Distance-based design and evaluation of heterogeneous cellular networks

Sinnen, Meshal

2019

Sinnen, M. (2019). Distance-based design and evaluation of heterogeneous cellular networks. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/84431>

<https://doi.org/10.32657/10356/84431>

**NANYANG
TECHNOLOGICAL
UNIVERSITY**

SINGAPORE

**DISTANCE-BASED DESIGN AND
EVALUATION OF
HETEROGENEOUS CELLULAR
NETWORKS**

MESHAL SINNEN

**SCHOOL OF ELECTRICAL & ELECTRONIC
ENGINEERING**

2019

DISTANCE-BASED DESIGN AND EVALUATION OF HETEROGENEOUS CELLULAR NETWORKS

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A thesis submitted to the Nanyang Technological University
in partial fulfilment of the requirement for the degree of
Doctor of Philosophy

2019

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

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Authorship Attribution Statement

This thesis contains material from 1 paper published as the following peer-reviewed manuscript where I was the first and/or corresponding author.

Material from Chapter 3 is published as M. Sinnen, K. H. Li, K. C. Teh, “Study of distance distributions and clustered user association in separation-based two-tier HCNs,” in *Proc. IEEE WCNC*, April 2018. The contributions of the co-authors are as follows:

- I, along with A/Profs Li and Teh, provided the initial direction and objectives of the project.
- I designed the system models, performed theoretical analysis, and conducted simulations to obtain results and outcomes pertaining to the manuscript.
- I prepared the manuscript drafts. The manuscript was revised by A/Profs Li and Teh.

Material from Chapter 5 has been accepted for publication in *IET Communications* as M. Sinnen, K. H. Li, K. C. Teh, “Modelling and analysis of two-tier HCNs with co-tier and cross-tier separation dependencies” and is available at DOI: 10.1049/iet-com.2018.5195. The contributions of the co-authors are as follows:

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- I prepared the manuscript drafts. The manuscript was revised by A/Profs Li and Teh.

12/01/2019

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Acknowledgements

I would like to express my gratitude to Associate Professor Li Kwok Hung, who guided and supervised my work through the entirety of my Ph.D, without whose patience and continued support this work would not have been possible. Furthermore, I would like to thank Associate Professor Teh Kah Chan, whose professional advice has proved invaluable throughout the course of this research.

There are a few more people who have helped me through the course of these years at NTU. I would like to thank my friends and housemates, Ali Zonoozi, Adam Hotra, Viktor Barat, and Biswa Jyoti Deb, who have been an integral part of keeping this process upbeat with their friendship, that I will value for a lifetime.

Special thanks goes out to my friends Olivier Cometto, Mervin Ng, and all the people from the NTU tennis team I met along the way, for providing a wonderful environment outside the lab, and helping create some unforgettable memories in the process.

Finally, I wish to convey my thanks and love to my parents and sister for their unconditional support during this period, and throughout my life.

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Abstract

In recent times, heterogeneous cellular networks (HCNs) have been proposed as an attractive solution to cater to the increased demands in mobile user capacity and could have widespread application in cellular networks deployed under 5th generation (5G) mobile standards. Deploying small cells to coexist with macrocell base transmitting stations (BTSs) from traditional cellular networks enables increased area spectral efficiency (ASE) and throughput through the enhanced reuse of available bandwidth. However, deployment of HCNs is a complex operation and ongoing research aims to solve some of the problems relating to it. In particular, the aggregation of carrier interference can create coverage holes if cell planning is not optimized. While increased ASE may still be achieved by simply deploying small cells in large numbers, the coverage of the system can suffer due to overwhelming interference at mobile terminals, with reduced transmission efficiency per BTS. Therefore, it is important to consider strategies to mitigate interference and increase BTS efficiency when designing BTS location and resource allocation.

Stochastic geometric models such as the Poisson Point Process (PPP) have been researched in detail to model the random nature of highly densified HCN BTS locations and uniformly distributed users. In most of this research, the emphasis is placed on the overall spatial performance metrics such as coverage and spectral efficiency. However, using the PPP to model users and BTSs in an HCN ignores practical factors such as the correlation between BTS and user locations, as well as the separation that usually exists between BTSs, where the latter is desirable for interference mitigation. In this thesis, we propose that these factors should be analyzed and accounted for when planning the locations of BTSs in two-tier HCNs and allocating resources for each BTS. Following a comprehensive literature review of stochastic geometric models, we introduce a novel framework that models user correlation to BTSs based on the Poisson Cluster Process (PCP). Furthermore, we incorporate a cross-tier separation rule between tier-1 macrocell BTSs and tier-2 picocell BTSs, by modelling the latter as a Poisson Hole Process

