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Bond-slip behaviour of deformed reinforcing bars embedded in well-confined concrete

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Precast concrete beam–column sub-assemblages may exhibit pull-out failure of reinforcing bars embedded in the middle joint when subjected to column removal scenarios. This failure mode has to be considered in analytical study through a component-based joint model. However, current bond-slip models incorporated in design guidelines are incapable of predicting the load capacity and potential failure mode of reinforcing bars with inadequate embedment length in beam–column joints. Therefore, it is necessary to re-evaluate the bond-slip behaviour of embedded reinforcement. This paper presents an analytical approach to predict the force-slip relationship of reinforcement. In the approach, bond stress at the post-yield stage of reinforcement is calculated from existing test data of deformed rebars embedded in well-confined concrete under pull-out loads. The approach is calibrated by experimental results of steel bars with various embedment lengths. Simplified approaches are also developed in accordance with the bond stress profile along the embedment length and the average bond stress. Finally, a component-based joint model is established for precast concrete beam–column sub-assemblages, in which the force-slip relationships of reinforcement are derived from the proposed approach. The joint model yields reasonably good predictions of the compressive arch action and catenary action capacities of sub-assemblages under progressive collapse scenarios.

Notation

A_s	cross-sectional area of reinforcement	s_f	slip at free end of reinforcement
d	diameter of embedded reinforcement	s_l	slip at loaded end of reinforcement
E_h	hardening modulus of steel reinforcement	s_y	slip at section where steel bar attains its yield strength
E_s	modulus of elasticity of reinforcement	s_1, s_2, s_3, s_4	slips of reinforcement to define bond-slip model
F	load applied to embedded reinforcement	Δl	length of steel segments
F_d	force at critical section	ε	strain of steel segments
F_e, F_h	forces at ends of steel segments	ε_d	strain at critical section
F_y	yield force of steel bars	$\varepsilon_e, \varepsilon_h$	strains at ends of steel segments
f_c	compressive strength of concrete	ε_l	steel strain at loaded end of reinforcement
$f_{\max}$	maximum tensile stress at loaded end of reinforcement	ε_y	yield strain of steel bars
f_s	tensile stress at loaded end of reinforcement	τ	bond stress of steel bars
f_u	ultimate strength of steel reinforcement	τ_d	bond stress at critical section of elastic steel segment
f_y	yield strength of reinforcement	τ_e	average bond stress along mobilised embedment length of reinforcement
l_d	length of debonded segment (same as yielded length of steel bar at its load capacity)	τ_f, τ_l	bond stresses at free and loaded ends, respectively
l_e, l_y	lengths of elastic and yielded steel segments, respectively	τ_y	bond stress at post-yield stage of reinforcement
l_{eff}	effective embedment length of reinforcement	τ_{ye}	bond stress before steel bar yields
l_r	required embedment length of reinforcement to develop ultimate strength	τ_1, τ_2	maximum bond stress and onset of frictional bond, respectively
l_s	length of straight portion in front of hook		
s	slip of steel reinforcement		
s_d	slip at critical section		
s_e, s_h	slips at ends of steel segments relative to surrounding concrete		

Introduction

Under column removal scenarios, bond-slip behaviour of reinforcing bars embedded in beam–column joints plays an instrumental role in their rotational capacities and has been incorporated in component-based joint models (Bao *et al.*, 2008;

Yu and Tan, 2010b). In reinforced concrete structures designed against seismic loading conditions, sufficient embedment length is provided for tension reinforcement, and failure is defined by rupture of steel bars (Sadek *et al.*, 2011; Yu and Tan, 2010a). However, in precast concrete structures with non-seismic design, bottom reinforcement in the middle joint may not be able to develop rupture due to inadequate embedment length (Kang and Tan, 2015). Therefore, it is essential to take account of pull-out failure when analysing the behaviour of beam-column sub-assemblages subjected to progressive collapse.

