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Time-mean structure of secondary flows in open channel with longitudinal bedforms

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ABSTRACT

Secondary motions are commonly present in open channel flows. This study aims to investigate, both experimentally and analytically, time-mean characteristics of cellular secondary flows generated by longitudinal bedforms. Experiments were conducted in a tilting, rectangular flume with six different longitudinal bedforms, including alternate bed strips with different roughness heights and bed ridges of wavy and rectangular shapes. Various flows were sampled using a two-dimensional laser Doppler anemometer (LDA) and a one-dimensional ultrasonic Doppler velocimeter (UDV). Experimental results demonstrate secondary flows appearing basically in cellular fashion over the modeled longitudinal bedforms. It is also shown that the cellular structures can be described analytically with kinematic considerations. The discrepancies between theoretical and measurement results are discussed. An empirical relationship between maximum vertical velocity and bed configuration is finally proposed based on the experimental data.

Keywords

Open channel flow; Secondary flow; Velocity distribution; Stream function; Bedforms; Roughness

1. INTRODUCTION

Open channel flows are three-dimensional and the primary motion could be significantly subject to secondary currents in the vertical and spanwise directions. Streamwise flow circulation is defined as secondary flow, which can be induced by different causes. According to Prandtl [23], there exist two mechanisms responsible for the generation of streamwise circulations. The first mechanism is associated with the skewing of mean flow, i.e., the non-uniformity of mean flow in the streamwise direction. For example, in curved channels or meandering rivers, flow may circulate in the cross-sectional plane either driven by centrifugal or transverse pressure gradient. Such a secondary flow is called Prandtl's first kind of secondary flow. The generation of this kind of secondary flow is essentially an inviscid process, and thus it can occur either in turbulent or laminar flow. The second mechanism for the formation of streamwise circulation is related to the non-homogeneity and anisotropy of turbulence. Due to the lateral imbalance of turbulent stresses, streamwise vortices may be formed. These

vortices may be stretched and amalgamated in the transverse direction, resulting in large-scale flow circulations. Secondary flow so generated is called Prandtl's second kind of secondary flow, or turbulence-induced secondary flow. There are a variety of turbulence-induced secondary flows in natural streams, which are caused by asymmetry of channel boundaries, free surface effects, variations in bed conditions and instabilities in turbulent flows.

This study is focused on turbulence-induced secondary flows that are particularly associated with longitudinal bed-forms. Such secondary flows are ubiquitous because natural rivers with mobile bed often possess longitudinal bedforms [9]. Generally, longitudinal bedforms refer to large-scale bed elements that are elongated parallel with the primary flow and characterized by alternate bed roughness or/and elevation variations in the spanwise direction. For convenience, longitudinal bedforms are classified as 'strips' and 'ridges' in this study (see Fig.1). The 'strips' are those characterised by dominant bed roughness variation in the spanwise direction, while the 'ridges' (or 'ridges/troughs') are those with significant lateral variation in bed elevation. Owing to the recurrence of bedforms in the spanwise direction, the generated secondary flows often appear as paired counter-rotating flow cells in the cross-stream plane, as sketched in Fig. 1. Therefore, such secondary flows are usually called cellular secondary flows. On the other hand, the appearance of cellular secondary flows may cause lateral sediment transport, which can in turn enhance and maintain longitudinal bedforms [2,6,8,18,27]. Therefore, there exists an interactive relationship between cellular secondary flows and longitudinal bedforms.

Nezu and Nakagawa [15,17] proposed that the evolution process of longitudinal bedform started with the presence of the corner vortex induced by sidewall effect. With the corner vortex, lateral variations in the bed shear stress and thus bedload transport appeared leading to the formation of a sand ridge. Following that, the bed shear stress varied further in the lateral direction, generating more vortices. If this process progressed repeatedly, sand ridges and cellular secondary currents will eventually occur in the entire cross-section. Nezu and Nakagawa [16] also reported that regular patterns of ridges and troughs could occur in the central region of the flow channel even when the first ridge near the sidewall was poorly developed. This observation implies that the sidewall effect is not necessary for the generation of longitudinal bedforms and cellular secondary flows. Ikeda [7], Colombini [4] and McLelland et al. [12] argued that the initiation of cellular secondary flows and sand ridges is an instability-related process. Small disturbances either from the bed surface or flow itself are able to induce streamwise vortices, which then produce lateral sediment transport and sorting. With the same feed-back mechanism as suggested by Nezu and Nakagawa [15], the secondary motions would enhance the initial bed disturbance, generating a longitudinal pattern of ridges or strips.

Besides the attempts at clarifying generation mechanisms, many efforts have also been extended to make clear the basic characteristics of cellular secondary flows and associated longitudinal bedforms. Field observations of longitudinal bedforms have been intensively reported by geologists and hydraulic engineers since 1930s [2]. Longitudinal bedforms can occur in natural rivers with clay, sand or gravel beds. Longitudinal

bedforms appear with periodic, spanwise variations in bed texture (roughness) and/ or bed elevation. In addition to the observations from the natural environment, longitudinal bedforms have also been observed in laboratory experiments. Casey [3] conducted an experiment using mixed grades of sand in a straight flume. He noticed that the finer particles moved over the coarser in forming longitudinal strips, with a spanwise wavelength of roughly twice the flow depth. Similar experimental results were also observed by Vanoni [28], Allen [1], Günter [5], Ikeda [7], Hirano and Ohmoto [6], Nezu and Nakagawa [16] and McLelland et al. [12]. In particular, Ikeda [7] studied the size and the shape of self-formed strips with uniform non-cohesive sands. His experiments show that longitudinal sediment ridges were formed over the entire movable bed, of which the crests were several millimeters high. Some of the ridges were very stable and persisted almost throughout the run time, and the distance between the neighboring crests was roughly twice the flow depth.

