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Formula for Viscosity of Glycerol-Water Mixture

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Abstract

An empirical formula is proposed for the calculation of the viscosity of glycerol-water mixture for mass concentrations in the range of 0-100% and temperatures varying from 0 to 100 ^{o}C. It compares well with three databases available in the literature and its application procedure is also simpler than other previously-developed correlations.

Introduction

Many industrial applications require knowledge of liquid viscosities, for example, for optimizing chemical processes and determining power supplies associated with pipeline transportation and pump operations. Aqueous glycerol solutions are widely used in experimental studies of flow phenomena. Experiments conducted with glycerol solutions facilitate investigation of flows in a wide range of Reynolds numbers. However, the approach used to estimate the viscosity of glycerol solutions is still incomplete in the literature. This is largely because there is no comprehensive theory on the viscosity of liquids at present. Some theoretical approaches including those developed based on

molecular dynamics could provide valuable understandings of relevant fundamentals but often cause large deviations from measured viscosity data[1, 2].

Practically, calculation of the mixture viscosity is often performed with empirical data-driven correlations between viscosity and other liquid properties. Such correlations are generally interpolative. Some are simple but applicable to limited conditions, while the others may involve application procedures that are tedious [1]. For example, Chen and Pearlstein [3] proposed a four-parameter correlation for the dynamic viscosity (μ) of glycerol-water mixtures,

$$\mu = A_1 \exp[A_2(T+273.15)^{-3} + A_3(T+273.15) + A_4(T+273.15)^{-1}] \tag{1}$$

where μ is in cP or 0.001Ns/m^2, T is in $^{\circ}\text{C}$, and the coefficients, A_1 to A_4, vary with the glycerol concentration in mass (C_m). Use of Eq. (1) is restricted because the values of A_1 to A_4 are only given for some discrete concentrations, i.e. $C_m = 40\%, 50\%, \ldots, 90\%, 99\%$, 100%. For other concentrations, numerical interpolations or extrapolations are necessary, which may induce additional computational errors.

Shankar and Kumar [4] suggested the kinematic viscosity (ν) of aqueous glycerol solutions be estimated as

$$\frac{\ln(\nu/\nu_w)}{\ln(\nu_g/\nu_w)} = C_m\left[1 + (1-C_m)(B_1 + B_2 C_m + B_3 C_m^2)\right] \tag{2}$$

where subscripts g and w denote glycerol and water, respectively, and B_1 to B_3 are the temperature-dependent coefficients. Similar difficulties are encountered in applying Eq. (2) because the B-coefficients were evaluated only for five discrete temperatures, i.e. T = 10, 20, 30, 40, 50 $^{\circ}\text{C}$.

A recent effort for determining kinematic viscosities of glycerol solutions was made by Chenlo et al [5]. They related the kinematic viscosity to the mole fraction of glycerol (C_{mol}) and temperature, i.e.

$$\frac{v}{v_w} = 1 + 0.125 C_{mol} \exp\left(\frac{C_{mol}^{0.219}}{2.291[(T + 273.1)/273.1]^3 - 1} \right) \quad (3)$$

where

$$v_w = 0.09607 \times 10^{-6} \exp\left(\frac{2.9}{[(T + 273.1)/273.1]^3} \right) \quad (4)$$

Eq. (3) does not engage any unknown coefficients and also reproduces Chenlo et al's data with small deviations. However, it cannot be applied for high glycerol concentrations. For example, when compared with the data by Shankar and Kumar [4], Eq. (3) yields significant deviations, which reach up to 9% for $C_m = 40\%$, and 50% for $C_m = 70\%$.

In this study, a formula for the dynamic viscosity of glycerol-water mixtures is proposed, which is applicable for a wide variety of conditions, i.e. C_m = 0-100% and T = 0-100 ^{o}C. Comparisons are also made between the proposed formula and three different sets of experimental data available in the literature.

Formulation proposed in this study

In the following, the analysis is conducted for temperatures varying from 0 to 100 ^{o}C at the atmospheric pressure. First, the dynamic viscosity of the glycerol-water mixture, μ, is related to those of the two components in the power form,

$$\mu = \mu_w^\alpha \mu_g^{1-\alpha} \quad (5)$$

where subscripts w and g denote water and glycerol, respectively, and α is the weighting factor varying from 0 to 1. Eq. (5) can be also rewritten as

$$\mu = \mu_g \exp(A\alpha) \tag{6}$$

where $A = \ln(\mu_w/\mu_g)$. Eq. (6), in its exponential form, resembles Eqs. (1) and (2). Other formulas similar to Eq. (5) were also reported previously, but the weighting factors involved were only associated with the concentration of glycerol [1]. Actually, as shown later, α varies with temperature as well as concentration.

