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Multivariate adaptive regression splines approach to estimate lateral wall deflection profiles caused by braced excavations in clays

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Abstract: Based on an expanded database including a total of 30 case histories for braced excavation in stiff, medium and soft clays, a multivariate adaptive regression splines (MARS) approach for estimating wall deflection profile caused by deep braced excavations is presented in this study. For each soil type, ten case histories with information on subsurface soil conditions, geometry characteristics, excavation support system details, the maximum wall deflections and the wall deflection profile against embedded depth are provided. Seven input variables, including wall length, excavation depth, excavation length, system stiffness, average unit weight and undrained shear strength of the soil, and the depth below the ground surface, are adopted as inputs to the MARS deflection profile model. Comparison with two more excavation case histories indicates that the developed MARS model can give an accurate graphical representation of the wall deflection profile. It is capable of not only predicting the value of maximum wall deflection but also estimating the possible depth at which maximum lateral deformation occurs.

Keywords: Multivariate adaptive regression splines; Wall deflection profile; Braced excavation; Case histories; Clays

1. Introduction

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With rapid growth in urban development, more and more excavations projects for high-rise buildings and subway lines are being executed and scheduled. In the design of a braced excavation, it is essential to consider not only the stability issue such as basal heave during construction, but also the potential serviceability problem of adjacent buildings caused by excessive wall deflections and ground movements. In order to ensure the stability of the excavation and reduce the effect on the neighboring buildings and underground utilities caused by excavation, continuous wall structures are often used. In these cases, the use of a multi-strutted structural system is generally required in order to reduce ground movements and also to achieve relatively high benefits. A reasonable estimation of the lateral wall deflection profiles caused by braced excavations and the corresponding maximum value is desirable for safe and economical design purpose.

There are two common techniques for estimating the horizontal wall displacements and ground settlements using either interpolation from a published database of different areas of the world or numerical analysis using either finite element methods or finite difference methods. Since soil is a complicated material that always displays non-linear and inelastic behavior, robust predictions of ground movements are difficult. Even though various aspects of soil are incorporated into many numerical models, many of these models are generally complex and the adopted parameters do not have a clear physical meaning, even the determination of them require special kind of testing technique and laboratory skills. In addition, these numerical models require huge amount of computational resources. Actually, the performance of deep excavations depends on a large number of geotechnical and geometrical influential parameters that are interdependent to different degrees. Therefore, practicing engineers are apt to avoid using numerical simulations and tend to use design charts which directly relate wall deflections to soil properties. It is a proven approach to identify the significant parameters affecting the deformation behavior of deep excavations through the empirical analysis of the measured displacements in a large number of case histories.

Currently, the most commonly used design chart for predicting lateral wall deflections in deep excavations in clay is the one proposed by Clough et al. (1989). It allows the estimation of lateral movements in terms of system stiffness which is principally represented by three factors including the bending stiffness of the retaining wall and supports, the configuration, location and

distance of the strut, the embedment length of the retaining wall and the factor of safety against basal heave. Long (2001) and Moormann (2004) have assessed the validity and applicability of the Clough design chart using databases of more than 296 and 530 case histories, respectively, plotting maximum lateral deformation, normalized with respect to the excavation height, versus system stiffness and compared the result with the curves proposed by Clough et al. (1989). For different factors of safety against basal heave. Long (2001) differentiated the data by low and high factor of safety while Moormann (2004) differentiated it by soft and stiff ground.

Several other empirical and semi-empirical methods commonly adopted for estimating maximum wall deflections can be referred to in Mana and Clough (1981), Wong and Broms (1989), Clough and O'Rourke (1990), Hashash and Whittle (1996), Addenbrooke et al. (2000), Kung et al. (2007), Bryson and Zapata (2012) and Zhang et al. (2015). However, most of these methods aim at solving for the lateral maximum wall deflection, without any attention on the lateral wall deflection profiles, which is thought to be of the same significance with the maximum value. This paper utilized a non-parametric, multivariate adaptive regression spline algorithm to derive the estimation models for the lateral wall deflection profiles and also the maximum wall deflections induced by excavations in clays, based on an expanded database with different clay types, excavation geometries, soil and support system parameters. Validations of the proposed MARS model were also carried out through comparisons with two more well-documented excavation case histories.