In contrast to the facility in identifying longitudinal bed-forms, secondary flows are much more difficult to measure because of their relatively small magnitude. Around 1950s, the existence of cellular secondary flows in rivers was only inferred from laterally periodic distributions of primary flow and sediment concentration, rather than being confirmed through direct velocity measurements, by geologists and river engineers [8,10,17,28]. The precise field measurement of cellular secondary flows was possible only after the advent of electromagnetic current meter (EM) and acoustic Doppler velocimeter (ADV). Nezu et al. [19] measured secondary flows using two sets of EM in a river 17.5 m wide and 2.2 m deep. Müller and Studerus [13] first conducted an exploratory experiment in an open channel flow with rough and smooth longitudinal strips. The velocity was measured using a laser Doppler anemometer (LDA) and a hot-film anemometer. They reported that upflows occurred over the smooth strips and downflows over the rough strips forming a pair of counter-rotating flow cells with a diameter the same as the flow depth. Nezu and Nakagawa [15] also carried out an experiment over artificial longitudinal ridges of 450 trapezoidal cross-section. The velocity was measured using X-type hot-film anemometers. A pair of counter-rotating flow cells was also recognized, the upflows occurring over the ridge and downflows over the trough. Laboratory studies of cellular secondary flows have been also conducted by Mclean [11], Studerus [26], Wang et al. [29,30] and Wang and Cheng [31], among others.

In spite of the previous efforts, quantitative velocity information of secondary flows is still very lacking. This rendered that secondary flow structures have been often described phenomenologically. In order to provide detailed flow measurements for relevant investigations, a series of flow experiments over various longitudinal bedforms have been carried out recently by the authors [32]. In this paper, the measured secondary flow velocities are presented. A simple analytical formulation is also provided to quantitatively describe the observed structures of cellular secondary flows. It is noted that the knowledge of the secondary flow structure is very essential for delineating distributions of other variables in the primary flow direction, such as velocity, Reynolds stresses, bed shear stress and even suspended sediment concentration, which are all affected by

secondary currents. This is because similarities often exist among transverse variations of the different variables [4,31].

Nomenclature			
$b_{\max}, b_{\min}$	highest and lowest bed elevation, respectively	Δb	amplitude of varying bed elevation
Fr	Froude number	$\Delta\tau_{bx}$	amplitude of the lateral variation in the stream-wise bed shear stress
h_m	mean flow depth	$\Delta(n^2)$	amplitude of varying squared Manning coefficient
$h_{\max}, h_{\min}$	maximum and minimum flow depths, respectively	γ	specific weight of fluid
J	mean flow energy slope	$\eta = (y - b)/h$	relative vertical coordinate
L_r	representative length	φ_y, φ_z	original phases in the vertical and transverse directions, respectively
m	integer numbers	λ	average longitudinal bedform width or mean secondary flow cell breadth
n_m	mean Manning coefficient	λ_r, λ_s	widths of rough and smooth strips, respectively
$n_{\max}, n_{\min}$	maximum and minimum Manning coefficients, respectively	$\lambda_{rg}, \lambda_{tr}$	widths of ridges and troughs, respectively
Re	Reynolds number	$\lambda_{dn}, \lambda_{up}$	breadths of the downflow and upflow zones, respectively
U_m	mean streamwise velocity in the central region of the flume	τ_b	bed shear stress in two-dimensional flow
$U_{\max}$	maximum streamwise velocity	τ_{bx}	bed shear stress in the x direction
U_{ms}	mean streamwise velocity over the entire cross-section of the flume	τ_{bxm}	mean bed shear stress in the x direction
V, W	time-averaged flow velocities in vertical and spanwise directions, respectively	ψ	stream function of fluid
$V_{\max}$	maximum upwelling velocity	$\zeta = z/\lambda$	relative transverse coordinate
V_r	representative velocity	$ V _m$	secondary flow index
ν_t	turbulent viscosity		
x, y, z	Cartesian coordinates in streamwise, vertical and spanwise directions, respectively		

2. EXPERIMENTR

2.1. Experimental set-up

The experiments were carried out in a straight rectangular tilting flume that was 14 m long, 0.6 m wide and 0.6 m deep. The slope of the flume was adjustable through jacks. Six types of rigid longitudinal bedforms were modeled successively in the flume. The cross-sectional shapes of these bedforms and the adopted coordinates are sketched in Fig. 2, where z is the transverse coordinate with $z = 0$ at the centreline of the flow channel; y is the vertical coordinate with $y = 0$ at the lowest bed surface at the section tested; λ is the average width of strips or ridges; and h_m and $h_{\max}$ are the mean and maximum flow depth, respectively. The details of the geometrical characteristics are also listed in Table 1. These bedforms could be categorized into bed strips and bed ridges. The bed strips comprised of longitudinal smooth strip and rough strip in an alternate manner. The rough strip was prepared by sediment particles that were generally uniform with a median diameter of 2.55 mm. The sediment was densely packed and not able to move for the flow conditions tested. PVC plate was used to form the smooth strip. The surface of all the strips was set nearly at the same level across the channel. However, the bed ridges (with rectangle and wavy shapes) were characterized by lateral periodic variations in the bed elevation. The rectangular ridges were formed by the PVC plates, and the wavy ridges was modeled by laterally corrugated iron sheets with painted surfaces. The wavy bedform adopted here was similar to that developed on a flat sediment bed (see [22]). As

shown in Fig. 2, all these bed configurations were symmetric with respect to the centerline of the flume.

The flow conditions for the experiments are given in Table 1. Flow depths were chosen to be nearly the same as the average width of strips or ridges. This choice is a typical case as reported in previous literature [12]. The bed slope varied within a small range. The similar flow conditions were adopted for different experiments to minimize variations in the bulk flow conditions, so that the role of bedforms in generating secondary flows can be highlighted.

2.2. Measurement apparatus

A Dantec LDA system, FlowLite 2D, was employed to measure the vertical and longitudinal velocity components. The system comprised of an integrated four-beam, two-component laser-optics unit, laser generator and signal processor, together with a traversing system and a PC installed with operation software, BSA 1.6. The LDA measurements were conducted over a half of the cross-section owing to the symmetric bed configuration. The sampling meshes, comprising of all sampling points at the test section, are plotted in Fig. 2. Cases S75, R75-10, R75-5 and WR were measured along 41 vertical lines evenly spaced 7.5 mm, while Cases S50 and R50-10 were measured along 31 vertical lines evenly spaced 10 mm. Because the two laser beams for measuring streamwise velocity were in the plane parallel to the flume bed and free surface, they could be moved as close as possible to the boundaries. In contrast, the two laser beams for measuring vertical velocity were in the cross-sectional plane and oblique to the bed and free surface, and thus they could be obstructed near the boundaries. Therefore, the measurable region for the vertical velocity varied with transverse location, and more points could be taken along the vertical lines closer to the sidewall (see Fig. 2). In the central part of the channel, the measurable region ranged approximately from $y = 10$ to 60 mm, and the neighboring points were spaced 5 mm. More points were added to the meshes for the lines closer to the sidewall, and 2 or 3 mm vertical spacing was used for these additional points. The sampling rate varied point by point. It ranged 100–300 Hz and 20,000 sets of data were recorded for most individual points. However, for the points very close to the bed, the sampling rate for some of such points reduced to only several decades, and the data collection automatically terminated once the sampling duration reached 300 s for saving time. The measured instantaneous velocities were analyzed together with the information of seeding particle arrival time and transit time. The data analysis yielded the mean velocities and other statistical moments, including turbulence intensities, skewness, flatness and cross-correlations. In order to visualize the mean flow pattern across the section, the sampling mesh were extended to a complete rectangular size, in which unknown values at the extended points were extrapolated from the collected data using kriging [12].