With Eq. (5), α is given by

$$\alpha = \frac{\ln(\mu/\mu_g)}{\ln(\mu_w/\mu_g)} \tag{7}$$

Eq. (7) predicts that α decreases from 1 to 0 with increasing concentration, as implied by experimental data. This is illustrated in Fig. 1 for $T = 20^{\circ}\mathrm{C}$ with the data given by Segur and Oberstar [6].

To examine the variation of α in detail, the above graph is re-plotted in Fig. 2 as β $(= \alpha - 1 + C_m)$ against C_m. Fig. 2 shows that β varies almost linearly as C_m approaches 0 and 100%. These linear variations are approximated as follows:

$$\beta_1 = aC_m \quad \text{for } C_m \to 0 \tag{8}$$

$$\beta_2 = b(1 - C_m) \quad \text{for } C_m \to 100\% \tag{9}$$

To estimate the non-linear change in β for $0 < C_m < 100\%$, a power-sum interpolating function is applied here,

$$\beta^n = \beta_1^n + \beta_2^n \tag{10}$$

where n is an exponent. The best-fit using Eq. (10) is almost achieved with $n = -1$, as shown in Fig. 2. For experimental data collected for other temperatures between 0 and 100 °C [6], Eq. (10) with $n = -1$ can also provide similar approximations. Substituting Eqs. (8) and (9) into Eq. (10) and noting that $\alpha = 1 - C_m + \beta$, we get

$$\alpha = 1 - C_m + \frac{abC_m(1 - C_m)}{aC_m + b(1 - C_m)} \tag{11}$$

Evaluation of coefficients, *a* and *b*

To estimate a and b, the data used are those collected by Segur and Oberstar [6] for $0\,^{\circ}\text{C} < T < 100\,^{\circ}\text{C}$. The relations of a and b to the temperature are shown in Fig. 3, which are further approximated by

$$a = 0.705 - 0.0017T \tag{12}$$

$$b = (4.9 + 0.036T)a^{2.5} \tag{13}$$

Evaluations of μ_w and μ_g

The proposed formula, Eq. (5), is interpolative in nature, so the viscosities of the two components, μ_w and μ_g, must be known.

Dynamic viscosity of water, μ_w

It is noted that μ_w generally reduces with increasing temperature, T. The reduction can be presented in the form similar to Eq. (5),

$$\mu_w = \mu_{w100}^{\varepsilon}\mu_{w0}^{1-\varepsilon} \tag{14}$$

where μ_{w0} is the dynamic viscosity at $T = 0\,^{\circ}\text{C}$, μ_{w100} is that at $T = 100^{\circ}\text{C}$, and ε is the weighting factor in the range of 0 to 1. From Eq. (14),

$$\varepsilon = \frac{\ln(\mu_w / \mu_{w0})}{\ln(\mu_{w100}/ \mu_{w0})} \tag{15}$$

With the data by Washburn et al. [7], the variation of ε with $T/100$ is computed and plotted in Fig. 4. It can be observed that the variations at T approaching 0 $^{\circ}\text{C}$ and 100 $^{\circ}\text{C}$ are approximately linear. Therefore, it is assumed that

$$\varepsilon = a_w\left(\frac{T}{100}\right) \quad \text{for } T \to 0\,^{\circ}\text{C} \tag{16}$$

$$1 - \varepsilon = b_w\left(1 - \frac{T}{100}\right) \quad \text{for } T \to 100\,^{\circ}\text{C} \tag{17}$$

where a_w and b_w are coefficients. Eqs. (16) and (17) can be also transformed to

$$\varepsilon - \frac{T}{100} = (a_w - 1)\left(\frac{T}{100}\right) \quad \text{for } T \to 0\,^{\circ}\text{C} \tag{18}$$

$$\varepsilon - \frac{T}{100} = (b_w - 1)\left(\frac{T}{100} - 1\right) \quad \text{for } T \to 100\,^{\circ}\text{C} \tag{19}$$

It is proposed here that the above two equations be generalized for $0\,^{\circ}\text{C} < T < 100\,^{\circ}\text{C}$ as follows

$$\left(\varepsilon - \frac{T}{100}\right)^{-1} = \left[(a_w - 1)\left(\frac{T}{100}\right)\right]^{-1} + \left[(b_w - 1)\left(\frac{T}{100} - 1\right)\right]^{-1} \tag{20}$$

Eq. (20) is plotted in Fig. 5, showing that the interpolating function is generally close to the data points reported by Washburn et al. [7]. It is noted that the interpolating technique is the same as that used in Eq. (10) and Fig. 2.