2. MARS methodology

Friedman (1991) introduced MARS as a statistical method for fitting the relationship between a set of input variables and dependent variables. It is a nonlinear and nonparametric regression method based on a divide and conquer strategy in which the training data sets are partitioned into separate piecewise linear segments (splines) of differing gradients (slope). No specific assumption about the underlying functional relationship between the input variables and the output is required. The end points of the segments are called knots. A knot marks the end of one region of data and the beginning of another. The resulting piecewise curves (known as basis functions), give greater flexibility to the model, allowing for bends, thresholds, and other departures from linear functions.

MARS generates basis functions by searching in a stepwise manner. An adaptive regression algorithm is used for selecting the knot locations. MARS models are constructed in a two-phase procedure. The forward phase adds functions and finds potential knots to improve the performance, resulting in an overfit model. The backward phase involves pruning the least effective terms. An open source code on MARS from Jekabsons (2010) is used in carrying out the analyses presented in this paper.

Let y be the target output and $X = (X_1, \dots, X_P)$ be a matrix of P input variables. Then it is assumed that the data are generated from an unknown “true” model. In case of a continuous response this would be

$$y = f(X_1, \dots, X_P) + e = f(X) + e \quad (1)$$

in which e is the distribution of the error. MARS approximates the function f by applying basis functions (BFs). BFs are splines (smooth polynomials), including piecewise linear and piecewise cubic functions. For simplicity, only the piecewise linear function is expressed. Piecewise linear functions are of the form $\max(0, x-t)$ with a knot occurring at value t . The equation $\max(.)$ means that only the positive part of $(.)$ is used otherwise it is given a zero value. Formally,

$$\max(0, x-t) = \begin{cases} x-t, & \text{if } x \geq t \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

The MARS model $f(X)$ is constructed as a linear combination of BFs and their interactions, and is expressed as

$$f(X) = \beta_0 + \sum_{m=1}^M \beta_m \lambda_m(X) \quad (3)$$

where each $\lambda_m(X)$ is a basis function. It can be a spline function, or the product of two or more spline functions already contained in the model (higher orders can be used when the data warrants it; for simplicity, at most second-order is assumed in this paper). The coefficients β are constants, estimated using the least-squares method.

Fig. 1 shows a simple example of how MARS would use piecewise linear spline functions to attempt to fit data. The MARS mathematical equation is expressed as

$$y = 4.467 + 1.104 \times BF1 - 3.997 \times BF2 + 1.967 \times BF3 \quad (4)$$

where $BF1 = \max(0, x-16)$, $BF2 = \max(0, 16-x)$, and $BF3 = \max(0, 25-x)$ and $\max$ is defined as: $\max(a, b)$ is equal to a if $a > b$, else b . The knots are located at $x = 16$ and 25 . They delimit three intervals where different linear relationships are identified. It is obvious that MARS approach is good at dealing with problems in which there are great scatters in both the explanatory independent variables and the target responses.

Fig. 1 Knots and linear splines for a simple MARS example

The MARS modeling is a data-driven process. To fit the model in Eq. (3), first a forward selection procedure is performed on the training data. A model is constructed with only the intercept, β_0 , and the basis pair that produces the largest decrease in the training error is added. Considering a current model with M basis functions, the next pair is added to the model in the form

$$\hat{\beta}_{M+1} \lambda_m(X) \max(0, X_j - t) + \hat{\beta}_{M+2} \lambda_m(X) \max(0, t - X_j) \quad (5)$$

with each β being estimated by the method of least squares. As a basis function is added to the model space, interactions between BFs that are already in the model are also considered. BFs are added until the model reaches some maximum specified number of terms leading to a purposely overfit model.