Eligehausen *et al.* (1983) tested short embedded reinforcement in beam-column joints under monotonic loadings, and proposed a local bond-slip model in accordance with the experimental results. Later, the model was incorporated in Model Code 1990 (CEB-FIP, 1991). The model was also modified and used for predicting the required embedment length of longitudinal reinforcement anchored in exterior beam-column joints (Soroushian and Choi, 1992). However, it is only valid for conventional steel bars exhibiting pull-out failure at the elastic stage without lateral pressure. Greater bond stress can be expected by increasing the lateral pressure (Wu *et al.*, 2013) and relative rib area (Metelli and Plizzari, 2014). When the model was applied for reinforcement with inelastic pull-out failure, the ultimate capacity failure mode of reinforcement could not be accurately predicted due to the overestimation of bond stress at the post-yield stage of embedded reinforcement (Monti *et al.*, 1993). Viwathanatepa *et al.* (1979) experimentally investigated the inelastic bond-slip behaviour and pull-out failure of ribbed bars embedded in reinforced concrete columns. However, little attention was paid to the post-yield bond stress in the proposed bond-slip model. Indeed, bond stress decreases dramatically at the post-yield stage of steel reinforcement (Shima *et al.*, 1987), due to contraction of the cross-section and shearing off of the concrete keys between steel lugs (Bigaj, 1995). Huang *et al.* (1996) defined a bilinear bond stress-slip relationship to take account of the post-yield bond stress, in which four sets of parameters were expressed in terms of bond conditions and compressive strength of concrete. This relationship provides a possible solution for assessing the pull-out behaviour of steel bars at the post-yield stage. With respect to reinforcement with sufficiently long embedment length to develop zero slip at the free end and rupture at the loaded end, the bond stress-strain-slip model developed by Shima *et al.* (1987) can be utilised to determine the bond stress distribution and force-slip relationship. Bigaj (1995) also derived the bond stress at the post-yield stage of embedded reinforcement based on experimental results. Nevertheless, significant discrepancies exist among the post-yield bond stresses provided by Huang *et al.* (1996), Shima *et al.* (1987) and Bigaj (1995). In the latest Model Code 2010 (FIB, 2013), the post-yield bond stress of reinforcement is not explicitly expressed. Instead, the inelastic bond-slip behaviour of reinforcement is implicitly considered by a reduction factor for elastic bond

stress, which is highly dependent on the inelastic strain of reinforcement. As a result, a nested iteration procedure is always needed for reinforcing bars embedded in concrete, which substantially increases the computational cost. Therefore, a simplified yet accurate approach has to be developed for evaluating the bond-slip behaviour of embedded reinforcement.

This paper presents an analytical approach for evaluating the bond-slip behaviour and potential failure mode of steel reinforcing bars embedded in concrete. Special attention is paid to the reduction of bond stress and pull-out failure at post-yield stage. Bond stress profiles along the embedment length of reinforcement can be obtained through the approach. In addition, simplified approaches are developed for reinforcement with various embedment lengths. Finally, the force-slip relationship of the reinforcement is incorporated in a component-based joint model for precast concrete beam-column sub-assemblages under column removal scenarios.

Analytical approach

Local bond-slip model

When subjected to tension force at one end, embedded reinforcement with various anchorage lengths exhibits different failure modes, as summarised in Table 1. A ‘sufficiently long’ embedment length enables a reinforcing bar to rupture, with zero slip at the free end at the ultimate strength of the bar. This can be found in column longitudinal reinforcement anchored in a reinforced concrete footing (Sezen and Setzler, 2008). Although reinforcement with a ‘long’ embedment length mobilises slips at the free end, it can still rupture in tension. Reinforcement with a ‘short’ embedment length develops inelastic behaviour at the loaded end and eventually exhibits pull-out failure in tension. With a further reduction in embedment length to ‘extremely short’, embedded reinforcement shows pull-out failure at the elastic stage.

In deriving the load-slip relationship of embedded reinforcing bars, the bond-slip relationship proposed by Huang *et al.* (1996) is used for elastic segments, as expressed in Equation 1. Compared to Eligehausen’s mode (Eligehausen *et al.*, 1983), it is capable of capturing the decreasing frictional bond

Embedment length	Free end slip	Stress state at the loaded end	Failure mode
Extremely short	Non-zero	Elastic	Pull-out
Short	Non-zero	Post-yield	Pull-out
Long	Non-zero	Post-yield	Rupture
Sufficiently long	Zero	Post-yield	Rupture

Table 1. Failure modes of embedded bars subject to pull-out force

stress at large slips. At post-yield stage, bond stress is approximately assumed to be uniform over yielded steel segments, as postulated in the beam-column joint model by Lowes *et al.* (2004).

$$1. \quad \begin{aligned} \tau &= \tau_1 \left(\frac{s}{s_1} \right)^{0.4} & \text{for } s \leq s_1, \quad \varepsilon \leq \varepsilon_y \\ \tau &= \tau_1 & \text{for } s_1 < s \leq s_2, \quad \varepsilon \leq \varepsilon_y \\ \tau &= \frac{\tau_2 + (\tau_1 - \tau_2)(s_3 - s)}{s_3 - s_2} & \text{for } s_2 < s \leq s_3, \quad \varepsilon \leq \varepsilon_y \\ \tau &= \frac{\tau_2(s_4 - s)}{s_4 - s_3} & \text{for } s_3 < s \leq s_4, \quad \varepsilon \leq \varepsilon_y \\ \tau &= \tau_y & \text{for } \varepsilon > \varepsilon_y \end{aligned}$$

where $s_1 = 1$ mm and $s_2 = 3$ mm; s_3 is the clear spacing of steel lugs and can be taken as 10.5 mm; s_4 is the slip when bond stress between concrete and reinforcement is zero and it is taken as $3s_3$ (Huang *et al.*, 1996).

