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2004

Cheng, N. S. (2004). Analysis of velocity lag in sediment-laden open channel flows. Journal of Hydraulic Engineering, 130(7), 657–666.

<https://hdl.handle.net/10356/83701>

[https://doi.org/10.1061/\(ASCE\)0733-9429\(2004\)130:7\(657\)](https://doi.org/10.1061/(ASCE)0733-9429(2004)130:7(657))

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[DOI:[http://dx.doi.org/10.1061/\(ASCE\)0733-9429\(2004\)130:7\(657\)](http://dx.doi.org/10.1061/(ASCE)0733-9429(2004)130:7(657))].

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Cheng, N. S. (2004). "Analysis of velocity lag in sediment-laden open channel flows." *Journal of Hydraulic Engineering*, ASCE, 130(7), 657–666.

ANALYSIS OF VELOCITY LAG IN SEDIMENT-LADEN OPEN CHANNEL FLOWS

Nian-Sheng Cheng¹

Abstract: Laboratory experiments have recently confirmed that the streamwise particle velocity is largely less than that of the fluid in sediment-laden flows. This velocity lag is investigated analytically in the present study based on the drag force exerting on a particle in the presence of other neighbors. The normalized drag force or the hindrance coefficient is found generally dependent on the particle concentration, particle Reynolds number and specific gravity. The velocity lag is then derived by relating the hindrance coefficient to the shear stress distribution for uniform sediment-laden open channel flows. The analysis shows that the profile of the velocity lag, when normalized by the shear velocity, is associated with the shear Reynolds number, dimensionless particle diameter and specific gravity. For the dilute condition, the velocity lag distribution varies only with the shear Reynolds number, and the lag can be ignored if the shear Reynolds number

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is less than unity. The theoretical predictions are comparable to the limited experimental results.

Keywords: velocity lag, relative velocity, slip velocity, drag, hindrance coefficient, two-phase flow, suspended sediment, open channel flow, fluid-particle interaction

Introduction

Many experimental and theoretical studies have been conducted to evaluate effects of suspended sediment on the fluid velocity profile; however, little effort among hydraulic engineers has been devoted to investigate the velocity of sediment particles, which is actually required to compute transport rates of suspended sediment with a known concentration distribution. For sediment-laden open channel flows, the streamwise velocity of sediment particles is usually assumed the same as that of the fluid, implying that the streamwise fluid-particle relative velocity would equal zero. This approximation has not been validated. It may be acceptable for small particles and practically useful for many cases, but not always hold for real situations. Aziz (1996) reported that the suspended load calculated by integrating the product of the fluid velocity and the particle concentration led to an overestimation of the sediment transport rate for most cases, and the computed results based on Einstein's method were 6 ~ 37% greater than the actual load. Some other investigators (e.g., Bagnold 1973) have recognized that the particle velocity is less than that of the carrier fluid, but encountering difficulties quantifying the difference.

Only recently has the velocity lag been observed experimentally in open channel flows with advanced measuring techniques in differentiating particle and fluid velocities (e.g., Muste and Patel 1997; Best et al. 1997). Muste and Patel (1997) combined a conventional LDV system with an auxiliary discriminator to distinguish signals originating from the fluid tracers from those arising from sediment particles. Their observations indicated that the streamwise velocity of suspended sediment in open channel experiments with 0.22

mm natural sand was less than that of water by as much as 4%. Best et al. (1997) employed phase Doppler anemometry (PDA) to determine the mean and turbulent characteristics of both the water and glass spheres with a mean grain diameter of 0.22 mm. They reported that the particle Reynolds number based on the resultant velocity lag ranged from 1 to 5 and became greater towards the bed. An implication of their result is that the drag force due to the velocity lag was beyond the range for the Stokes flow and thus the flow separation might occur around the suspended particles. Similar lag results have also been obtained by Rashidi et al. (1990) in tracing the sediment particles in open channel flows with the visualization technique, and Taniere et al. (1997), who conducted experiments in a wind tunnel to study the lag behavior of solid particles in a horizontal boundary layer. The most recent attempt is due to Kiger and Pan (2002), who utilized an image separation technique to make simultaneous PIV measurements of both particle and fluid phases in a pressurized channel flow. Their results clearly show that the particles exhibited a lag in the mean streamwise velocity. In particular, they found that the measurement bias was not considerable, which was at least an order of magnitude smaller than the observed velocity lag. Their results also suggested that the lag be caused by the organization of the upward moving particles into the flow ejection events rather than a purely stochastic gradient transport process. Given the current measurement techniques, experiments can only be performed for very low sediment concentrations, as did by the above-mentioned investigators. Therefore, the reported velocity lag is very small compared to the mean streamwise velocity of fluid or particles. However, as the concentration becomes large, the velocity lag may become more significant due to interactions of particles with boundaries and with each other.

