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Shared Steering Torque Control for Lane Change Assistance: a Stochastic Game-Theoretic Approach

Abstract—The challenging issue of ‘human-machine co-pilot’ opens up a new frontier to enhance driving safety. However, driver-machine conflicts and uncertain driver/external disturbances are significant problems in cooperative steering system, which degrades the system’s path-tracking ability and lowers driving safety. This paper proposes a novel stochastic game-based shared control framework to model steering torque interaction between driver and intelligent electric power steering (IEPS) system. A six-order driver-vehicle dynamic system including driver/external uncertainty is established for path-tracking. Then the affine-linear quadratic (LQ) based path-tracking problem is put forward to model the maneuvers of driver and IEPS, respectively. Particularly, the closed-loop Nash/Stackelberg shared frameworks to the affine-quadratic problem are derived through stochastic dynamic programming (SDP). Two cases of co-pilot lane change driving scenarios are studied via computer simulation. Simulation results confirm the validity of the proposed shared control scheme. The intrinsic relation between stochastic Nash and Stackelberg strategies are investigated based on the results. Finally, the steering-in-the-loop (SIL) experiment is conducted to show the potential of applying the proposed shared control framework to dealing with driver-IEPS conflicts and uncertain driver/external turbulence.

Index Terms— Shared steering control, driver-machine conflicting, affine-quadratic game, Stochastic dynamic programming.

I. INTRODUCTION

THE past three decades have witnessed enormous advances in driver assistance system (DAS) and autonomous driving [1]. The latest state-of-art has made it possible to produce affordable driverless vehicles, which frees human driver from heavy driving workload. However, during the transition from fully automated to fully manual driving, human driver is subjected to a great stress of adaptation, which raises the likelihood of accidents [2]. Therefore, to keep human driver in the control-loop while reducing his or her workload and improving his steering efficiency [3], the idea of ‘human-machine co-pilot’ is proposed as an eclectic scheme [4]. The steering co-pilot vehicle is equipped with active front steering system (AFS) [5] or IEPS. Composed of perception, decision, and electric power steering (EPS) modules, IEPS system can produce an extra steering torque to assist the driver. Driving simulation results indicated that the shared control framework led to a reduction in perceptual demands and an enhanced driving performance [6].

The prior works [7, 8] designed lateral shared control scheme by simply mapping the correcting steering angle to the

guidance torque. In order to take driver behavior into consideration, many model-based control methods were given to model the driver within shared control scheme [9-11], of which the shared model predictive control (MPC) [12, 13] and shared (linear-quadratic) LQ [14] method were most frequently used in driver-machine shared control system. These researches give a variety of model-based methods to design shared controller, but insufficient attention has been paid to co-pilot conflicts, in which the driver pursues a different target path from that by the machine. Driver-machine conflict is an inevitable problem during co-piloting, which is particularly severe in torque-overlay-based shared control scheme due to the mechanical coupling between driver and machine. In a sense, the investigation of conflicting co-pilot is in critical need. This paper aims to establish a novel stochastic game-based shared controller to reduce steering torque conflicts while ensuring the stability of the co-pilot system.

In the field of chassis control, game-based control was early used in driver’s interaction with direct yaw controller (DYC) [15]. Cole et.al applied the non-cooperative game theory to model the driver’s steering interaction with the AFS [16-18]. Reference [16] established the steering angle-overlay-based control framework to deal with conflicts in driver-machine target paths. It was found that the open-loop Stackelberg equilibrium-based MPC and LQ methods tend to achieve the same control gains. And the proposed game strategy can reasonably represent human-machine interaction during collision avoidance maneuver. However, the angle-overlay-based scheme may result in the total loss of the driver’s ultimate control and involve complex steering feel calibration [19]. Noticing the weakness of the angle overlay method, Flad et al. used the switching move method to fit the driver behavior and achieved torque-overlay-based shared steering control with open-loop Stackelberg strategy [20], but The situation where the driver pursues a different target path from that by the machine was not considered in the work. The driver’s idiosyncrasies in driving behavior also need to be studied. Enlightened by this, some model-based researches have begun to incorporate driver uncertainty [14, 21]. Still, insufficient effort has been made in coping with uncertain driver behavior during co-pilot conflicts. In this paper, a stochastic game-based method, which has been used in demand-side management for smart grid [22] and charging strategy for plug-in HEV [23], is applied to enables uncertain driver behavior to be included in the shared control framework.

This paper addresses the modeling of the driver-IEPS’s torque interaction with driver uncertainty being considered. Aiming to enhance safety and reduce driver-machine conflicts

in co-pilot, this research goes one step further in investigating stochastic driver-IEPS shared control system. And the stochastic dynamic programming (SDP) method [24] is employed to obtain the shared control strategy. The main contributions of this paper are:

1) A novel stochastic game-based control framework is provided to model the driver-machine steering interaction with the presence of uncertain driver behavior. This framework provides an improved game-based shared steering torque control methodology to handle the driver-machine conflicts.

2) The proposed strategy of IEPS features adjusting the driver's steering feel, allocating control authority flexibly and further diminishes torque conflicts during co-piloting. By modeling the uncertain driver behavior and the external disturbance, IEPS's compensation strategy also stabilizes the vehicle in the presence of uncertain driver input.

The remaining parts of this paper are arranged as follows: modeling of shared control system dynamics with driver uncertainty is elaborated in section II. Section III gives the design philosophy of stochastic game-based shared control algorithm for co-pilot path-tracking maneuver. Next, section VI introduces the computation platform of simulation and presents the simulation results. In section V the validity of the proposed shared control algorithm is proved in the SIL experiment. Finally, section VI concludes the paper.

II. MODELING OF THE SHARED CONTROL SYSTEM

Considering the cascade relation between the driver-IEPS system and the 2-DoF vehicle lateral dynamics, the six-order vehicle model can be defined as follows [20, 25]:

$$\begin{aligned} \dot{\mathbf{x}}_c &= \mathbf{A}_c \mathbf{x}_c + \mathbf{B}_h \hat{\tau}^1 + \mathbf{B}_h \tau^{1s} + \mathbf{B}_m \tau^2 + \mathbf{N}_c w \\ \mathbf{y}_c &= \mathbf{C}_c \mathbf{x}_c \end{aligned} \quad (1)$$

where $\mathbf{x} = [\theta_{sw} \dot{\theta}_{sw} v_y \dot{\psi} Y \psi]^T$, which represent steering wheel angle, angular velocity, vehicle lateral speed, yaw rate, lateral displacement, and yaw angle, respectively, and $\mathbf{y}_c = [Y \psi]^T$. Driver's steering torque consists of deterministic driver input $\hat{\tau}^1$ and uncertain driver input τ^{1s} , respectively. And the disturbance variable $w = \tau_f \text{sgn}(\dot{\theta}_{sw}) + \tau_{dis}$ where τ_f refers to friction torque of the steering system, and τ_{dis} refers to steering resistance torque. The parameters of (1) are listed in Table I, and the parameter matrices can be:

$$\begin{aligned} \mathbf{A}_c &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -\frac{B_{eq}}{J_{eq}} & 0 & 0 & 0 & 0 \\ -\frac{C_f}{mN_s} & 0 & \frac{C_f + C_r}{mN_s} & \frac{aC_f - bC_r}{mN_s} - v_x & 0 & 0 \\ \frac{aC_f}{I_z N_s} & 0 & \frac{aC_f - bC_r}{I_z N_s} & \frac{a^2 C_f + b^2 C_r}{I_z N_s} & 0 & 0 \\ \dot{\theta}_{sw} & 0 & \frac{I_z v_x}{1} & \frac{I_z v_x}{0} & 0 & v_x \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad \mathbf{C}_c = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}^T \\ \mathbf{B}_h &= \begin{bmatrix} 0 & \frac{1}{J_{eq}} & 0 & 0 & 0 & 0 \end{bmatrix}^T, \quad \mathbf{B}_m = \begin{bmatrix} 0 & \frac{N_m}{J_{eq}} & 0 & 0 & 0 & 0 \end{bmatrix}^T, \quad \mathbf{N} = \begin{bmatrix} 0 & -\frac{1}{J_{eq}} & 0 & 0 & 0 & 0 \end{bmatrix}^T. \end{aligned}$$