With respect to a reinforcing bar with ‘extremely short’ embedment length, the bond-slip model shown in Figure 1 can be directly employed in the whole loading process. Nonetheless, reinforcement with ‘short’ embedment length, as defined in Table 1, exhibits post-yield behaviour when subjected to pull-out force. Thus, post-yield bond stress has to be quantified for yielded steel segments, whereas the model shown in Figure 1 is still valid for elastic steel segments. Besides, beyond its ultimate load capacity, a descending branch exists and unloading of yielded steel segments occurs (Engström *et al.*, 1998). At the descending branch, bond stress along yielded steel segments is assumed to be identical to the post-yield bond stress, as concrete keys between steel lugs have been sheared off and bond stress cannot be restored, even though steel strains decrease with increasing loaded end slip. When the embedment length is ‘sufficiently long’ to fracture the embedded reinforcement, the same bond stress can also be used for the steel segments at post-yield stage. It is worth mentioning that the bond-slip relationship holds only for well-confined concrete, namely, thick concrete cover or closely spaced stirrups are provided in the concrete specimens, such that tension splitting failure can be averted.

Figure 1. Bond-slip model for embedded reinforcing bars at elastic stage

Determination of bond stresses

In previous studies, Eligehausen *et al.* (1983) recommended the maximum elastic bond stress and the frictional bond through pull-out tests on short reinforcement embedded in concrete. Once reinforcing bars enter the inelastic stage under pull-out loads, bond stress at the post-yield stage has to be determined in order to evaluate the bond-slip behaviour and failure mode of the embedded reinforcement.

Upon yielding of reinforcement, bond stress is suddenly reduced due to stress redistribution at the interface of the steel bar and surrounding concrete (Bigaj, 1995). Simultaneously, concrete keys between steel lugs are sheared off due to inelastic elongation of the bar. This is similar to the elastic pull-out phase of short reinforcement controlled by frictional bond stress (Alsiwat and Saatcioglu, 1992; Pochanart and Harmon, 1989), except for the Poisson effect on the steel bars, namely, contraction of steel cross-section due to inelastic tensile elongation. Therefore, the frictional bond stress given by Eligehausen *et al.* (1983) can be used to calculate the bond stress at the post-yield stage of reinforcement if the Poisson effect is considered. It is reported that the Poisson effect reduces the bond stress by 20–30% when steel bars yield in tension (Eligehausen *et al.*, 1983; Viwathanatepa *et al.*, 1979). Hence, bond stress at the post-yield stage can be taken as 70–80% of the frictional bond stress of reinforcement with short embedment length. In Eligehausen’s model, the maximum elastic bond stress is $2.46(f'_c)^{1/2}$ and the frictional bond stress is $0.91(f'_c)^{1/2}$ when they are normalised by the compressive strength f'_c of concrete. Correspondingly, the post-yield bond stress can be quantified as $0.68(f'_c)^{1/2}$ if a reduction factor of 0.75 is selected to consider the Poisson effect. Thus, the ratio of the calculated post-yield bond stress to the maximum elastic bond stress is 0.28.

The post-yield bond stress can also be estimated from the embedded bars featuring pull-out failure at the post-yield stage. When the maximum load is reached at the loaded end of reinforcement, a stepwise uniform bond stress profile is assumed over the whole embedment length, as shown in Figure 2. The maximum elastic bond stress τ_1 is attained along

Figure 2. Bond stress distribution of reinforcing bar at the peak pull-out force

the elastic steel segments and the post-yield bond stress τ_y is over the yielded segments. As the ratio of τ_3 to τ_1 is fixed at 0.28, bond stresses τ_1 and τ_y can be determined in accordance with the force equilibrium of the elastic and yielded steel segments, as expressed in Equations 2–4.

$$2. \quad \tau_1 \pi d l_e = f_y A_s$$

$$3. \quad \tau_y \pi d l_y = (f_{\max} - f_y) A_s$$

$$4. \quad l_e + l_f = l_{\text{eff}}$$

Table 2 shows the bond stresses of embedded reinforcement tested by Viwathanatepa *et al.* (1979), Ueda *et al.* (1986) and Engström *et al.* (1998). It is worth noting that the failure cone close to the loaded end of reinforcement is considered by deducting its length from the total embedment length, and therefore the effective embedment length is used in calculating the bond stresses. The average post-yield bond stress under pull-out forces can be quantified as $0.66(f'_c)^{1/2}$, close to the value calculated from Eligehausen's model, by considering the Poisson effect.

