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A New Navigation Function Based Decentralized Control of Multi-Vehicle Systems in Unknown Environments

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Abstract This paper deals with navigation for a group of vehicles while avoiding collisions and ensuring global network connectivity in unknown environments using a new decentralized navigation function. It is pointed out that the traditional navigation function is not effective in the situation where vehicles work in a large environment. It is shown that in this situation velocity of the vehicle would be extremely small, which is not realistic in practical applications. This paper proposes a new decentralized navigation function with a novel goal function based on which a decentralized control law that is along the negative gradient of the decentralized navigation function is derived. Finally the proposed decentralized control law is applied in a multi-vehicle navigation scenario. Based on the properties of the proposed navigation function and dual Lyapunov theorem, a sufficient condition is derived for vehicles to converge to regions surrounding their corresponding goal positions in a collision-free and connectivity-keeping manner. Simulation results demonstrate the efficacy of the proposed method.

Keywords Multi-vehicle systems · Decentralized navigation function · Collision avoidance · Connectivity maintenance · Multi-objective control

1 Introduction

Recent years have witnessed emerging use of multi-vehicle systems in various complicated applications [1]-[2], which has imposed researchers to pay more attentions to control design for vehicle groups to achieve various goals. Usually, vehicles are

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required to achieve some pre-defined motion behaviors (e.g., formation stabilization, formation tracking, rendezvous, navigation, flocking etc.). However, in practice, control performance of vehicles may be affected by environmental constraints, sensing/communication limitations and actuation capabilities. Thus, despite the goal behavior, there are some other requirements that should be satisfied as well. For example, when vehicles work in unknown environments, there may be some pop-up or previously undetected obstacles in the workspace, which will threaten the safety of vehicles, resulting in reactions of vehicles to avoid collisions with them. Such reactive motions may cause collisions among neighbor vehicles. Therefore, vehicles should have the capability of avoiding obstacles and inter-vehicle collisions. Furthermore, vehicle motions while avoiding collisions with obstacles and neighbor vehicles may split communication links, which may lead to failure of information exchange among vehicles and even breakdown of the whole task. In this way, connectivity of the communication network is necessary. Therefore, for vehicles working in unknown environments, some other objectives such as collision avoidance and connectivity maintenance should be achieved as well. Up to now, multi-objective control of vehicle groups has been and still remains an active and challenging research topic due to its practicality and complexity [3].

Currently, there are already some literatures along this direction [4]-[15]. Among the existing approaches, navigation function is the most popular tool because of its capability of dealing with multiple constraints and boundedness of its gradient which is critical for systems with actuation limitations [4]-[9]. [4] and [5] deal with decentralized navigation and formation stabilization problem while avoiding inter-vehicle collisions respectively. [6] solves rendezvous problem for multi-agent systems with connectivity maintenance. [7]-[9] design controllers that can achieve specific objectives while avoiding obstacles and maintaining connectivity. [10] incorporates constraints including workspace boundary avoidance, obstacle avoidance, inter-vehicle avoidance and connectivity maintenance into the constraint function of a decentralized navigation function to design a formation stabilization control law satisfying these constraints simultaneously. Among the existing literatures, when dealing with obstacle avoidance, they usually simplify the shape of obstacles to point or sphere or need some prior knowledge of the number or size of obstacles. In addition, to handle connectivity maintenance, they only consider local connectivity preserving problem, aiming at keeping the initial communication links over time. However, maintaining each link of the initial communication graph is rather restrictive. As a matter of fact, keeping only the global connectivity of the communication graph is enough, where some links could be removed while some others may be added to guarantee that the overall connectivity of the communication graph is preserved.

Although having been investigated for many years, convergence of the closed loop system using navigation function is still a non-trivial problem. Some researchers ([8], [9] and [10]) analyze the convergence based on the method provided in the seminal work of navigation functions [16]. Decentralized navigation function is utilized to deal with formation stabilization problem for multi-agent systems with connectivity preserving and obstacle and inter-vehicle collision avoidance in [10]. However, this paper is under a very conservative assumption that no obstacles or other agents are assumed to stay within the collision region of one agent, when this agent is close to breaking the communication link with its neighbor. Actually, this assumption seems easy to be violated in practice. [8] and [9] employ

navigation function to solve multi-robot navigation and rendezvous task respectively both with collision avoidance and connectivity requirements. Although both of them provide convergence analysis, their conclusions seems impractical as the exponential parameter of the navigation function will go to infinity in order to guarantee global convergence. Inspired by [17], some people try to find the connection between dual Lyapunov stability theorem and the convergence property of the system using navigation functions and get some valuable results ([4]-[5] and [18]-[20]). [18] presents a scheme for constructing density functions to guarantee systems almost globally asymptotically stable using navigation functions. [4] and [5] derive a sufficient condition for almost global convergence based on dual Lyapunov theorem. [19] and [20] try to release the conservation of the convergence condition derived in [4] and [5]. They define a new form of constraint function and get a less conservative sufficient condition, however, the constraint function they define cannot guarantee the safety when one vehicle is close to colliding with more than one obstacle or neighbor agents or breaking the communication link with more than one neighbor.

As stated above, all of the existing literatures of navigation function focus on the construction of constraint function and convergence analysis of the system. In order to guarantee safety and convergence, people mainly concentrate on how to define new forms of constraint functions. However, to the best of our knowledge, since the seminal work of navigation function [16], people always use the same goal function. As will be explained in the following section, there exists a serious problem with the traditional goal function when the function value and the exponential parameter is very large.

In this paper, a new control scheme is proposed to address decentralized navigation problem for a group of vehicles while avoiding collisions in unknown environments and preserving global connectivity among vehicles. First, it is pointed out that the traditional navigation function is not effective in the situation where vehicles work in a large environment. It is shown that in this situation vehicle velocity would be extremely small, which is not realistic in practical applications. Then a new navigation function with a novel goal function is proposed to solve this problem based on which a decentralized control law that is along the negative gradient of the decentralized navigation function is derived. Finally, the proposed decentralized control law is applied to a multi-vehicle navigation task with constraints of obstacle and inter-vehicle collision avoidance and network connectivity maintenance. Safety and convergence analyses of the system are also provided.

Compared with the existing literatures investigating similar problems, this paper has two contributions. First, it is pointed out that the traditional navigation function is not effective in the situation where vehicles work in a large environment. In that situation control inputs would be extremely small, which means that vehicles will almost stop at their initial positions for a long time. Then a solution is proposed by introducing a new decentralized navigation function. However, existing literatures mainly consider how to construct constraint functions and analyze convergence of the closed loop system. To the best of our knowledge, this is the first paper that have formulated and solved this problem. Second, the proposed decentralized navigation function is applied to address a decentralized navigation problem for a group of vehicles under constraints including obstacle avoidance, inter-vehicle avoidance and global connectivity maintenance. In addition, mathematically rigorous safety and convergence analyses are derived as well. However,

existing approaches such as [6]-[10] only consider local connectivity maintenance constraint when dealing with similar multi-objective problems. Furthermore, the proposed method can help vehicles to avoid obstacles in unknown environments with limited sensing capabilities, while existing literatures like [8]-[10] need some prior knowledge of the size or number of obstacles in the workspace.

The remainder of this paper is organized as follows. In Section 2, a new decentralized navigation function is constructed. Then the control objective is described. Section 3 provides safety and convergence analysis. Some simulation results are given in Section 4 and final conclusions are stated in Section 5.