To reduce the number of terms, a backward deletion sequence follows. The aim of the backward deletion procedure is to find a close to optimal model by removing extraneous variables. The backward pass prunes the model by removing the BFs with the lowest contribution to the model until it finds the best sub-model. Thus, the BFs maintained in the final optimal model are selected from the set of all candidate BFs, used in the forward selection step. Model subsets are compared using the less computationally expensive method of Generalized Cross-Validation (GCV). The GCV equation is a goodness of fit test that penalizes large numbers of BFs and serves to reduce the chance of overfitting. For the training data with N observations, GCV for a model is calculated as follows (Hastie 2009)

$$GCV = \frac{\frac{1}{N} \sum_{i=1}^N [y_i - f(x_i)]^2}{[1 - \frac{M + d \times (M-1)/2}{N}]^2} \quad (6)$$

in which M is the number of BFs, d is the penalizing parameter, N is the number of observations, and $f(x_i)$ denotes the predicted values of the MARS model. The numerator is the mean squared error of the evaluated model in the training data, penalized by the denominator. The denominator accounts for the increasing variance in the case of increasing model complexity. Note that $(M-1)/2$ is the number of hinge function knots. The GCV penalizes not only the number of the model's basis functions but also the number of knots. A default value of 3 is assigned to penalizing parameter d (Friedman 1991). At each deletion step a basis function is removed to minimize Eq. (3), until an adequately fitted model is found. MARS is an adaptive procedure because the selection of BFs and the variable knot locations are data-based and specific to the problem at hand.

After the optimal MARS model is determined, by grouping together all the BFs that involve one variable and another grouping of BFs that involve pairwise interactions (and even higher level interactions when applicable), the procedure known as analysis of variance (ANOVA) decomposition (Friedman 1991) can be used to assess the contributions from the input variables and the BFs through comparing (testing) variables for statistical significance. Previous applications of MARS algorithm in civil engineering can be found in Attoh-Okine et al. (2009), Zarnani et al. (2011), Samui and Karup (2011), Lashkari (2012), Zhang and Goh (2013), Adoko et al. (2013), Goh and Zhang (2014), Khoshnevisan et al. (2015), Zhang et al. (2015).

3. Case histories

The databases developed by Long (2001) and Moormann (2004) are mostly used. Based on the results of his empirical study, Moormann (2004) concluded that the data for deep excavations in soft clays scatter in a wide range and there is no clear dependency of the system stiffness factor proposed by Clough et al. (1989) on the lateral deflections. For stiff clays, the results are similar to those presented by Long (2001) where the displacements are not influenced by factor of safety against basal heave and their dependency on the system stiffness is not observed. Moormann

(2004) regarded the lack of dependency of lateral wall deflections on system stiffness to factors including the soil conditions at the embedment portion of the wall, the ground water conditions, the surrounding buildings or geometrical irregularities, workmanship, excavation sequence, pre-load of struts, etc. However, the quantification of these factors mentioned above is difficult since they are not reported and documented in detail in most cases. Due to this and also in view of the lack of information in the case histories presented by Long (2001) and Moormann (2004), a detailed database is essentially needed for investigating the aforementioned factors that may influence the lateral wall deflections in deep excavation in clays.

Zapata (2007) compiled the expended database with case histories distinguished by soil type defined by the undrained shear strength found at the dredge level of the excavation. Table 1 lists the locations and the references of these case histories. Note that 10 case histories are presented for each soil type, giving a total of 30 case histories. For detailed information about the subsurface soil conditions, geometry characteristics, excavation support system details, raw inclinometer data, and maximum ground movements for each case history, Zapata (2007) is referred.

Table 1 Case histories of the expended database

Table 2 summarizes the geometric (H_w , H_e , and B), soil ($\gamma_{s,avg}$ and s_u), and support system ($EI/\gamma_w S_v^4$) parameters for the case histories on stiff, medium and soft clay, respectively. In addition, the maximum horizontal wall movements recorded at the end of excavation are also included.

Table 2. Summary of the excavation case histories examined (adapted from Zapata 2007)

Fig. 2 shows typically the subsurface soil conditions, geometry characteristics, excavation support system details, raw inclinometer data, and maximum ground movements for case history No. 1: Lion Yard Development in Cambridge, UK (Ng, 1992).