Solution procedure

After determining the bond stresses, a nested iteration procedure is employed to derive the force-slip relationship of embedded bars from the local bond-slip relationship. In the solution procedure, the embedded reinforcement is discretised into segments with length equal to its diameter. Therefore, the equilibrium and compatibility of each segment (see Figure 3)

have to be satisfied, as expressed in Equations 5 and 6. It is worth noting that bond stress τ is assumed to be constant over each segment, and it can be calculated from the slip at the middle point of the segment. By summing up the bond force and slip on each segment along the whole embedment length, total force and slip at the loaded end can be computed.

$$5. \quad F_e = \tau \pi d \Delta l + F_h$$

$$6. \quad s_e = s_h + \frac{1}{2}(\varepsilon_h + \varepsilon_e)\Delta l$$

Note that the foregoing analytical approach applies to straight bars. In the case of reinforcing bars with hooked anchorage, an equivalent length of $l_s + 5d$ is suggested (Filippou *et al.*, 1983), where l_s is the length of the straight portion in front of the hook and d is the diameter of the steel reinforcement.

Calibration of analytical approach

Pull-out failure of reinforcement at post-yield stage

The proposed analytical approach is calibrated through relevant experimental results (Engström *et al.*, 1998; Ueda *et al.*, 1986; Viwathanatepa *et al.*, 1979), in which the bilinear

Figure 3. Equilibrium and compatibility of steel segments

Reference	Specimen	Effective embedment length: mm	Compressive strength of concrete: MPa	Bond stress		
				$\tau_1/\sqrt{f'_c}$	$\tau_2/\sqrt{f'_c}$	$\tau_y/\sqrt{f'_c}$
Viwathanatepa <i>et al.</i> (1979)	#3	546	32.5	2.93	1.08	0.82
	S61	330	23.7	2.48	0.91	0.69
	S62	330	23.4	2.66	0.96	0.74
	S63	330	19.2	1.88	0.69	0.53
	S101	532	19.9	3.13	1.16	0.88
Ueda <i>et al.</i> (1986)	S102	532	25.0	2.58	0.95	0.72
	S104	532	28.3	1.62	0.59	0.45
	S105	532	35.3	2.15	0.79	0.60
	S106	532	25.9	2.43	0.90	0.68
	S107	532	18.2	1.87	0.69	0.52
Engström <i>et al.</i> (1998)	N290b	260	30.6	2.19	0.80	0.61
Mean value				2.36	0.87	0.66
Coefficient of variation				19.8%		

Table 2. Bond stress of embedded steel bars under pull-out loads

stress-strain relationship of the steel reinforcement is adopted. At the descending phase of applied load, the unloading stiffness of reinforcement is assumed to be equal to the initial elastic modulus. Table 3 includes the material properties of reinforcement. Figure 4 shows the force-slip relationships of reinforcement under pull-out loads. As bond stresses are directly computed from experimental results, the maximum loads sustained by embedded reinforcing bars are approximately identical to the experimental values (see Figure 4). Comparisons between experimental and analytical force-slip

Reference	Steel bar	Diameter of bar: mm	Elastic modulus: GPa	Yield strength: MPa	Hardening modulus: MPa	Ultimate strength: MPa
Viwathanatepa <i>et al.</i> (1979)	#3	20.2	201.3	468.5	2275	737.8
	S61	19.1	199.8	438.2	5925	775
Ueda <i>et al.</i> (1986)	S101	32.3	203.9	414.1	5822	660.8
Engström <i>et al.</i> (1998)	N290b	16	200.0	569.0	921	648.0

Table 3. Material properties of embedded bars

Figure 4. Comparison between experimental and analytical results under pull-out loads: (a) #3 by Viwathanatepa *et al.* (1979); (b) N290b by Engström *et al.* (1998); (c) S61 by Ueda *et al.* (1986); (d) S101 by Ueda *et al.* (1986)

relationships demonstrate that the proposed analytical approach yields reasonably accurate predictions of the ascending phase of the load-slip curves. However, with respect to the descending phase due to pull-out of embedded reinforcement from the surrounding concrete, only the experimental result of N290b by Engström *et al.* (1998) is provided, and the analytical result is in good agreement with the test result.

Rupture of reinforcement with zero slip at free end

Besides pull-out failure of embedded reinforcement at the post-yield stage, the analytical approach can also be utilised to simulate pull-out failure of reinforcement at the elastic stage and fracture of reinforcement with or without free end slip. In the case of 'short' reinforcement with elastic pull-out failure, reinforcement does not develop post-yield behaviour at the loaded end and the bond-slip model expressed in Equation 1 can be used for the whole steel segment at the elastic stage. When 'long' embedment length is provided for embedded reinforcement, fracture of reinforcement occurs instead of pull-out failure, as defined in Table 1. Accordingly, slip at the free end is gradually reduced with increasing embedment length. If the embedment length of reinforcement is 'sufficiently long' to eliminate slip at the free end, the proposed approach can also be verified by experimental results.