A DOP2000 ultrasonic pulsed Doppler velocimeter (UDV, model 2125) manufactured by Signal Processing (Lausanne, Switzerland) was used to measure the transverse velocity. This instrument measures instantaneous velocities at a number of points simultaneously along the ultrasonic beam. The transverse velocity was measured by

placing the UDV transducer perpendicular to the outer side of the flume sidewall without any intrusion into the flow. The sampling lines over the cross-section were vertically spaced 5 mm, which matched the LDA sampling mesh. The lowest line was just 1 mm above the top of the bed surface and the highest line was 2 or 3 mm below the free surface. To avoid effects of air at the interface between the transducer front and the flume sidewall, the interface was also sealed using an appropriate gel. Nearly 5000 instantaneous velocity profiles were recorded at each elevation at an invariant sampling rate of 18.5 Hz. For each transverse profile, the spatial resolution for sampling points was 0.75 mm. Thus, approximately 800 sampling points were taken for each line.

In the fluid dynamics community, LDA is considered an absolute flow measurement method. It has a very high accuracy, measuring velocity ranging from zero to supersonic order. As the optics of the apparatus was calibrated by the manufacturer, errors associated with them are considered negligible. In fact, LDA has been successfully employed in measuring secondary flows in the previous studies [13,18,26,29]. Therefore, no particular accuracy analysis is given here for the LDA measurement.

The velocity resolution of the UDV was up to 0.0007 mm/s. This resolution satisfies the accuracy requirement considering that the magnitude of secondary velocity was usually in the order of 1 mm/s. The accuracy of the UDV has also been examined by comparing its data with the LDA results in the author's recent work [32]. It has been demonstrated that the mean primary velocities in the central region could be consistently measured by the LDA and the UDV.

However, the UDV has some measurement uncertainties, resulting from water–glass interface (or sidewall), beam expansion, and bed and free surface effects. The side-wall effect is not important since only the data collected in the central region are analyzed in this study, where are free from the interface influence. The other two effects may cause some errors. It is known that ultrasonic beam of UDV gradually expands along the beam path, which means that sampling volume increases in size with distance from the transducer. For the transducer used in the experiments that had an emitting frequency of 8 MHz and a Piezo diameter of 5 mm, the sampling volume height even reached about 6 mm at the centerline of the channel. The potential consequence of the beam expansion was that the magnitude of the measured transverse velocity could be reduced or amplified since velocity in the proximity could be also read. The beam expansion could present most serious problems either near the bed or the flow surface where echoes from the stationary boundary were likely to bias the velocity measurement. The errors caused by this factor will be discussed in Section 4.5.

3. EXPERIMENTAL RESULTS

3.1. Case S75

Fig. 3 shows the contour map of V for Case S75. In the central zone of $-2 < z/\lambda < 0$, downflow occurs over the rough strips while upflow over the smooth strips. The cores of the contours are roughly located at the middle of the flow depth above the centreline of

each strip. The vertical division between downflow and upflow is located at the strip interface. In the central zone, the maximum vertical velocity is about 2.0% of the average primary velocity.

The contours of V are gradually skewed while approaching the sidewall, especially in the zone of $-4 < z/\lambda < -3.5$. This indicates that the sidewall also plays a role in generating secondary flows. However, the results shows that the sidewall effect is limited and only significant in the near-wall zone, which should be differentiated from those induced by longitudinal bedforms, the latter being concerned in this study. On the other hand, it is known that the vertical velocity beside the sidewall is upward for common corner flows. As shown in Fig. 3, this upward velocity was weakened in the presence of the rough strip attached to the sidewall. This is because secondary flows generated by longitudinal bedforms are usually directed downward over rough strips. For open channel flows with uniform bed, sidewall effect is usually restricted within a lateral region $(2-3)h$ [20,21]. Therefore, it is safe to suppose here that in the central zone, the sidewall effect is insignificant and the secondary flow observed is mainly related to the bed strips. This point is evidenced by the shape of V -contours, being well-positioned in the central region but skewed near the sidewall. In the following, only the flow data collected in the central zone of $-2 < z/\lambda < 0$ for each case will be presented and analyzed.

Fig. 4 plots the typical transverse profiles of W measured at four elevations in the central zone. Positive W -value indicates the motion from left to right, while the negative W -value means the opposite motion. Similar to the vertical velocity, W also varies with the bed configuration. It undulates transversely in a sinusoidal form with a wavelength identical to that of the bed strips. The measurements also indicate that the highest lateral velocity usually occurs over the interface between the rough and smooth strip (see Fig. 4). This may be due to the sudden change in the bed roughness. From the channel bottom to the middle of the flow depth, the amplitude of W first decreases gradually to zero; it then increases while approaching the free surface. It should be mentioned that the W -variations for the lower and upper flow portions have different phases (e.g., the two profiles measured at $y = 1 \text{ mm}$ and $y = 70 \text{ mm}$). This indicates that secondary flow circulates in the cross-sectional plane under the two-dimensional consideration.

Using the measured V and W , the velocity vectors at the test section are plotted in Fig. 5, showing a pair of counter-rotating flow circulations. The lower portions of cross-flows meet over the smooth strip, while the upper portions meet above the rough strip. The breadth of the flow cell is equal to the strip width, and the height of the flow cell is the same as the flow depth. The mean magnitude of the velocity vectors plotted in Fig. 5 is about $0.01U_m$ and the maximum magnitude is about $0.02U_m$, where U_m is the average streamwise velocity in the central zone of the channel flow.