Fig. 2. A typical case history of case No. 1

Based on each case history listed in Table 2 and the corresponding wall deflection profiles against depth below the ground surface, the MARS model is developed. A total of 7 input

variables comprising the geometric, soil, support system and depth parameters as well as the output lateral wall deflections are listed in Table 3. Of all the wall deflection observations, 75% samples were randomly selected as the training data and the remaining 25% samples were used for testing. For illustration of the data structures, Table 4 lists some sample testing data sets.

Table 3 Summary of input variables and outputs.

Table 4 Sample testing data sets

4. MARS modeling results

Various MARS models with different number of BFs were developed for comparison. The optimal MARS model is determined based on the smallest relative error (defined as the ratio of the difference between the MARS predicted and target values divided by the target value, in percentage). Figs. 3 and 4 shows the relative error for both the location where maximum wall deflection occurs and the magnitude of the maximum wall deflection, respectively. From Figs. 3 and 4, it is obvious that the developed MARS model with 29 BFs is the most accurate.

Fig. 3. Relative error for location where maximum wall deflection occurs

Fig. 4. Relative error for magnitude of the maximum wall deflection

The training and testing predictions of δ_h at certain depth using the optimal MARS model are shown in Fig. 5. It is obvious that the MARS model has been able to learn the complicated relationship between the wall deflection and the the geometric, soil, support system and depth parameters. For a majority of the data points especially those with significant wall deflections (i.e., greater than 100 mm), the relative errors fell within $\pm 20\%$ of the target values. Fig. 6 plots the comparison between the target and the MARS predicted δ_{hm} , together with the corresponding location where δ_{hm} occurs. It can be observed that the developed MARS model is also capable of providing accurate estimations of the maximum wall deflection values, as well as the depth at which this maximum wall deflection occurs.

Fig. 5 Comparison between target and predicted δ_h .

Fig. 6 Comparison between target and predicted δ_{hm} and location.

For better illustration of the MARS modeling accuracy in wall deflection profile estimations, Fig. 7 shows the comparison of wall deflection profile between the observed and MARS predicted for most of the cases considered. It is obvious that the developed MARS modeling results fit the observed wall deflection profile well.

Fig. 7 Comparison of wall deflection profile between observed and MARS predicted

Table 5 displays the ANOVA decomposition of the developed MARS model. The first column lists the ANOVA function number. The second column gives an indication of the importance of the corresponding ANOVA function, by listing the GCV score for a model with all BFs corresponding to that particular ANOVA function removed. The third column provides the standard deviation of the function. It gives an indication of its relative importance to the overall model and can be interpreted in a manner similar to the standardized regression coefficient in a linear model. The fourth column gives the number of BFs comprising the ANOVA function. The last column gives the particular input variables associated with the ANOVA function. Fig. 8 gives the plot of the relative importance of the input variables for the MARS model, which is evaluated by the increase in the GCV value caused by removing the considered variables from the developed MARS model. The results indicate that the wall deflection at certain depth is more sensitive to the excavation geometries and the depth compared with the soil and support system parameters. In addition, Fig. 8 indicates that the wall deflection profiles are not influenced by the soil undrained shear strength s_u . However, it does not mean that the maximum wall deflection value is independent of s_u .

Table 5 ANOVA decomposition for MARS model

Fig. 8 Parameter relative importance of the developed MARS model

Table 6 lists the BFs and their corresponding equation for the developed MARS model. It is observed from Table 6 that interactions have occurred between BFs (21 of the 29 BFs are interaction terms). The presence of interactions suggests that the built MARS model is not simply additive and that interactions play a significant role in building an accurate model for lateral wall deflection. This again indicates that MARS is capable of capturing the nonlinear and complex relationships between wall deflection profile and a multitude of influential parameters with interactions among each other without making any specific assumption about the underlying

functional relationship between the input variables and the dependent response. The equation of MARS lateral wall deflection model is given by