Table 4 includes the material properties of concrete and embedded reinforcement tested by Bigaj (1995). Owing to grooving of reinforcement on both sides, the effective circumference and cross-sectional area of steel bars are used instead of the nominal values. It should be noted that little information is available on the force-slip curves of reinforcement at the elastic stage. Thus, the slip-strain model proposed by Soltani and Maekawa (2008) is used to calibrate the force-slip relationships of reinforcement before the yield strength of reinforcement is attained. Figure 5 shows the comparisons between analytical results according to the proposed approach and Soltani's slip-strain model. It is observed that the force-slip relationship predicted by the proposed approach agrees well with that by Soltani's model at the elastic stage. At the post-yield stage, comparisons are made between experimental results and estimations through the analytical approach, as

shown in Figure 6. The force-slip curves predicted by the model are in good agreement with experimental results at the post-yield stage. Thus, the proposed model represents a generic model that covers the bond-slip behaviour of reinforcement with different failure modes.

Behaviour of reinforcement with various embedment lengths

Using the analytical approach, the behaviour of embedded reinforcement with various embedment lengths can be estimated under pull-out loads. Specimen N290b tested by Engström *et al.* (1998) is selected as the control case and the ratio of the embedment length to diameter of steel bars varies from 10 to 42. Force-slip relationships of reinforcement and bond stress profiles along the embedment length at different stages can be obtained analytically.

Force-slip relationship

Figure 7(a) shows the force-slip relationships of reinforcement with different ratios of embedment length to rebar diameter. When the ratio varies from 10 to 16, the load capacity of the embedded reinforcement increases with increasing embedment length-diameter ratio. Correspondingly, the failure mode of reinforcement is transformed from pull-out failure at the elastic stage of reinforcement to that at the post-yield stage. By increasing the ratio from 16 to 18, the failure mode is shifted from pull-out failure to fracture of reinforcement at the loaded end (see Figure 7(a)). Besides, the free end slip is less than unity when the load capacity of the embedded reinforcement is attained, as shown in Figure 7(b). Therefore, to develop the ultimate strength of steel reinforcement subjected to tension, the minimum embedment length is 18 times the rebar diameter. Further increases in the embedment length substantially reduce the free end slip and associated bond stress at the ultimate strength of reinforcement, as shown in Figures 7(b) and 7(c). When the ratio of embedment length to rebar diameter is 32, the free end slip is negligibly small (only 0.01 mm). However, bond stress at the free end is nearly 1.92 MPa. To fully eliminate the slip at the free end when the ultimate strength of reinforcement is attained, the embedment length has to be at least 42 times its diameter.

Steel bar	Concrete compressive strength: MPa	Effective circumference of bar: mm	Cross-sectional area: mm ²	Elastic modulus: GPa	Yield strength: MPa	Hardening modulus: MPa	Ultimate strength: MPa
P-16-16-1	27.0	13.1	174.2	128.5	539.7	945	624.4
P-16-16-2	28.4						
P-20-16-1	28.4	17.1	280.9	150.4	526.2	952	612.9
P-20-16-2	26.8						

Table 4. Properties of concrete and embedded reinforcement (Bigaj, 1995)

Figure 5. Comparison between analytical results predicted by proposed model and Soltani's model: (a) P-16-16-1; (b) P-16-16-2; (c) P-20-16-1; (d) P-20-16-2

Bond stress distribution

In addition to the force-slip relationship of embedded reinforcement, the distribution of bond stress along the embedment length can also be obtained from the analytical approach. Figure 8 shows the bond stress profile of reinforcing bar N290b with an effective embedment length of 16.25 times its diameter and exhibiting pull-out failure at the post-yield stage. At the elastic stage, bond stresses vary almost linearly along the embedment length, attaining the minimum value at the free end and the maximum value at the loaded end (see Figure 8(a)). Once plasticity initiates close to the loaded end, bond stress drops suddenly to its post-yield bond stress. At the load capacity of reinforcement, the maximum elastic bond stress is attained along the elastic steel segments, as shown in Figure 8(b), which agrees well with the assumption made in calculating the bond stresses. Thereafter, further increases in slip reduce the bond

stresses along the elastic steel segments and make the bond stress at the free end slightly larger than that at the loaded end, until pull-out of the reinforcement from concrete occurs.

When the ratio of embedment length to diameter is 42, slip at the free end is always zero. Figure 9(a) shows the bond stress profile along the embedment length at the ultimate strength of reinforcement. Similarly to reinforcement N290b, bond stresses vary almost linearly along elastic segments, with zero value at the free end. In the case of yielded steel segments, post-yield bond stress remains constant, as defined in Equation 1. Furthermore, bond stress at each steel segment is correlated to the tensile stress of the segment, as shown in Figure 9(a). Thus, if the tensile stress at the loaded end of reinforcement is less than its ultimate strength, a shorter length of reinforcement is mobilised to resist the applied force.