2 Problem Formulation

This paper considers decentralized navigation while guaranteeing collision avoidance and global connectivity maintenance in unknown environments for n nonholonomic vehicles. It is assumed that each vehicle is equipped with a laser rangefinder and wireless communication devices. This paper is under the assumption that two vehicles can communicate if they are within a specified maximum communication range and lose connectivity if they are outside of that range. Therefore, considering the limited sensing and communication capability, two vehicles can communicate with each other if they are within a distance R_c and a vehicle can sense obstacles within a distance R_s . For simplicity and without loss of generality, assume that all vehicles have equal communication and actuation capabilities.

In the following, nonholonomic vehicle group will be modeled. Then the new decentralized navigation function will be constructed.

2.1 Modeling of nonholonomic vehicle group

2.1.1 Modeling of nonholonomic vehicle

Consider n nonholonomic vehicles in a workspace $W \in \mathbb{R}^2$. For simplicity and without loss of generality, the workspace W is assumed to be circular with its center located at the origin and bounded with radius R_w . The kinematic model of the i th vehicle is given as follows.

$$\begin{cases} \dot{x}_i = v_i \cos \theta_i \\ \dot{y}_i = v_i \sin \theta_i \\ \dot{\theta}_i = \omega_i \end{cases}, \quad i \in \{1, 2, \dots, n\}, \quad (1)$$

where $\mathbf{p}_i^c = [x_i, y_i]^T$ are the position coordinates of the center of mass, θ_i is the heading angle, and v_i and ω_i are the linear velocity and angular velocity respectively. Using the input-output feedback linearization method proposed in [21], the following kinematic model can be derived

$$\dot{\mathbf{p}}_i = \mathbf{u}_i, \quad (2)$$

where $\mathbf{p}_i = \begin{bmatrix} p_{i1} \\ p_{i2} \end{bmatrix} = \begin{bmatrix} x_i + L \cos \theta_i \\ y_i + L \sin \theta_i \end{bmatrix}$, $\mathbf{u}_i = \begin{bmatrix} u_{i1} \\ u_{i2} \end{bmatrix} = \begin{bmatrix} \cos \theta_i & -L \sin \theta_i \\ \sin \theta_i & L \cos \theta_i \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix}$, $\mathbf{p}_i$ is the operation point of vehicle i (see Fig. 1) located in the front of vehicle i and

L is the distance between $\mathbf{p}_i$ and $\mathbf{p}_i^c$. Let $\mathbf{p}_i^d \in \mathbb{R}^2$ be the goal position for vehicle i . The configuration space is spanned by $\mathbf{p} = [\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_n]^T$. For simplicity, it is assumed that each vehicle can be treated as a circular disc.

Fig. 1 Multi-vehicle navigation

2.1.2 Modeling of Communication Graph

The communication graph among vehicles is modeled by an undirected graph $\bar{\varsigma} = (\bar{\mathbf{V}}, \bar{\mathbf{W}})$, where $\bar{\mathbf{V}} = \{1, 2, \dots, n\}$ and $\bar{\mathbf{W}}(t) = \{(i, j) \in \bar{\mathbf{V}} \times \bar{\mathbf{V}} | d_{ij} \leq R_c\}$ represent, respectively, the vehicle set and the edge set, where $d_{ij} = \|\mathbf{p}_i - \mathbf{p}_j\|_2$. The set of neighbor vehicles of vehicle i is defined as $N(i) = \{j | (i, j) \in \bar{\mathbf{W}}\}$. In $\bar{\varsigma}$, edge (i, j) implies that the distance between vehicle i and vehicle j is smaller than R_c and can communicate with each other.

The following notations and symbols are adopted in this paper. $\mathbf{x} \triangleq [x_i]_{n \times 1}$ is a $n \times 1$ column vector with its i th entry being x_i . $\text{diag}\{x_i |_{i=1}^n\}$ denotes a diagonal matrix with its i th diagonal entry being x_i . $\mathbf{I}_n$ denotes the $n \times n$ identity matrix. $\mathbf{1}_n$ denotes a $n \times 1$ vector with all one elements. $\nabla_{\mathbf{x}} V = [\frac{\partial V}{\partial x_i}]_{n \times 1}$ is the gradient of V with respect to $\mathbf{x}$. $\nabla_{\mathbf{x}} \cdot V \triangleq \sum_{i=1}^n \frac{\partial V}{\partial x_i}$ is the divergence of V with respect to $\mathbf{x}$. $\|\cdot\|$ denotes Euclidean norm in the following without special notation.

2.2 Construction of decentralized navigation function

In this subsection, the traditional navigation function (3) is reviewed and its limitation is analyzed.

$$\varphi_i = \frac{\gamma_i}{(\gamma_i^\alpha + G_i)^{\frac{1}{\alpha}}}, \quad (3)$$

where γ_i is goal function, G_i is constraint function and α is exponential parameter.

In almost all the existing works, goal function is generally defined as $\gamma_i = \frac{1}{2} \|\mathbf{p}_i - \mathbf{p}_i^d\|^2$ and the value of constraint function G_i varies from 0 to 1. The control law is along the negative gradient of φ_i and is given as

$$\mathbf{u}_i = -k_i \nabla_{\mathbf{p}_i} \varphi_i, \quad (4)$$

where k_i is a positive constant and $\nabla_{\mathbf{p}_i} \varphi_i = \frac{\alpha G_i \nabla_{\mathbf{p}_i} \gamma_i - \gamma_i \nabla_{\mathbf{p}_i} G_i}{\alpha (\gamma_i^\alpha + G_i)^{\frac{1}{\alpha} + 1}}$. Furthermore, in order to make undesirable critical points locate in a narrow set near the boundary of constraints, the exponential parameter α will take a large value as will be illustrated in the Section 3. Thus, problem occurs. Consider the situation in which all constraints are satisfied, i.e., $G_i = 1$. Then the control law can be derived as follows.

$$\mathbf{u}_i = -k_i \frac{\mathbf{p}_i - \mathbf{p}_i^d}{(\gamma_i^\alpha + 1)^{\frac{1}{\alpha} + 1}} \quad (5)$$

Taking 2-norm on both sides of (5), it can be obtained that $\|\mathbf{u}_i\| = \frac{\sqrt{2}k_i \gamma_i^{\frac{1}{2}}}{(\gamma_i^\alpha + 1)^{\frac{1}{\alpha} + 1}}$.

Generally, γ_i would be very large in a large workspace. Moreover, α should also be very large as stated above. Thus it is obvious that γ_i^α is much larger than 1, resulting in $\|\mathbf{u}_i\| \cong \frac{\sqrt{2}k_i}{\gamma_i^{\alpha + \frac{1}{2}}}$, which is extremely small. In this way, vehicles will almost stop at their initial positions forever, which is unrealistic and cannot be employed in practical applications. Fig. 2 depicts the figure of navigation function φ_i and its gradient $\nabla_{\mathbf{p}_i} \varphi_i$ with respect to γ_i , when α takes value of 5 and 30 respectively. From Fig. 2, it is obvious that when γ_i takes values larger than 2, $\nabla_{\mathbf{p}_i} \varphi_i$ is close to zero and as α grows, $\nabla_{\mathbf{p}_i} \varphi_i$ approaches zero with a smaller γ_i .

Generally, γ_i will have a large value if vehicles work in a large environment, and α should be a large constant in order to constrain undesirable critical points away from safe regions. So the problem stated above would happen almost definitely. In the sequel, a novel goal function is introduced to solve this issue.