(PHP). The first part of the analysis is a comprehensive study of distance distributions and the association of different user groups to BTSs from each tier, as well as the utilization of BTSs under constrained resources. These theoretical models are reused for the analysis of coverage, with some detail on how this affects the rate and ASE. Using numerical results, we analyze the effect of centralizing BTSs to user hotspots and demonstrate what the optimal separation between tier-1 and tier-2 BTSs must be in situations when the network has enough resources for users, and conversely, when resources are limited.

The HCN model is extended by incorporating a co-tier separation rule that prohibits tier-2 BTSs from existing within a minimum distance from each other. The proposed model is analyzed for the case of uniform users, such that we measure the effectiveness of deploying tier-2 BTSs to fill coverage holes that exist naturally. We begin the analysis by focusing on distance PDFs and association probability, while showing that a minimum radius of association can be guaranteed based on the setting of co-tier and cross-tier separations. This is followed by a comprehensive analysis of coverage probability, and its relationship to ASE. Limits on both separation categories are explored for optimizing spatial coverage and ASE, while maintaining a viable tier-2 BTS density to form an HCN. The discussion highlights trade-offs that exist between coverage and ASE according to different separation settings and suggests the use of thresholds on each performance parameter to determine the optimal combination of co-tier and cross-tier separation. We showcase how this proposed planning technique works better as a rule of thumb in deploying HCNs when compared to benchmark results and include practical measures of throughput to support this claim.

The work is concluded with a summary of contributions and recommendations for future work. One major conclusion is that careful placement of tier-2 BTSs is needed in relation to hotspots modelled by a PCP, and that the coverage of an HCN is maximized when the BTSs are near the centers of hotspots. Furthermore, optimally separating tier-2 BTSs from each other and tier-1 BTSs results in optimized coverage and ASE for the HCN. The work can be extended towards incorporating the mixed user distribution for the analysis of the HCN modelled according to both co-tier and cross-tier separations. Extension towards the use of alternate path loss and fading models such as the ones used to measure path loss indoors are also recommended.

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List of Mathematical Notations

$\mathbb{R}^d$	General Euclidean space of d dimensions.
$N(B)$	Number of points of a point process falling inside the Euclidean space B .
$\#\{\cdot\}$	Alternate definition of number of points of a point process that intersect a defined Euclidean space inside the curved brackets.
Φ	Set of points defined by a point process.
Φ_a	Specific point process formalized by the subscript a .
Φ_{SA}	Set of social attractors found at the centres of clustered user groups.
Φ_k	Set of active base transmitting stations (BTSs) from the k -th tier of a two-tier HCN, where $k = 1,2$.
$\Phi_{2,b}$	Set of potential tier-2 BTS locations that may be used for deployment, conditioned on cross-tier separation from tier-1 BTSs in a two-tier HCN.
λ	Density of points of an arbitrary point process.
λ_a	Density of points of specific point process formalized by the subscript a .
λ_{SA}	Density of social attractors found at the centres of clustered user groups.
λ_k	Density of active base transmitting stations (BTSs) from the k -th tier of a two-tier HCN, where $k = 1,2$.
$\lambda_{2,b}$	Density of potential tier-2 BTS locations.
$\mathbf{x}, \mathbf{y}$	Arbitrary vector describing the location of a point inside a point process.
$\mathbf{o}$	Vector defining the position of the origin.
R_U	Limiting radius around an SA to restrict the location of clustered users modelled by the Matérn Cluster Process (MCP), or Truncated Thomas Cluster Process (TTCP).
σ_U	Standard deviation for the location of clustered users modelled by the TTCP.