Figure 6. Comparison between experimental and analytical results of embedded reinforcement by Bigaj (1995): (a) P-16-16-1; (b) P-16-16-2; (c) P-20-16-1; (d) P-20-16-2

Average bond stress

When the embedment length of reinforcement is ‘sufficiently long’ to eliminate slip at the free end, the average bond stress along elastic steel segments can be calculated based on the analytical approach. At each load level, tensile stress and slip of reinforcement can be determined at the loaded end. Thus, the average bond stress can be quantified from Equation 7 (Lowes *et al.*, 2004). Figure 9(b) shows the variation of average bond stress with tensile stress of reinforcement at the loaded end. It can be observed that the calculated average bond stress increases with the tensile stress of reinforcement. Therefore, to take account of the effect of the compressive strength of concrete and the tensile stress of reinforcement, the average bond stress is normalised by $(f'_c f_s)^{1/2}$, where f'_c is the cylinder compressive strength of concrete and f_s is the tensile stress at the loaded end of reinforcement. The normalised average bond stress increases with increasing tensile stress at the loaded end.

However, a plateau stage exists on the normalised average bond stress–tensile stress curve. When the tensile stress is greater than 100 MPa, the normalised bond stress remains nearly constant. The maximum value is 0.05 when the yield strength of reinforcement is attained at the loaded end. For reinforcement with a yield strength of 569 MPa, the maximum average bond stress is $1.2(f'_c)^{1/2}$. The value is greater than that provided by Lehman and Moehle (2000) and Sezen and Setzler (2008). Once post-yield behaviour commences at the loaded end, the bond stress τ_y calculated in Table 2 can be used for yielded steel segments. The post-yield bond stress is $0.66(f'_c)^{1/2}$.

$$7. \quad \tau_c = \frac{f_s^2 A_s}{2s_1 E_s \pi d}$$

(a)

(b)

(c)

Figure 7. Effect of embedment length on the force-slip relationship of reinforcing bars: (a) force-slip curve of embedded reinforcement; (b) variation of free end slip at the load capacity of reinforcement; (c) variation of bond stress at the free end at the load capacity of reinforcement

(a)

(b)

Figure 8. Variation of bond stresses along the embedment length of reinforcement N290b: (a) at elastic and plastic stages; (b) at load capacity and descending stage

Simplified approach

Pull-out failure of reinforcement at post-yield stage

The bond stress profile along the embedment length of reinforcement varies with loads applied to the embedded steel bar, as shown in Figure 8. Hence, three stages, namely, the elastic ascending stage, post-yield ascending stage and descending stage (see Figure 10), are classified for reinforcement with 'short' embedment length, so as to simplify the analytical approach correspondingly in accordance with bond stress distribution.

Elastic ascending stage

At the elastic ascending stage, a linear bond stress profile is assumed along the whole embedment length of a steel bar, as shown in Figure 10(a). For a given slip at the loaded end, a

Figure 9. Variation of bond stresses along sufficiently long reinforcement ($L = 42d$): (a) profiles of bond stress and steel stress; (b) variation of average bond stresses

slip at the free end is assumed and bond stresses at the two ends of the steel bar can be determined through the bond-slip relationship defined in Equation 1. Thus, the force sustained by the steel bar can be obtained through equilibrium, as expressed in Equation 8. Correspondingly, steel strain at the loaded end can be determined through the constitutive model for the steel bar.

$$8. \quad F = \frac{(\tau_f + \tau_l)l_e \pi d}{2}$$

$$9. \quad s_l = s_f + \frac{\pi d(\tau_f + 2\tau_l)l_e^2}{6E_s A_s}$$

Figure 10. Bond stress and slip of embedded reinforcing bars at different stages: (a) at elastic ascending stage; (b) at plastic ascending stage; (c) at descending stage

As bond stress varies linearly along the embedment length, distribution of steel strain is a parabolic function, with zero strain at the free end and the maximum strain at the loaded end. Accordingly, slip at the loaded end can be taken as a summation of the free end slip and the integration of steel strains along the embedment length (Shima *et al.*, 1987). Thus, slip at the loaded end can be calculated from Equation 9. Once compatibility of the embedded reinforcement is satisfied, in other words, the calculated slip at the loaded end is equal to the initially assumed value, slips at the free and loaded ends are obtained.

Post-yield ascending stage

Once plasticity is initiated at the loaded end, the associated bond stress is substantially reduced at the post-yield stage of the steel bar, as shown in Figure 8(a). Hence, equilibrium and compatibility have to be appropriately modified to take account of the post-yield bond stress.