2.2.1 Construction of goal function

According to the discussions above, one intuitive idea to solve the problem is to normalize the goal function, i.e., to shrink the value of goal function to the interval between 0 and 1. As in the workspace W defined in this paper, the distance between vehicle i and its goal position could not be larger than $2R_w$. Thus, a novel goal function which is plotted in Fig. 3 is proposed as follows.

$$\gamma_i = \begin{cases} a_1 d_{id}^3 + b_1 d_{id}^2 & 0 \leq d_{id} < r \\ a_2 d_{id}^3 + b_2 d_{id}^2 + c_2 d_{id} + d_2 & r \leq d_{id} < 2R_w \\ 1 & d_{id} \geq 2R_w \end{cases}, \quad (6)$$

Fig. 2 Navigation function and its gradient with respect to goal function

where

$$\begin{aligned}
 d_{id} &= \|\mathbf{p}_i - \mathbf{p}_i^d\|, \\
 a_1 &= \frac{4\varepsilon R_w^2 + 4\varepsilon R_w r - 3r^2}{4R_w r^3 (r - 2R_w)}, \\
 b_1 &= \frac{3r^2 - 12\varepsilon R_w^2}{4R_w r^2 (r - 2R_w)}, \\
 a_2 &= \frac{4R_w r (r - 2R_w)^3}{4\varepsilon R_w r - 3r^2 + 8R_w r - 12\varepsilon R_w^2}, \\
 b_2 &= \frac{-12\varepsilon R_w^2 r - 24R_w^2 r + 3r^3 + 48\varepsilon R_w^3}{4R_w r (r - 2R_w)^3}, \\
 c_2 &= \frac{9R_w r^2 - 12\varepsilon R_w^3 - 3r^3}{r (r - 2R_w)^3}, \\
 d_2 &= 1 - \frac{R_w (-4\varepsilon R_w^2 r - 8R_w^2 r + 12R_w r^2 - 3r^3)}{r (r - 2R_w)^3}, \\
 r &= 2\varepsilon R_w.
 \end{aligned}$$

It can be verified that γ_i is twice continuously differentiable in $(0, 2R_w)$.

Remark 1 To shrink the value of goal function to the interval between 0 and 1, the following goal function using simple normalization operation, $\gamma_i = \frac{\|\mathbf{p}_i - \mathbf{p}_i^d\|^2}{4R_w^2}$, can also be used. It can be easily found that the norm of $\nabla_{\mathbf{p}_i} \gamma_i$ decreases as vehicle i approaches its goal position. However, if the proposed goal function (6) is employed, the norm value of $\nabla_{\mathbf{p}_i} \gamma_i$ can be adjusted by tuning r and ε , resulting in tuning vehicle speed according to the distance to its goal position.

2.2.2 Construction of constraint function

First, the constraint functions corresponding to collision avoidance are constructed.

To guarantee that vehicles always stay in the workspace W , the regions outside of W can be treated as an obstacle. Thus, the potential collision of one vehicle contains collisions with workspace boundary, obstacles and neighbor vehicles. To

Fig. 3 Proposed goal function

deal with potential collisions, the following constraint function is designed.

$$f(x) = \begin{cases} 0 & x < R \\ a_f x^3 + b_f x^2 + c_f x + d_f & R \leq x < R + \delta \\ 1 & x \geq R + \delta \end{cases}, \quad (7)$$

where $a_f = \frac{1}{\delta^3}$, $b_f = -\frac{3(R+\delta)}{\delta^3}$, $c_f = \frac{3(R+\delta)^2}{\delta^3}$ and $d_f = 1 - \frac{(R+\delta)^3}{\delta^3}$. It can be found that $f(x)$ is twice continuously differentiable in $(R, +\infty)$, which is described in Fig. 4. In the following, constraint functions for avoidance of workspace boundary, obstacles and neighbor vehicles are defined.

To describe workspace boundary approaching process, the workspace boundary region, $\Omega_i^B \subset \mathbb{R}^2$, is defined for vehicle i as $\Omega_i^B \triangleq \{\mathbf{p}_i | R_b \leq d_{ib} \leq R_b + \delta_b\}$, where R_b is the safe distance, δ_b is a buffer distance and $d_{ib} = R_w - \|\mathbf{p}_i\|$. In this way, any vehicle i inside Ω_i^B is considered as a potential collision with workspace boundary. Furthermore, a critical workspace boundary region, $\Psi_i^B \subset \mathbb{R}^2$, is defined for vehicle i as $\Psi_i^B \triangleq \{\mathbf{p}_i | R_b \leq d_{ib} \leq R_b + \xi\}$, where ξ is a small positive constant. Based on the definitions above, the constraint function for workspace boundary collision avoidance is designed as $G_{ib} = f(d_{ib})$, where $R = R_b$ and $\delta = \delta_b$.

To describe obstacle avoidance, a model is designed to characterize the obstacle encountering process. A nearest-distance based method is employed to characterize the current encountered obstacle, that is, the information of the obstacle point with the nearest distance from the vehicle is used to construct the virtual obstacle, as shown in Fig. 5. In Fig. 5, obstacle ς is represented by an arbitrary shape with a bold chain line. The virtual obstacle $v(\varsigma)$ is represented by a circle with a bold solid line centered at $\mathbf{p}_o$, the point with the nearest distance from vehicle i on the boundary of ς , whose radius R_o is the safe distance between vehicles and obstacles. The sensing range of vehicle i is represented by a sector $A\mathbf{p}_iB$ with a bold dotted line whose radius is R_s . Intuitively, the radius of sensing range is much larger than safe distance between vehicles and obstacles. When obstacle ς locates

Fig. 4 Constraint function for collision avoidance

outside the sensing range of vehicle i , it is assumed that a virtual obstacle $v(\varsigma)$ is located on the boundary of the sensing range of vehicle i to make the constraint function continuous. In order to deal with obstacle avoidance problem, an obstacle collision region, $\Omega_{io}^O \subset \mathbb{R}^2$, is defined for vehicle i and virtual obstacle $v(\varsigma)$ as $\Omega_{io}^O \triangleq \{\mathbf{p}_i | R_o \leq d_{io} \leq R_o + \delta_o\}$, where δ_o is a buffer distance and $d_{io} = \|\mathbf{p}_i - \mathbf{p}_o\|$, such that any vehicle i inside this region is considered as a potential collision with obstacle $v(\varsigma)$. In addition, a critical obstacle collision region, $\Psi_i^O \subset \mathbb{R}^2$, is also defined for vehicle i and virtual obstacle $v(\varsigma)$ as $\Psi_i^O \triangleq \{\mathbf{p}_i | r \leq d_{io} \leq r + \xi\}$. Thus, the constraint function for obstacle avoidance can be constructed as $G_{io} = f(d_{io})$, where $R = R_o$ and $\delta = \delta_o$.

Compared with existing approaches (such as [8], [9], [15] and [22]), where they simplify the shape of obstacles to point or sphere or need some prior knowledge of the number or size of obstacles, the proposed method only requires local information provided by onboard sensors (such as laser rangefinder), which makes it more practical for implementation.

Inter-vehicle collision avoidance can be described in a way similar to obstacle avoidance. Thus, inter-vehicle collision region between vehicle i and vehicle k , depicted in Fig. 6, is defined as $\Omega_{ik}^N \triangleq \{\mathbf{p}_i | R_n \leq d_{ik} \leq R_n + \delta_n\}$, where R_n is the safe distance between vehicles, δ_n is a buffer distance and $d_{ik} = \|\mathbf{p}_i - \mathbf{p}_k\|$, $k \in N(i)$. Additionally, the critical inter-vehicle collision region between vehicle i and vehicle k can be also defined as $\Psi_{ik}^N \triangleq \{\mathbf{p}_i | R_n \leq d_{ik} \leq R_n + \xi\}$. Thus, the constraint function for inter-vehicle collision avoidance can be given as $G_{in} = \prod_{k \in N(i)} n_{ik}$, where $n_{ik} = f(d_{ik})$ with $R = R_n$ and $\delta = \delta_n$.

