Multi-Dimensional Pairs Trading Using Bernstein Copula

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Multi-Dimensional Pairs Trading Using Bernstein Copula

Introduction & Motivation

- Pairs trading is a statistical arbitrage strategy that involves the simultaneous long/short of 2 relatively mispriced stocks (2 dimensions) which have strong historical co-movements.
- In 2-dimensional pairs trading, the copula method is proposed to overcome the limitations of the most popular distance method. In this research, we propose a multi-dimensional pairs trading framework to overcome the limitation of 2-dimensional pairs trading methods.

Table 1: Comparison Between Distance, 2-Dimensional Copula and Multi-Dimensional Copula Methods

Method	Associations Captured	Form of Marginal Distribution Required	Relative Pricing
Distance	**Linear associations** as linear correlation is used	**Normal marginal distributions** of financial data (rarely the case in reality)	Solely **based on the common portion of full dependency information** and **ignores the uncommon portion** (Figure 1)
2-Dimensional Copula	**Linear and non-linear associations**	Gives an explicit function of joint distribution **regardless of the forms of marginal distributions**	
Multi-Dimensional Copula			**Reveals the uncommon portion to increase dependency information** and measure relative pricing more comprehensively (Figure 2)

Figure 1: Two-Dimensional Pairs Trading

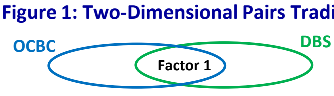
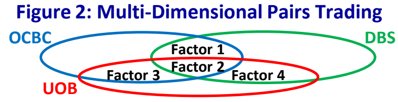


Figure 2: Multi-Dimensional Pairs Trading



Methodology

1. Apply Sklar's theorem in 3 dimensions.

$$H(r_t^X, r_t^Y, r_t^Z) = C(F_X(r_t^X), F_Y(r_t^Y), F_Z(r_t^Z))$$

where H is the joint distribution, r_t^X, r_t^Y and r_t^Z are stock returns, C is the copula.

2. Estimate the best-fitted marginal distributions.

3. Apply Bernstein Copula to estimate the joint distribution.

4. Take first derivative of the copula function to calculate conditional probability (e.g. A on B & C) and measure mispricing degree.

Note: If conditional probability > 0.5, the underlying stock is overvalued relative to the rest and vice versa.

Empirical Results

Table 2: Optimization of Trigger Point and Stop-Loss Threshold for the Distance Method

S\T	1.5	1.6	1.7	1.8	1.9	2.0	2.1
3.4	0.3170%	0.3110%	0.3140%	0.2270%	0.2300%	0.2760%	0.2490%
3.6	0.3590%	**0.3630%**	**0.3660%**	0.2810%	0.2840%	0.3300%	0.3030%
3.8	0.3590%	0.3520%	0.3560%	0.2710%	0.2740%	0.3190%	0.3160%
4.0	**0.3630%**	**0.3660%**	**0.3690%**	0.2850%	0.2870%	0.3330%	0.3300%
4.2	**0.3610%**	**0.3630%**	**0.3660%**	0.2810%	0.2840%	0.3300%	0.3270%
4.4	**0.3600%**	0.3540%	0.3570%	0.2720%	0.2750%	0.3210%	0.3180%
4.6	0.3570%	0.3510%	0.3540%	0.2690%	0.2720%	0.3180%	0.3150%

Table 3: Optimization of Trigger Point and Stop-Loss Threshold for 2D Copula Method

S\T	0.55	0.56	0.57	0.58	0.59	0.60	0.61
1.6	-0.4040%	-0.3140%	-0.1940%	-0.2270%	-0.0530%	-0.1050%	-0.1370%
1.7	-0.1160%	-0.0420%	-0.0620%	-0.0410%	0.0770%	0.0100%	0.0060%
1.8	0.0090%	0.0740%	0.0790%	0.0810%	0.0790%	0.0860%	0.0900%
1.9	0.3250%	**0.4180%**	**0.4510%**	**0.4670%**	**0.4490%**	0.3400%	0.3400%
2.0	-0.0050%	0.0610%	0.1660%	0.1950%	0.2930%	0.2660%	0.2700%
2.1	-0.0970%	-0.0800%	0.0600%	0.1510%	0.2640%	0.3020%	0.2550%
2.2	-0.2280%	-0.2180%	0.0320%	0.1500%	0.2760%	0.2710%	0.2500%