In addition to the slips at the free and loaded ends, the length of yielded steel segment has also to be assumed at the

post-yield ascending stage, as shown in Figure 10(b). Therefore, the force at the loaded end can be calculated from the force equilibrium of the yielded steel segment, as expressed in Equation 10. Through a bilinear constitutive model of reinforcement, the steel strain at the loaded end is calculated accordingly and the slip at the yielded section can be determined from Equation 11. Thereafter, the analytical bond-slip relationship is used to determine the bond stress at the yielded section. With respect to the elastic steel segment, the same procedure as that used for the elastic ascending stage is followed. As force at the section where the steel bar yields is known, the calculated force from Equation 12 must be equal to the yield force of the reinforcement. Moreover, slips computed from Equations 11 and 13 must be equal to one another so as to satisfy compatibility.

$$10. \quad F = F_y + \pi d \tau_y l_y$$

$$11. \quad s_y = s_l - \frac{(\varepsilon_y + \varepsilon_l) l_y}{2}$$

$$12. \quad F_y = \frac{(\tau_f + \tau_{ye}) l_e \pi d}{2}$$

$$13. \quad s_y = s_f + \frac{\pi d (\tau_f + 2\tau_{ye}) l_e^2}{6 E_s A_s}$$

Descending stage

Beyond the maximum load that the embedded bar is able to sustain, the applied force starts to decrease with increasing slip at the loaded end, as shown in Figure 4. Hence, a different analytical procedure is proposed to take account of the descending stage.

Once the descending stage commences, the strains of the embedded bar decrease with increasing slip at the loaded end. The maximum strain at each section should be used to determine the steel stress at descending stage. Based on the stress state of reinforcement at the post-yield ascending stage, the whole embedment length of reinforcement is divided into two segments, namely, the elastic and debonded segments. The length of the debonded segment can be treated as the length of the yielded steel segment at the post-yield ascending stage, over which bond stress remains the same as the post-yield bond stress. Nonetheless, a linear bond stress distribution can still be assumed along the elastic segment, as shown in Figure 10(c).

With regard to the elastic steel segment, the analytical procedure at the elastic ascending stage is applied, and slips at the free end and critical section between the elastic and debonded segments are correlated by Equation 14. It is notable that the force at the critical section, which can be calculated from equilibrium in Equation 15, should be smaller than the yield force of the steel bar as a result of unloading of the reinforcement. The procedure used for the yielded steel segment at the post-yield ascending stage can be employed for the debonded segment, as expressed in Equations 16 and 17.

$$14. \quad s_d = s_f + \frac{\pi d (\tau_f + 2\tau_d) l_d^2}{6 E_s A_s}$$

$$15. \quad F_d = \frac{(\tau_f + \tau_d) l_e \pi d}{2}$$

$$16. \quad s_l = s_d + \frac{(\varepsilon_d + \varepsilon_l) l_d}{2}$$

$$17. \quad F = F_d + \pi d \tau_y l_d$$

Rupture of reinforcement with zero slip at free end

When the embedment length of reinforcement is ‘sufficiently long’ to ensure zero slip at the free end, the required length of reinforcement to mobilise bond stress increases with increasing load at its end at the elastic stage. When the yield strength of the reinforcement is attained at the loaded end, the mobilised length of the elastic steel segment and the slip at its yielded section remain constant. Further increase in force and slip at the loaded end of the reinforcement only increases the length of the yielded steel segment. Since average bond stresses at the elastic and post-yield stages are derived from the analytical approach, uniform bond stress distribution is assumed along the elastic and yielded steel segments, as shown in Figure 11. Thus, the bond-slip model proposed by Lowes *et al.* (2004) is employed for ‘sufficiently long’ reinforcement. In the model, only force equilibrium of reinforcement is considered, and slip at the loaded end is calculated as the summation of steel strains along the mobilised embedment length.

At the elastic stage, a constant bond stress of $0.05(f'_c f_s)^{1/2}$ is distributed along the elastic steel segment, as shown in Figure 11(a), and slip at the loaded end can be calculated from Equation 7. Once reinforcement enters its post-yield stage at the loaded end, a piecewise bond stress profile is assumed

Figure 11. Bond stress along the embedment length of sufficiently long reinforcement: (a) at elastic stage; (b) at post-yield stage

along the whole embedment length of reinforcement (see Figure 11(b)). The bond stress is $0.05(f'_c f_y)^{1/2}$ for the elastic steel segment, whereas post-yield bond stress of $0.66(f'_c)^{1/2}$ is used for the yielded segment. Correspondingly, slip at the loaded end can be calculated from Equation 18.

$$18. \quad s_l = \left[\frac{f_y^2}{2\tau_{ye}E_s} + \frac{(f_s - f_y)f_y}{\tau_y E_s} + \frac{(f_s - f_y)^2}{2\tau_y E_h} \right] \frac{A_s}{\pi d}$$

Verification of simplified approach

The simplified approach is verified by experimental and analytical results of embedded reinforcement with pull-out failure at post-yield stage and zero free end slip at fracture, respectively. For steel reinforcement with pull-out failure at the post-yield stage, the overall force-slip relationships predicted by the simplified model yield good agreement with experimental results, as shown in Figure 4. However, when the embedment length of reinforcement is 'sufficiently long' to ensure zero slip at the free end, the estimated slip of reinforcement by the simplified approach is slightly greater than that predicted by Soltani's model (Soltani and Maekawa, 2008) at the elastic stage, as shown in Figure 5. However, the force-slip relationships of reinforcement at the post-yield stage agree reasonably well with experimental results (see Figure 6).