Solving for ε from Eq. (20) and substituting into Eq. (14) yields

$$\mu_w = 1.790 \exp\left(\frac{(-1230-T)T}{36100+360T}\right) \tag{21}$$

where μ_w is in cP or 0.001Ns/m^2, T is in the range of 0 - 100 $^\circ$C, and the four constants are estimated by applying the data given by Washburn et al. [7]. Eq. (21) represents the Washburn et al.'s data with deviations less than 0.4%. Additional calculations show that Eq. (21) can also reproduce the data reported by Linstrom and Mallard [8] with deviations less than 0.5%.

Dynamic viscosity of glycerol, μ_g

Similarly, the following exponential formula is proposed for calculating the dynamic viscosity of glycerol,

$$\mu_g = 12100 \exp\left(\frac{(-1233+T)T}{9900+70T}\right) \tag{22}$$

where μ_g is in cP or 0.001Ns/m^2 and T is in $^\circ$C. Eq. (22) is calibrated using the data, which were given by Segur and Oberstar [6] for $T = 0\text{-}100^\circ$C, with differences being less than 3%. In addition, Eq. (22) also compares well with the following five-parameter fit by Stengel et al. [9],

$$\begin{aligned} \nu_g = \exp(&4.549 - 0.12309T + 9.1129\times10^{-4}T^2 \\ &- 4.7562\times10^{-6}T^3 + 1.3296\times10^{-8}T^4) \end{aligned} \tag{23}$$

To convert ν_g to μ_g, the density of glycerol (ρ_g in kg/m^3) is computed as

$$\rho_g = 1277 - 0.654T \tag{24}$$

which gives the best fit of the measurements for $T = 17\text{-}83$ $^\circ$C by Adamenko et al. [10]. The predicted dynamic viscosities using Eqs. (22) and (23) differ at most by 1.6%.

Comparisons and discussions

Three databases are used for comparison purposes, as summarized in Table 1. The first one was provided by Segur and Oberstar [6], with which the two coefficients, a and b, used for Eq. (5) are evaluated. The other two databases available in the literature are due to Shankar and Kumar [4] and Chenlo et al. [5], respectively. Differences between the formula predictions and these databases are examined in this section.

Shankar and Kumar [4] measured the kinematic viscosity of glycerol-water solutions for glycerol mass fractions ranging from $C_m = 0$ to 100% in the temperature range of 10 to 50 ^{0}C. Chenlo et al's data were collected in the glycerol fraction of $C_{mol} = 0$ - 5 mol (per kg of water) or $C_m = 0 - 31.5\%$ at temperatures from 20 to 50 ^{o}C. The glycerol concentration in mass (C_m) is related to the fraction in mol (C_{mol}) as $C_m = M_g C_{mol}/(1000 + M_g C_{mol})$, where M_g is the molecular weight of glycerol (= 92.09 grams). To make conversion between μ and ν, the density of the mixture (ρ) is calculated approximately as

$$\rho = \rho_g C_m + \rho_w (1 - C_m) \tag{25}$$

where ρ_g is the glycerol density and ρ_w is the water density. Theoretically, Eq. (25) is considered inexact because of possible interactions of the two components. However, the error induced is generally very small. It varies within ±0.8% based on the data provided by Adamenko et al. [10], who measured the mixture density for the condition of $T = 17$ - 83 ^{o}C and $C_m = 60\%$ - 86%. In Eq. (25), ρ_g is computed using Eq. (24), and ρ_w is computed with the empirical formula proposed as follows

$$\rho_w = 1000 \left(1 - \left| \frac{T-4}{622} \right|^{1.7} \right) \tag{26}$$

where ρ_w is in kg/m^3 and T is in ^{o}C. Eq. (26) predicts the water density for T = 0-100 ^{o}C

with errors less than 0.02%, when compared with the data by Linstrom and Mallard [8].