TABLE I
PARAMETERS OF STEERING SYSTEM AND VEHICLE SYSTEM

Steering system		2-DoF Vehicle system	
J_{eq}	0.1 kg·m ²	m	1406 kg
B_{eq}	0.8 Nm·s/rad	I_z	1802 kg·m ²
K_f	0.0559 Nm/A	C_f	-140000 N/rad
N_m	16.021	C_r	-100000 N/rad
N_s	15.8	a, b	1.016 m, 1.562 m

III. CONTROLLER DESIGN FOR THE STOCHASTIC SHARED CONTROL SYSTEM BASED ON AFFINE-QUADRATIC NON-COOPERATIVE GAME

A. Modeling of the driver-IEPS shared control framework based on LQ approach

This subsection formulates the driver and the IEPS steering behavior using discrete LQ optimal control approach. By discretizing the proposed continuous-time system (1) at a sample time of T_s , the discrete-time system used for shared controller design is obtained:

$$\mathbf{x}_{c(k+1)} = \mathbf{A}_{cd} \mathbf{x}_{ck} + \mathbf{B}_{hd} \hat{\tau}_k^1 + \mathbf{B}_{md} \tau_k^2 + \mathbf{N}_{cd} w_k + \mathbf{B}_{hd} \tau_k^{1s} \quad (2)$$

$$\mathbf{y}_{ck} = \mathbf{C}_c \mathbf{x}_{ck} \quad (3)$$

where

$$\mathbf{A}_{cd} = e^{\mathbf{A}_c T_s}, \quad \mathbf{B}_{hd} = \mathbf{B}_h \int_0^{T_s} e^{\mathbf{A}_c \tau} d\tau, \quad \mathbf{B}_{md} = \mathbf{B}_m \int_0^{T_s} e^{\mathbf{A}_c \tau} d\tau, \quad \mathbf{N}_{cd} = \mathbf{N}_c \int_0^{T_s} e^{\mathbf{A}_c \tau} d\tau.$$

To model LQ path-tracking problem, the updating of the driver/IEPS's path preview should be included. As depicted in Fig. 2, at each time step, the driver and the IEPS will remember the next n_p preview points ahead of the vehicle in their respective target paths. This dynamic process can be expressed as a shift register, and (4) represents the updating dynamics of the driver/IEPS's target path preview [26]:

$$\mathbf{R}_{x(k+1)} = \mathbf{A}_r \mathbf{R}_{xk} + \mathbf{B}_r \mathbf{r}_{xk}^{pre} \quad (4)$$

where

$$\mathbf{A}_r = \begin{bmatrix} \mathbf{0} & \mathbf{I}_{2 \times 2} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I}_{2 \times 2} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix}, \quad \mathbf{B}_r = \begin{bmatrix} \mathbf{0} \\ \vdots \\ \mathbf{0} \\ \mathbf{I}_{2 \times 2} \end{bmatrix},$$

$$\mathbf{r}_x(k+i) = [Y_x(k+i) \quad \psi_x(k+i)]^T, \quad \mathbf{r}_x^{pre} = [Y_x(k+n_p) \quad \psi_x(k+n_p)]^T,$$

$$\mathbf{R}_{xk} = [\mathbf{r}_x(k) \quad \mathbf{r}_x(k+1) \quad \cdots \quad \mathbf{r}_x(k+n_p-1)]^T, \quad x \in \{h, m\}$$

Fig. 2. Driver and IEPS's target path preview.

Then the global shared control model can be derived by incorporating equation (4) into (2):

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}_1\hat{\mathbf{r}}_k^1 + \mathbf{B}_2\boldsymbol{\tau}_k^2 + \mathbf{N}\mathbf{w}_k + \mathbf{B}_1\boldsymbol{\tau}_k^{1s} + \mathbf{F}_c\mathbf{R}_k^{pre} \quad (5)$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{cd} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_r & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_r \end{bmatrix}, \mathbf{B}_1 = \begin{bmatrix} \mathbf{B}_{hd} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} \mathbf{B}_{md} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \mathbf{N} = \begin{bmatrix} \mathbf{N}_{cd} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix},$$

$$\mathbf{F}_c = \begin{bmatrix} \mathbf{0} & \mathbf{B}_r & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_r \end{bmatrix}^T, \mathbf{x}_k = [\mathbf{x}_{ck} \quad \mathbf{R}_{hk} \quad \mathbf{R}_{mk}]^T, \mathbf{R}_k^{pre} = [\mathbf{r}_h^{pre} \quad \mathbf{r}_m^{pre}]^T.$$

In equation (5), setting $\mathbf{c}_k = \mathbf{N}\mathbf{w}_k$, and $\boldsymbol{\theta}_k = \mathbf{B}_1\boldsymbol{\tau}_k^{1s}$, which refer to driver/external uncertain disturbance, and neglecting the preview input $\mathbf{F}_c\mathbf{R}_k^{pre}$ [17], the global shared control model can be rewritten as:

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}_1\hat{\mathbf{r}}_k^1 + \mathbf{B}_2\boldsymbol{\tau}_k^2 + \mathbf{c}_k + \boldsymbol{\theta}_k \quad (6)$$

In the driver-IEPS-road global shared control system (6), for driver and IEPS, their LQ cost functions that evaluate path-tracking performance from the current time step k to the next n_u steps can be established respectively as:

$$L_1 = \frac{1}{2}\mathbf{x}_{k+n_u}^T \mathbf{Q}_1 \mathbf{x}_{k+n_u} + \frac{1}{2} \sum_{j=k}^{n_u-1} (\mathbf{x}_{k+j}^T \mathbf{Q}_1 \mathbf{x}_{k+j} + \hat{\mathbf{r}}_{k+j}^{1T} \mathbf{R}_{11} \hat{\mathbf{r}}_{k+j}^1 + \boldsymbol{\tau}_{k+j}^{2T} \mathbf{R}_{12} \boldsymbol{\tau}_{k+j}^2) \quad (7)$$

$$L_2 = \frac{1}{2}\mathbf{x}_{k+n_u}^T \mathbf{Q}_2 \mathbf{x}_{k+n_u} + \frac{1}{2} \sum_{j=k}^{n_u-1} (\mathbf{x}_{k+j}^T \mathbf{Q}_2 \mathbf{x}_{k+j} + \hat{\mathbf{r}}_{k+j}^{1T} \mathbf{R}_{21} \hat{\mathbf{r}}_{k+j}^1 + \boldsymbol{\tau}_{k+j}^{2T} \mathbf{R}_{22} \boldsymbol{\tau}_{k+j}^2) \quad (8)$$

where

$$\mathbf{Q}_1 = \mathbf{H}_1^T \mathbf{W}_1 \mathbf{H}_1, \mathbf{Q}_2 = \mathbf{H}_2^T \mathbf{W}_2 \mathbf{H}_2, ,$$

$$\mathbf{H}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & \dots & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & \dots & 0 & 0 & \dots \end{bmatrix}, \mathbf{W}_1 = \begin{bmatrix} q_Y^1 & 0 \\ 0 & q_\psi^1 \end{bmatrix},$$

$$\mathbf{H}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & \dots & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & \dots & 0 & 0 & \dots \end{bmatrix}, \mathbf{W}_2 = \begin{bmatrix} q_Y^2 & 0 \\ 0 & q_\psi^2 \end{bmatrix}.$$