Applications to component-based joint models

Based on previous study (Bao *et al.*, 2008; Yu and Tan, 2013), a component-based model can be developed for the analysis of precast concrete beam-column joints under column removal scenarios. In the model, fibre elements are used to model the beam. A rigid element is used for the beam-column joint, as shear deformation in the joint is insignificant. Interactions between beam and joint are simplified as a series of springs. The properties of tensile and compressive springs can be defined by the force-slip relationships of reinforcement embedded in the joint. Besides, an elastic shear spring with infinite stiffness is assumed at the interface of beam and joint, as shear failure is not expected through the model (Lowes and Altoontash, 2003).

Behaviour of precast concrete beam-column sub-assemblages tested by Kang and Tan (2015) and Kang *et al.* (2015) under column removal scenarios are simulated through the joint model. In the middle joint of sub-assemblage MJ-B-0-88/0-59R, the beam bottom reinforcement exhibited pull-out failure at the post-yield stage (Kang and Tan, 2015), and thus vertical load on the middle joint varies continuously with middle joint displacement (see Figure 12(a)). However, MJ-B-1-19/0-59 developed fracture of the bottom reinforcement near the middle joint (Kang *et al.*, 2015), which results in a sudden drop of vertical load at around 234 mm vertical displacement, as shown in Figure 12(c). Comparisons between experimental and numerical results suggest that the joint model is capable of estimating the compressive arch action and catenary action capacities of the beam with reasonably good accuracy. Besides, horizontal compression and tension forces developed in the beam are also successfully captured at the compressive arch action and catenary action stages, respectively, as shown in Figures 12(b) and 12(d).

Discussion

In existing design codes and standards, the required embedment length of steel reinforcement is calculated as a function of average bond stress and yield strength of steel reinforcement. However, experimental results indicate that bond stresses at elastic and post-yield stages differ considerably from one another (Bigaj, 1995; Shima *et al.*, 1987). In the current study, post-yield bond stress is quantified from available test data and incorporated in the bond-slip model. Also, average elastic and post-yield bond stresses are determined for steel reinforcement with adequate embedment length to ensure zero slip at the free end when steel bars attain their ultimate strength. Correspondingly, the embedment length of reinforcement to develop ultimate strength can be calculated from Equation 19.

$$19. \quad l_r = \left(\frac{f_y}{\tau_{ye}} + \frac{f_u - f_y}{\tau_y} \right) \frac{d}{4}$$

The proposed bond-slip model can also be used for evaluating the load capacity of steel bars when their embedment length is

Figure 12. Comparison between experimental and numerical results: (a) vertical load–displacement curve of MJ-B-0-88/0-59R; (b) horizontal force–displacement curve of MJ-B-0-88/0-59R;

(c) vertical load–displacement curve of MJ-B-1-19/0-59; (d) horizontal force–displacement curve of MJ-B-1-19/0-59

less than the value specified by the relevant design codes to develop the ultimate strength. This is of crucial importance for middle beam–column joints under progressive collapse scenarios, as the bottom reinforcement may not develop its full strength when subjected to moment reversal. Furthermore, under cyclic loading conditions, slip of anchored reinforcement contributes to a significant portion of the total lateral deformations of reinforced concrete members. A stiffer response of structural members can be estimated if reinforcement slip is neglected (Sezen and Setzler, 2008). By means of the bond–slip model, cyclic behaviour of reinforced concrete beams, column, walls and joints can also be simulated with low computational cost.

Conclusions

In this paper, an analytical approach is proposed to determine the bond–slip behaviour of embedded reinforcing bars in

well-confined concrete, and published experimental results are re-evaluated to calculate the bond stress at elastic and post-yield stages. The analytical approach is calibrated against test data. The bond–slip behaviour of reinforcement with various embedment lengths is also investigated through the analytical approach, and the required length for reinforcement to develop fracture with zero slip at the free end is quantified. In addition, the bond stress profile along the embedment length and average bond stress at each load level are obtained for reinforcement with pull-out failure at post-yield stage and zero free end slip at fracture, respectively.

Simplified approaches are derived in accordance with the bond stress distribution along the embedment length and the average bond stress. For reinforcement with pull-out failure at the post-yield stage, nearly linear bond stress distribution is assumed along the embedment length. When the embedment length is

‘sufficiently long’ to enable zero slip at fracture, the average bond stress is formulated as a function of concrete compressive strength and tensile stress at the loaded end of reinforcement. Comparisons between experimental and analytical results indicate that the simplified model yields reasonably good prediction of the force-slip curves of reinforcement.

Finally, a component-based joint model is developed for precast concrete beam-column sub-assemblages under column removal scenarios. In the model, interactions between beam and joint are simulated by non-linear springs with zero length, of which the constitutive relationship is defined by the force-slip curves of reinforcement. The model is capable of predicting the compressive arch action and catenary action capacities and associated horizontal tension forces with good accuracy.

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