Table 4: Optimization of Trigger Point and Stop-Loss Threshold for 3D Copula Method

S/T	0.57	0.58	0.59	0.60	0.61	0.62	0.63
1.2	0.4746%	**0.5151%**	0.4990%	**0.5340%**	**0.5170%**	**0.5240%**	0.4950%
1.3	0.3629%	0.4082%	0.3640%	0.3760%	0.3730%	0.3800%	0.3570%
1.4	**0.5096%**	**0.5285%**	0.4730%	0.4510%	0.4890%	0.4910%	0.4820%
1.5	0.4930%	**0.5195%**	0.4960%	0.4820%	**0.5220%**	0.4810%	0.3820%
1.6	0.4341%	0.4532%	0.4294%	0.4248%	0.4408%	0.4564%	0.4563%
1.7	0.2934%	0.2967%	0.3046%	0.3297%	0.3303%	0.3245%	0.3009%
1.8	0.1948%	0.2226%	0.1825%	0.1908%	0.2137%	0.2327%	0.2061%

Figure 3: OCBC, DBS & UOB (2010-2014)

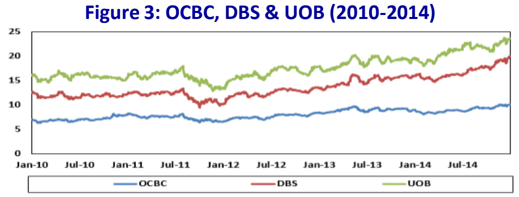


Figure 4: Cumulative Returns

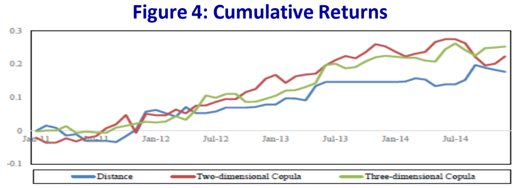


Table 5: Excess Returns and Trading Statistics

	Distance	2D Copula	3D Copula
Average Excess Returns	0.0037*	0.0047	**0.0052***
t-statistic	1.6715	1.6592	**1.7322**
Average Number of Trades	1.0417	6.7917	**6.7932**
S.D. of Number of Trades	0.7506	2.1260	2.6121
Average Time of Open Position (month)	2.2321	4.2639	3.3330
S.D. of Time of an Open Position	1.8106	0.5384	**0.7300**

* Represents 10% significance level.

Table 6: Alpha, Fama-French 3 Factors and Carhart Momentum Factor

	Distance		2D Copula		3D Copula	
	Coefficient	t-statistic	Coefficient	t-statistic	Coefficient	t-statistic
Alpha	0.0027	1.0306	0.0037	1.0403	**0.0050***	**1.8570**
Mkt-Rf	-0.0001	-0.3160	0.0002	0.3709	-0.0003	-0.7186
SMB	-0.0003	-0.3143	-0.0010	-0.6861	0.0006	0.4997
HML	0.0006	0.5150	0.0010	0.5668	0.0006	0.4391
WML	0.0012	1.5546	-0.0001	-0.1074	-0.0011	-1.3113

* Represents 10% significance level.

Conclusion

1. By capturing both linear and non-linear associations, as well as increasing dependency information, the multi-dimensional copula method is able to generate a higher average excess return that is not due to any market risk factors, as shown by a high and significant alpha.

2. It is also able to identify more trades and it is more certain of the duration of an open position.

3. Future research will focus on large sample analysis of the proposed strategy and implementation of stocks selection process.

Main References

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Project Title: Multi-Dimensional Pairs Trading Using Bernstein Copula
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