$\mathbf{W}_1$ and $\mathbf{W}_2$ denote the weighting matrices of the driver and IEPS respectively, where the q_Y^1 and q_ψ^1 represent the weighting values of lateral displacement error and the yaw angle error of the driver; q_Y^2 and q_ψ^2 represent the weighting values of lateral displacement error and the yaw angle error of IEPS. $\mathbf{Q}_1$ and $\mathbf{Q}_2$ refer to the driver and IEPS's weighting matrices for the states variables of the global shared control system. $\mathbf{R}_{11}$ and $\mathbf{R}_{22}$ denote the driver and IEPS's weightings on their own steering torque input. The cross term $\mathbf{R}_{12}$ and $\mathbf{R}_{21}$ represent the driver and the IEPS's consideration about each other's steering torque input. The non-zero $\mathbf{R}_{21}$ in IEPS's cost function can adjust the driver's steering feel and decrease driver-IEPS conflicts, which will be discussed in section 5.

B. Feedback Nash equilibrium solution for the Stochastic shared control system

Equation (7) and (8) establish the cost functions of the driver-IEPS n_u stage path-tracking game. The "information non-uniqueness" can be eliminated due to the stochastic input $\boldsymbol{\theta}_k$, which gives the equilibrium solution the "feedback" feature [27]. According to the feedback information pattern, the driver and the IEPS have access to only the current value of the driver-IEPS-road global state $\mathbf{x}_k$ at every stage of the game. With this feedback information pattern, the Nash equilibrium solution can be derived by SDP, which originates from the Bellman optimal police [28]. Also note that at time step k , $\mathbf{c}_k$

remains invariant at all game stages and $\{\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \dots, \boldsymbol{\theta}_{np}\}$ is a set of statistically independent Gaussian random vectors in different game stages. The value functions for the driver and the IEPS are defined respectively as:

$$V_{k+j}^1 = \frac{1}{2}\mathbf{x}_{k+j}^T \mathbf{Z}_{k+j}^1 \mathbf{x}_{k+j} + \boldsymbol{\zeta}_{k+j}^{1T} \mathbf{x}_{k+j} + n_{k+j}^1; V_{k+n_u}^1 = \frac{1}{2}\mathbf{x}_{k+n_u}^T \mathbf{Q}_1 \mathbf{x}_{k+n_u} \quad (9)$$

$$V_{k+j}^2 = \frac{1}{2}\mathbf{x}_{k+j}^T \mathbf{Z}_{k+j}^2 \mathbf{x}_{k+j} + \boldsymbol{\zeta}_{k+j}^{2T} \mathbf{x}_{k+j} + n_{k+j}^2; V_{k+n_u}^2 = \frac{1}{2}\mathbf{x}_{k+n_u}^T \mathbf{Q}_2 \mathbf{x}_{k+n_u} \quad (10)$$

where $\mathbf{Z}_{k+j}^i$ is the quadratic matrices of the value functions, $\boldsymbol{\zeta}_{k+j}^i$ and n_{k+j}^i are produced by driver/external uncertain disturbance ($\mathbf{c}_k + \boldsymbol{\theta}_k$). The above value functions respectively represent the optimal cost functions (7) and (8) from the current step $k+j$ to step n_u . Based on SDP, the value functions (9) and (10) satisfy the following recursive relation:

$$V_{k+j}^{1N} = \min_{\hat{\mathbf{r}}_{k+j}^1 \in \mathbf{R}} E_{\boldsymbol{\theta}_{k+j}} \left[g(\mathbf{x}_{k+j}, \hat{\mathbf{r}}_{k+j}^1, \boldsymbol{\tau}_{k+j}^{2N}) + V_{k+j+1}^{1N} \right] \quad (11)$$

$$V_{k+j}^{2N} = \min_{\boldsymbol{\tau}_{k+j}^2 \in \mathbf{R}} E_{\boldsymbol{\theta}_{k+j}} \left[g(\mathbf{x}_{k+j}, \hat{\mathbf{r}}_{k+j}^{1N}, \boldsymbol{\tau}_{k+j}^2) + V_{k+j+1}^{2N} \right] \quad (12)$$

where the subscript "N" refers to Nash game, and the statistic cost function at each stage is expressed as:

$$g(\mathbf{x}_{k+j}, \hat{\mathbf{r}}_{k+j}^1, \boldsymbol{\tau}_{k+j}^2) = \frac{1}{2}(\mathbf{x}_{k+j}^T \mathbf{Q}_1 \mathbf{x}_{k+j} + \hat{\mathbf{r}}_{k+j}^{1T} \mathbf{R}_{11} \hat{\mathbf{r}}_{k+j}^1 + \boldsymbol{\tau}_{k+j}^{2T} \mathbf{R}_{12} \boldsymbol{\tau}_{k+j}^2).$$

Using state equation (6), solutions for recursive equations (9) and (10) are yielded:

$$\mathbf{R}_{11}\hat{\mathbf{r}}_{k+j}^{1N} + \mathbf{B}_1^T \mathbf{Z}_{k+j+1}^1 (\mathbf{A}_d \mathbf{x}_{k+j} + \mathbf{B}_1 \hat{\mathbf{r}}_{k+j}^{1N} + \mathbf{B}_2 \boldsymbol{\tau}_{k+j}^{2N} + \mathbf{c}_{k+j} + \mathbf{B}_1 \boldsymbol{\zeta}_{k+j}^1) + \mathbf{B}_1^T \boldsymbol{\zeta}_{k+j+1}^1 = 0 \quad (13)$$

$$\mathbf{R}_{22}\boldsymbol{\tau}_{k+j}^{2N} + \mathbf{B}_2^T \mathbf{Z}_{k+j+1}^2 (\mathbf{A}_d \mathbf{x}_{k+j} + \mathbf{B}_1 \hat{\mathbf{r}}_{k+j}^{1N} + \mathbf{B}_2 \boldsymbol{\tau}_{k+j}^{2N} + \mathbf{c}_{k+j} + \mathbf{B}_1 \boldsymbol{\zeta}_{k+j}^1) + \mathbf{B}_2^T \boldsymbol{\zeta}_{k+j+1}^2 = 0 \quad (14)$$

Corollary 1: With the feedback information pattern, the close-loop Nash equilibriums which satisfy (13) and (14) have the following structure [29]:

$$\boldsymbol{\tau}_{k+j}^{2N} = -\mathbf{P}_{k+j}^2 \mathbf{x}_{k+j} - \boldsymbol{\alpha}_{k+j}^2 \quad (15)$$

$$\hat{\mathbf{r}}_{k+j}^{1N} = -\mathbf{P}_{k+j}^1 \mathbf{x}_{k+j} - \boldsymbol{\alpha}_{k+j}^1 \quad (16)$$