Next, the constraint function related with connectivity maintenance is constructed. To deal with connectivity maintenance, each edge of the communication

Fig. 5 Schematic diagram for obstacle avoidance

Fig. 6 Schematic diagram for inter-vehicle collision avoidance

graph is endowed a weight a_{ij} , which can be defined as

$$a_{ij} = \begin{cases} 1 & d_{ij} \leq R_c - \delta_c \\ h_1(d_{ij}) & R_c - \delta_c < d_{ij} \leq R_c - \frac{\delta_c}{2} \\ h_2(d_{ij}) & R_c - \frac{\delta_c}{2} < d_{ij} \leq R_c \\ 0 & d_{ij} > R_c \end{cases}, \quad (8)$$

where

$$\begin{aligned}
h_1(x) &= a_{h_1}x^4 + b_{h_1}x^3 + c_{h_1}x^2 + d_{h_1}x + e_{h_1}, \\
h_2(x) &= a_{h_2}x^4 + b_{h_2}x^3 + c_{h_2}x^2 + d_{h_2}x + e_{h_2}, \\
a_{h_1} &= \frac{8}{\delta_c^4}, \\
b_{h_1} &= -\frac{8(4R_c - 3\delta_c)}{\delta_c^4}, \\
c_{h_1} &= \frac{24(2R_c - \delta_c)(R_c - \delta_c)}{\delta_c^4}, \\
d_{h_1} &= -\frac{8(R_c - \delta_c)^2(4R_c - \delta_c)}{\delta_c^4}, \\
e_{h_1} &= 1 + \frac{8R_c(R_c - \delta_c)^3}{\delta_c^4}, \\
a_{h_2} &= -\frac{8}{\delta_c^4}, \\
b_{h_2} &= \frac{8(4R_c - 3\delta_c)}{\delta_c^4}, \\
c_{h_2} &= -\frac{24R_c(2R_c - \delta_c)}{\delta_c^4}, \\
d_{h_2} &= \frac{8R_c^2(4R_c - 3\delta_c)}{\delta_c^4}, \\
e_{h_2} &= -\frac{8R_c^3(R_c - \delta_c)}{\delta_c^4},
\end{aligned}$$

δ_c is a buffer distance and $d_{ij} = \|\mathbf{p}_i - \mathbf{p}_j\|$. It can be verified that a_{ij} is twice differentiable with respect to d_{ij} , which is depicted in Fig. 7.

Fig. 7 Edge weight

Then the connectivity escape region can be defined as $\Omega^C \triangleq \{\mathbf{p} | \underline{\lambda} \leq \lambda_2(\mathbf{p}) \leq \bar{\lambda}\}$, where λ_2 is the second smallest eigenvalue of the Laplace matrix of the communication graph, $\underline{\lambda}$ and $\bar{\lambda}$ are the lower and upper bound for λ_2 respectively. Current configuration of the vehicle group belonging to the connectivity escape region has a risk of connectivity broken. Furthermore, the critical connectivity escape region can also be defined as $\Psi^C \triangleq \{\mathbf{p} | \underline{\lambda} \leq \lambda_2(\mathbf{p}) \leq \underline{\lambda} + \xi\}$. Based on the

definitions above, a constraint function is defined as follows.

$$g(x) = \begin{cases} 0 & x \leq \underline{\lambda} \\ a_g x^3 + b_g x^2 + c_g x + d_g & \underline{\lambda} < x \leq \bar{\lambda} \\ 1 & x > \bar{\lambda} \end{cases}, \quad (9)$$

where $a_g = \frac{1}{(\bar{\lambda} - \underline{\lambda})^3}$, $b_g = -\frac{3\bar{\lambda}}{(\bar{\lambda} - \underline{\lambda})^3}$, $c_g = \frac{3\bar{\lambda}^2}{(\bar{\lambda} - \underline{\lambda})^3}$ and $d_g = 1 - \frac{\bar{\lambda}^3}{(\bar{\lambda} - \underline{\lambda})^3}$. It is easy to find that $g(x)$ is twice differentiable in $(\underline{\lambda}, +\infty)$, which is depicted in Fig. 8. Thus, the constraint function for connectivity maintenance can be designed as $G_{ic} = g(\lambda_2)$.

Fig. 8 Constraint function for connectivity maintenance

However, computing λ_2 is a centralized operation. Inspired by [24], a local estimate of the communication graph structure can be built for each vehicle and then λ_2 can be estimated in a decentralized way. Define v_2 as the eigenvector corresponding to λ_2 . Then using the following power iteration procedure, v_2 can be estimated.

$$\hat{v}_2 = -k_1 \text{Ave}(\{\tilde{v}_2^i\}) \mathbf{1} - k_2 L \tilde{v}_2 - k_3 \left(\text{Ave}(\{(\tilde{v}_2^i)^2\}) - 1 \right) \tilde{v}_2, \quad (10)$$

where $\tilde{v}_2$ is the estimate of v_2 , $\tilde{v}_2^i$ is the i th element of $\tilde{v}_2$ and $\text{Ave}(\cdot)$ is the averaging operator. To implement (10) in a decentralized manner, the following PI average consensus estimator proposed in [26] is employed.

$$\begin{aligned} \dot{z}^i &= \gamma (\alpha^i - z^i) - K_p \sum_{j \in N(i)} (z^i - z^j) + K_i \sum_{j \in N(i)} (w^i - w^j), \\ \dot{w}^i &= -K_i \sum_{j \in N(i)} (z^i - z^j) \end{aligned} \quad (11)$$

As has been illustrated in [24], two PI consensus estimators are utilized: (1) with input $\alpha^{i,1} = \tilde{v}_2^i$, estimator 1 provides $z^{i,1}$ as the i th vehicles estimate of

Ave($\{\tilde{v}_2^i\}$); (2) with input $\alpha^{i,2} = (\tilde{v}_2^i)^2$, estimator 2 provides $z^{i,2}$ as the i th vehicles estimate of Ave($\{(\tilde{v}_2^i)^2\}$).

Then the following decentralized estimator is given,

$$\dot{\tilde{v}}_2^i = -k_1 z^{i,1} - k_2 \sum_{j \in N(i)} a_{ij} (\tilde{v}_2^i - \tilde{v}_2^j) - k_3 (z^{i,2} - 1) \tilde{v}_2^i - k_4 |\tilde{v}_2^i| \tilde{v}_2^i. \quad (12)$$

In the above estimator, each vehicle estimates a component of v_2 , namely $\tilde{v}_2^i$. Let $\tilde{\lambda}_2$ be the eigenvalue if $\tilde{v}_2$ were the corresponding eigenvector. Thus $\tilde{\lambda}_2$ can be computed as $\tilde{\lambda}_2 = \frac{k_3}{k_2} (1 - \text{Ave}(\{(\tilde{v}_2^i)^2\}))$. It can be found that the computation of $\tilde{\lambda}_2$ is not a decentralized operation due to the existence of Ave($\{(\tilde{v}_2^i)^2\}$). In order to make it decentralized, $z^{i,2}$ is utilized to replace Ave($\{(\tilde{v}_2^i)^2\}$). Then the i th vehicle can compute its own estimate of λ_2 as $\lambda_2^i = \frac{k_3}{k_2} (1 - z^{i,2})$, which is actually an estimate of $\tilde{\lambda}_2$. Moreover, as shown in [24], $\frac{\partial \tilde{\lambda}_2}{\partial \mathbf{p}_i}$ can be computed as

$$\frac{\partial \tilde{\lambda}_2}{\partial \mathbf{p}_i} = \sum_{j \in N(i)} \frac{\partial a_{ij}}{\partial \mathbf{p}_i} (\tilde{v}_2^i - \tilde{v}_2^j)^2. \quad (13)$$

2.3 Control Objectives

To illustrate the control objectives of this paper, the concept of feasible workspace is defined first.