3.2. Case S50

Fig. 6 shows the contours of V in the central region ($-2 < z/\lambda < 0$) for Case S50, for which the rough strip is twice wider than the smooth strip. The distribution of V is different from that for Case S75 (Fig. 3). For Case S50, the upflow zone is narrower than

the downflow zone, which causes that the magnitude of upflow velocity to be larger than that of downflow velocity. The maximum upflow velocity is about 2.5% of the average primary velocity, while the maximum downflow velocity is about 1.5%. The maximum upflow and downflow velocities occur above the respective centrelines of smooth and rough strips. The vertical velocity reduces to zero at the vertical line over the strip interface. Fig. 7 plots the typical transverse profiles of W measured at four elevations for Case S50. These profiles vary periodically in the transverse direction, but the positive part and negative part are not identical, and the shape of the curve is skewed. However, the maximum-and minimum-values of W still occur over the strip interface.

The velocity vectors are plotted in Fig. 8(a), in which the mean magnitude of the velocity vectors is about $0.011U_m$ and the maximum magnitude is about $0.025U_m$. A pair of secondary flow cells can be clearly recognized. However, the shape of the flow cells is not symmetric with respect to the circulation centre nearly locating over the strip interface. Corresponding to the unequal widths of smooth and rough strips, the upflow zone is narrower than the downflow zone. Similar secondary flow structures over unequal-width strips have also been reported by Studerus [26], for which the smooth strip is wider than the rough strip, having a ratio of 5:3. His result is shown in Fig. 8(b), in which flow cells can be also seen.

The experiment of Studerus [26], together those presented for Case 75 and Case S50, indicate that secondary flow cells can be generated independent of any particular strip widths, but the detail of the cellular structure may vary with the width ratio.

3.3. Case WR

Fig. 9 shows the contour map of V for Case WR. The most significant feature for this case is that the downflow occurs over the trough while the upflow over the crest. The upflow zone is concentrated around the ridge cusp and thus the upflow is nearly twice stronger than the down-flow. The maximum upwelling velocity is about 1.6% of the average primary velocity. Fig. 10 plots the typical transverse profiles of W at four elevations. It shows that W varies wavyly in the transverse direction, with a spatial period the same as the wavelength of bedform. In the lower portion of the flow the transverse motion is directed from trough to crest, while in the upper portion the transverse motion occurs in the opposite direction.

The cross-sectional velocity vectors for Case WR are plotted in Fig. 11(a). The counter-rotating secondary flow cells are very similar to those of Cases S75 and S50. The near-bed flow separates over the bed trough and converges over the bed ridge, while the opposite situation occurs near the free surface. The flow cell measures as wide as half of the ridge spacing, and as high as the flow depth. The mean magnitude of the velocity vectors is $0.007U_m$ and the largest magnitude is about $0.016U_m$. Some similar results were also obtained previously by Nezu and Nakagawa [15]. However they used a different ridge element, of which the cross-sectional shape was a 45° trapezoid (5 mm thick and 20 mm wide). The ridge elements were placed with a spacing of 80 mm, which was twice the flow depth. Their results show that secondary flow cells were also

generated over such ridges, as clearly demonstrated in Fig. 11(b). But the secondary flow structure is not strictly the same as that presented here for Case WR. This may further imply that secondary flow cells can be generated by all kinds of ridges, but the detail of the flow structure may vary for different ridge geometries.

3.4. Cases R75-10, R75-5 and R50-10

Fig. 12 shows three contour maps of V plotted for Cases R75-10, R75-5 and R50-10, respectively. The vertical velocity magnitudes for Cases R75-10 and R50-10 are almost the same since the ridge height for the two cases are identical. The maximum upwelling velocity is about 1.5% of the average primary velocity, and the maximum downwelling velocity is about 0.5%. For R75-5, the vertical velocity is relatively smaller because the ridge height is smaller. The relative maximum upwelling velocity is about 1.0%, and the relative maximum downwelling velocity is about 0.4%.

Figs. 13 and 14 show that the typical transverse velocity profiles for Cases R75-10 and R50-10, respectively. Compared with the W -profiles for the earlier cases, the W -profiles for these two cases exhibit more fluctuations with smaller amplitudes. Apparent transverse velocity occurs only in the lower portion of the flow. Generally, W varies with the bed configuration. It appears to be largest at the location above the transition from the trough to the crest, and becomes zero above the midpoints of the ridge and trough zones. When approaching the free surface, the variation amplitude decreases gradually. Small random variations are observed near the free surface, which may be due to the free-surface fluctuations.

The velocity vectors measured at the test section for Cases R75-10 and R50-10 are plotted in Fig. 15. It can be seen that the relatively strong upflow occurs over the vertical side of the ridge, while the downflow occurs over the crest as well as the trough. The flow cells appear in pairs both above ridge zone and trough zone. The vertical dimension of the flow cells is smaller than the flow depth; for the upper portion of the flow, the secondary flow becomes insignificant.

4. ANALYSIS OF SECONDARY FLOW STRUCTURE

As shown in the foregoing section, the secondary flows associated with various bed configurations appear complex but with some regular patterns, which could be further simplified for analytical considerations. This section aims to provide a quantitative description of these flow structures. Base on the measured phenomenological features and some kinematic arguments, stream functions of cellular secondary flows are first proposed and mathematical expressions of secondary flow velocities are then derived.

4.1. Symmetric flow cells (Case S75)

The time-mean secondary flows over longitudinal bed strips appear as paired-cells in the cross-sectional plane, as shown in Figs. 5, 8 and 11. These flow cells have a vertical dimension of the flow depth and a transverse dimension of the mean strip width. They

also have a circle-like shape, and the streamlines in the cross-sectional plane should appear as closed curves because of the continuity condition. By noticing these characteristics, we may propose a stream function $\psi(y, z)$, for secondary flows just based on kinematic considerations.

The recurrence of cellular secondary flows in the transverse direction suggests that $\psi(y, z)$ is a periodic function with respect to the transverse coordinate z . For example, for Case S75, the widths of upwelling and downwelling flow portions are identical, and also the same as the width of each strip. The transverse variation of W generally possesses a simple wavy pattern which is almost sinusoid, as illustrated in Fig. 4. Therefore, $\sin(\pi z / \lambda + \varphi_z)$ is proposed here for describing this variation, where λ is the average strip width (or half of the wavelength of bedforms), and φ_z is the initial phase in the transverse direction that is dependent on the adopted coordinates. It should be mentioned that the sinusoidal function has already been employed for cellular secondary flows in some other studies [4,7,17,29].