R_{BTS}	Limiting radius around an SA to restrict the location of the potential tier-2 BTS when modelled by the MCP or TTCP.
σ_{BTS}	Standard deviation for the location of potential tier-2 BTSs when modelled by the TTCP.
$\delta_{1,2}$	Cross-tier separation parameter.
δ_2	Co-tier separation parameter.
P_k	Transmission power level of the k^{th} tier, where $k = 1, 2$.
α	Path loss exponent
$\bar{P}_1$	Power ratio $(P_1/P_2)^{1/\alpha}$.
$\bar{P}_2$	Power ratio $(P_2/P_1)^{1/\alpha}$
Ψ_1	Set of independent uniform users across the two-tier HCN.
$\Psi_{2,A}$	Subset of clustered users provisioned with corresponding conditioned tier-2 BTSs.
$\Psi_{2,B}$	Subset of clustered users not provisioned with tier-2 BTSs.
Ψ_2	Superset of all clustered users.
ρ_1	Density of independent uniform users across the two-tier HCN.
$\rho_{2,A}$	Density of clustered users provisioned with corresponding conditioned tier-2 BTSs.
$\rho_{2,B}$	Density of clustered users not provisioned with tier-2 BTSs.
ρ_2	Density of all clustered users.
$\mathbf{z}_1$	Location of user that belongs to the set Ψ_1 .
$\mathbf{z}_{2,A}$	Location of user that belongs to the set $\Psi_{2,A}$.
$\mathbf{z}_{2,B}$	Location of user that belongs to the set $\Psi_{2,B}$.
$\mathbf{z}_2$	Location of user that belongs to the set Ψ_2 .
$\mathbf{x}_{ki}$	Location of the i^{th} nearest independent BTS from the k^{th} tier of a two-tier HCN with respect to a user at the origin, where $i = 1, 2, \dots, \infty$ if $k = 1$ and $i = 0, 1, 2, \dots, \infty$ if $k = 2$.
$\bar{\mathbf{x}}_{20}$	Potential location of a tier-2 BTS with respect to a user $\mathbf{z}_2$, where this location is conditioned on the location of the corresponding SA

$\mathbf{x}_{20}$	Location of a deployed tier-2 BTS with respect to a user transformed to the origin, $\mathbf{z}_{2,A}$, where this location is conditioned on the location of the corresponding SA.
X_{ki}	Distance of the i^{th} nearest BTS from the k^{th} tier of a two-tier HCN with respect to a user at the origin, where $i = 1, 2, \dots, \infty$ if $k = 1$ and $i = 0, 1, 2, \dots, \infty$ if $k = 2$.
$\bar{X}_{20}$	Distance of a potential tier-2 BTS with respect to a user transformed to the origin, $\mathbf{z}_2$, where this location is conditioned on the location of the corresponding SA.
X_{20}	Distance of a deployed tier-2 BTS with respect to a user transformed to the origin, $\mathbf{z}_{2,A}$, where this location is conditioned on the location of the corresponding SA.
$h_{\mathbf{x}_{ki}}$	Instantaneous fading gain on signal received by the user from BTS at $\mathbf{x}_{ki}$.
$\mathcal{T}_{\bar{U}}^{\mathbf{x}_{ki}}$	Event describing association of user from $\Psi_{\bar{U}}$ to BTS at $\mathbf{x}_{ki}$, where $\bar{U} = 1, \bar{U} = 2, A$ or $\bar{U} = 2, B$.
$\mathbf{R}_b$	Set of orthogonal resource blocks allocated to a BTS.
N_b	Number of resource blocks allocated to a BTS.
$\bar{\mathcal{T}}_{\bar{U}}^{\mathbf{x}_{ki}}$	Event describing association of user from $\Psi_{\bar{U}}$ to BTS at $\mathbf{x}_{ki}$, where $\bar{U} = 1, \bar{U} = 2, A$ or $\bar{U} = 2, B$, when finite resource blocks are available for each BTS.
$\mathcal{L}_{\{\bar{U}\}}^{\mathbf{x}_{ji}}(\mathbf{x}_{ki})$	Laplace transform of interference from single interferer located at $\mathbf{x}_{ji}$ when the user from $\Psi_{\bar{U}}$ is served by $\mathbf{x}_{ki}$, where $\bar{U} = 1, \bar{U} = 2, A$ or $\bar{U} = 2, B$.
$\mathcal{L}_{\{\bar{U}\}}^j(\mathbf{x}_{ki})$	Laplace transform of interference from tier- j when the user from $\Psi_{\bar{U}}$ is served by $\mathbf{x}_{ki}$, where $\bar{U} = 1, \bar{U} = 2, A$ or $\bar{U} = 2, B$.
$\mathcal{C}_{\bar{U}}^{\mathbf{x}_{ki}}$	Event describing coverage of user from $\Psi_{\bar{U}}$ when associated with BTS at $\mathbf{x}_{ki}$, where $\bar{U} = 1, \bar{U} = 2, A$ or $\bar{U} = 2, B$.
$\bar{\mathcal{C}}_{\bar{U}}^{\mathbf{x}_{ki}}$	Event describing coverage of user from $\Psi_{\bar{U}}$ when associated with BTS at $\mathbf{x}_{ki}$, when limited resource blocks are available for each BTS, where $\bar{U} = 1, \bar{U} = 2, A$ or $\bar{U} = 2, B$.