$$\begin{aligned} \delta_h (\text{mm}) = & 155.01 + 0.303 \times \text{BF1} + 34.249 \times \text{BF2} - 27.487 \times \text{BF3} + 0.064 \times \text{BF4} + 39.826 \times \text{BF5} \\ & + 0.569 \times \text{BF6} + 10.376 \times \text{BF7} - 19.312 \times \text{BF8} - 0.329 \times \text{BF9} - 21.135 \times \text{BF10} + 132.6 \times \text{BF11} \\ & - 8.454 \times \text{BF12} - 177.65 \times \text{BF13} - 0.012 \times \text{BF14} - 25.352 \times \text{BF15} - 14.802 \times \text{BF16} - 5.977 \times \text{BF17} \\ & + 22.486 \times \text{BF18} + 10.129 \times \text{BF19} - 6.479 \times \text{BF20} - 4.318 \times \text{BF21} - 7.723 \times \text{BF22} + 1.422 \times \text{BF23} \\ & - 4.003 \times \text{BF24} + 29.485 \times \text{BF25} - 1.832 \times \text{BF26} - 6.107 \times \text{BF27} - 1.181 \times \text{BF28} - 1.54 \times \text{BF29} \end{aligned} \quad (7)$$

Table 6. Expressions of BFs for MARS model

5. Case validations

To validate the proposed MARS deflection model, two well-documented excavation case histories as listed in Table 7 were analyzed. It should be noted that for simplicity, only the maximum wall deflection of the two case histories are listed and compared in Table 7 (δ_{hm_m} denotes the measured maximum wall deflection while δ_{hm_MARS} represents the MARS model predicted maximum wall deflection). Fig. 9 shows the predicted lateral wall deflection at different depth/profiles versus the measured ones. The plot indicates that the developed MARS wall deflection profile model is able to predict reasonable well both the excavation-induced wall deflection profiles and the maximum deflection, as well as the depth at which the maximum wall deflection occurs for case histories considered.

Table 7 Summary of two excavation case histories for model validation

Fig. 9 Measured wall deflection profiles versus MARS predicted

6. Summary and conclusions

This paper adopted a non-parametric, multivariate adaptive regression spline algorithm to derive the predictive models for lateral wall deflections and deflection profiles induced by excavations in clays, based on an expanded database with different clay types, excavation geometries, soil and support system parameters. Two more well-documented excavation case histories were also used to validate the proposed MARS deflection model. Major findings obtained in this research include:

- (1) MARS is capable of capturing the nonlinear relationships involving a multitude of variables with interaction among each other without making any specific assumption about the underlying functional relationship between the input variables and the response.
- (2) The developed MARS model is easy to interpret and can thus be used for estimation of the lateral wall deflections and deflection profiles with satisfactory predictive accuracy.
- (3) MARS technique is able to provide the relative importance of the input variables.

However, it should be noted that there are still two main limitations with regard to use of this built MARS model:

- (1) This model is based on a limited number of detailed excavation case histories. The database should be further expanded to include more well-documented case histories. Consequently, it is not recommended that the model be applied for values of input parameters beyond the specific ranges in this study.
- (2) Even the information about the subsurface soil conditions, geometry characteristics, excavation support system is very detailed, nevertheless, the groundwater level change during construction is excluded in the database. Therefore, the groundwater lowering effect on wall deflection is neglected in the MARS model. The developed MARS model cannot be used for case in which there is significant groundwater drawdowns.

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Figure captions

Fig. 1 Knots and linear splines for a simple MARS example

Fig. 2 A typical case history of case No. 1: (a) subsurface soil conditions, geometry characteristics and excavation support system details, (b) raw inclinometer data and wall deflection profile (adapted from Zapata 2007).