Definition 1 (Feasible Workspace). The feasible workspace of vehicle i , F_i , can be defined as

$$F_i = \{\mathbf{p}_i | G_i = G_{ib} G_{io} G_{in} G_{ic} > 0\}. \quad (14)$$

From the above definitions, it can be found that vehicle i can avoid collisions with workspace boundary, obstacles and neighbor vehicles while preserving connectivity if it is located in F_i . Then the multi-objective control problem can be formulated as follows.

Problem 1 (Multi-Objective Control). Given the goal position for vehicle i as $\mathbf{p}_i^d$, $i \in \{1, 2, \dots, n\}$, multi-objective control problem can be divided into the following two sub-problems.

- (a) (Navigation). Design a bounded control law such that the position of vehicle i , $\mathbf{p}_i$, converges to its goal position $\mathbf{p}_i^d$.
- (b) (Collision Avoidance and Connectivity Maintenance). Assuming that the initial position of vehicle i locates in its feasible workspace, i.e., $\mathbf{p}_i(0) \in F_i$, $\forall i \in \{1, 2, \dots, n\}$, the proposed control law in (a) can also guarantee that $\mathbf{p}_i \in F_i$, $\forall i \in \{1, 2, \dots, n\}$, all the time.

To deal with the control problem, the subsequent development is based on the following assumption.

Assumption 1 The goal position of each vehicle, $\mathbf{p}_i^d$ ($\forall i \in \{1, 2, \dots, n\}$), is achievable, which means that all the goal positions locate in their corresponding feasible workspaces. Mathematically, $\mathbf{p}_i^d \in F_i$, $\forall i \in \{1, 2, \dots, n\}$.

3 Control Design

Based on the construction of decentralized navigation function introduced above, the control law can also be designed as a negative gradient of the decentralized navigation function just as (4), where $\nabla_{\mathbf{p}_i} \gamma_i$ and $\nabla_{\mathbf{p}_i} G_i$ can be expressed as

$$\nabla_{\mathbf{p}_i} \gamma_i = \begin{cases} (3a_1 d_{id} + 2b_1) (\mathbf{p}_i - \mathbf{p}_{id}) & 0 \leq d_{id} < r \\ (3a_2 d_{id}^2 + 2b_2 d_{id} + c_2) \frac{\mathbf{p}_i - \mathbf{p}_{id}}{d_{id}} & r \leq d_{id} < 2R \\ \mathbf{0} & d_{id} \geq 2R \end{cases} \quad (15)$$

and

$$\nabla_{\mathbf{p}_i} G_i = G_i \left(\frac{\nabla_{\mathbf{p}_i} G_{ib}}{G_{ib}} + \frac{\nabla_{\mathbf{p}_i} G_{io}}{G_{io}} + \sum_{k \in N(i)} \frac{\nabla_{\mathbf{p}_i} n_{ik}}{n_{ik}} + \frac{\nabla_{\mathbf{p}_i} G_{ic}}{G_{ic}} \right) \quad (16)$$

respectively. In the sequel, safety and convergence properties of the closed loop system are analyzed.

3.1 Safety Analysis

The following theorem presents the conclusion that the control law (4), (15) and (16) can guarantee collision avoidance with workspace boundary, obstacles and neighbor vehicles for each vehicle and maintain connectivity of the whole communication graph.

Theorem 1 Consider vehicle i , $\forall i \in \{1, 2, \dots, n\}$, described as (1). Give that the initial position of vehicle i locates in its own feasible workspace, i.e., $\mathbf{p}_i(0) \in F_i$. When control laws (4), (15) and (16) are applied, collision avoidance can be guaranteed and connectivity can be maintained all the time, i.e., $\mathbf{p}_i \in F_i, \forall t > 0$.

Proof Considering the critical regions defined in Section 2, there exist sub-regions of five types relating to the following five cases.

Case 1: Vehicle i almost hits the workspace boundary. Mathematically, $\mathbf{p}_i \in \Psi_i^B$.

Case 2: Vehicle i almost hits obstacle $v(\zeta)$. Mathematically, $\mathbf{p}_i \in \Psi_i^O$.

Case 3: Vehicle i almost hits vehicle k , $k \in N(i)$. Mathematically, $\mathbf{p}_i \in \Psi_{ik}^N$.

Case 4: Vehicle group almost lose connectivity of the communication graph. Mathematically, $\mathbf{p} \in \Psi^C$.

Case 5: Vehicle i almost hits more than one objects including workspace boundary, obstacles and neighbor vehicles or vehicle group almost lose connectivity at the same time. Briefly, more than two cases above, including two same cases, happen at the same time. In this work, for simplicity and without loss of generality, consider only the simple case where vehicle group almost lose connectivity and vehicle i almost hits obstacle simultaneously. The result in this case is easy to be extended to a more general case.

Next, it will be shown that vehicle i will locate inside its feasible workspace F_i navigated by the proposed controller.

In case 1, $\lim_{\xi \rightarrow 0} G_{ib} = 0$, $G_{io} > 0$, $n_{ik} > 0$ for $\forall k \in N(i)$ and $G_{ic} > 0$. Thus, $\mathbf{u}_i \cong \beta_i G_{io} G_{in} G_{ic} \nabla_{\mathbf{p}_i} G_{ib}$, where $\beta_i = \frac{k_i \gamma_i}{\alpha(\gamma_i^\alpha + G_i)^{\frac{1}{\alpha} + 1}}$ and $\nabla_{\mathbf{p}_i} G_{ib}$ can be computed

as

$$\nabla_{\mathbf{p}_i} G_{ib} = \begin{cases} \mathbf{0} & d_{ib} \leq R_b \text{ or} \\ & d_{ib} > R_b + \delta_b \\ -\frac{(3a_f d_{ib}^2 + 2b_f d_{ib} + c_f)\mathbf{p}_i}{\|\mathbf{p}_i\|} & R_b < d_{ib} \leq R_b + \delta_b \end{cases}. \quad (17)$$

Since β_i , G_{io} , G_{in} and G_{ic} are all positive, it can be found that the controller is along the direction of $-\mathbf{p}_i$ when vehicle i locates in Ψ_i^B , which implies that vehicle i is forced to move away from the workspace boundary. Therefore, vehicle i will never reach Ψ_i^B .

In case 2, $\lim_{\xi \rightarrow 0} G_{io} = 0$, $G_{ib} > 0$, $n_{ik} > 0$ for $\forall k \in N(i)$ and $G_{ic} > 0$. Thus, $\mathbf{u}_i \cong \beta_i G_{ib} G_{in} G_{ic} \nabla_{\mathbf{p}_i} G_{io}$, where

$$\nabla_{\mathbf{p}_i} G_{io} = \begin{cases} \mathbf{0} & d_{io} \leq R_o \text{ or} \\ & d_{io} > R_o + \delta_o \\ \frac{(3a_f d_{io}^2 + 2b_f d_{io} + c_f)(\mathbf{p}_i - \mathbf{p}_o)}{d_{io}} & R_o < d_{io} \leq R_o + \delta_o \end{cases}. \quad (18)$$

Similarly, it can be concluded that the controller is along the direction of $\mathbf{p}_i - \mathbf{p}_o$ when vehicle i locates in Ψ_i^O , which results in the phenomenon that vehicle i is forced to move away from the obstacle. Hence, vehicle i will never reach Ψ_i^O .