$\mathcal{L}^j(\mathbf{x}_{k1})$	Laplace transform of interference from tier- j to an independent user served by the BTS at $\mathbf{x}_{k1}$, in a scenario where only independent uniform users are considered
$b(\mathbf{a}, r)$	Circle centred at the coordinate $\mathbf{a}$ with radius r .
$\cap(\mathbf{a}, r)$	Circumference or full arc of a circle centred at the coordinate $\mathbf{a}$ with radius r .
$\theta_{R_2, R_1, X}$	Functional angle depending on the general distances R_1 , R_2 , and X .
$\bar{\theta}_{R_2, R_1, X}$	Functional angle depending on the general distances R_1 , R_2 , and X , where $\bar{\theta}_{R_2, R_1, X} = \pi - \theta_{R_2, R_1, X}$.

Chapter 1

Introduction

1.1 Heterogeneous Cellular Networks

1.1.1 Evolution of Cellular Networks

The mobile and wireless telecommunications industry has undergone rapid transformations in recent times and are set to progress from existing 4th generation (4G) Long Term Evolution (LTE) standards [1]-[9] to ratifying the 5th generation (5G) mobile standard [8]-[15]. Many mobile telecommunications operators are moving towards the first 5G deployments as early as the year 2019 [10]. The key feature of this evolution has been transforming voice-oriented mobile cellular networks in their inception to current high capacity data-centric platforms [7], [8]. 5G attempts to facilitate a larger volume of subscriptions that includes human users and connected machines for the Internet of Things (IOT) [11]. Furthermore, 5G is expected to feature networks that deliver stable coverage and high-speed, low-latency data transfer for vehicular devices [10].

Cellular networks are infrastructure-based wireless access systems, where mobile users connect to base transmitting stations (BTSs) to obtain information services [17]. The BTSs are connected to a wired core network of servers and exchanges through a last-mile connection called the backhaul. The BTSs are access points (APs) that facilitate the temporal wireless delivery of services to mobile users at different locations. Other technologies that also feature APs include wireless local area networks (WLANs) based on the IEEE 802.11 standard (WiFi), and longer-range wireless access networks based as the IEEE 802.16 standard (also known as WiMAX) [18]-[21]. Cellular networks were originally designed to serve large populations of users spread sporadically across a region, and therefore their traditional focus in the earliest analogue 1G standard was to maximize the

capability of accessing services such as voice calls and short-message service (SMS) [7], [8]. Therefore, it was important to maximize spatial coverage. Due to wireless signal attenuation, large regions were covered by combining multiple BTSs, each covering a smaller region surrounding it. These regions are called cells, and each cell reused the available bandwidth, a concept known as frequency reuse. Each user was able to communicate to and from the BTS via a downlink (from BTS to user) and uplink (from user to BTS) channel [17].

The subsequent 2G Global System for Mobile Communications (GSM) standard used digital access techniques such as time-division multiple access (TDMA) and frequency division duplex (FDD) were used. The BTSs coordinated with each other to manage users and handovers were featured as a mechanism by which users move between cells [8]. Due to the higher density of deployed BTSs under 2G, transmission on overlapping frequencies created inter-cell interference, i.e. the reception of interfering signals by a mobile terminal from interfering BTSs. To be in coverage, a mobile terminal required sufficient received signal strength, and an acceptable signal-to-noise-plus-interference ratio (SINR). Inter-cell interference mitigation was made possible by splitting the available bandwidth into fractions according to a frequency reuse factor, such that adjacent cells would transmit at non-overlapping frequencies. The most common 2G GSM networks were deployed using directional BTS antennas with a frequency reuse factor of 7 [8], where seven different frequency channels were reused across BTSs. One of the potential drawbacks (at the time) with this type of frequency reuse was that the available bandwidth per cell (or cell-sector) was divided by the reuse factor. The cost of improved coverage in the cellular network was reduced spectral efficiency [22].