Fig. 3 Relative error for location where maximum wall deflection occurs

Fig. 4 Relative error for magnitude of the maximum wall deflection

Fig. 5 Comparison between target and predicted δ_h : (a) training patterns, (b) testing patterns

Fig. 6 Comparison between target and predicted: (a) δ_{hm} , and (b) depth at which δ_{hm} occurs

Fig. 7 Comparison of wall deflection profile between observed and MARS predicted

Fig. 8 Parameter relative importance of the developed MARS model

Fig. 9 Measured wall deflection profiles versus MARS predicted: (a) Case 1, and (b) Case 2

Fig. 1 Knots and linear splines for a simple MARS example

$$\begin{aligned}
 \text{Wall Type} &= \text{Diaphragm} \\
 \text{Thickness (m)} &= 0.6 \\
 H \text{ (m)} &= 16.3 \\
 H_e \text{ (m)} &= 9.6 \\
 B \text{ (m)} &= 45 \\
 I \text{ (m}^4\text{/m)} &= 0.018 \\
 E \text{ (GPa)} &= 31 \\
 S_v \text{ (m)} &= 3.2 \\
 S_H \text{ (m)} &= 1.5 \\
 \delta_{H(\max)} (\delta_{v(\max)}) \text{ (mm)} &= 17.66 \text{ (10.13)} \\
 EI/\gamma_w S_v^4 &= 543.01 \\
 \gamma_{s, \text{avg}} \text{ (kN/m}^3\text{)} &= 20 \\
 s_u \text{ (kPa)} &= 120 \\
 FS \text{ (Eq. 2-26 and 2-27)} &= 4.40 \text{ and } 3.73
 \end{aligned}$$

(a)

Depth (m)	δ_H (mm)
0.00	9.04
1.04	10.30
2.50	12.04
3.50	13.73
4.48	15.01
5.50	16.26
6.48	17.02
7.47	17.67
8.47	17.64
10.01	16.18
11.51	14.76
12.49	11.73
13.51	9.62
15.03	6.24
16.57	3.57

(b)

Fig. 2 A typical case history of case No. 1: (a) subsurface soil conditions, geometry characteristics and excavation support system details, (b) raw inclinometer data and wall deflection profile (adapted from Zapata 2007).

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Table captions

Table 1 Case histories of the expended database

Table 2 Summary of the excavation case histories examined (adapted from Zapata 2007)

Table 3 Summary of input variables and outputs

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Table 1 Case histories of the expended database

Case No.	Soil type	Location	Reference
1	stiff	Lion Yard Development, Cambridge	Ng (1992)
2		New Palace Yard Park Project, London	Burland and Hancock (1977)
3		Far-East Enterprise Center Project, Taipei	Hsieh and Ou (1998)
4		Oxley Rise Development, Singapore	Poh et al. (1997)
5		Central Insurance Building, Taipei	Ou and Shiau (1998)
6		Post Office Square Garage, Boston	Whittle et al. (1993)
7		National Taiwan University Hospital, Taiwan	Liao and Hsieh (2002)
8		Taipei County Administration Center, Taiwan	Liao and Hsieh (2002)
9		75 State Street, Boston	Becker and Haley (1990)
10		Smith Tower, Houston	Ulrich (1989)
11	medium	Taipei National Enterprise Center TNEC	Ou et al. (1998)
12		Lurie Medical Building, Chicago (East Wall)	Finno and Roboski (2005)
13		Lurie Medical Building, Chicago (West Wall)	Finno and Roboski (2005)
14		Taiwan Formosa, Taipei	Hsieh and Ou (1998)
15		Tokyo Subway Excavation Project, Japan	Miyoshi (1977)
16		HDR - 4 Project for the Chicago Subway	Finno et al. (1989)
17		Oslo Subway Excavation Project	NGI (1962)
18		Embarcadero BART Zone 1, San Francisco	Clough and Buchignani (1981)
19		Metro Station South Xizan Road, Shanghai	Wang et al. (2005)
20		Open Cut in Oslo	Peck (1969)
21	soft	Chicago and State Street Excavation, Chicago	Finno et al. (2002)
22		Mass Rapid Transit Line, Singapore	Goh et al. (2003)
23		Excavation adjacent to Shanghai Metro Tunnels	Hu et al. (2003)
24		Excavation in Downtown Chicago	Gill and Lukas (1990)
25		Peninsula Hotel Project, Bangkok	Teparaksa (1993)
26		AT&T Corporate Center, Chicago	Baker et al. (1989)
27		Chicago Museum of Science & Industry Parking	Konstantakos (2000)
28		One Market Plaza Building, San Francisco	Clough and Buchignani (1981)
29		Sheet Pile Wall Field Test, Rotterdam	Kort (2002)
30		MUNI Metro Turnback Project, San Francisco	Koutsoftas et al. (2000)