The computed dynamic viscosities are plotted in Fig. 6, which simulate well all

three databases. To further assess the predictions using the present approach, the

predicting error for each data point is calculated as

$$Error\,(\%) = \frac{\left| \mu_{calculated} - \mu_{measured} \right|}{\mu_{measured}} \times 100$$

As summarized in Table 1, among more than 400 data points, only 6 are predicted with

errors greater than 5%. The maximum prediction error is less than 3.5%, 8.5% and 2.2%

for the databases given by Segur and Oberstar [6], Shankar and Kumar [4] and Chenlo et al. [5],

respectively. The assessment also gives that the average errors, computed as |*Error*(%)|,

are 1.3%, 2.3% and 0.7% for the three databases, respectively. In Fig. 7, the percentage

of data points reproduced is plotted against the associated maximum error, Error (%). It

shows that the predicting errors using the proposed approach are generally small. For

example, to reproduce 95% of the data points, the maximum error induced is about 2%

for the measurements by Chenlo et al.[5], 3% for those by Segur and Oberstar [6] and 5% for

those by Shankar and Kumar [4].

Conclusions

An exponential formula is developed in this study to calculate the viscosity of

glycerol-water mixture. A power-sum interpolating function is employed to evaluate

factors involved. The derived formula applies for glycerol concentration in mass in the range of 0-100% and temperatures varying from 0 to 100 $^{\circ}$C. It is simple to use, in comparison with similar relations developed previously, and able to reproduce 95% of measured viscosities from the three databases with deviations less than 5%.

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Table 1.   Summary of experimental data used for comparisons.