Fig. 5 shows that the shape of secondary flow cell appears to be symmetric with respect to its circulation center, which locates at the middle of the flow depth above the interface of rough and smooth strips. The measurements indicate that the circulation centre is nearly at $y/h = 0.5$, where y is the vertical coordinate and h is the flow depth. Slight shift of the circulation centre can be observed for the measured situations, but here for simplicity it is taken at $y/h = 0.5$. With this assumption, the vertical variation can also be described by a sinusoidal function, say, $\sin(\pi y / \lambda + \varphi_y)$, where φ_y is the initial phase in the vertical direction. Combining the two sinusoidal functions, the stream function applied in the cross-section for the secondary flow may be written as

$$\psi = \frac{-V_r L_r}{\pi} \sin(\pi \eta + \varphi_y) \sin(\pi \zeta + \varphi_z) \quad (1)$$

in which V_r, L_r is the typical velocity and length scales, respectively; $\eta = y/h$; and $\xi = z/\lambda$. By differentiating the stream function, the vertical and transverse velocity, V and W , can be obtained

$$V = \frac{\partial \psi}{\partial z} = -\frac{V_r L_r}{\lambda} \sin(\pi \eta + \varphi_y) \cos(\pi \zeta + \varphi_z) \quad (2)$$

$$W = -\frac{\partial \psi}{\partial y} = \frac{V_r L_r}{h} \cos(\pi \eta + \varphi_y) \sin(\pi \zeta + \varphi_z) \quad (3)$$

For the idealized conditions, $V(\xi = 0.0, \eta = 0.5) = V_{\max}$, $V(\xi = 0.0, \eta = 0.0) = 0$, $W(\eta = 0.5, \xi = 1.0) = W_{\max}$, and $W(\eta = 0.0, \xi = 0.0) = 0$, and then we obtain $V_r = V_{\max}$, $L_r = \lambda$ and $\varphi_z = \varphi_y = 0$. As a result, the expressions for stream function and secondary velocities are

$$\psi = -\frac{\lambda V_{\max}}{\pi} \sin(\pi\eta) \sin(\pi\zeta) \quad (4)$$

$$V = \frac{\partial\psi}{\partial z} = -V_{\max} \sin(\pi\eta) \cos(\pi\zeta) \quad (5)$$

$$W = -\frac{\partial\psi}{\partial y} = \frac{\lambda V_{\max}}{h} \cos(\pi\eta) \sin(\pi\zeta) \quad (6)$$

Fig. 16(a) shows the theoretical streamlines for secondary flow cell of symmetric pattern, which are obtained using Eq. (4) for the condition of $h = 0.075$ m, $\lambda = 0.075$ m, and $V_{\max} = 0.02 U_m = 0.0094$ m/s. The secondary flow structure illustrated in Fig. 16(a) is very similar to the measurement shown in Fig. 5. With the computations using Eqs. (5) and (6), Fig. 17 shows the comparison between the measured and theoretical results of W and V . In the plots, all the results are normalized by $0.02 U_m$. The computed results are denoted by solid lines, and the crosses represent the measured velocities. It can be seen that the overall velocity distribution pattern is generally well predicted, but some significant discrepancies from the measurements exist for certain locations that will be explained in Section 4.5.

4.2. Laterally skewed flow cells (Case S50)

The experimental results for Case S50 indicate that the widths for the upflow zone and downflow zone are not identical. As shown in Fig. 8, the circulation centre shifts laterally from the geometrical centre of flow cell, and the shape of the flow cell is also laterally skewed. It is noticed from Fig. 7 that the transverse profiles of W are also wavelike, but the lengths for descending (view from right to left) and ascending parts within a spatial period are different. Thus, the usual sinusoidal function, $\sin(\pi\xi)$, is not applicable for describing the transverse variation of secondary flows for such cases. Alternatively, a piece-wise sinusoidal function is proposed

$$\text{PS}(\zeta) = \begin{cases} \frac{\lambda_{\text{up}}}{\lambda} \sin \left[\frac{(\zeta - 2m)\lambda}{\lambda_{\text{dn}}} \pi \right] & 2m - \frac{\lambda_{\text{dn}}}{2\lambda} \leq \zeta \leq 2m + \frac{\lambda_{\text{dn}}}{2\lambda} \\ \frac{\lambda_{\text{up}}}{\lambda} \sin \left[\frac{(\zeta - 2m - \frac{\lambda_{\text{dn}}}{2\lambda})\lambda}{\lambda_{\text{up}}} \pi + \frac{\pi}{2} \right] & 2m + \frac{\lambda_{\text{dn}}}{2\lambda} \leq \zeta \leq 2m + \frac{\lambda_{\text{dn}}}{2\lambda} + \frac{\lambda_{\text{up}}}{\lambda} \end{cases} \quad (7)$$

where $m = 0, \pm 1, \pm 2, \dots$; $\lambda_{\text{dn}}, \lambda_{\text{up}}$ = widths of the downflow and upflow zones, respectively; and $\lambda = (\lambda_{\text{dn}} + \lambda_{\text{up}})/2$, which is the same as the average width of strip. It is noted that this equation reduces to $\sin(\pi\xi)$ when $\lambda_{\text{dn}} = \lambda_{\text{up}}$.

Regardless of the lateral shift of the circulation centre, there is no evident shift of the circulation centre in the vertical direction, as shown in Fig. 8. Therefore, it is supposed that the circulation centre remains at $y/h = 0.5$, and $\sin(\pi\eta)$ is also applicable. Therefore, the stream function in a more general form for laterally skewed cellular flows is

$$\psi = -\frac{\lambda V_{\max}}{\pi} \sin(\pi\eta) \text{PS}(\zeta) \quad (8)$$

This stream function can be used for the cases either of equal strip width or unequal strip width. By differentiating this stream function, V and W are given by

$$\frac{V}{V_{\max}} = -\sin(\pi\eta) \text{PC}(\zeta) \quad (9)$$

$$\frac{W}{V_{\max}} = \frac{\lambda}{h} \cos(\pi\eta) \text{PS}(\zeta) \quad (10)$$

where $\text{PC}(\xi)$ is the derivative of $\text{PS}(\xi)$, and given by

$$\text{PC}(\zeta) = \begin{cases} \frac{\lambda_{\text{up}}}{\lambda_{\text{dn}}} \cos \left[\frac{(\zeta - 2m)\lambda}{\lambda_{\text{dn}}} \pi \right] & 2m - \frac{\lambda_{\text{dn}}}{2\lambda} \leq \zeta \leq 2m + \frac{\lambda_{\text{dn}}}{2\lambda} \\ \cos \left[\frac{(\zeta - 2m - \frac{\lambda_{\text{dn}}}{2\lambda})\lambda}{\lambda_{\text{up}}} \pi + \frac{\pi}{2} \right] & 2m + \frac{\lambda_{\text{dn}}}{2\lambda} \leq \zeta \leq 2m + \frac{\lambda_{\text{dn}}}{2\lambda} + \frac{\lambda_{\text{up}}}{\lambda} \end{cases} \quad (11)$$