Substituting (15), (16) into (13) and (14), and by requiring that the results satisfy all possible $\mathbf{x}_{k+j}$, the following equations are derived:

$$\mathbf{A}_{k+j} \bar{\mathbf{P}}_{k+j} = \mathbf{C}_{k+j}^{\alpha} \mathbf{A} \quad (17)$$

$$\mathbf{A}_{k+j} \bar{\boldsymbol{\alpha}}_{k+j} = \mathbf{B}_{k+j}^{\alpha} \bar{\boldsymbol{\zeta}}_{k+j+1} + \mathbf{C}_{k+j}^{\alpha} (\mathbf{c}_k + \mathbf{B}_1 \boldsymbol{\zeta}_k) \quad (18)$$

where

$$\bar{\boldsymbol{\alpha}}_{k+j} = [\boldsymbol{\alpha}_{k+j}^1 \quad \boldsymbol{\alpha}_{k+j}^2]^T, \bar{\mathbf{P}}_{k+j} = [\mathbf{P}_{k+j}^1 \quad \mathbf{P}_{k+j}^2]^T, \bar{\boldsymbol{\zeta}}_{k+j+1} = [\boldsymbol{\zeta}_{k+j+1}^1 \quad \boldsymbol{\zeta}_{k+j+1}^2]^T$$

$$\mathbf{A}_{k+j} = \begin{bmatrix} \mathbf{R}_{11} + \mathbf{B}_1^T \mathbf{Z}_{k+j+1}^1 \mathbf{B}_1 & \mathbf{B}_1^T \mathbf{Z}_{k+j+1}^1 \mathbf{B}_2 \\ \mathbf{B}_2^T \mathbf{Z}_{k+j+1}^2 \mathbf{B}_1 & \mathbf{R}_{22} + \mathbf{B}_2^T \mathbf{Z}_{k+j+1}^2 \mathbf{B}_2 \end{bmatrix},$$

$$\mathbf{B}_{k+j}^{\alpha} = \text{diag}(\mathbf{B}_1^T, \mathbf{B}_2^T), \mathbf{C}_{k+j}^{\alpha} = [\mathbf{B}_1^T \mathbf{Z}_{k+j+1}^1 \quad \mathbf{B}_2^T \mathbf{Z}_{k+j+1}^2]^T.$$

The Nash equilibrium solution at stage j can be formed by (17) and (18), and the solution at the first stage is taken as the control strategy for the co-pilot system at time step k . By substituting (9), (10) into the left side of (11), (12), and (15), (16) into the right side of (11), (12), the so-called discrete Nash Riccati equation [15] can be derived:

$$\begin{bmatrix} \mathbf{Z}_{k+j}^1 \\ \mathbf{Z}_{k+j}^2 \end{bmatrix} = \begin{bmatrix} \mathbf{P}_{k+j}^{1T} \mathbf{R}_{11} & \mathbf{P}_{k+j}^{2T} \mathbf{R}_{12} \\ \mathbf{P}_{k+j}^{1T} \mathbf{R}_{21} & \mathbf{P}_{k+j}^{2T} \mathbf{R}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{P}_{k+j}^1 \\ \mathbf{P}_{k+j}^2 \end{bmatrix} + \begin{bmatrix} \mathbf{F}_{k+j}^T \mathbf{Z}_{k+j+1}^1 \mathbf{F}_k + \mathbf{Q}_1 \\ \mathbf{F}_{k+j}^T \mathbf{Z}_{k+j+1}^2 \mathbf{F}_k + \mathbf{Q}_2 \end{bmatrix} \quad (19)$$

$$\bar{\zeta}_{k+j} = \mathbf{F}_{k+j}^{\zeta} \bar{\zeta}_{k+j+1} + \mathbf{B}_{k+j}^{\zeta} \bar{a}_{k+j} + \mathbf{C}_{k+j}^{\zeta} (\mathbf{c}_k + \mathbf{B}_1 \xi) \quad (20)$$

where

$$\begin{aligned} \bar{\zeta}_{k+n_u} &= [\mathbf{0} \quad \mathbf{0}]^T, \quad \mathbf{Z}_{k+n_u}^1 = \mathbf{Z}_{k+n_u}^2 = \mathbf{0}, \quad \mathbf{F}_{k+j} = \mathbf{A} - \mathbf{B}_1 \mathbf{P}_{k+j}^1 - \mathbf{B}_2 \mathbf{P}_{k+j}^2, \\ \mathbf{F}_{k+j}^{\zeta} &= \text{diag}(\mathbf{F}_{k+j}^T, \mathbf{F}_{k+j}^T), \quad \mathbf{C}_{k+j}^{\zeta} = [\mathbf{F}_{k+j}^T \mathbf{Z}_{k+j+1}^1 \quad \mathbf{F}_{k+j}^T \mathbf{Z}_{k+j+1}^2]^T, \\ \mathbf{B}_{k+j}^{\zeta} &= \begin{bmatrix} \mathbf{P}_{k+j}^{1T} \mathbf{R}_{11} - \mathbf{F}_{k+j}^T \mathbf{Z}_{k+j+1}^1 \mathbf{B}_1 & \mathbf{P}_{k+j}^{2T} \mathbf{R}_{12} - \mathbf{F}_{k+j}^T \mathbf{Z}_{k+j+1}^2 \mathbf{B}_2 \\ \mathbf{P}_{k+j}^{1T} \mathbf{R}_{21} - \mathbf{F}_{k+j}^T \mathbf{Z}_{k+j+1}^1 \mathbf{B}_1 & \mathbf{P}_{k+j}^{2T} \mathbf{R}_{22} - \mathbf{F}_{k+j}^T \mathbf{Z}_{k+j+1}^2 \mathbf{B}_2 \end{bmatrix}. \end{aligned}$$

Then the feedback terms $\mathbf{P}_k^1, \mathbf{P}_k^2$ of the Nash equilibrium solution (15), (16) at the first stage of the game is derived as:

$$\bar{\mathbf{P}}_k = \mathbf{A}_k^{-1} \mathbf{C}_k^{\alpha} \mathbf{A} \quad (21)$$

where $\mathbf{A}_k, \mathbf{C}_k^{\alpha}$ can be calculated via equation (17). For the zeroth order terms α_k^1, α_k^2 , it is difficult to derive their expression via (18) directly due to driver uncertainty and external disturbance $(\mathbf{c}_k + \mathbf{B}_1 \xi)$. To find the affine relationship between $(\mathbf{c}_k + \mathbf{B}_1 \xi)$ and $\bar{\zeta}_{k+j}$, we substitute equation (18) into (20) to eliminate the zeroth order terms α_k^1, α_k^2 :

$$\bar{\zeta}_{k+j} = (\mathbf{F}_{k+j}^{\zeta} + \mathbf{B}_{k+j}^{\zeta} \mathbf{A}_{k+j}^{-1} \mathbf{B}_{k+j}^{\alpha}) \bar{\zeta}_{k+j+1} + (\mathbf{B}_{k+j}^{\zeta} \mathbf{A}_{k+j}^{-1} \mathbf{C}_{k+j}^{\alpha} + \mathbf{C}_{k+j}^{\zeta}) (\mathbf{c}_k + \mathbf{B}_1 \xi) \quad (22)$$

at n_u-1 stage, we have:

$$\bar{\zeta}_{k+n_u-1} = (\mathbf{B}_{k+n_u-1}^{\zeta} \mathbf{A}_{k+n_u-1}^{-1} \mathbf{C}_{k+n_u-1}^{\alpha} + \mathbf{C}_{k+n_u-1}^{\zeta}) (\mathbf{c}_k + \mathbf{B}_1 \xi) \quad (23)$$