As subscriber bases grew along with the demand for higher-bandwidth data services, efficient utilization of bandwidth became important. To maximize reusable bandwidth, 3G and 4G systems had a frequency reuse factor of 1, such that all cells transmitted on overlapping frequencies, on different channels for the downlink and uplink [23], [24]. The wideband code division multiple access (W-CDMA) scheme, rake receivers to counter fading from multipath propagation, enhanced inter-cell interference coordination, BTS power control and improved handover mechanisms were the key technological advancements that enabled this modification in 3G [8], [22]-[27]. In 4G LTE, orthogonal frequency division multiple access (OFDMA) was used for downlink transmission [2]-[6], [28] with

increased robustness to intra-cell interference and multipath fading than WCDMA [4]. Similarly, single carrier frequency division multiple access (SC-FDMA) was designated for uplink transmission because of the high peak-to-average power ratio of OFDMA and its effect on mobile terminal batteries [5]. Additionally, 4G featured multiple-input multiple-output (MIMO) antenna transmission schemes [2]-[4], [29]-[31], and resource allocation [2]-[4], [6]. Most 4G cellular systems operated near 3G frequency bands, although operation at higher frequencies was ratified in the standard [5].

Evolution to 5G features innovations to push the boundaries of system capacity and coverage even further. Among these are device-centric architectures [9], Massive-MIMO [9], [12], millimetre-wave (MM-wave) transmission [15], [16], usage of unlicensed bands, and transmission at higher frequency bands [14].

1.1.2 Concept of HCNs and Related Challenges

In this thesis, we focus on device-centric cellular network architectures in the form of heterogeneous cellular networks (HCNs) [22], [28]. In recent times, various HCN models have been researched in abundance, and the concept has been touted with great potential to increase the spectral efficiency of cellular networks and target specific user groups as may be required in a network that carries a non-uniform spatial distribution of users [9], [22], [28].

Traditionally, cellular networks comprise high-powered BTSs designed to serve users located at large distances from them; which was sufficient when the required service capacity was relatively low. On average, the transmitted power of such cellular networks is homogeneous [22], [28], and the BTSs tend to be well separated in terms of distance. Such cellular networks are often referred to as macrocell networks. While most macrocell deployments were adequate to serve requirements of legacy networks and existing user populations during their respective implementation phases, two key factors render them relatively obsolete; thereby necessitating changes in cellular layout. Primarily, increase in subscriber volumes and evolution to higher capacity data services for mobile users compels operators to further reuse available spectrum. Moreover, infrastructural changes made in cities; especially in consistently developing urban environments, results in changing user patterns that usually weren't forecast during the original planning

phase of the cellular networks. Older macro-cell deployments may not suffice as optimal solutions to accommodate user environments that change over time; hence, a more flexible deployment method deserves serious consideration.

Furthermore, received signal strength on the uplink and downlink in cellular transmission is heavily dependent on the distance between the BTS and the mobile terminal [17], [28]. Consequently, users that are located near their serving BTSs tend to experience higher received signal strengths, and due to the spacing between BTSs, higher SINR levels due to lower levels of interference. The reverse is true for users that are distant from their serving BTSs and simultaneously near other interfering BTSs. Such users are usually located at the boundaries of cells, also known as cell-edges. A higher SINR and received signal strength for the user ensures that the user will be able to receive information bits at a higher rate [17].

Therefore, densifying cellular networks with more BTSs that are also planned to target expected user patterns can reduce distances between BTSs and users, and consequently, improve the coverage and rate of users. Moreover, the addition of BTSs leads to increased area spectral efficiency, as available bandwidth is reused across an increased number of transmission sites per unit area. However, simply increasing the number of high-powered macrocell BTSs in a cellular network is usually infeasible due to practical constraints in installation, higher capital and operating expenditures, and energy consumption constraints. Instead, optimizing the existing macrocell network using techniques such as adding spectrum to each BTS, sectorizing antennas, or improved modulation schemes are often considered as an intermediate solution to improving coverage and rate among users [28], [32]. A more feasible solution is to deploy smaller BTSs that transmit at lower power levels, to coexist with macrocell infrastructure. These small-cell BTSs are often termed picocell or femtocell BTSs, depending on the ratio of their transmit power levels to those of a conventional macrocell BTS [28], [33]-[42]. Small-cell BTSs can be further categorized according to the power levels at which they transmit. Macrocell BTSs usually transmit at of 40 W and are almost exclusively deployed outdoors. Microcell and picocell BTSs transmit in the range of 5 – 10 W, and 250 mW – 2W respectively, with the majority of their applications also being outdoors, or in large indoor spaces, such as shopping malls or stadiums. Femtocell BTSs are proposed as an alternate solution to indoor WiFi APs and transmit at 100 mW or less.