For practical application, the parameters, λ_{dn} and λ_{up} , need to be determined first. They can be roughly estimated from the strip width. For example, the circulation centre for Case S50 is roughly over the strip interface, as shown in Fig. 8. This may hold for the other cases considering that the strongest cross-flow usually occurs in the transitional location due to the sudden change of the bed roughness. Therefore, it can be taken that $\lambda_{\text{dn}} = \lambda_r$, the width of the rough strip; and $\lambda_{\text{up}} = \lambda_s$, the width of the smooth strip. With the measured conditions for Case S50 that $h = 0.075$ m, $\lambda_r = 0.1$ m, $\lambda_s = 0.05$ m, $\lambda = 0.075$ m, and $V_{\max} = 0.025$, $U_m = 0.01$ m/s, the idealized streamlines for laterally skewed flow cells are obtained using Eq. (8) and plotted in Fig. 16(b). Fig. 18 plots the comparison of V and W computed using Eqs.(9) and (10) and the experimental data. In general, the distribution pattern of the secondary flow velocities is well predicted.

4.3. Vertically distorted flow cells (Case WR)

For the last two cases, the cellular secondary flows are induced solely by lateral variations in bed roughness. In comparison, the experimental results for Case WR demonstrate that cellular secondary flows can also be generated by the wavy ridges without transverse roughness variations. In spite of the different bed configurations, the secondary flow structures are very similar as shown in Figs. 5, 8 and 11. This similarity implies that the bed trough for Case WR could serve equivalently as the rough strip, while the bed ridge could be comparable to the smooth strip. Actually, lateral bed perturbations either in bed roughness or elevation will always cause lateral gradient of bed shear stress. For the location with relatively deeper flow depth or greater roughness,

the local bed shear stress may increase, and vice versa. Thus, downflow occurs over rough strips or troughs and upflow occurs over smooth strips or ridges.

Considering the above-mentioned similarity, we extend the use of the previously proposed stream function to the wavy bed surface simply by introducing the following vertical-coordinate transformation

$$\eta = \frac{y - b}{h} \quad (12)$$

where b is the local bed height; $y - b$ is the local vertical distance from the bed surface; and h is the local flow depth. Therefore, the stream function for the case of wavy ridge is given by

$$\psi = -\frac{\lambda V_{\max}}{\pi} \sin\left(\pi \frac{y - b}{h}\right) \text{PS}(\zeta) \quad (13)$$

It should be noted that λ , which was formerly defined as the average strip width, here is the half spacing between two adjacent ridge cusps. Then, V and W are derived

$$\frac{V}{V_{\max}} = -\frac{1}{\pi} \sin(\pi\eta) \text{PC}(\zeta) - \frac{\lambda(1-\eta)}{h} \frac{\partial h}{\partial z} \cos(\pi\eta) \text{PS}(\zeta) \quad (14)$$

$$\frac{W}{V_{\max}} = \frac{\lambda}{h} \cos(\pi\eta) \text{PS}(\zeta) \quad (15)$$

Eq. (14) indicates that V is also affected by the lateral bed slope, i.e., $\partial h / \partial z$.

With the measured conditions for Case WR that $h_{\max} = 0.075$ m, $\lambda = 0.075$ m, $\lambda_{\text{dn}} = 0.09$ m, $\lambda_{\text{up}} = 0.06$ m, and $V_{\max} = 0.016$, $U_m = 0.0083$ m/s, the idealized streamlines for the flow cells of vertically distorted pattern over wavy ridges can be obtained, as plotted in Fig. 16(c). Using Eqs.(14) and (15), the theoretical results of V and W for Case WR are computed and compared with the measurements, as shown in Fig. 19. It is demonstrated that the velocity distribution pattern is well predicted, but the significant discrepancy exists at some locations between the computation and measurements that will be discussed in Section 4.5.

4.4. Secondary flow structure over rectangular bed ridges (Cases R75-10, R75-5 and R50-10)

For the secondary flows induced by transverse variations in the bed elevation, the cross-flow near the bed is guided by lateral bed slope. If the bed elevation ascends or descends gradually in transverse direction, such as Case WR, a complete flow cell can extend from the trough bottom to the ridge cusp. However, if the bed elevation changes suddenly, such as rectangular bed ridges, the cross-flow near the bed that would be directed from trough to ridge may be impeded by the vertical connection between ridge and trough. Meanwhile, the corner near the side of the ridge may induce upwelling flows,

which also tends to alter the direction of the near-bed cross-flow. Two separate flow cells are thus formed from the trough centerline to the ridge centerline, as demonstrated in Fig. 15. This is greatly different from the secondary flow structure formed over the wavy or trapezoidal ridges.

Because the two sets of paired cells appear in a wavelength of rectangular ridges, the ratio of the width to height of the flow cell would be less than unity if the cell develops over the entire flow depth, which is half of the ridge wavelength. Actually, the secondary flow in the upper portion attenuates gradually while approaching the free surface, as shown in Fig. 12. Consequently, the secondary flows so generated are much weaker, appearing almost disconnected over the entire cross-section if compared with those generated by wavy ridges or rough/smooth strips. This result may further imply that the cellular flow cell generated by the bed configuration tends to attach spatially to the bed surface, and its shape can be effectively measured by the ratio of width to height that is approximately unity regardless of the flow depth.

Because the secondary flow structure over rectangular ridges is more complex and less organized (see Fig. 15), no attempt has been done in this study to quantify it analytically.

4.5. Discussions

The analytical formulations proposed in this study for describing the structures of the secondary flows are simple but practically capable of providing reasonable predictions. It should be noted that the formulations, which are empirically derived only on kinematic grounds, could serve as good alternatives to theoretical solutions to secondary flow velocities. The latter can hardly be established given the inherent mathematical difficulties [17].