By substituting the initial value (23) into the recursive formula (21), the following relation holds:

$$\Psi_{k+j}^N = (\mathbf{F}_{k+j}^{\zeta} + \mathbf{B}_{k+j}^{\zeta} \mathbf{A}_{k+j}^{-1} \mathbf{B}_{k+j}^{\alpha}) \Psi_{k+j+1}^N + (\mathbf{B}_{k+j}^{\zeta} \mathbf{A}_{k+j}^{-1} \mathbf{C}_{k+j}^{\alpha} + \mathbf{C}_{k+j}^{\zeta}) \quad (24)$$

$$\bar{\zeta}_{k+j} = \Psi_{k+j} (\mathbf{c}_k + \mathbf{B}_1 \xi), \quad j \in [0, n_u - 1] \quad (25)$$

where

$$\Psi_{k+n_u-1}^N = \mathbf{B}_{k+n_u-1}^{\zeta} \mathbf{A}_{k+n_u-1}^{-1} \mathbf{C}_{k+n_u-1}^{\alpha} + \mathbf{C}_{k+n_u-1}^{\zeta}.$$

With equation (25) and (18), the zeroth order terms α_k^1, α_k^2 of the Nash equilibrium solution can be expressed as:

$$\bar{a}_k = \mathbf{A}_k^{-1} (\mathbf{B}_k^{\alpha} \Psi_{k+1}^N + \mathbf{C}_k^{\alpha}) (\mathbf{c}_k + \mathbf{B}_1 \xi) \quad (26)$$

Finally, the control strategy based on Nash equilibrium at time step k can be expressed using (21) and (26):

$$\hat{\tau}_{k+1}^1 = -\mathbf{P}_k^1 \mathbf{x}_k - \alpha_k^1 \quad (27)$$

$$\hat{\tau}_{k+1}^2 = -\mathbf{P}_k^2 \mathbf{x}_k - \alpha_k^2 \quad (28)$$

C. Feedback Stackelberg equilibrium solution for the stochastic shared control system

In a Stackelberg game, a leader-follower relationship exists between driver and the IEPS. In each stage of the game, the driver, moves first, and IEPS the follower observes this move and moves next. Hence, the Stackelberg equilibrium solution is in general achieved by backward induction [30]. Based on SDP, the value functions of driver and IEPS $V_{k+j}^{1S}, V_{k+j}^{2S}$ are identical to (9) and (10), respectively. A pair of feedback strategies $\{\tau_{k+j}^1 \in \Gamma_{k+j}^1, \tau_{k+j}^2 \in \Gamma_{k+j}^2(\tau_{k+j}^1)\}$ would constitute a feedback Stackelberg equilibrium solution for a stochastic game with driver being the leader if the following relation is established:

$$V_{k+j}^{1S} = \min_{\hat{\tau}_{k+j}^1 \in \mathbf{R}, \tau_{k+j}^{2S} \in \Gamma^2(\hat{\tau}_{k+j}^1)} E_{\theta_{k+j}} \left[g(\mathbf{x}_{k+j}, \hat{\tau}_{k+j}^1, \tau_{k+j}^{2S}) + V_{k+j+1}^{1S} \right] \quad (29)$$

where the subscript ‘‘S’’ refers to Stackelberg game, and Γ^2 is a singleton set including the steering torque strategy for the IEPS:

$$\Gamma^2 = \left\{ \tau_{k+j}^{2S}(\tau_{k+j}^1) \mid V_{k+j}^{2S} = \min_{\tau_{k+j}^2 \in \mathbf{R}} E_{\theta_{k+j}} \left[g(\mathbf{x}_{k+j}, \hat{\tau}_{k+j}^1, \tau_{k+j}^2) + V_{k+j+1}^{2S} \right] \right\} \quad (30)$$

By solving the recursive equation in (30) with the help of equations (6) and (10), the Stackelberg equilibrium solution for the IEPS can be formulated as:

$$\begin{aligned} \tau_{k+j}^{2S} &= -(\mathbf{R}_{22} + \mathbf{B}_2^T \mathbf{Z}_{k+j+1}^2 \mathbf{B}_2)^{-1} \mathbf{B}_2^T \times \\ &\quad \left[\mathbf{Z}_{k+j+1}^2 (\mathbf{B}_1 \hat{\tau}_{k+j}^1 + \mathbf{A} \mathbf{x}_{k+j} + \mathbf{c}_{k+j} + \mathbf{B}_1 \xi) + \zeta_{k+j+1}^2 \right] \end{aligned} \quad (31)$$

By substituting equations (6), (9) and (31) into (29), the Stackelberg equilibrium solution of the recursive equation (29) for the driver satisfies the following relation:

$$\begin{aligned} \mathbf{A}_L \hat{\tau}_{k+j}^{1S} &+ \left[\mathbf{A}_{S4} \mathbf{Z}_{k+j+1}^2 + \mathbf{A}_{S3}^T \mathbf{Z}_{k+j+1}^1 (\mathbf{I} - \mathbf{B}_2 \mathbf{A}_{S2} \mathbf{Z}_{k+j+1}^2) \right] \mathbf{A} \mathbf{x}_{k+j} \\ &+ (\mathbf{A}_{S4} \mathbf{Z}_{k+j+1}^2 - \mathbf{A}_{S3}^T \mathbf{Z}_{k+j+1}^1 \mathbf{B}_2 \mathbf{A}_{S2} \mathbf{Z}_{k+j+1}^2 + \mathbf{A}_{S3}^T \mathbf{Z}_{k+j+1}^1) \\ &\times (\mathbf{c}_k + \mathbf{B}_1 \xi) + \mathbf{A}_{S3}^T \zeta_{k+j+1}^1 + (\mathbf{A}_{S4} - \mathbf{A}_{S3}^T \mathbf{Z}_{k+j+1}^1 \mathbf{B}_2 \mathbf{A}_{S2}) \zeta_{k+j+1}^2 = 0 \end{aligned} \quad (32)$$

where

$$\begin{aligned} \mathbf{A}_{S2} &= (\mathbf{R}_{22} + \mathbf{B}_2^T \mathbf{Z}_{k+j+1}^2 \mathbf{B}_2)^{-1} \mathbf{B}_2^T, \quad \mathbf{A}_{S3} = \mathbf{B}_1 - \mathbf{B}_2 \mathbf{A}_{S2} \mathbf{Z}_{k+j+1}^2 \mathbf{B}_1, \\ \mathbf{A}_{S4} &= \mathbf{B}_1^T \mathbf{Z}_{k+j+1}^2 \mathbf{A}_{S2}^T \mathbf{R}_{12} \mathbf{A}_{S2}, \quad \mathbf{A}_L = \mathbf{R}_{11} + \mathbf{A}_{S4} \mathbf{Z}_{k+j+1}^2 \mathbf{B}_1 + \mathbf{A}_{S3}^T \mathbf{Z}_{k+j+1}^1 \mathbf{A}_{S3}. \end{aligned}$$

Corollary 2: With the feedback information pattern, the control laws of the proposed affine-quadratic Stackelberg game have the following structure, which is similar to those of a Nash game:

$$\hat{\tau}_{k+j}^{1S} = -\mathbf{S}_{k+j}^1 \mathbf{x}_{k+j} - s_{k+j}^1 \quad (33)$$

$$\tau_{k+j}^{2S} = -\mathbf{S}_{k+j}^2 \mathbf{x}_{k+j} - s_{k+j}^2 \quad (34)$$