When small cell BTSs are overlaid on macrocell networks, they effectively form an HCN due to heterogeneous nature of BTS transmission power levels. HCNs have drastically different cell patterns, with a lot of smaller cells forming within the large macrocell area. The deployment of HCNs presents several challenges that cell operators need to tackle. Chief among them is the mitigation of inter-cell interference, which when unaccounted for can introduce coverage holes. Reduced coverage leads to a reduction in maximum achievable spectral efficiency, even though this metric is usually boosted by simply introducing small cell BTSs due to increased frequency reuse. Therefore, optimized planning or activation of BTSs in HCNs is important, if the overall performance of the HCN is to be maintained at acceptable levels. Other challenges include inter-cell interference coordination, resource allocation, as well as mobility and handover management, each of which becomes more complex with the introduction of many small-cell BTSs into the existing network.

As such, the analysis of models to represent a well-planned HCN is useful in aiding cell planners to understand the rules of thumb that need to be followed during design and implementation.

1.1.3 Stochastic Geometry for the Modelling of HCNs

Several mathematical models exist in the literature for this exercise. One of the most tractable models that exist is the Wyner model [43] – [47], which assumes a unit gain from a serving BTS to its user, and a fractional gain as interference to users of two neighbouring cells. This model is very limited in terms of analysing the effect of interference from a large number of surrounding BTSs, unless a large average interference can be easily accounted for [46], [47]. As such this model could be applied to the analysis of CDMA networks, but does not work well for systems that use orthogonal resources such as OFDMA on the downlink [48]. Another model that works quite well for single-tier macrocell networks is the lattice – or grid – model, of which the most popular variant is the hexagonal lattice [17], [49]. The model has long been used in the literature to analyse interference, coverage, and even model frequency reuse patterns for macrocell networks. While the exact spacing between BTSs leads to an optimistic analytical framework for single-tier cellular networks, the model is quite applicable to a single localized

situation such as an urban area. This is because most macrocell BTSs tend to be well separated from each other, and quite homogeneous in terms of placement within a locality. However, analysis of these networks can be quite complex mathematically if multiple random user locations need to be considered, and the model is more suitable for simulation purposes [48]. Another drawback of the aforementioned models is that they are not suitable for highly densified HCNs, primarily because the placement of BTSs tends to be relatively random and based on factors such as user distribution. Therefore, a more suitable framework is required to model spatial entities such as BTSs and users for the analysis of HCNs.

Stochastic geometry [50] – [56] is the study of randomly placed points across spatial dimensions and can be applied to the analysis of many upcoming wireless communication technologies, including HCNs. In recent times, there have been many comprehensive studies conducted into stochastic geometric models that can represent BTSs and users in a cellular network. Many of these studies are based on the most popular model, the Poisson Point Process (PPP). While the analysis of a PPP-based spatial BTS model is tractable and yields pessimistic results for coverage and rate of users, a few factors can make it less applicable to HCNs than other models in literature.

Firstly, the PPP implies that BTSs are located uniformly at random across the space, without any spatial dependencies between each other. While this does not have a major effect on low-density macrocell networks, distance dependency becomes significant in a densified HCN. In practical situations, it is rather uncommon to see two BTSs deployed next to each other, unless they are spatially sectorized. Furthermore, deployment of two BTSs tends to increase the interference to a user that lies near both BTSs, thus reducing the performance of the user in terms of coverage and rate. This situation can be particularly harmful if a low-powered small-cell BTS resides in the vicinity of a high-powered macrocell BTS. When this situation is extrapolated over multiple BTSs and users over a large area, the overall performance of the HCN is affected adversely. Secondly, while the use of the PPP is suited to analysing overall spatial measures such as coverage and spectral efficiency, it does not account for the correlation between users and BTSs. This correlation is particularly important if small-cell BTSs are overlaid on a macrocell network to cater to dense user hotspots. As such, some spatial coupling between BTSs and users needs to be accounted for in modelling HCNs.

Fortunately, several other stochastic geometric models, that are usually derived from the PPP, exist to overcome this problem. For example, the Poisson Hole Process (PHP), or Matérn Hardcore Process (MHP) are suitable for modelling BTSs that are planned with some distance-based separation between them. The Poisson Cluster Process (PCP) is well-suited to modelling the distance-based clustering of BTSs in populous locations, or for the correlation of hotspot users to user-centric BTSs. In this thesis, we use and combine some of these models with appropriate modifications, to provide a few tractable frameworks to analyse HCNs that follow more practical trends. The outcomes of the analysis are the numerical assessments of various performance parameters. These parameters could be then used to provide a few thumbs of rule for the spatial activation or planning of BTSs in an HCN. The scope and motivation for the research conducted is provided in greater detail in the next subsection.