Table 2 Summary of the excavation case histories examined (adapted from Zapata 2007)

Case No.	Soil type	H_w (m)	H_e (m)	B (m)	$EI / \gamma_w S_v^4$	$\gamma_{s,avg}$ (kN/m ³)	s_u (kPa)	δ_{hm} (mm)
1	stiff	16.3	9.6	45.0	543	20.0	120	17.7
2		30.0	18.5	18.5	1632	20.0	170	24.1
3		33.0	20.0	63.8	1443	19.0	76.5	124.8
4		14.0	11.1	33.0	149	20.8	80	10.0
5		23.0	11.4	33.7	186	19.7	50	44.5
6		25.6	20.2	61.0	1760	20.2	91	53.6
7		27.0	15.7	140.0	2433	20.0	77.5	81.3
8		38.0	20.0	93.0	13760	20.0	65	54.3
9		26.0	20.0	45.7	660	18.0	70	47.3
10		20.0	12.2	36.6	2748	20.1	140	14.8
11	medium	35.0	19.7	40.0	1280	18.9	50	106.5
12		16.5	10.0	68.0	20	19.0	36	43.2
13		19.0	12.8	68.0	20	19.0	36	63.5
14		31.0	18.4	35.0	1821	19.0	47.5	62.6
15		32.0	17.0	30.0	2261	19.0	42	176.6
16		19.2	12.2	12.2	420	19.0	30	172.6
17		16.0	11.0	11.0	901	17.0	30	223.6
18		30.5	21.3	21.3	2624	18.0	44	28.3
19		38.0	20.6	22.8	510	19.0	35	48.1
20		14.0	8.5	11.0	945	19.1	27.5	228.9
21	soft	18.3	12.2	22.0	376	19.1	20	38.1
22		31.0	16.0	20.0	3344	17.6	10	38.6
23		21.0	11.5	28.5	630	18.0	22	15.4
24		16.8	7.0	7.0	144	19.0	22.7	83.3
25		18.0	8.0	65.0	132	16.0	13.5	123.7
26		18.3	8.5	25.0	1697	19.0	21.5	37.4
27		13.7	10.3	85.0	547	19.0	45	3.6
28		30.5	11.0	11.0	1152	17.0	25	107.1
29		19.0	8.0	12.2	2	14.0	20	385.4
30		41.0	13.1	16.0	1491	16.5	27.5	48.1

Table 3 Summary of input variables and outputs

Inputs	Parameters	Physical meaning	Values or ranges
1	H_w (m)	Wall length	13.7 - 41.0
2	H_e (m)	Excavation depth	8.0 - 20.6
3	B (m)	Excavation width	11 - 140
4	$EI / \gamma_w S_v^4$	System stiffness	1.2 - 13760
5	$\gamma_{s,avg}$ (kN/m ³)	average unit weight of the soil	14.0 - 20.2
6	s_u (kPa)	undrained shear strength	10 - 140
7	h (m)	depth	0 - H_w
Output	δ_h (mm)	Lateral wall deflection profile	