Substituting (33), (34) into (31) and (32), and by setting the results to satisfy all possible $\mathbf{x}_{k+j}$, the following equations are derived:

$$\mathbf{Q}_{k+j} \bar{\mathbf{s}}_{k+j} = \mathbf{C}_{k+j}^s \mathbf{A} \quad (35)$$

$$\mathbf{Q}_{k+j} \bar{\mathbf{s}}_{k+j} = \mathbf{B}_{k+j}^s \bar{\zeta}_{k+j+1} + \mathbf{C}_{k+j}^s (\mathbf{c}_k + \mathbf{B}_1 \xi) \quad (36)$$

where

$$\begin{aligned} \bar{\mathbf{s}}_{k+j} &= [\mathbf{S}_{k+j}^1 \quad \mathbf{S}_{k+j}^2]^T, \quad \bar{\mathbf{s}}_{k+j} = [s_{k+j}^1 \quad s_{k+j}^2]^T \\ \mathbf{Q}_{k+j} &= \begin{bmatrix} \mathbf{A}_L & \mathbf{0} \\ \mathbf{A}_{S2} \mathbf{Z}_{k+j+1}^2 \mathbf{B}_1 & \mathbf{I} \end{bmatrix}, \quad \mathbf{B}_{k+j}^s = \begin{bmatrix} \mathbf{A}_{S3}^T & \mathbf{A}_{S4} - \mathbf{A}_{S3}^T \mathbf{Z}_{k+j+1}^1 \mathbf{B}_2 \mathbf{A}_{S2} \\ \mathbf{0} & \mathbf{A}_{S2} \end{bmatrix}, \\ \mathbf{C}_{k+j}^s &= \begin{bmatrix} \mathbf{A}_{S4} \mathbf{Z}_{k+j+1}^2 + \mathbf{A}_{S3}^T \mathbf{Z}_{k+j+1}^1 (\mathbf{I} - \mathbf{B}_2 \mathbf{A}_{S2} \mathbf{Z}_{k+j+1}^2) \\ \mathbf{A}_{S2} \mathbf{Z}_{k+j+1}^2 \end{bmatrix}. \end{aligned}$$

Similar to the Nash equilibrium, the control strategy at the first stage of the stochastic game can be obtained by the following recursive formula:

$$\begin{bmatrix} \mathbf{Z}_{k+j}^1 \\ \mathbf{Z}_{k+j}^2 \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{k+j}^{1T} \mathbf{R}_{11} & \mathbf{S}_{k+j}^{2T} \mathbf{R}_{12} \\ \mathbf{S}_{k+j}^{1T} \mathbf{R}_{21} & \mathbf{S}_{k+j}^{2T} \mathbf{R}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{S}_{k+j}^1 \\ \mathbf{S}_{k+j}^2 \end{bmatrix} + \begin{bmatrix} \mathbf{G}_{k+j}^T \mathbf{Z}_{k+j+1}^1 \mathbf{G}_k + \mathbf{Q}_1 \\ \mathbf{G}_{k+j}^T \mathbf{Z}_{k+j+1}^2 \mathbf{G}_k + \mathbf{Q}_2 \end{bmatrix} \quad (37)$$

$$\bar{\zeta}_{k+j} = \mathbf{G}_{k+j}^{\zeta} \bar{\zeta}_{k+j+1} + \mathbf{M}_{k+j}^{\zeta} \bar{\mathbf{s}}_{k+j} + \mathbf{N}_{k+j}^{\zeta} (\mathbf{c}_k + \mathbf{B}_1 \xi) \quad (38)$$

where

$$\begin{aligned} \mathbf{G}_{k+j} &= \mathbf{A} - \mathbf{B}_1 \mathbf{S}_{k+j}^1 - \mathbf{B}_2 \mathbf{S}_{k+j}^2, \quad \mathbf{Z}_{k+n_u}^1 = \mathbf{Z}_{k+n_u}^2 = \mathbf{0}, \quad \tilde{\boldsymbol{\zeta}}_{k+n_u} = [\mathbf{0} \quad \mathbf{0}]^T, \\ \mathbf{M}_{k+j}^\zeta &= \begin{bmatrix} \mathbf{S}_{k+j}^{1T} \mathbf{R}_{11} - \mathbf{G}_{k+j}^T \mathbf{Z}_{k+j+1}^1 \mathbf{B}_1 & \mathbf{S}_{k+j}^{2T} \mathbf{R}_{12} - \mathbf{G}_{k+j}^T \mathbf{Z}_{k+j+1}^1 \mathbf{B}_2 \\ \mathbf{S}_{k+j}^{1T} \mathbf{R}_{21} - \mathbf{G}_{k+j}^T \mathbf{Z}_{k+j+1}^2 \mathbf{B}_1 & \mathbf{S}_{k+j}^{2T} \mathbf{R}_{22} - \mathbf{G}_{k+j}^T \mathbf{Z}_{k+j+1}^2 \mathbf{B}_2 \end{bmatrix}, \\ \mathbf{G}_{k+j}^\zeta &= \begin{bmatrix} \mathbf{G}_{k+j}^T & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_{k+j}^T \end{bmatrix}, \quad \mathbf{N}_{k+j}^\zeta = \begin{bmatrix} \mathbf{G}_{k+j}^T \mathbf{Z}_{k+j+1}^1 \\ \mathbf{G}_{k+j}^T \mathbf{Z}_{k+j+1}^2 \end{bmatrix}, \end{aligned}$$

With the recursive formula (37), the feedback terms in the Stackelberg equilibrium solution (31), (34) at the first stage of the game can be derived as:

$$\bar{\mathbf{s}}_k = \mathbf{Q}_k^{-1} \mathbf{C}_k^s \mathbf{A} \quad (39)$$

Similarly, the zeroth order terms s_k^1, s_k^2 of the Stackelberg equilibrium solution at the first stage can be expressed as:

$$\bar{\mathbf{s}}_k = \mathbf{Q}_k^{-1} (\mathbf{B}_k^s \boldsymbol{\Psi}_{k+1}^s + \mathbf{C}_k^s) (\mathbf{c}_k + \mathbf{B}_1 \boldsymbol{\zeta}) \quad (40)$$

where the $\boldsymbol{\Psi}_{k+1}^s$ can be derived recursively by (39) with the initial condition (42):

$$\boldsymbol{\Psi}_{k+j}^s = (\mathbf{G}_{k+j}^\zeta + \mathbf{M}_{k+j}^\zeta \mathbf{Q}_{k+j}^{-1} \mathbf{B}_{k+j}^s) \boldsymbol{\Psi}_{k+j+1}^s + (\mathbf{M}_{k+j}^\zeta \mathbf{Q}_{k+j}^{-1} \mathbf{C}_{k+j}^s + \mathbf{N}_{k+j}^\zeta) \quad (41)$$

$$\boldsymbol{\Psi}_{k+n_u-1}^s = \mathbf{M}_{k+n_u-1}^\zeta \mathbf{Q}_{k+n_u-1}^{-1} \mathbf{C}_{k+n_u-1}^s + \mathbf{N}_{k+n_u-1}^\zeta \quad (42)$$

In summary, the control strategy based on Stackelberg equilibrium at time step k can be expressed using (39) and (40):

$$\hat{\tau}_k^{1S} = -\mathbf{S}_k^1 \mathbf{x}_k - s_k^1 \quad (43)$$

$$\hat{\tau}_k^{2S} = -\mathbf{S}_k^2 \mathbf{x}_k - s_k^2 \quad (44)$$

The control strategies (43) and (44) are adopted by the driver and IEPS respectively. Table II shows the basic structure of the stochastic Nash and Stackelberg algorithm.