In addition to the limited experimental information, theoretical attempts made on the velocity lag are also extremely lacking. As a typical two-phase problem, the sediment-laden open channel flow could be investigated, in principle, using two-phase or two-fluid models, in which the solid phase is considered as another fluid governed by its own conservation laws. Unfortunately, difficulties are unavoidably encountered because of the poor understanding of fluid-particle interactions, which is essential for determining some unknowns inherent in the two-phase model. Therefore, a solution to the velocity lag, which can theoretically be obtained using the two-phase model, may be subjected to empirical evaluations of various parameters. In spite of such limitations, a first attempt in this respect was made by Greimann et al. (1999) for very small Stokes numbers which characterize the time scale of the particle response to fluid turbulence. In their analysis, some characteristics of clear water flows were assumed applicable for sediment-laden flows with the particle concentration less than 0.001. Their results indicated that the velocity lag was of the order of the settling velocity and decreased towards the free surface. They finally derived a simple formula for estimating the velocity lag, u_L , in the sediment-laden open channel flow, i.e.

$$\frac{u_L}{w} = 0.66 \left(1 - \frac{y}{h} \right) \exp \left(1.34 \frac{y}{h} \right) \quad (1)$$

where w = settling velocity of particles; y = distance from the channel bed; and h = flow depth. Unfortunately, the predictions given by Eq. (1) deviate obviously from experimental results. Another two-phase analysis of the velocity lag is due to Jiang et al. (2002), whose model is similar to that used by Greimann et al. (1999) but provides different interpretations of the two-phase flow mechanism.

For some cases encountered in the practical engineering, the velocity lag may be small so that its effect on the computation of sediment transport rates would be less significant than those caused by other unknowns. However, the velocity lag is a very important parameter and concept when conducting an analysis of sediment transport using two-phase fluid models. Being consistent with these considerations, the initiative of this study is not to develop a novel approach to computation of sediment transport rates; rather it is to provide an analytical framework for showing how the streamwise velocity lag in uniform sediment-laden open channel flows could be formulated. This paper starts with an analysis of the drag force exerting on a particle in the fluid-particle mixture. In comparison with the unhindered situation, this drag force can be characterized in terms of the hindrance coefficient. The first part of the analysis shows that the hindrance coefficient generally varies with the Reynolds number, particle specific gravity, and particle concentration. Following that, the derived hindrance coefficient is then related to the shear stress associated with the sediment particles. With this relationship, the time-mean velocity lag is computed by assuming that the concentration distribution follows the Rouse function. The result obtained shows that the velocity lag, when normalized by the shear velocity, generally depends on the shear Reynolds number, dimensionless particle size and particle specific gravity. The computations are finally compared to limited experimental data available in the literature.

Drag Force in Suspension

The unhindered drag force experienced by a single particle in a flow is modified in the presence of other neighboring particles. This modification can simply be expressed in the form (Di Felice 1994; Gibilaro 2001):

$$F_{Dm} = HF_D \quad (2)$$

where F_{Dm} = drag force exerting on a particle in a fluid-particle mixture, F_D = unhindered drag force, and H = hindrance coefficient. Here, it is understood that the difference between F_{Dm} and F_D is evaluated based on the same flow rate of fluid, which suggests that regardless of the particle concentration, the cross-sectional average fluid velocity, U , remains unchanged for the mixture. This is illustrated in Fig. 1. Considering that the drag force is quantified in terms of the fluid-particle relative velocity, the particles in Fig. 1 can be simply assumed to be stationary while the fluid is flowing.

The hindrance coefficient has been empirically related to only the void fraction of the mixture in some previous studies. For example, Gibilaro et al. (1985) reported that for the condition of fluidization, it could be approximated as

$$H = (1 - c)^{-3.8} \quad (3)$$

where c = particle concentration. In comparison, Di Felice (1994) found that H is not only related to the particle concentration but also the Reynolds number, $Re = UD/\nu$, where D = particle diameter and ν = kinematic viscosity of fluid. The empirical expression proposed by Di Felice is given by

$$H = (1 - c)^{-3.7 + 0.65 \exp\left[\frac{-(1.5 - \log Re)^2}{2}\right]} \quad (4)$$

In the following, a more general formula for computing the hindrance coefficient is derived based on the unhindered drag coefficient.