Table 4 Sample testing data sets

H_w (m)	H_e (m)	B (m)	$EI / \gamma_w S_v^4$	$\gamma_{s,avg}$ (kN/m ³)	s_u (kPa)	h (m)	δ_h (mm)
16.3	9.6	45.0	543	20.0	120	7.5	17.7
16.3	9.6	45.0	543	20.0	120	12.5	11.7
33.0	20.0	63.8	1443	19.0	77	5.7	77.6
33.0	20.0	63.8	1443	19.0	77	9.6	101.0
33.0	20.0	63.8	1443	19.0	77	14.0	122.3
33.0	20.0	63.8	1443	19.0	77	18.6	120.1
33.0	20.0	63.8	1443	19.0	77	22.8	85.6
33.0	20.0	63.8	1443	19.0	77	28.6	22.2
23.0	11.4	33.7	186	19.7	50	2.0	22.9
23.0	11.4	33.7	186	19.7	50	6.7	36.1
23.0	11.4	33.7	186	19.7	50	10.0	44.1
23.0	11.4	33.7	186	19.7	50	13.4	40.2
23.0	11.4	33.7	186	19.7	50	16.6	28.2
23.0	11.4	33.7	186	19.7	50	20.0	15.6
25.6	20.2	61.0	1760	20.2	91	3.6	33.8
25.6	20.2	61.0	1760	20.2	91	8.8	49.0

Table 5 ANOVA decomposition for MARS model

Function No.	GCV	STD	#basis	Variable(s)
1	3533.72	60.85	4	H_w
2	1596.48	33.51	2	H_e
3	2987.19	53.96	2	B
4	1259.46	21.83	2	H_w, H_e
5	1248.76	21.81	6	$H_w, 7$
6	1902.33	25.51	5	H_e, h
7	1522.03	20.35	1	$B, EI / \gamma_w S_v^4$
8	1647.64	29.37	5	B, h
9	881.58	15.89	2	$\gamma_{s,avg}, h$

Table 6 Expressions of BF_s for MARS model

BF	Equation	BF	Equation
1	$\max(0, B - 16)$	16	$\max(0, H_w - 31) \times \max(0, h - 19.71)$
2	$\max(0, 16 - B)$	17	$\max(0, H_w - 31) \times \max(0, 19.71 - h)$
3	$BF2 \times \max(0, 7.51 - h)$	18	$BF12 \times \max(0, h - 19.71)$
4	$BF2 \times \max(0, 420.57 - EI / \gamma_w S_v^4)$	19	$BF12 \times \max(0, 19.71 - h)$
5	$\max(0, 33 - H_w)$	20	$\max(0, H_w - 33) \times \max(0, h - 19.71)$
6	$BF5 \times \max(0, H_e - 8.5)$	21	$\max(0, H_w - 33) \times \max(0, 19.71 - h)$
7	$BF5 \times \max(0, 8.5 - H_e)$	22	$BF2 \times \max(0, h - 14.25)$
8	$\max(0, h - 9.438) \times \max(0, 8.5 - H_e)$	23	$BF2 \times \max(0, 14.25 - h)$
9	$\max(0, 9.438 - h) \times \max(0, H_e - 8.5)$	24	$\max(0, H_e - 10.3)$
10	$\max(0, 9.438 - h) \times \max(0, 8.5 - H_e)$	25	$\max(0, 10.3 - H_e)$
11	$\max(0, 31 - H_w)$	26	$BF25 \times \max(0, h - 4.53)$
12	$\max(0, H_w - 32)$	27	$BF25 \times \max(0, 4.53 - h)$
13	$\max(0, 32 - H_w)$	28	$\max(0, h - 9.438) \times \max(0, \gamma_{s,avg} - 18.9)$
14	$\max(0, h - 9.438) \times \max(0, B - 12.2)$	29	$\max(0, h - 9.438) \times \max(0, 18.9 - \gamma_{s,avg})$
15	$\max(0, h - 9.438) \times \max(0, 12.2 - B)$		

Table 7 Summary of two excavation case histories for model validation

Case ID	Case name	H_w (m)	H_e (m)	B (m)	$EI / \gamma_w S_v^4$	$\gamma_{s,avg}$ (kN/m ³)	h (m)	δ_{hm_m} (mm)	δ_{hm_MARS} (mm)	References
1	TNEC	35	19.7	41	1507	18.5	0-35	107	89	Hsieh and Ou 1998
2	Shanghai CBD	35	20.0	65	1897	18.3	0-35	81	87	Lau et al. 2010