TABLE II

BASIC STRUCTURE OF THE STOCHASTIC NASH AND STACKELBERG ALGORITHM

	Nash	Stackelberg
Initialization:	$j=n_u-1, \mathbf{Z}_{k+n_u}^1=\mathbf{Z}_{k+n_u}^2=\mathbf{0}, \tilde{\boldsymbol{\zeta}}_{k+n_u}=\tilde{\boldsymbol{\zeta}}_{k+n_u}=\mathbf{0}$	
1	while $j \geq 0$ do	while $j \geq 0$ do
2	Compute $\mathbf{A}_k$	Compute $\mathbf{A}_{S2}, \mathbf{A}_{S3}, \mathbf{A}_{S4}, \mathbf{A}_L$
3	Compute $\mathbf{P}_{k+j}^1, \mathbf{P}_{k+j}^2$	Compute $\mathbf{S}_{k+j}^1, \mathbf{S}_{k+j}^2$
4	Compute $\mathbf{B}_{k+j}^s, \mathbf{C}_{k+j}^s$	Compute $\mathbf{B}_{k+j}^s, \mathbf{C}_{k+j}^s, \mathbf{Q}_{k+j}$
5	Compute $\mathbf{F}_{k+j}, \mathbf{F}_{k+j}^\zeta, \mathbf{B}_{k+j}^\zeta, \mathbf{C}_{k+j}^\zeta$	Compute $\mathbf{G}_{k+j}, \mathbf{G}_{k+j}^\zeta, \mathbf{M}_{k+j}^\zeta, \mathbf{N}_{k+j}^\zeta$
6	If $j < n_u - 1$	If $j < n_u - 1$
7	Iterate $\boldsymbol{\Psi}_{k+j}^N$	Iterate $\boldsymbol{\Psi}_{k+j}^S$
8	If $j=1$	If $j=1$
9	$\boldsymbol{\Psi}_{k+j+1}^N = \boldsymbol{\Psi}_{k+j}^N$	$\boldsymbol{\Psi}_{k+j+1}^S = \boldsymbol{\Psi}_{k+j}^S$
10	end	end
11	end	end
12	Iterate $\mathbf{Z}_{k+j}^1, \mathbf{Z}_{k+j}^2, j=j-1$	Iterate $\mathbf{Z}_{k+j}^1, \mathbf{Z}_{k+j}^2, j=j-1$
14	end	end
15	Obtain $\mathbf{P}_k^1 = \mathbf{P}_{k+j}^1, \mathbf{P}_k^2 = \mathbf{P}_{k+j}^2 (j=0)$	Obtain $\mathbf{S}_k^1 = \mathbf{S}_{k+j}^1, \mathbf{S}_k^2 = \mathbf{S}_{k+j}^2 (j=0)$
16	$\bar{\mathbf{a}}_k = \mathbf{A}_k^{-1} (\mathbf{B}_k^s \boldsymbol{\Psi}_{k+1}^N + \mathbf{C}_k^s) (\mathbf{c}_k + \mathbf{B}_1 \boldsymbol{\zeta})$	$\bar{\mathbf{s}}_k = \mathbf{Q}_k^{-1} (\mathbf{B}_k^s \boldsymbol{\Psi}_{k+1}^S + \mathbf{C}_k^s) (\mathbf{c}_k + \mathbf{B}_1 \boldsymbol{\zeta})$

IV. NUMERICAL SIMULATION AND RESULTS ANALYSIS

A. Simulation configuration and driving scenario

The proposed stochastic game-based shared control algorithm is executed in Simulink to visually demonstrate the driver-IEPS steering torque interaction under the consistent and conflicting circumstances. And the proposed algorithm is encoded using Matlab S-Function to run the shared path-tracking simulation in a 3-DoF nonlinear vehicle model, which includes lateral, yaw and roll motion. Two driving conditions are designed for investigation:

Case 1: Cooperative double lane change (DLC) maneuver. This case simulates an overtaking process to validate the cooperative lane keeping performance of the shared control system. In this case, the driver and the IEPS have the same target. The target paths are described in solid lines in Fig. 3.

Case 2: Conflicting single lane change (SLC) maneuver. This case simulates a conflicting situation, in which the IEPS controller makes an SLC maneuver, while the driver keeps driving ahead. This condition is to study the steering torque interaction under conflicting driving targets. The target paths of the driver and IEPS are depicted in dashed lines in Fig. 3.

Fig. 3. Driver-IEPS target path, where x-axis denotes the longitudinal displacement and y-axis denotes the lateral displacement of the roads.

TABLE III
SIMULATION PARAMETERS

Case 1: Identical target path		Case 2: Conflicting target path	
v_x	20 m/s	v_x	20 m/s
n_p	100	n_p	100
n_c	100	n_c	100
q_y^1, q_y^2	20, 20	q_y^1, q_y^2	6, 6
q_v^1, q_v^2	100, 100	q_v^1, q_v^2	1200, 1200
q_u^1, q_u^2	0.04	q_u^1, q_u^2	1, 1

The proposed control algorithms based on feedback (FB) Nash and Stackelberg equilibrium are implemented in the above two driving cases and the simulation time frequency is set to be 100 Hz. Corresponding simulation parameters are shown in Table III.

B. Analysis of simulation results

The path following performance and the driver-IEPS torque interaction results are shown in Fig. 4 and Fig. 5. Here, it should be noted that the IEPS steering torque shown in the following figures is the equivalent torque acted on the steering wheel. To better reveal the internal relation between the FB Nash and the

(a) Case 1: Cooperative DLC (b) Case 2: Conflicting SLC
Fig. 4. Comparison of the tracking performances and torque interactions between OL and FB Nash strategies.

(a) Case 1: Cooperative DLC (b) Case 2: Conflicting SLC
Fig. 5. Comparison of the tracking performances and torque interactions between OL and FB Nash strategies.

Fig. 6. Statistic results of the RMS driver-IEPS torque interaction given by OL and FB Nash/Stackelberg strategies. (a) Comparison between Nash and Stackelberg strategies in case 1, where the ordinate represents driver's RMS input. (b) Comparison between Nash and Stackelberg strategies in case 2, where the ordinate represents the RMS steering input difference between driver and IEPS.

FB Stackelberg strategies, simulation results of the proposed FB algorithms and those under the open-loop (OL) Nash/Stackelberg strategies [18] are also shown in Fig. 4 and Fig. 5. In Fig. 4(a), it can be seen that all the schemes give similar steering torque interaction results. But in Fig. 4(b), the FB Nash algorithms produce much smaller driver-IEPS steering torque interaction than that produced by the OL algorithms. This means that within FB framework, the conflicting between both agents can be effectively mitigated.

The simulation results by FB Stackelberg algorithms and the OL Stackelberg algorithm are shown in Fig. 5. When the driver and the IEPS hold conflicting target paths, as shown in Fig. 5(b), little difference between the driver/IEPS interaction torque with FB or OL method can be noticed.

The Root mean square (RMS) input of the FB game-based algorithm with different simulating frequencies and the 100Hz OL game-based strategy are given in Fig. 6. The statistic results of RMS driver input in case 1 (see Fig. 6(a)) indicates that all Nash strategies lead to identical driver steering input. The proposed FB Nash and Stackelberg equilibrium solutions gradually approach each other with the increase of simulation frequency, and the same rule can be found in Fig. 6(b). In fact, when the simulation frequency approaches infinity, the FB Nash and Stackelberg equilibrium solutions become identical, which can be strictly proved in [29]. Therefore, the proposed FB Stackelberg game serves as the shared control scheme of the experiment in the next section.