Derivation of hindrance coefficient

The unhindered drag for a single, isolated spherical particle in the otherwise clear fluid is given by

$$F_D = C_D \frac{\pi D^2}{4} \frac{\rho U^2}{2} \quad (5)$$

where C_D = drag coefficient, ρ = fluid density, and U = fluid velocity approaching the particle. By analogy, the drag force exerting on the particle in the presence of other neighbors can be expressed as

$$F_{Dm} = C_{Dm} \frac{\pi D^2}{4} \frac{\rho_m U_r^2}{2} \quad (6)$$

where C_{Dm} = drag coefficient in the mixture, ρ_m = average density of the mixture, and U_r = average fluid-particle relative velocity. The mixture density ρ_m can be related to the particle concentration as

$$\rho_m = \rho_p c + \rho(1 - c) \quad (7)$$

where ρ_p = particle density. With the assumption of the same flow rate of fluid, Q , for both cases as shown in Fig. 1, the average fluid velocity through the pores in the mixture or the relative velocity, U_r , is equal to $Q/(A - A_p) = U/(1 - c_A)$, where A = total cross-sectional area, A_p = particle-occupied area, and $c_A = A_p/A$ = areal concentration of the particles. If c_A is equal to the volumetric concentration, c , then one can get

$$U_r = \frac{U}{1 - c} \quad (8)$$

The replacement of the local areal concentration with the local volumetric concentration can be illustrated as follows. Consider a prismatic volume of fluid containing randomly distributed particles, of which the length is L and the cross-sectional area is A . If the particle distribution is statistically uniform, and for any cross section, the particle-

occupied area is A_p , then the total volume (V_p) of the particles in the volume is LA_p . Therefore, the volumetric concentration of the particles is $c = V_p/(LA) = (LA_p)/(LA) = c_A$.

Substituting Eqs. (5) and (6) into Eq. (2) and manipulating leads to the hindrance coefficient to be expressed as

$$H = \frac{C_{Dm}}{C_D} \frac{1 + (s-1)c}{(1-c)^2} \quad (9)$$

where $s = \rho_p/\rho =$ specific gravity.

The drag coefficient, C_D , can be generally related to the Reynolds number in the form (Cheng 1997)

$$C_D = \left[\left(\frac{A_1}{Re} \right)^{1/n} + A_2^{1/n} \right]^n \quad (10)$$

where A_1 , A_2 and $n =$ constants. For well worn natural sediment particles, Eq. (10) fits well to experimental data by taking $A_1 = 32$, $A_2 = 1$ and $n = 1.5$, as shown in Fig. 3 (Cheng 1997). For spherical particles, Fig. 4 indicates that for $Re < 1200$, Eq. (10) with $A_1 = 24$, $A_2 = 0.28$, and $n = 2$ agrees with the standard drag curve. The latter is plotted according to a set of correlation formulas proposed by Clift et al. (1978), which is found to the most closely match experimental data (Brown and Lawler 2003).

For the fluid-particle mixture, the modified drag coefficient, C_{Dm} , can be expressed in terms of the modified Reynolds number, Re_m , in the similar fashion to Eq. (10), i.e.

$$C_{Dm} = \left[\left(\frac{A_1}{Re_m} \right)^{1/n} + A_2^{1/n} \right]^n \quad (11)$$

The modified Reynolds number Re_m is defined as

$$Re_m = \frac{U_r D}{\nu_m} \quad (12)$$

where ν_m = kinematic viscosity of the mixture. Using Eqs. (7) and (8), Re_m can be related to Re in the form

$$Re_m = \frac{1}{\mu_r} \frac{1+(s-1)c}{1-c} Re \quad (13)$$

where $\mu_r = \rho_m \nu_m / \mu$ = relative effective viscosity. Cheng and Law (2003) related μ_r to c in the following exponential function:

$$\mu_r = \exp \left[\frac{2.5}{\beta} \left(\frac{1}{(1-c)^\beta} - 1 \right) \right] \quad (14)$$

where β = constant. Comparing Eq. (14) with other relevant relationships available in the literature indicates that the β -value varies approximately from 1.0 to 4.0. For shear-free conditions such as sedimentation and fluidization, the β -value is relatively small. Furthermore, if inter-particle collision and turbulence effects are negligible, β approaches zero and then Eq. (14) reduces to

$$\mu_r = (1-c)^{-2.5} \quad (15)$$

With Eqs. (10), (11), (13) and (14), Eq. (9) can be re-written as

$$H = \left[\frac{\left(\frac{1-c}{1+(s-1)c} \right)^{1/n} \exp \left[\frac{2.5}{n\beta} \left((1-c)^{-\beta} - 1 \right) \right] + \left(\frac{A_2}{A_1} Re \right)^{1/n}}{1 + \left(\frac{A_2}{A_1} Re \right)^{1/n}} \right]^n \frac{1+(s-1)c}{(1-c)^2} \quad (16)$$

