work schedule choices and travel behaviors of mixed travelers (individual and household travelers with daycare trip purpose) are explicitly considered in the analysis of flextime/SWH. In the long run, our study indicates that in the case of short flex-interval, commuters prefer to agglomerate in work schedule choice at equilibrium. We also prove that there are optimal proportions of different groups of commuters adopting different types of work schedule. By appropriate policy on work schedules' selection, the travel demand can be temporally redistributed to smooth commute traffic without impairing the total net benefit.

7.2 Future research

Till now, the management of commute traffic has been explored from the perspectives of both supply and demand separately. However, some extensions are expected to be further addressed in future studies.

For the supply management of parking spaces and ride-sourcing vehicles, foremost, we abstract all parking spaces at CBD into one point to better disclose the temporal effects of parking in private autos driving. When this assumption is relaxed to incorporate parking location, commuters in private autos can park in different locations depending on the parking fee and the arrival time (Arnott et al., 1991; Liu, 2018). Meanwhile, ride-sourcing riders usually drop off at workplace directly. In the case with outward parking preference (Anderson et al., 2004; Qian et al., 2012), ride-sourcing becomes more favorable as no walking from parking space to the workplace is needed. The parking location is well worth studying in the next step of study. Next, another limitation stems from the cruising for parking. We assume the searching time of homogeneous commuters increasing with the effective parking occupancy. However, commuters in different types of parking might behave differently in cruising and the searching time is also relevant to the perceived parking information of drivers. Not to mention that the cruising traffic can interact with the other in-transit traffic (e.g., the private autos not yet parked or the eFHV) in particular when the on-street parking is considered. The searching time for parking can be affected by the in-transit traffic and conversely, the traffic congestion near CBD can be exacerbated due to parking search. Recent studies (Geroliminis, 2015; Liu & Geroliminis, 2016a) encapsulated this phenomenon in an MFD model with on-street parking only. It is of our interest to explore the searching time in a more detailed and comprehensive context. Besides, while only homogeneous travelers are considered in the thesis, extensions should incorporate the user heterogeneity in future studies. For instance, the travelers with high value of time may prefer ride-sourcing service compared to private autos driving as they can grasp the time on road to finish their work.

For the demand management via appropriate work schedule designs, firstly, we primarily focused on the situation when normal schedule and FWA (CW or Flextime/SWH) workers share no common departure period in analyzing the

equilibrium and optimum. Departure conflict between workers in these work schedules can significantly increase traveler's commute cost and thereby lead to different equilibrium and optimum. Furthermore, the FWA schedules adopted by various firms may not be homogeneous. Though the same schemes of FWA are assumed, employers and employees have the rights to design their own FWA schemes freely. In addition, the heterogeneity of employees can affect the equilibrium work schedule choices as well as the system optimum. Besides, for the study of flextime with mixed commuters, possible extensions include: 1. Detailed exploration of the differentiated Pigouvian policy according to employees' trip purposes, so that its potential merit in maximizing total net benefit can be fully recognized; 2. The spatial effects of both workplace and school location in household travelers' decision making of work schedule, in addition to the temporal effects; 3. Extension of the modeling framework to take more general shared trip purposes (e.g., ridesharing in the family) into account.

Besides, from city planners' perspective, other than the supply and demand, the land use can affect the long-run commuting behavior substantially. It is worthwhile to examine the land use (e.g., the location the CBD) in commute traffic management in future studies. In addition, while the thesis focuses on analytical study, it is necessary to conduct a case study in a more realistic network with reasonable parameters calibrated from the field.

Last but not the least, while this thesis discusses the commute traffic management thoroughly in two different directions, the writer firmly believes that joint management from supply to demand can lead to best system performance of commute traffic. Therefore, in the next step of studies, the questions can be addressed on how the supply of transportation facility (parking spaces, fleet size of ride-sourcing service, railway/bus frequency, *etc.*) should adapt to the redistributed demand resulted from

the FWAs, and the other way around, i.e., how the FWAs and their policies should design to adapt to current supply management.

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