In case 3, there is only one vehicle $k \in N(i)$ such that $\lim_{\xi \rightarrow 0} n_{ik} = 0$ and $n_{il} > 0$ for $\forall l \in N(i)$, $l \neq k$. Whats more, $G_{ib} > 0$, $G_{io} > 0$ and $G_{ic} > 0$. Thus $\mathbf{u}_i \cong \beta_i G_{ib} G_{io} \bar{n}_{ik} G_{ic} \nabla_{\mathbf{p}_i} n_{ik}$, where $\bar{n}_{ik} = \prod_{l \in N(i), l \neq k} n_{il}$ and $\nabla_{\mathbf{p}_i} n_{ik}$ can be computed as

$$\nabla_{\mathbf{p}_i} n_{ik} = \begin{cases} \mathbf{0} & d_{ik} \leq R_n \text{ or} \\ & d_{ik} > R_n + \delta_n \\ \frac{(3a_f d_{ik}^2 + 2b_f d_{ik} + c_f)(\mathbf{p}_i - \mathbf{p}_k)}{d_{ik}} & R_n < d_{ik} \leq R_n + \delta_n \end{cases}. \quad (19)$$

Similar to case 2, it can be derived that the controller is along the direction of $\mathbf{p}_i - \mathbf{p}_k$ when vehicle k is in Ψ_{ik}^N , which implies that vehicle i is forced to move away from vehicle k . Hence, vehicle i never reach Ψ_{ik}^N .

In case 4, $\lim_{\xi \rightarrow 0} G_{ic} = 0$, $G_{ib} > 0$, $G_{io} > 0$ and $n_{ik} > 0$ for $\forall k \in N(i)$. Thus $\mathbf{u}_i \cong \beta_i G_{ib} G_{io} G_{in} \nabla_{\mathbf{p}_i} G_{ic}$, where

$$\nabla_{\mathbf{p}_i} G_{ic} = \begin{cases} \mathbf{0} & \lambda_2 \leq \underline{\lambda} \text{ or} \\ & \lambda_2 > \bar{\lambda} \\ (3a_g \lambda_2^2 + 2b_g \lambda_2 + c_g) \frac{\partial \lambda_2}{\partial \mathbf{p}_i} & \underline{\lambda} < \lambda_2 \leq \bar{\lambda} \end{cases}. \quad (20)$$

It can be found that, the controller is along the direction of $\frac{\partial \lambda_2}{\partial \mathbf{p}_i}$ when vehicle group locates in Ψ^C , which implies that vehicle i is forced to move to make λ_2 increase in order to maintain connectivity. Thus vehicle group never reach Ψ^C .

In case 5, $\lim_{\xi \rightarrow 0} G_{io} = 0$, $\lim_{\xi \rightarrow 0} G_{ic} = 0$, $G_{ib} > 0$ and $n_{ik} > 0$ for $\forall k \in N(i)$. Thus, $G_i \cong 0$ and $\nabla_{\mathbf{p}_i} G_i \cong \mathbf{0}$. According to (3) and (4), $\mathbf{p}_i$ is a critical point of φ_i (i.e., $\nabla_{\mathbf{p}_i} \varphi_i \cong \mathbf{0}$), and φ_i achieves its maximum value at the critical point (i.e., $\varphi_i \cong 1$). Since φ_i is maximized at $\mathbf{p}_i$, no open set of initial conditions can be attracted to

$\mathbf{p}_i$ along its negative gradient. Therefore, case 5 never happens under the proposed controller.

According to the discussions above, the proposed control law can guarantee that possible collisions with workspace boundary, obstacles and neighbor vehicles are avoided for each vehicle and connectivity is maintained for the vehicle group.

3.2 Convergence Analysis

In this part, convergence to the goal position for each vehicle navigated by the proposed control law is analyzed through the following three theorems.

Theorem 2 *The decentralized navigation function φ_i is minimized at the goal position $\mathbf{p}_i^d$.*

Proof From the definition of the goal function γ_i in (6), it can be derived that $\gamma_i|_{\mathbf{p}_i^d} = 0$. Moreover, according to the expression of $\nabla_{\mathbf{p}_i}\gamma_i$ in (15), $\nabla_{\mathbf{p}_i}\gamma_i|_{\mathbf{p}_i^d} = \mathbf{0}$. Thus, it can be concluded that $\nabla_{\mathbf{p}_i}\varphi_i|_{\mathbf{p}_i^d} = \mathbf{0}$. Therefore, the goal position $\mathbf{p}_i^d$ is a critical point of φ_i . Moreover, the Hessian matrix of γ_i is $\nabla_{\mathbf{p}_i}^2\gamma_i = 3a_1 \frac{(\mathbf{p}_i - \mathbf{p}_i^d)(\mathbf{p}_i - \mathbf{p}_i^d)^T}{d_{id}^{\frac{1}{\alpha}+2}} + 3a_1 d_{id}\mathbf{I} + 2b_1\mathbf{I}$, thus $\nabla_{\mathbf{p}_i}^2\gamma_i|_{\mathbf{p}_i^d} = 2b_1\mathbf{I}$. Additionally, the Hessian matrix of φ_i can be derived as

$$\begin{aligned} \nabla_{\mathbf{p}_i}^2\varphi_i = & \frac{1}{\alpha(\gamma_i^\alpha + G_i)^{\frac{1}{\alpha}+2}} \{ (\gamma_i^\alpha + G_i) [\alpha \nabla_{\mathbf{p}_i}\gamma_i \nabla_{\mathbf{p}_i}^T G_i \\ & + \alpha G_i \nabla_{\mathbf{p}_i}^2\gamma_i - \nabla_{\mathbf{p}_i} G_i \nabla_{\mathbf{p}_i}^T \gamma_i - \gamma_i \nabla_{\mathbf{p}_i}^2 G_i] - \frac{\alpha+1}{\alpha} \cdot \\ & (\alpha G_i \nabla_{\mathbf{p}_i}\gamma_i - \gamma_i \nabla_{\mathbf{p}_i} G_i) (\alpha \gamma_i^{\alpha-1} \nabla_{\mathbf{p}_i}^T \gamma_i + \nabla_{\mathbf{p}_i}^T G_i) \} \end{aligned} \quad (21)$$

As it has been obtained that at the goal position $\mathbf{p}_i^d$, $\gamma_i|_{\mathbf{p}_i^d} = 0$, $\nabla_{\mathbf{p}_i}\gamma_i|_{\mathbf{p}_i^d} = \mathbf{0}$ and $\nabla_{\mathbf{p}_i}^2\gamma_i|_{\mathbf{p}_i^d} = 2b_1\mathbf{I}$, the Hessian matrix of φ_i evaluated at $\mathbf{p}_i^d$ can be given as $\nabla_{\mathbf{p}_i}^2\varphi_i|_{\mathbf{p}_i^d} = 2G_i^{-\frac{1}{\alpha}}b_1\mathbf{I}$. As b_1 is a positive constant and $G_i > 0$ at the goal position according to Assumption 1, $\nabla_{\mathbf{p}_i}^2\varphi_i|_{\mathbf{p}_i^d}$ is positive definite. It is well known that the decentralized navigation function φ_i is minimized at a critical point if the Hessian matrix of φ_i evaluated at that point is positive definite. Thus the proof completes.