Eq. (16) shows that the hindrance coefficient generally varies with the particle concentration, Reynolds number, and particle specific gravity. It is also noted that the Reynolds number effect can be ignored if its value is either very large or very small. For example, if $Re < 1$, the viscous effect is dominantly important, and Eq. (16) can thus be simplified as

$$H = \frac{1}{1-c} \exp \left[\frac{2.5}{\beta} \left((1-c)^{-\beta} - 1 \right) \right] \quad (17)$$

In particular, if β can also be assumed vanishingly small, i.e., the effective viscosity can be computed using Eq. (15), then Eq. (17) can further be expressed as

$$H = (1-c)^{-3.5} \quad (18)$$

It is interesting to note that Eq. (18) is the same as Eq. (3) except for the slight change in the exponent. On the other hand, for very large Re , Eq. (16) reduces to

$$H = \frac{1+(s-1)c}{(1-c)^2} \quad (19)$$

Comparison with other studies

Eq. (16) is plotted in Fig. 2 by taking $A_1 = 32$, $A_2 = 1$ and $n = 1.5$ for natural sediment particles. The relevant computation is first made for the case of $\beta \rightarrow 0$, for which the term $\exp \left[\frac{2.5}{n\beta} \left((1-c)^{-\beta} - 1 \right) \right]$ included in Eq. (16) can be replaced with $(1-c)^{-2.5 / n}$ according to Eq. (15). Also superimposed on this figure are the two empirical formulas, Eqs. (3) and (4). Fig. 2 shows that the results computed using the three equations are quite close for $c < 0.1$, but the discrepancy becomes noticeable with increasing concentration. It can also be noted that for some cases, for example, $Re = 1$ and 100, Eq. (16) can be approximated by Eq. (4). Additional computations using Eq. (16) indicate that the computed hindrance coefficient becomes larger with increasing β -values particularly for $c > 0.1$.

Evaluation of Velocity Lag

To compute the streamwise velocity lag in sediment-laden open channel flows, the drag force in the mixture discussed in the previous section can further be connected with the shear stress in the fluid-particle mixture. First, consider a particle on the bed, as shown in Fig. 5(a). The streamwise component of the driving force for this particle is known as the drag force. According to its definition, this force is supposed to exert on a projected area of the particle, which is perpendicular to the flow direction. If the projected area is chosen to be horizontal, i.e. parallel to the bed surface, then the streamwise driving force can simply be related to the bed shear stress. This yields the magnitude of the driving force to be taken as the product of the bed shear stress and the projected area of the particle. It is known that both methods lead to the equivalent result when applied for evaluating the critical shear velocity for the threshold condition of the incipient sediment motion, where the driving force just balances the resistance (e.g., Chien and Wan 1999). By analogy, for a suspended particle which is away from the bed, the drag force, F_{Dm} , may be also related to the local shear stress, τ_m , in the similar approach. This yields that at any elevation in the open channel (see Fig. 5(b)),

$$F_{Dm} = HF_D = \tau_m \frac{\pi D^2}{4} \quad (20)$$

Substituting Eqs. (5), (8) and (16) into Eq. (20), replacing U_r with the velocity lag, u_L , and manipulating leads to that u_L , when normalized by the shear velocity, u_* , can be expressed as

$$\frac{u_L}{u_*} = \left(\sqrt{\frac{1}{4} \left(\frac{N}{A_2} \right)^{\frac{2}{n}} + \left(\frac{M}{A_2} \right)^{\frac{1}{n}}} - \frac{1}{2} \left(\frac{N}{A_2} \right)^{\frac{1}{n}} \right)^n \quad (21)$$

or

$$\frac{u_L}{u_*} = M \left(\sqrt{\frac{1}{4} N^{\frac{2}{n}} + (MA_2)^{\frac{1}{n}}} + \frac{1}{2} N^{\frac{1}{n}} \right)^{-n} \quad (22)$$

where

$$M = \frac{2\tau_m}{\rho_m u_*^2} \quad (23)$$

$$N = \frac{A_1}{Re_*} \frac{1}{1+(s-1)c} \exp \left[\frac{2.5}{\beta} \left(\frac{1}{(1-c)^\beta} - 1 \right) \right] \quad (24)$$

and $Re_* = u_* D / \nu$ = particle Reynolds number based on the shear velocity or shear Reynolds number. Eq. (21) or (22) shows that for particles with given properties, the dimensionless velocity lag can be computed provided that the shear stress distribution is known.