1.2 Scope and Contributions of Research

The research in this thesis focuses on downlink performance of a two-tier HCN, where a macrocell BTS is overlaid with small-cell BTSs to form an HCN. We consider a standard distance-based path loss model, with small-scale Rayleigh fading gain throughout our analysis. Therefore, the research presented in this thesis is best suited to modelling outdoor HCN scenarios, which usually imply that the small-cell BTSs are microcell or picocell BTSs. For brevity, we consider the latter, which is reflected by the ratio of the picocell BTS power level to that of the macrocell BTSs.

The primary factor studied in this research is the separation between any pair of BTSs in the network. First, we consider the effect of separating tier-1 macrocell BTSs from tier-2 picocell BTSs, by introducing a *cross-tier* (or inter-tier) separation parameter, which will be discussed in greater detail. This separation between tier-1 and tier-2 BTSs can be tractably modelled using a PHP. As BTSs are expected to serve users around them, we consider the correlation between users and the BTSs by employing a PCP. This spatial coupling of users is possible with both tiers of BTSs, although the radius of users surrounding tier-1 BTSs is expected to be much larger than those around tier-2 BTSs. This is because macrocell BTSs are traditionally deployed to serve larger areas around them. In the case of smaller user

hotspots that are to be served by picocell BTSs, the location of the deployed BTS in relation to the hotspot tends to be more critical. As it is extremely difficult to deploy picocell BTSs central to small user hotspot patterns all the time, we model an offset between the tier-2 BTS and the centre of the user cluster. One of the contributions we provide with this framework is that it can be generalized to relatively random distances between a pair of BTSs, or users and BTSs by simply varying a few numerical parameters. As such, analysis of the HCN is not limited to a few abstract situations concerning these distance dependencies. The primary results of this part of the study concern spatial coverage and spectral efficiency for clustered user patterns served by these distance-based HCN deployments. These measures apply to a network that has sufficient resources in terms of bandwidth to serve users across the network. In order to complement these preliminary results, we showcase how applying resource constraints to each BTS affects network performance, depending on the densities of the clustered user groups that need to be served.

A question that arises when considering separation between BTSs in an HCN is the separation between a pair of tier-2 BTSs. An overly proximal tier-2 interferer can also deplete SINR, and consequently, coverage of a picocell user group. Although the levels of interference aren't expected to be as detrimental as those create by a very close tier-1 BTS, it is worth analysing the advantage that having a minimum separation between two tier-2 BTSs can provide. We extend the cross-tier separation model by incorporating a *co-tier separation* between tier-2 BTSs only. For tractability, the set of tier-1 BTSs are analysed as a PPP, with distance-based justification provided for this assumption. Furthermore, for focus on the effect of adding the co-tier separation parameter, we focus on spatial statistics by using uniformly distributed PPP users for the analysis. Under this assumption, we view the deployment of tier-2 BTSs as a potential solution to tackle existing coverage holes in a macrocell network and showcase how they can effectively be reduced. The results demonstrate improvements in coverage, and trade-offs against spectral efficiency. A significant outcome of this study is that both co-tier and cross-tier separation parameters have limits to which they can be increased if they are to improve overall HCN performance. It can also be seen that the density of deployed tier-2 BTSs is limited by each separation parameter, which is useful if there are

constraints on density due to cost or energy consumption. We present the exact breakdown of the thesis by chapter in the following section.

1.3 Organization of Thesis

This report is divided into six chapters including the introduction. Chapter 2 presents a comprehensive overview of related work in Section 2.1, with focus on the mathematical models used in each study, their outcomes and limitations. Section 2.2 then presents an introduction to important stochastic geometric models that are related to the work in this thesis. Characteristics of these classic point process models, and important measures such as distance distribution, association probability, cumulative interference, and probability of coverage are explained in detail.

Chapter 3 introduces a novel framework for analysis clustered user groups in an HCN, and the effect of two distance dependencies on its performance. Firstly, a distance offset parameter is introduced, to account for the relatively random distance between the centre of small user hotspots, and potential tier-2 BTS locations. Tier-2 BTSs are only deployed (or activated), conditioned on a minimum distance to the nearest tier-1 BTS, which is termed the cross-tier separation parameter. The analysis begins with deriving expressions that describe distance distributions from users to both serving BTSs, as well as conditioned BTSs that influence association, interference and coverage. This is followed this up with a comprehensive analysis of association probabilities for the system model. The chapter provides a discussion section for the validation of theoretical analysis with numerical results.