I. STEERING-IN-THE-LOOP EXPERIMENT VALIDATION

In this section, the proposed FB Stackelberg strategy is implemented in a steering-in-the-loop (SIL) test bench to investigate the real-time performance of the shared controller.

Fig. 7 shows the testing facilities, which mainly consists of a DS1501 Micro Autobox from dSPACE, an Allegro A3941 motor driver, a steering assembly from an A-class production car, a Panasonic MDME202GCG AC-servo motor cylinder for steering resistance loading, a BOSCH steering angular sensor and a PCAN for data acquisition.

Fig. 7. Basic facilities of the SIL test bench.

Fig. 9. Path tracking performances and the driver/IEPS torque interactions results with zero and non-zero cross term. (a) Path tracking performance, driver/IEPS torque input and total steering motor current with $R_{21}=0$ and $R_{21}=1$ in case 1. (b) Path tracking performance, driver/IEPS torque input and total steering motor current with $R_{21}=0$ and $R_{21}=1$ in case 2.

And a 3-DoF nonlinear vehicle model and the road information are encoded into the Micro Autobox to simulate the lane change situation. The shared controller, which is also implemented in the Micro Autobox, receives the vehicle model states and calculates the steering torque command of driver and IEPS, respectively. Then the sum torque command is sent to a proportional-integral (PI) motor current controller to operate the steering assembly, and the angular sensor feeds steering column angle and angular speed back to the nonlinear vehicle model to run the SIL.

Due to the uncertain friction of the SIL system, steering resistance torque cannot be obtained directly. A Luenberger disturbance observer, which was used in our previous work [25] is built to obtain this torque. The poles of the observer p_1 , p_2 and p_3 are assigned to be -90 , $-20+30i$ and $-20+30i$.

A. Co-pilot with a certain driver steering input

Note that the observer enables the proposed algorithm to operate in real-life, and the proposed Stackelberg game strategies (43) and (44) are adopted to conduct the SIL experiment with the same parameters as those used in section 4. The FB Stackelberg strategies with different cross term R_{21} , which was eliminated in the OL game strategy, are investigated in the absence of uncertain driver input. Fig. 9(a) and Fig. 9(b) show the results of the cooperative DLC and conflicting SLC maneuver cases, as described in Section 4. In Fig. 9(a), it is found that the increase of the cross term R_{21} of the IEPS controller helps driver to save more steering effort. And the steering motor current represents the sum of the driver-IEPS interaction torque, which powers the steering assembly and operates the vehicle to track the target path.

In Fig. 9(b), the existence of non-zero R_{21} , which represents the IEPS's consideration for driver's steering input makes the driver-IEPS torque conflicts decrease significantly, which indicates that the proposed algorithm relieves torque conflicts

effectively by adjusting the cross term R_{21} of the IEPS controller.

Fig. 10 exhibits the RMS values of the steering interaction torque with different cross term. On cooperative DLC in Fig. 10(a), it is found that the driver steering torque varies with different R_{21} , which means the proposed control strategy can tune the driver's steering feel and regulate the control authorities of both agents continuously by adjust R_{21} . During conflicting situation in Fig. 10(b), it indicates that the cross term R_{21} can decrease the conflicting interacting torque remarkably.

B. Co-pilot with a Stochastic driver steering input

The driver's uncertain behavior during co-piloting is an important issue pending for further discussed [14, 21]. With the presence of the stochastic driver input, the presented algorithm is discussed in DLC and SLC conditions in the SIL system. To illustrate the role of the compensation strategy in the Stackelberg game-based method, two kinds of control strategies for the IEPS control algorithms are adopted:

1) State-feedback-based strategy (no compensation strategy). In order to eliminate some information that is required in the proposed strategies in theory but cannot be measured directly by sensors, the zeroth-order term in (44) is neglected. Consequently, the plant state is the only information that IEPS controller needs.

2) Compensation-based FB strategy as described in (44). Besides state information, the external disturbance is also required by the IEPS, which is obtained by the disturbance observer.

During cooperative DLC maneuver, a random steering torque input $\tau^{ls} \sim N(0,1)$ is added to the driver input to simulate the driver uncertainty. The path tracking performance is shown in Fig. 11(a); if the IEPS uses the observer information and takes the compensation scheme, the tracking performance of

Fig. 10. Statistic results of the RMS driver-IEPS torque interaction with a different cross term. (a) RMS steering torque input of driver, IEPS and the difference RMS steering torque between driver and IEPS with different R_{21} in case 1. (b) RMS steering torque input of driver, IEPS and the difference RMS steering torque between driver and IEPS with different R_{21} in case 2.

(a) Case 1: Cooperative DLC maneuver

(b) Case2: Conflicting SLC maneuver

Fig. 11. Path tracking performances with different control strategies. (a) Path tracking performance comparison between state feedback-based strategy (no compensation strategy) and compensation based FB strategy in case 1. (b) Path tracking performance comparison between state feedback-based strategy (no compensation strategy) and compensation-based FB strategy in case 2.

the co-pilot system can be improved evidently and immune to the stochastic driver input. During conflicting SLC maneuver, while the driver's input is interfered by IEPS, the driver may exert extra force to resist the interference[31], so a random steering torque input $\tau^{ls} \sim \mathcal{N}(-3, 3)$ is added into the driver input to simulate the circumstance. Fig. 11(b) shows that, without compensation strategy, the IEPS's tracking performance deteriorates gravely and the co-pilot system cannot stabilize the vehicle.

On condition that the compensation strategy is adopted, the driver-IEPS torque interaction and the external disturbance results observed are shown in Fig. 12. Note that the cross term R_{21} has been set to be zero in this subsection, the torque interaction results are similar to those in Fig. 9. The driver-IEPS interaction gradually reaches a steady-state, despite driver uncertainty and external disturbance. In Fig. 12(b), the proposed compensation strategy enables the IEPS to use /exert extra steering torque to compensate the non-zero mean steering input of the driver and stabilize the vehicle.

II. CONCLUSION

This paper provides a novel feedback stochastic game-based steering torque control framework for driver-machine co-pilot. The co-pilot problem is described by the affine-LQ method on the basis of a six-order stochastic driver-vehicle dynamic system. The Stochastic Nash and Stackelberg equilibrium solutions of the proposed LQ problem, which is characterized by feedback information pattern, are derived using stochastic dynamic programming. Simulation results indicate that the proposed FB Nash strategy will save about 50% steering effort than the OL Nash strategy during a conflicting SLC maneuver, and the proposed stochastic Nash and Stackelberg strategies become identical as simulation frequency increases. Finally, experiment results reveal that the proposed strategy of the IEPS

(a) Case 1: Cooperative DLC maneuver

(b) Case 2: Conflicting SLC maneuver

Fig. 12. The driver-IEPS torque interactions and observer results with compensation-based FB Stackelberg strategy. (a) Driver/IEPS torque command, total steering motor current and external disturbance observer results in the presence of uncertain driver behavior in case 1. (b) Driver/IEPS torque command, total steering motor current and external disturbance observer results in the presence of uncertain driver behavior in case 2.

can tune the driver's steering feel during cooperative DLC maneuver and decrease the driver-IEPS steering torque evidently during conflicting SLC maneuver. While the co-pilot system is disturbed by uncertain driver behavior, the proposed stochastic game-based strategy enables the IEPS to compensate the uncertain driver/external disturbance and stabilize the vehicle during the lane change maneuver. Future work involves validation of the proposed strategy through an experiment vehicle with a human driver, and consideration of vehicle nonlinearity during co-piloting in some extreme driving conditions.

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