On the other hand, it is noted that Ikeda [7] also proposed an analytical model for secondary flows, which is similar to the result presented here. However, his model is subject to several assumptions. For example, he assumed that the transverse Reynolds shear stress could be expressed in terms of secondary velocity gradients, i.e., $v_t \left(\frac{\partial V}{\partial z} + \frac{\partial W}{\partial y} \right)$, where v_t is turbulent viscosity. This assumption may not hold for the flow condition concerned here because any turbulence model based on the eddy viscosity concept fails to reproduce turbulence-induced secondary flows, as pointed out by Naot and Rodi [14]. Moreover, the Ikeda model requires several parameters to be determined experimentally, and also cannot be applied for the case of distorted secondary flow cells. Given these restrictions, no attempts were made here to compare the Ikeda model with the present study.

In performing the analysis, the non-slip condition for W is not considered in this study. However, this inadequacy does not impair considerably the practical applicability of the solution. The experimental results show that W has large magnitudes even very close to the bed. Note that the lowest and second lowest measuring position for W is only 1 mm from the bed. Since the overall trend of the W -distributions can be well predicted, the analytical expression proposed in this study can be considered valid for the major flow

depth (e.g., $0.05 < y/h < 1$). Flow close to the bed is very complex owing to the direct effect of bed roughness, which is often referred to as roughness layer with a thickness 1–4 times the roughness element size [24]. Investigation of the flow in this layer is beyond the scope of this study.

A systematic deviation of the predicted W from the measurements for certain locations can be found in Figs. 17–19. It may result partially from the limitation of the UDV measurement. This is because ultrasonic beam of UDV gradually expands along the beam path, which means that sampling volume increases in size with distance from the transducer. The potential consequence of the increasing sampling volume is that the magnitude of the measured W component may be reduced or amplified in errors, particularly near the bed or the free surface where echoes from the stationary boundary were also likely to bias the velocity measurement (see those lowest points of W in Figs. 17 and 18). In comparison, the discrepancy between the LDA data and the predicted velocities are smaller and more randomly since the LDA sampling volume is of a tiny size of 0.1 mm scale. On the other hand, the present model is derived based on the simple kinematic consideration and thus some limitations are expected when applying it for computing secondary flow velocities.

5. MAXIMUM VERTICAL VELOCITY AND BED CONFIGURATION

In the proposed analytical solution, the maximum vertical velocity $V_{\max}$ is required as a basic parameter. This section is to develop an empirical relationship between $V_{\max}$ and bed configurations.

From the experimental observations, it is understood that transverse variations either in bed elevation or bed roughness can cause perturbations to streamwise bed shear stress τ_{bx} . Simultaneously, the lateral imbalance of turbulent stresses of the flow is also induced, which consequently generates cellular secondary flows. Therefore, one can expect that the intensity of secondary flows is closely related to the maximum differences in bed elevation and/or roughness height, i.e., their variation amplitudes. In addition, the secondary flow intensity should be also affected by the primary flow since the overall turbulence strength is determined by the bulk flow properties. To quantify the secondary flow intensity, we introduce a secondary flow index, i.e., the spatially averaged magnitude of vertical velocity

$$|V|_m = \frac{\int \int |V| dy dz}{\int \int dy dz} \quad (16)$$

Based on the considerations mentioned above, this index should depend on the following factors:

$$|V|_m = f(\Delta b, \Delta(n^2), h_m, n_m^2, U_m) \quad (17)$$

where $\Delta b = b_{\max} - b_{\min}$, the amplitude of varying bed elevation; $b_{\max}$ is the highest bed elevation; $b_{\min}$ is the lowest bed elevation; $\Delta(n^2) = n_{\max}^2 - n_{\min}^2$, the amplitude of varying squared Manning coefficient that represents the variation of bed roughness; $n_{\max}$ is the maximum Manning coefficient; $n_{\min}$ is the minimum Manning coefficient; h_m is the mean flow depth; n_m is the mean Manning coefficient and U_m is the space-averaged mean primary velocity. The choice of the squared Manning coefficient is explained in the subsequent paragraph.

In Eq. (17), the three mean parameters, h_m , n_m and U_m , are included for characterizing the overall properties of the primary flow, while the rest two factors, Δb and $\Delta(n^2)$, are considered to be responsible for the generation of the secondary flows. With h_m , n_m^2 and U_m , the mean streamwise bed shear stress may be expressed as

$$\tau_{bxm} = \gamma J h_m \quad \text{or} \quad (18)$$

$$\tau_{bxm} = \frac{\gamma U_m^2}{h_m^{1/3}} n_m^2 \quad (19)$$

where γ is the specific weight of fluid and J is the mean flow slope. In considering the shear stress variation induced dominantly by the bed surface elevation, we may assume that the flow slope is constant. Using Eq. (18), one gets

$$\Delta \tau_{bx} = \gamma J \Delta b \quad \text{or} \quad \frac{\Delta \tau_{bx}}{\tau_{bxm}} = \frac{\Delta b}{h_m} \quad (20)$$

Similarly, for the case of the perturbation caused largely by the roughness variation, it may be assumed that effects of the flow depth and depth-averaged velocity are negligible. Then the application of Eq. (19) yields

$$\Delta \tau_{bx} = \frac{\gamma U_m^2}{h_m^{1/3}} \Delta(n^2) \quad \text{or} \quad \frac{\Delta \tau_{bx}}{\tau_{bxm}} = \frac{\Delta(n^2)}{n_m^2} \quad (21)$$

Furthermore, if assuming that the stress variation, $\Delta \tau_{bx}$, induced by the perturbations either related to the bed roughness or the bed elevation is additive, Eq. (17) may be rewritten in the following dimensionless form:

$$\frac{|V|_m}{U_m} = f \left(\frac{\Delta b}{h_m} + \frac{\Delta(n^2)}{n_m^2} \right) \quad (22)$$

If using the general expression of V , i.e., Eq.(9), Eq. (16) becomes

$$\begin{aligned} |V|_m &= \frac{\int_0^\lambda \int_0^h \left| -\frac{V_{\max}}{\pi} \sin(\pi\eta) \text{PC}(\zeta) \right| dy dz}{\int_0^\lambda \int_0^h dy dz} = \frac{\frac{4\lambda_{\text{up}} h V_{\max}}{\pi^2}}{\lambda h} \\ &= \frac{\lambda_{\text{up}}}{\lambda} \frac{4V_{\max}}{\pi^2} \end{aligned} \quad (23)$$

This equation indicates that $V_{\max}$ linearly varies with the ratio between upflow width and downflow width of secondary flow cells

$$V_{\max} \propto \frac{\lambda}{\lambda_{\text{up}}} |V|_{\text{m}} \quad (24)$$