The following theorem guarantees that all the critical points of the decentralized navigation function locate in the sub-space where G_i is arbitrarily small. Before going on, some sets are defined for vehicle i . The set of goal configurations for vehicle i is defined as $F_{id} \triangleq \{\mathbf{p}|\gamma_i = 0\}$. The set $F_{ib} \triangleq \{\mathbf{p}|0 < G_{ib} < \zeta, \zeta > 0\} - F_{id}$ describes the regions near the workspace boundary. The set $F_{io} \triangleq \{\mathbf{p}|0 < G_{io} < \zeta, \zeta > 0\} - F_{id}$ denotes the regions near the potential obstacle collision constraint. The set $F_{in} \triangleq \bigcup_{k \in N(i)} \{\mathbf{p}|0 < n_{ik} < \zeta, \zeta > 0\} - F_{id}$ describes the regions near the potential inter-vehicle collision constraint. The set $F_{ic} \triangleq \{\mathbf{p}|0 < G_{ic} < \zeta, \zeta > 0\} - F_{id}$ denotes the regions near the connectivity constraint. The set $F_{iu} \triangleq F_i - F_{id} \cup F_{ib} \cup F_{io} \cup F_{in} \cup F_{ic}$ represents the regions away from all constraints.

Theorem 3 *For every $\zeta > 0$, there exists a number $\Gamma(\zeta)$ such that if $\alpha > \Gamma(\zeta)$, no critical points of φ_i are in the set F_{iu} .*

Proof At a critical point, it can be derived that $\alpha G_i \nabla_{\mathbf{p}_i} \gamma_i = \gamma_i \nabla_{\mathbf{p}_i} G_i$. A sufficient condition for this equality not to hold in F_{iu} is that $\alpha > \sup \frac{\gamma_i \|\nabla_{\mathbf{p}_i} G_i\|}{\|\nabla_{\mathbf{p}_i} \gamma_i\| G_i}, \forall \mathbf{p} \in F_{iu}$. As in F_{iu} , $G_{ib} \geq \zeta$, $G_{io} \geq \zeta$, $n_{ik} \geq \zeta$ for $\forall k \in N(i)$ and $G_{ic} \geq \zeta$, it can be obtained that

$$\begin{aligned} \Gamma(\varepsilon) &= \sup \frac{\gamma_i \|\nabla_{\mathbf{p}_i} G_i\|}{\|\nabla_{\mathbf{p}_i} \gamma_i\| G_i} \\ &\leq \sup \frac{\gamma_i}{\|\nabla_{\mathbf{p}_i} \gamma_i\|} \left(\frac{\|\nabla_{\mathbf{p}_i} G_{ib}\|}{\zeta} + \frac{\|\nabla_{\mathbf{p}_i} G_{io}\|}{\zeta} + \sum_{k \in N(i)} \frac{\|\nabla_{\mathbf{p}_i} n_{ik}\|}{\zeta} + \frac{\|\nabla_{\mathbf{p}_i} G_{ic}\|}{\zeta} \right). \end{aligned} \quad (22)$$

As $\frac{\gamma_i}{\|\nabla_{\mathbf{p}_i} \gamma_i\|}$, $\|\nabla_{\mathbf{p}_i} G_{ib}\|$, $\|\nabla_{\mathbf{p}_i} G_{io}\|$, $\|\nabla_{\mathbf{p}_i} n_{ik}\|$ and $\|\nabla_{\mathbf{p}_i} G_{ic}\|$ are all bounded terms in F_{iu} , $\Gamma(\zeta)$ exists.

The next result is to utilize dual Lyapunov theory [27] to analyze convergence conditions. Before going on, the closed loop kinematics of the vehicle group (2) under the control law (4), (15) and (16) can be expressed as

$$\dot{\mathbf{p}} = f(\mathbf{p}) = [-k_i \nabla_{\mathbf{p}_i} \varphi_i]_{n \times 1}. \quad (23)$$

Then define $\varphi = \sum_{i=1}^n \varphi_i$ and $\rho = \varphi^{-1}$. The following theorem gives the convergence condition.

Theorem 4 Consider the n -vehicle system navigated by control law (4), (15) and (16). A sufficient condition for the system to satisfy condition $[\nabla_{\mathbf{p}} \cdot (f\rho)](x) > 0$ at the undesirable critical points $\{\mathbf{p} | \nabla_{\mathbf{p}_i} \varphi_i = \mathbf{0}, \gamma_i \neq 0, \forall i \in \{1, 2, \dots, n\}\}$ is given by $\gamma_i > \gamma_{\min}, \forall i \in \{1, 2, \dots, n\}$.

Proof It can be derived that $\nabla_{\mathbf{p}} \rho = -\varphi^{-2} \nabla_{\mathbf{p}} \varphi$ and $\nabla_{\mathbf{p}} \cdot (f\rho) = \nabla_{\mathbf{p}} f \cdot \rho + \rho \nabla_{\mathbf{p}} \cdot f = -\varphi^{-2} \nabla_{\mathbf{p}} \varphi \cdot f + \varphi^{-1} \nabla_{\mathbf{p}} \cdot f$. As at the critical point $\nabla_{\mathbf{p}_i} \varphi_i = \mathbf{0}$ for all i , $f = \mathbf{0}$, $\nabla_{\mathbf{p}} \cdot (f\rho)$ can be simplified as $\nabla_{\mathbf{p}} \cdot (f\rho) = \varphi^{-1} \nabla_{\mathbf{p}} \cdot f = -\varphi^{-1} \sum_{i=1}^n k_i (\nabla_{x_i}^2 \varphi_i + \nabla_{y_i}^2 \varphi_i)$. A sufficient condition for $\nabla_{\mathbf{p}} \cdot (f\rho)$ to be positive is $\nabla_{x_i}^2 \varphi_i + \nabla_{y_i}^2 \varphi_i < 0$ for all i .

Moreover, at the critical points

$$\nabla_{x_i}^2 \varphi_i = \frac{1}{(\gamma_i^\alpha + G_i)^{\frac{1}{\alpha} + 1}} ((\alpha - 1) \frac{G_i}{\gamma_i} (\nabla_{x_i} \gamma_i)^2 + G_i \nabla_{x_i}^2 \gamma_i - \frac{\gamma_i}{\alpha} \nabla_{x_i}^2 G_i). \quad (24)$$

Similarly, it can be obtained that at the critical points

$$\nabla_{y_i}^2 \varphi_i = \frac{1}{(\gamma_i^\alpha + G_i)^{\frac{1}{\alpha} + 1}} ((\alpha - 1) \frac{G_i}{\gamma_i} (\nabla_{y_i} \gamma_i)^2 + G_i \nabla_{y_i}^2 \gamma_i - \frac{\gamma_i}{\alpha} \nabla_{y_i}^2 G_i). \quad (25)$$

Thus $\nabla_{x_i}^2 \varphi_i + \nabla_{y_i}^2 \varphi_i < 0$ implies that

$$G_i (\nabla_{x_i}^2 \gamma_i + \nabla_{y_i}^2 \gamma_i) - \frac{\gamma_i}{\alpha} (\nabla_{x_i}^2 G_i + \nabla_{y_i}^2 G_i) + (\alpha - 1) \frac{G_i}{\gamma_i} [(\nabla_{x_i} \gamma_i)^2 + (\nabla_{y_i} \gamma_i)^2] < 0. \quad (26)$$