Databases	Range of temperature, T (^{o}C)	Range of mass percentage of glycerol in mixture, C_m (%)	Maximum prediction error (%)	Average prediction error (%)	Number of data points predicted with error		
					>5%	<5%	<3%
Segur and Oberstar [6]	0 - 100	0 - 100	3.5	1.3	0	256	246
Shankar and Kumar [4]	10 - 50	0 - 100	8.5	2.3	6	102	79
Chenlo et al. [5]	20 - 50	0 - 31.5	2.2	0.7	0	42	42
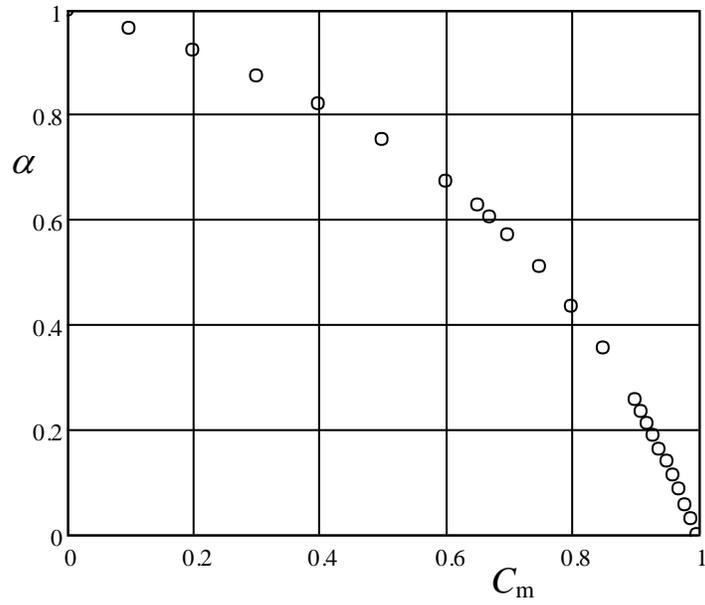
Fig. 1. Dependence of α on glycerol concentration in mass at T = 20 $^{\circ}$C. Data are taken from Segur and Oberstar [6].
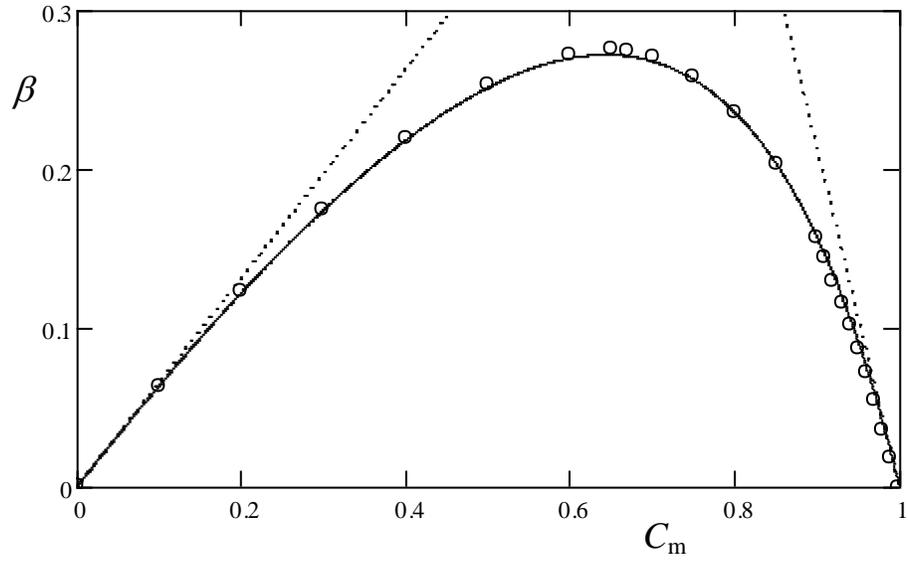
Fig. 2. Relation of β-C_m at T = 20 ^{o}C in comparison with interpolating function. Data are taken from Segur and Oberstar [6].
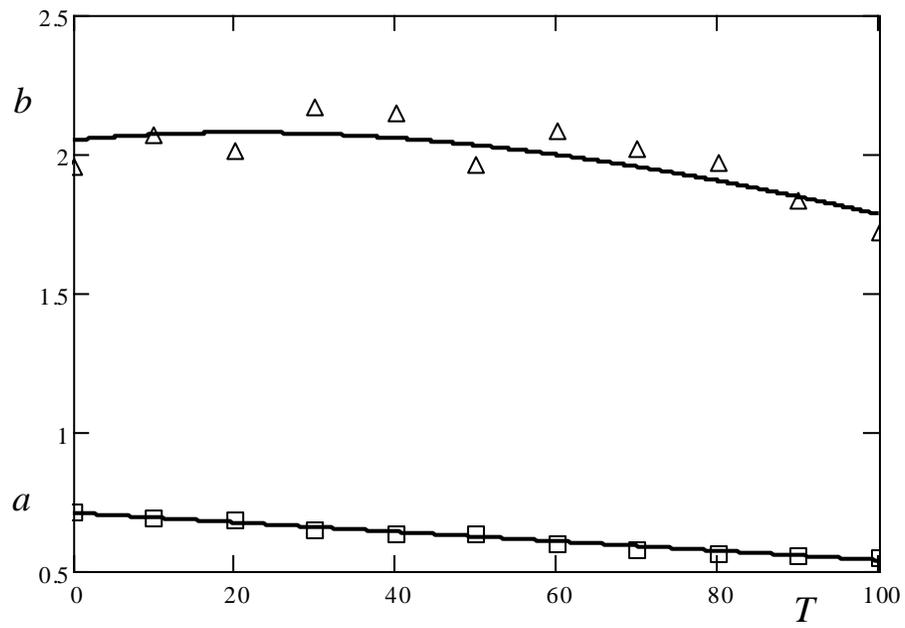