Shear stress distribution

For a uniform, equilibrium sediment-laden open channel flow, the shear stress, τ_m , at the distance y from the bed, equals

$$\tau_m = \int_y^h g S_b \rho_m dy \quad (25)$$

where S_b = bed slope and h = flow depth. Substituting Eq. (7) into Eq. (25) gives

$$\tau_m = \rho g S_b (h - y) [1 + (s - 1) \bar{c}_y] \quad (26)$$

or

$$\frac{\tau_m}{\rho_m u_*^2} = \frac{h - y}{h} \frac{1 + (s - 1) \bar{c}_y}{1 + (s - 1)c} \quad (27)$$

where $u_* = (ghS_b)^{0.5}$ and $\bar{c}_y$ is the concentration averaged from $y = y$ to $y = h$, i.e.

$$\bar{c}_y = \frac{1}{h - y} \int_y^h c dy \quad (28)$$

If the concentration profile is given by the Rouse distribution,

$$c = c_a \left(\frac{h-y}{y} \frac{a}{h-a} \right)^z \quad (29)$$

where a = reference level, c_a = reference concentration at $y = a$, and $z = w/(0.4u_*) =$

Rouse number, then $\overline{c_y}$ can be further expressed as

$$\overline{c_y} = c_a \frac{h}{h-y} \left(\frac{a}{h-a} \right)^z \int_{y/h}^1 \left(\frac{1-\zeta}{\zeta} \right)^z d\zeta \quad (30)$$

In addition, the reference concentration, c_a , can be computed for $a = 0.05h$ using the following empirical relationship (Garcia and Parker 1991):

$$c_a = EZ \left(1 + \frac{EZ}{0.3} \right)^{-1} \quad (31)$$

where $E = 1.3 \times 10^{-7}$, $Z = Re_*^5 D_*^{4.5} (wD/\nu)^{-5}$ and $D_* = [(s-1)g/\nu^2]^{1/3} D$ = dimensionless particle diameter. Eq. (31) indicates that the reference concentration c_a depends only on Re_* and D_* since the settling Reynolds number wD/ν can be expressed in terms of D_* (Cheng 1997).

Finally, from Eq. (21) together with Eqs. (27), (29) to (31), it follows that the dimensionless velocity lag distribution varies generally with the three dimensionless parameters, Re_* , D_* and s . Such variations for natural sediment particles are delineated in Figs. 6 to 8 for some cases. The results plotted in Fig. 6 are obtained by taking $\beta = 1$, $s = 2.5$ and $D_* = 10$, showing that the velocity lag distribution can be affected considerably by the variations in the shear Reynolds number, Re_* . In comparison, the effects of the dimensionless particle diameter and specific gravity are not so significant, as shown in Figs. 7 and 8, respectively. The curves plotted in Fig. 7 are computed for $\beta = 1$, $s = 2.5$, $Re_* = 100$ and $D_* = 1 \sim 100$, and those in Fig. 8 for $\beta = 1$, $D_* = 10$, $Re_* =$

100 and $s = 1 \sim 5$. As shown in Fig. 8, the specific gravity effect is not considerable and the slight variation can be observed only as the bed is approached.

Velocity lag for three extreme conditions

Case 1: $u_L D / \nu < 1$

If the fluid-particle velocity difference, compared with the mean flow velocity of particles or fluid, is quite small, it may be practical to assume that the viscous effect is dominantly significant or the particle Reynolds number based on the velocity lag is less than 1. For this condition, N^2 is much greater than MA_2 and thus Eq. (22) reduces to

$$\frac{u_L}{u_*} = \frac{M}{N} = \frac{2}{A_1} Re_* \frac{h-y}{h} \left[1 + (s-1)\bar{c}_y \right] \exp \left[-\frac{2.5}{\beta} \left((1-c)^{-\beta} - 1 \right) \right] \quad (32)$$

In particular, for $y \rightarrow 0$, Eq. (32) can be re-written as

$$u_L|_{y \rightarrow 0} = \frac{2}{A_1} \frac{\tau_b D}{\mu} \exp \left[-\frac{2.5}{\beta} \left((1-c_o)^{-\beta} - 1 \right) \right] \quad (33)$$

where

$$\tau_b = \rho u_*^2 [1 + (s-1)\bar{c}] \quad (34)$$

is the bed shear stress, $\bar{c}$ = depth-averaged concentration and c_o = concentration at the bed. Eq. (33) indicates that the velocity lag near the bed is proportional to the particle diameter and the bed shear stress for the case of $u_L D / \nu < 1$.