From the definition of γ_i , it can be obtained that $\nabla_{x_i} \gamma_i$ and $\nabla_{y_i}^2 \gamma_i$ are upper bounded. Thus it can be assumed that $\sup (\nabla_{x_i} \gamma_i)^2 = M_1$ and $\sup \nabla_{x_i}^2 \gamma_i = M_2$. In this way, a sufficient condition of (26) is

$$2G_i M_2 - \frac{\gamma_i}{\alpha} (\nabla_{x_i}^2 G_i + \nabla_{y_i}^2 G_i) + 2(\alpha - 1) \frac{G_i}{\gamma_i} M_1 < 0, \quad (27)$$

which can be simplified as

$$\gamma_i^2 - 2\alpha M_2 \gamma_i \frac{G_i}{\nabla_{x_i}^2 G_i + \nabla_{y_i}^2 G_i} - 2\alpha(\alpha - 1) M_1 \frac{G_i}{\nabla_{x_i}^2 G_i + \nabla_{y_i}^2 G_i} > 0. \quad (28)$$

As all the undesirable critical points locate in a narrow set near the constraints according to Theorem 3, it can be obtained that G_i approaches 0, which implies that at least one constraint function approaches 0. For simplicity and without loss of generality, assume that $G_{ib} \rightarrow 0$. The conclusions thereafter can be easily extended to other situations. Additionally, it can be derived that

$$\nabla_{x_i}^2 G_i = \bar{G}_{ib} \nabla_{x_i}^2 G_{ib} + 2\nabla_{x_i} G_{ib} \nabla_{x_i} \bar{G}_{ib} + G_{ib} \nabla_{x_i}^2 \bar{G}_{ib}, \quad (29)$$

where $\bar{G}_{ib} = \frac{G_i}{G_{ib}}$. Thus,

$$\frac{\nabla_{x_i}^2 G_i}{G_i} = \frac{\nabla_{x_i}^2 G_{ib}}{G_{ib}} + \frac{2\nabla_{x_i} G_{ib} \nabla_{x_i} \bar{G}_{ib}}{G_i} + \frac{\nabla_{x_i}^2 \bar{G}_{ib}}{\bar{G}_{ib}}. \quad (30)$$

Consequently it can be obtained that

$$\lim_{G_{ib} \rightarrow 0} \frac{\nabla_{x_i}^2 G_i}{G_i} = +\infty. \quad (31)$$

Similarly, $\lim_{G_{ib} \rightarrow 0} \frac{\nabla_{y_i}^2 G_i}{G_i} = +\infty$. From the continuity of the function $\frac{\nabla_{x_i}^2 G_i}{G_i}$ and $\frac{\nabla_{y_i}^2 G_i}{G_i}$, it can be concluded that there exists a $M_3 > 0$, such that $0 < \frac{G_i}{\nabla_{x_i}^2 G_i + \nabla_{y_i}^2 G_i} \leq M_3$ for $0 < G_{ib} \leq \zeta$. Thus, (28) can be further simplified as

$$\gamma_i^2 - 2\alpha M_2 M_3 \gamma_i - 2\alpha(\alpha - 1) M_1 M_3 > 0. \quad (32)$$

Generally, α is chosen larger than 1. It is obvious that equation $\gamma_i^2 - 2\alpha M_2 M_3 \gamma_i - 2\alpha(\alpha - 1) M_1 M_3 = 0$ has two roots R_1 and R_2 , where $R_1 < R_2$ and $R_2 > 0$. So a sufficient condition for (32) to hold is that $\gamma_i > \gamma_{\min}$, where $\gamma_{\min} = R_2$. This completes the proof.

From the discussions above, it can be concluded that for vehicle group navigated by the control law (4), (15) and (16), there exists a positive $\gamma_{\min}$ such that the closed loop system will not converge to the undesirable critical points when $\gamma_i > \gamma_{\min}$, $\forall i \in \{1, 2, \dots, n\}$. As a result, the closed loop system will converge to the set $\{\mathbf{p} | \gamma_i \leq \gamma_{\min}, \forall i = 1, 2, \dots, n\}$.

4 Simulation Results

In this section, a series of numerical simulations are conducted to demonstrate the efficacy of the proposed method. Consider 6 vehicles and 6 obstacles in the simulations. For simplicity and without loss of generality, all the obstacles are denoted by circles and the radii are set as $0.2m$. The initial positions, goal positions of vehicles and the positions of obstacles are given in Table 1. The radius of the workspace is set as $R_w = 10m$ and the parameter ε of goal function is chosen as $\varepsilon = 0.1$. The safe distances for workspace boundary avoidance, obstacle avoidance and inter-vehicle avoidance are set as $R_b = 0.5m$, $R_o = 0.5m$ and $R_n = 0.6m$,

respectively. The maximum communication distance is set as $R_c = 2.5\text{m}$. The buffer distances are $\delta_b = \delta_o = \delta_n = \delta_c = 0.5\text{m}$. The parameters of the decentralized estimator are chosen as $\bar{\lambda} = 3$, $\underline{\lambda} = 0$, $\gamma = 1.5$, $K_p = 20$, $K_i = 6$, $k_1 = 1.5$, $k_2 = 0.15$, $k_3 = 1.5$ and $k_4 = 0.0001$. The control parameters are set as $k_i = 10$ and $\alpha = 60$. The sampling period is chosen as 10ms in the simulations.

Table 1 Configurations of vehicles and obstacles

Position (m)	1	2	3	4	5	6
$\mathbf{p}_i(0)$	(-6, -3)	(-4.5, -3)	(-4.5, -4.5)	(-6, -4.5)	(-6, -6)	(-4.5, -6)
$\mathbf{p}_i^d$	(6, 6)	(4.5, 6)	(4.5, 4.5)	(6, 4.5)	(6, 3)	(4.5, 3)
$\mathbf{p}_o$	(-3, -0.5)	(0, 2)	(1, 0)	(-1, -1.5)	(-2, -4)	(1, -2)

Fig. 9 depicts the trajectories of all vehicles navigated by the proposed control law. From this figure, it can be concluded that all vehicles reach their goal positions from their initial positions successfully while avoiding collisions with obstacles. Fig. 10 shows the distance between each vehicle and its corresponding goal position. Fig. 11 describes the distances between each vehicle and the workspace boundary. Fig. 12 shows the distances between each vehicle and the virtual obstacle. Fig. 13 depicts the distances between any two vehicles. From the above three figures, it can be found that all curves are above the dotted red lines denoting the safe distance, which implies that all vehicles can avoid collisions with workspace boundary, obstacles and neighbor vehicles in the process. Fig. 14 illustrates the connectivity performance, where λ_2 is depicted. From this figure, it can be concluded that connectivity is maintained all the time as λ_2 is always larger than zero. Fig. 15 plots the control inputs of all vehicles.

Fig. 9 Vehicle trajectories

Fig. 10 Distance between vehicles and goal positions

Fig. 11 Distance between vehicles and workspace boundary

5 Conclusion

This paper presents a decentralized control scheme to navigate a group of vehicles from their initial positions to the goal positions while avoiding collisions with workspace boundary, obstacles and neighboring vehicles as well as preserving global connectivity of the communication graph at the same time. Compared with the existing literatures, limitations of the traditional navigation function which has long been ignored is pointed out, then a new navigation function with a novel form of goal function is proposed based on which a decentralized control law is derived. In addition, this paper incorporates the global connectivity maintenance approach

Fig. 12 Distance between vehicles and obstacles

Fig. 13 Distance between neighbor vehicles

into the decentralized navigation function. Safety and convergence property of the closed loop system using the proposed method are also analyzed rigorously. Future work will consider robustness of the multi-vehicle network under vehicle failures.

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Fig. 14 Connectivity indicator

Fig. 15 Control inputs

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