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Fig. 3   Variations of *a* and *b* with temperature, *T* (^{o}C). The symbols denote the values estimated from the data by Segur and Oberstar [6].
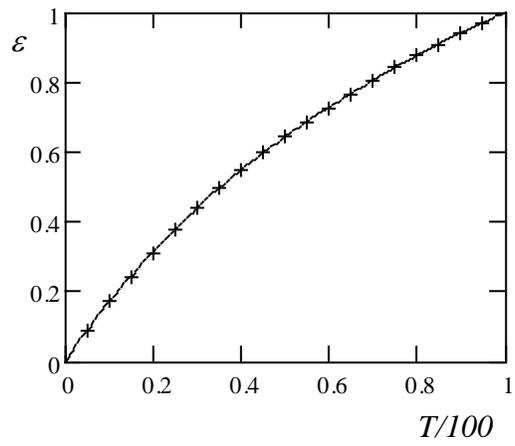
Fig. 4. **Variation of ε with T/100 for water. T is in ^{o}C. Data are taken from Washburn et al. [7].**
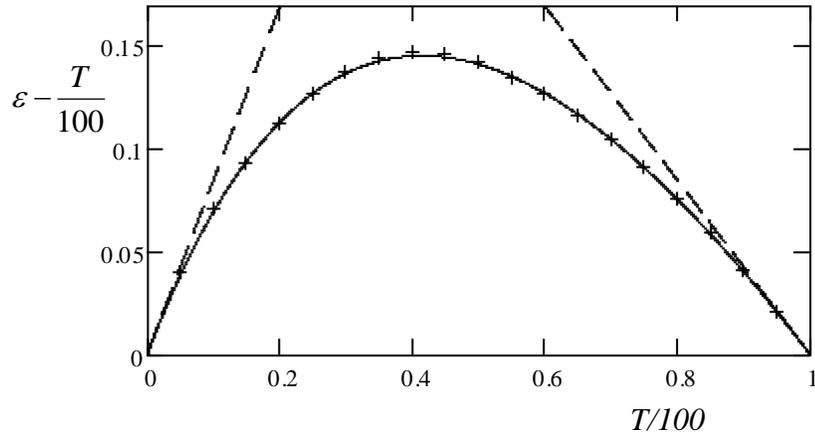
Fig. 5. Variation of ε-T/100 with T/100 in comparison with interpolating function denoted by solid line. T is in °C. The dash lines represent linear asymptotes. Data are taken from Washburn et al. [7].
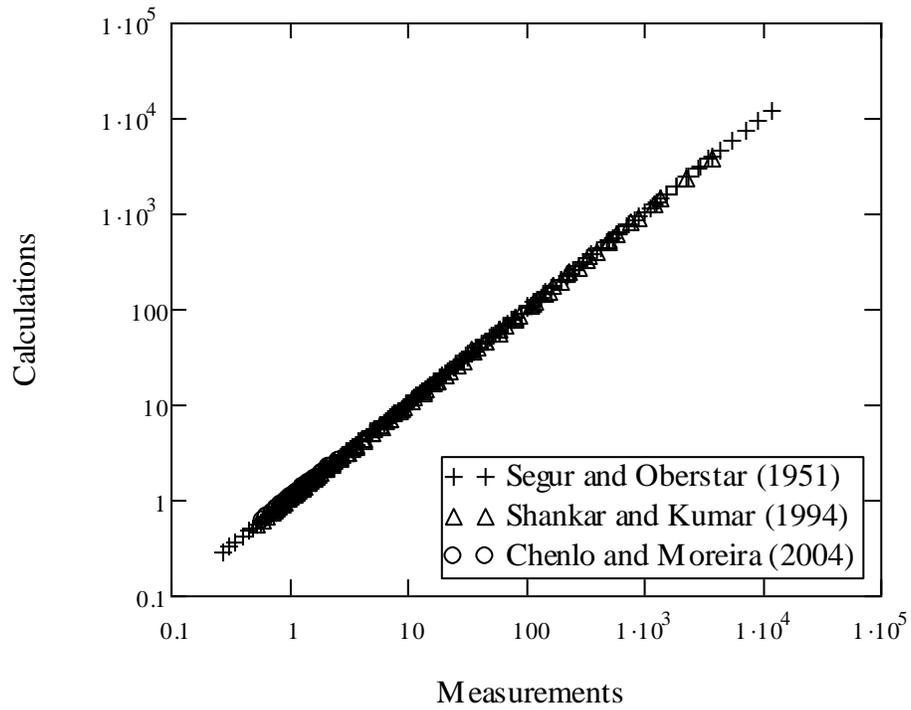
Fig. 6 Calculated dynamic viscosities in comparison with measurements. The unit is cP.
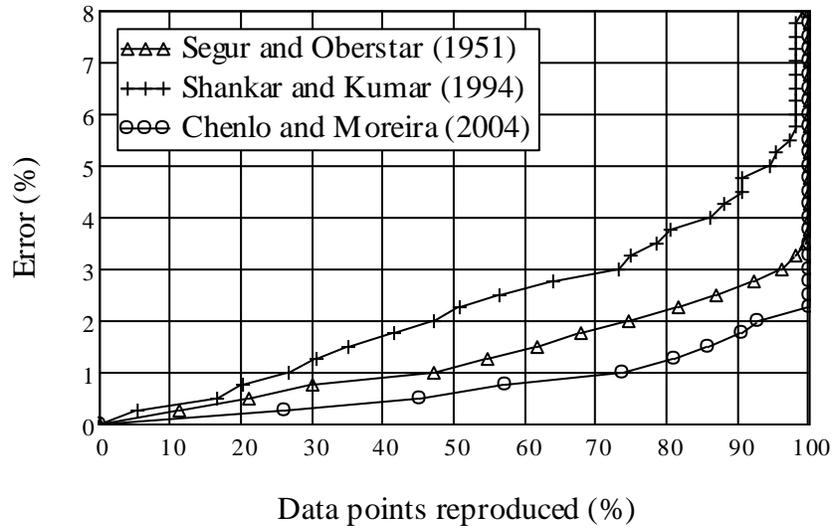
Fig. 7 Percentage of data points reproduced and associated predicting error.