Case 2: $c \rightarrow 0$

Under the dilute condition, it can be assumed that $1 + (s-1)c \approx 1$, $1 + (s-1)\bar{c}_y \approx 1$, and $\mu_r \approx 1$. This leads to $M = 2(1-y/h)$ and $N = A_1 / Re_*$, and Eq. (21) therefore reduces to

$$\frac{u_L}{u_*} = \left(\sqrt{\frac{1}{4} \left(\frac{A_1}{A_2 Re_*} \right)^{\frac{2}{n}} + \left(2 \frac{h-y}{A_2 h} \right)^{\frac{1}{n}} - \frac{1}{2} \left(\frac{A_1}{A_2 Re_*} \right)^{\frac{1}{n}}} \right)^n \quad (35)$$

Eq. (35) suggests that the velocity lag distribution can be affected only by the shear Reynolds number for the dilute condition. Such distributions are further plotted in Fig. 9 for various shear Reynolds numbers, demonstrating that the dimensionless velocity lag decreases gradually with decreasing shear Reynolds number.

From Eq. (35), we can see that the velocity lag becomes vanishingly small if

$$\frac{1}{4} \left(\frac{A_1}{A_2 Re_*} \right)^{\frac{2}{n}} \gg \left(2 \frac{h-y}{A_2 h} \right)^{\frac{1}{n}} \quad (36)$$

Note that Eq. (36) is always true for $y = h$. However, to ensure that the above inequality holds for any other y -values, Eq. (36) can be placed in the form

$$Re_* \ll \frac{A_1}{4^{n/2} \sqrt{2A_2}} \quad (37)$$

since $(h-y)/h \leq 1$. Substituting the values of A_1 and A_2 given previously into Eq. (37) yields $Re_* \ll 8$ for natural sediment particles, and $Re_* \ll 8.02$ for spherical particles. Such limiting conditions in terms of the shear Reynolds number suggest that if a flat sediment bed, which is comprised of the same particles as those suspended in the flow, is not hydraulically smooth, then the velocity lag of the suspended particles may become appreciable even under the dilute condition. In other words, for sediment-laden flows with a constant shear velocity, the velocity lag can be ignored only if the particle size is very fine. This is quite consistent with the flow visualization results presented by Rashidi et al. (1990), who found that the particle velocity in the dilute condition was very close to that of fluid for $Re_* = 1.3$ but became noticeably smaller when Re_* increased up to 11.8.

Fig. 10 is a plot of the variation of the depth-averaged velocity lag, $\overline{u_L}/u_*$, with the shear Reynolds number, Re_* , which can be computed numerically using Eq. (35). The figure shows that for the dilute condition, the depth-averaged lag is very small for $Re_* < 1$ but increases rapidly if Re_* varies from 1 to 1000.

Case 3: $Re_ \rightarrow \infty$ and $c \rightarrow 0$*

From Eq. (22), it follows that the dimensionless velocity lag, u_L/u_* , is maximized if $N \rightarrow 0$ or $Re_* \rightarrow \infty$. For this extreme condition, Eq. (22) can be simplified as

$$\frac{u_L}{u_*} = \sqrt{\frac{M}{A_2}} \quad (38)$$

Then, with Eqs. (23) and (27), it can be found that the maximum M equals $2(1-y/h)$, which occurs for $c \rightarrow 0$. Therefore, substituting this M -value into Eq. (38) leads to the maximum dimensionless velocity lag to be expressed as

$$\frac{u_L}{u_*} = \sqrt{\frac{2}{A_2} \frac{h-y}{h}} \quad (39)$$

Eq. (39) shows that the profile of the maximum dimensionless velocity lag is unique, being independent of the abovementioned three dimensionless parameters, Re_* , D_* and s .

Comparisons with Limited Experimental Data

Because the relevant experimental results are limited in the literature, a complete verification of the derived formula, Eq. (21) or (22), is impossible to make at the current stage. In the following, five datasets obtained in the case of uniform open channel

flows, as summarized in Table 1, are used for the purposes of comparison, of which three were collected by Muste and Patel (1997) and the rest by Best et al. (1997).

Muste and Patel (1997) conducted their experiments in uniform open channel flows with aspect ratios ranging from 7.05 to 7.11. Natural sediment with 0.22 mm diameter was employed to conduct three series of experiments with different sediment concentrations. The shear velocity extrapolated from the measured Reynolds shear stress varied from 3.02 to 3.13 cm/s. The measurements were made for the conditions that no ripples formed on the channel bed. Their experimental results concerning the velocity lag, which are used for the present comparisons, are those detailed in Greimann et al. (1999). Best et al.'s (1997) experiments are similar to those performed by Muste and Patel (1997). They used glass spheres as sediment to facilitate fluid-sediment discrimination using the PDA technique. Two tests were done with the same shear velocity and flow depth, and the test, Run3, was conducted at the maximum sediment transport, beyond that bedforms started to form on the bed. However, it should be noted that the velocity lag defined by Best et al. (1997) was the difference between the resultant fluid velocity and the resultant particle velocity, and its magnitude might be very close to the streamwise velocity lag, which is used in the present study, as the vertical velocity is usually much smaller than the streamwise component in uniform open channel flows.