This relationship is physically understandable. For a flow cell with the same intensity, a constricted upflow zone will lead to an increase in the maximum upward velocity, which is required by the flow continuity, and vice versa. Substituting Eq. (24) into Eq. (22) yields

$$\frac{V_{\max}}{U_m} \frac{\lambda_{\text{up}}}{\lambda} = f \left(\frac{\Delta b}{h_m} + \frac{\Delta(n^2)}{n_m^2} \right) \quad (25)$$

In the following, Eq. (25) is substantiated using the experimental data. The Manning coefficient is taken as 0.01 for the smooth PVC bed strip used for the experiment. For the rough bed strips, it can be estimated from the relevant roughness size with the Strickler-type relationship [25], which states

$$n = \frac{d_{50}^{1/6}}{K} \quad (26)$$

where $K = 21.1$. The average Manning coefficient n_m can be calculated using Eqs. (18) and (19) given the flow slope, J , and the average flow velocity, U_m .

Fig. 20 shows the empirical relationship of $V_{\max}$ and $\Delta b/h_m + \Delta(n^2)/n_m^2$. The experimental data used are given in Table 2, which include the results presented in this study and those by Nezu and Nakagawa [15]. For the latter data, U_m is roughly taken as $U_{\max}/1.15$ since U_m is not provided. The ratio of $U_{\max}/U_m$ is estimated using the logarithmic distribution of the primary velocity for 2D open channel flows. The figure shows that the upper range of the relationship appears as linear with a positive intercept. However, the intensity of secondary flows should reduce to zero if the lateral variations in the bed roughness and surface elevation are vanishingly small. To describe this reduction, an exponential function is employed here. Finally, using the experimental data, an empirical relationship is proposed as follows:

$$\frac{V_{\max}}{U_m} \frac{\lambda_{\text{up}}}{\lambda} = (0.0067E + 0.0122)[1 - \exp(-15.4E)] \quad (27)$$

where $E = \Delta b/h_m + \Delta(n^2)/n_m^2$. Eq. (27) is also plotted in Fig. 20, showing that $V_{\max}$ increases rapidly with E for $E < 0.2$, and increases moderately with E in a linear fashion for $E > 0.2$.

6. CONCLUSIONS

This study is focused on time-mean structures of cellular secondary flows generated by various longitudinal bed-forms that were imposed in open channel flows. A series of laboratory experiments was conducted with six artificial, rigid longitudinal bedforms. The bedforms included alternate bed strips with different roughness heights and bed ridges of wavy and rectangular shapes. Detailed flow measurements were conducted using a two-component laser Doppler anemometer (LDA) and a one dimensional ultrasonic Doppler velocimeter (UDV).

The generated secondary flows can be viewed as pairs of counter-rotating in the cross-sectional plane. The vertical and spanwise dimensions of flow cells measured the same as flow depth and average width of bedforms, respectively. Downflow occurred over rough strip or trough whereas upflow occurred over smooth strip or ridge. Near the bed, cross-flow was directed from the rough strip (or trough bottom) to smooth strip (or ridge cusp). Cross-flow near the free surface was directed oppositely. The strongest cross-flow usually occurred over the interface between the rough and smooth strips due to the sudden change in the bed roughness. The formed flow cell appeared with a symmetric pattern for the case of strips of equal widths, but being laterally skewed for the strips of unequal width. Over the wavy ridges, the flow cells appeared to be vertically distorted. However, doubled secondary flow cells were generated over the rectangular ridges because of the vertical connection between the ridge and trough.

This study suggests that time-mean secondary flows could be described reasonably by simple analytical expressions in the sinusoidal form, which are applicable for various secondary flow structures with different patterns. The analytical results provide not only a better understanding of secondary flow characteristics, but also an insight into modifications of other flow quantities caused by secondary flows. To complete the analytical solution, an empirical relationship between the maximum vertical velocity and bed configuration is also proposed based on some physically reasoning and the experimental data. Comparisons between the experimental data and analytical results for secondary velocities show that the cellular pattern of the secondary flows can be generally predicted but with discrepancies due to theoretical limitations and measurement uncertainties.

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Figure. 20 Relationship between maximum vertical velocity and bed configuration.

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Fig. 14. Transverse profiles of W at various elevations for Case R50-10.

Figure 14

Figure 15

Figure 16

Figure 17

Figure 18

Figure 19

Figure 20

Case no.	Bedform type (–) (1)	Width of rough strips (ridges), λ_r (mm) (2)	Width of smooth strips (troughs), λ_s (mm) (3)	Average width of strips or ridges, λ (mm) (4)	Height of ridges, h_{rg} (mm) (5)	Mean flow depth, h_m (m) (6)	Section- averaged flow velocity, U_{ms} (m/s) (7)	Central-region- averaged flow velocity, U_m (m/s) (8)	Mean energy slope, J (‰) (9)	Froude number, Fr (–) (10)	Reynolds number, Re (–) (11)
S75	Rough/smooth strips	75	75	75	–	0.0750	0.4691	0.4739	1.2	0.55	44,206
S50	Rough/smooth strips	100	50	75	–	0.0750	0.4074	0.4147	1.0	0.48	38,684
WR	Wavy ridges	–	–	75	11.7	0.0782	0.5078	0.5167	0.8	0.60	48,358
R75-10	Rectangular ridges/troughs	75	75	75	10	0.0800	0.4745	0.5218	0.6	0.63	45,149
R50-10	Rectangular ridges/troughs	100	50	75	10	0.0800	0.4989	0.5549	0.7	0.67	49,013
R75-5	Rectangular ridges/troughs	75	75	75	5	0.0775	0.4898	0.5228	0.9	0.62	46,881

Table 1

Case no.	$\Delta b/h_m$	$\Delta(n^2)/n_m^2$	$\frac{V_{max}}{U_m} \frac{\lambda_{ap}}{\lambda}$
S75	—	1.137	0.0200
S50	—	1.044	0.0167
Wang and Cheng [31]	—	1.749	0.0250
Müller and Studerus [13]	—	1.754	0.0242
WR	0.153	—	0.0128
S75-10	0.125	—	0.0120
S75-5	0.065	—	0.0056
S50-10	0.122	—	0.0120
Nezu and Nakagawa [15]			
(Case K)	0.128	—	0.0112
(Case I)	0.217	—	0.0129
(Case J)	0.128	—	0.0120

Table 2