The experimental data plotted in Figs. 11 and 12 exhibit appreciable noises. This is largely due to the difficulties encountered in discriminating the particle and fluid velocities. It may also be associated, in part, with the flow condition of the suspended load, which was determined in both studies as the critical situation beyond which

bedforms started to form. It is not clear if sediment transport under this condition was in equilibrium of demand and supply, and if non-equilibrium transport has effects on the velocity lag. Obviously, further studies in this respect are needed.

Both studies have been performed with very low particle loadings. The maximum volumetric concentration recorded by Muste and Patel was 6.53×10^{-5} at $y/h = 0.15$, while that reported by Best et al. was 2.49×10^{-4} at $y/h = 0.07$. Therefore, the theoretical velocity lag results to be discussed in the following are computed simply using Eq. (35) that is derived for $c \ll 1$. The constants taken are $A_1 = 32$, $A_2 = 1$ and $n = 1.5$ for the Muste and Patel's tests for which natural sediment particles were used, and $A_1 = 24$, $A_2 = 0.28$ and $n = 2$ for the Best et al.'s measurements with glass spheres. Besides these constants, no other free parameters are employed to perform the theoretical computations for Figs. 11 to 13.

Fig. 11 shows that the theoretical curves obtained from Eq. (35) agree with the measurements for test SL1, but generally underestimates the velocity lag for tests SL2 and SL3. In comparison, the predictions given by Greimann et al.'s formula, Eq. (1), are much larger than those obtained using the present approach as well as the measurements. Fig. 12 indicates that both Eqs. (1) and (35) overestimate the relative velocity in comparison with the mean values measured by Best et al. (1997).

To observe the particle size effect on the velocity lag, Best et al. (1997) further subdivided their experimental data into four categories based on four different particle size fractions (i.e., 0.1-0.15; 0.15-0.2; 0.2-0.25; 0.25-0.3 mm). As expected, the data scatter for each size fraction, as shown in Fig. 13 for Run2 as an example, is more evident than that of the mean relative velocity plotted in Fig. 12. Fig. 13 indicates that

the velocity lag increases generally with increasing particle diameter. The measured results are in qualitative agreement with the predictions given by Eq. (35), in comparison with those obtained from Eq. (1), in spite of the small variation of the particle size and marked data scatter. Similar comparison results as given in Fig. 13 can also be obtained for Run 3, but not shown in this study.

The experimental results of the depth-averaged velocity lag are superimposed on Fig. 10, which are comprised of those integrated from the profiles as shown in Figs. 11 and 12 as well as those obtained from Best et al.'s subdivided data series for the four size fractions. When taking the average, the negative values of the velocity lag which were measured near the free surface by Best et al. (1997) are also taken into account. Fig. 10 shows that the depth-averaged values of the velocity lag calculated from the vertical distributions given in Figs. 11 and 12 seem closer to the theoretical curve than those obtained from the subdivided data series.

Concluding Remarks

An analytical approach is presented in this paper for computing the fluid-particle relative velocity in the streamwise direction for uniform sediment-laden open channel flows. The drag force exerting on a particle in the fluid-particle mixture is first formulated based on the unhindered drag expression. The derived hindrance coefficient or dimensionless drag force is found to be related to the particle Reynolds number, concentration and specific gravity. This result is then connected with the shear stress

distribution in sediment-laden open channel flows, so that the streamwise velocity lag of the suspended particles can be evaluated. The velocity lag normalized by the shear velocity is generally a function of three dimensionless parameters, i.e., the dimensionless particle diameter, shear Reynolds number and specific gravity; however, for the dilute condition, it is related only to the shear Reynolds number and the lag can be ignored if the shear Reynolds number is less than 1. The analysis also shows that the maximum normalized velocity lag distribution has nothing to do with the three dimensionless parameters. The theoretical predictions of the velocity lag agree reasonably with the limited experimental results which are available in the literature.

The drag force exerting on a particle in sediment-laden flows derived in this study is based on the assumption that the fluid-particle mixture can be considered as a pseudo-fluid with the modified properties. The drag force so obtained is theoretically applicable for non-shear flows with homogeneous mixtures, so it may deviate for the cases where significant velocity or concentration gradients exist. In addition, the drag force would be changed when sediment particles experience intensive boundary collisions in the region near the bed. Therefore, the computation of the velocity lag presented in this study can be improved if possible changes in the drag force are included in the analysis. On the other hand, further research is needed to precisely measure the relative velocity in the two-phase open channel flows for substantiating relevant analysis and theory.

Appendix I: References

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Appendix II: Notation

The following symbols are used in this paper:

A = cross-sectional area;

A_1 = constant;
 A_2 = constant;
 A_p = particle-occupied area;
 a = reference distance from the bed;
 C_D = drag coefficient;
 C_{Dm} = drag coefficient in the mixture;
 c_o = concentration at the bed;
 c_a = reference concentration at $y = a$;
 c_A = areal concentration of particles;
 $\bar{c}$ = depth-averaged concentration;
 $\overline{c_y}$ = concentration averaged from $y = y$ to $y = h$;
 D = particle diameter;
 D_* = $D[(s-1)\rho^2 g/\mu^2]^{1/3}$ = dimensionless particle diameter;
 E = constant;
 F_D = unhindered drag force;
 F_{Dm} = drag force exerting on a particle in a fluid-particle mixture;
 g = gravitational acceleration;
 H = hindrance coefficient or voidage function;
 h = flow depth;
 M = a function given by Eq. (23);
 N = a function given by Eq. (24);
 n = constant;
 Re = UD/ν ,

- $Re_* = u_* D/\nu$ = shear Reynolds number;
- Re_m = modified Reynolds number in the presence of other particles;
- $s = \rho_p/\rho$ = specific gravity;
- S_b = bed slope;
- U = cross-sectional average fluid velocity;
- U_r = average fluid-particle relative velocity;
- $u_* = (ghS_b)^{0.5}$ = shear velocity;
- u_L = relative velocity or velocity lag in the streamwise direction;
- w = particle settling velocity;
- y = distance from the channel bed;
- $Z = Re_*^{-5} D_*^{4.5} (wD/\nu)^{-5}$;
- $z = w/(0.4u_*)$ = Rouse number;
- β = constant;
- μ = dynamic viscosity of fluid;
- $\mu_r = \rho_m \nu_m/\mu$ = relative effective viscosity;
- ν = kinematic viscosity of fluid;
- ν_m = kinematic viscosity of the mixture;
- ρ = fluid density;
- ρ_m = average density of the fluid-particle mixture;
- ρ_p = particle density;
- τ_b = bed shear stress; and
- τ_m = shear stress in the fluid-particle mixture.

CAPTION FOR FIGURES

- Fig. 1** Definition sketch for drag forces. The flow rate remains unchanged for both cases
- Fig. 2** Variations of hindrance coefficient with concentration and Reynolds number
- Fig. 3** Drag coefficient of natural sediment particles
- Fig. 4** Drag coefficient of spheres
- Fig. 5** Drag force exerting on a particle (a) on the bed, and (b) away from the bed
- Fig. 6** Shear Reynolds number effect on the velocity lag distribution
- Fig. 7** Effect of dimensionless particle diameter on the velocity lag distribution
- Fig. 8** Specific gravity effect on the velocity lag distribution
- Fig. 9** Variations of the velocity lag profile in the dilute condition
- Fig. 10** Relationship of the depth-averaged velocity lag and shear Reynolds number for the dilute condition
- Fig. 11** Comparison with Muste and Patel's (1997) experimental data
- Fig. 12** Comparison with Best et al.'s (1997) measured mean velocity lags
- Fig. 13** Comparison with Best et al.'s (1997) results (Run2) for various size fractions

TABLE 1. Summary of Experimental Data by Best et al. (1997) and Muste and Patel (1997)

Researcher	Run No.	Particle diameter (mm)	Particle specific gravity	Shear velocity (cm/s)	Flow depth (cm)	Aspect ratio	Kinematic Viscosity (cm ² /s)
Best et al. (1997)	Run2	0.22	2.60	3.40	5.75	5.22	0.0084
	Run3	0.22	2.60	3.40	5.75	5.22	0.0084
Muste and Patel (1997)	SL01	0.22	2.65	3.02	12.9	7.05	0.0102
	SL02	0.22	2.65	3.05	12.9	7.05	0.0104
	SL03	0.22	2.65	3.13	12.8	7.11	0.0105

Fig. 1

Fig.2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11(a)

Fig. 11(b)

Fig. 11(c)

Fig. 12

Fig. 13(a)(b)

Fig. 13(c)(d)