Chapter 4 is used to extend the analysis provided in Chapter 3 towards coverage under Rayleigh fading conditions. We provide equations that model conditional coverage for all different possible association events and use unconditioning techniques based on standard probability theory to showcase overall coverage probabilities. The link between coverage and ASE is explained in this chapter. Furthermore, the performance of the HCN under constrained resources is analysed and discussed in terms of coverage, ASE and BTS utilization.

In Chapter 5, the additional co-tier separation parameter that stipulates a minimum distance between a pair of tier-2 BTSs is incorporated into the BTS model

presented in chapter 3. This is followed by a comprehensive analysis of distance distributions, association probabilities, and coverage, and spectral efficiency. Additionally, practical performance parameters such as packet throughput and delay are included to give the reader a better understanding of the implications of improved coverage when the proposed planning technique is used.

In Chapter 6, the research presented in this thesis is concluded with a summary of the studies provided, a reiteration of the outcomes, and more recommendations for future work.

Chapter 2

Literature Review

2.1 Related Work

In this section, an overview of work related to the analysis of cellular networks, the use of stochastic geometry in wireless networks, related results and outcomes are presented. Some of these studies expound on techniques that will be used in the chapters to follow.

The literature presented in [Chap. 15, 17], [Chap. 3, 49], [44], [57] and [58] use the hexagonal grid to model traditional macrocell networks. In [59]-[62], the authors simplify this model further by assuming each cell to be circular with a designated radius. Each body of work presents the coverage of orthogonal cellular systems – which in some cases refer to traditional TDMA/FDMA systems – in terms of the probability that the SIR at a user's mobile terminal is above a designated SIR threshold. Distance dependent path loss models are used to account for signal attenuation, where the loss in an environment is quantified by a path loss exponent. The SIR is also tied to the Shannon-theoretic rate of users, which is the theoretical maximum possible rate at which information can be transmitted over wireless networks without arbitrary loss of packets. While the Shannon-theoretic rate is useful in evaluating the performance of a wireless network, it must be stated that actual transmission rates in a wireless network depend on modulation techniques that are used, as well error-correction code rate. From this measure, the ASE of a cellular network can be derived, and it quantifies the number of information bits that can be transmitted for a unit time and frequency, over a unit spatial area for the cellular network.

A lot of work using stochastic geometric models considers the homogeneous PPP to model BTSs and users. The authors in [56] describe the use of the PPP and stationary point process to model macrocell networks. The work initially describes important properties of the PPP and characterizes interference for cellular networks

modelled using this method. Using distance-based path loss and Rayleigh fading, the coverage probability of the cellular network is analyzed, where the coverage is dependent on the user's SIR meeting a pre-determined threshold. An important result from this paper is the use of the Laplace transform of interference when Rayleigh fading is considered, which is directly derived using the probability generating functional (PGFL) of the PPP (this is discussed in section 2.2). In this case, the small-scale Rayleigh fading gain is approximated as an exponential random variable, which leads to the Laplace transform. Furthermore, the paper states that a trade-off exists between SIR threshold, coverage and ASE, and it may important to find the optimal threshold to maximize ASE. This is because, although increasing the SIR threshold of the network reduces overall coverage, the average rate of users in coverage is increased.

In [48], the authors present a tractable framework for the analysis of single-tier macrocell networks using the PPP and showcase tractable analysis for coverage and rate for different cases of fading, based on a minimum SINR threshold for connectivity. Once again, the case for Rayleigh fading is demonstrated using the Laplace transform of surrounding interference to the user. The results demonstrate that the PPP is a pessimistic model for the measurement of coverage and the average rate of users when assessed against the simulated performance of cellular networks that consider actual BTS locations. Conversely, the work also shows that using a square grid to simulate performance of a cellular network yields optimistic results. Another important finding from this paper is the fact that noise tends to influence coverage minimally, and that cellular networks modelled by the PPP are interference-limited. As such, noise can be discounted during analysis, with a signal-to-interference ratio (SIR) threshold used to approximate coverage, and consequently, rate.

The work in [48] has been extended to the analysis of HCNs comprising K tiers of transmission power levels in [38] and [39]. The analysis is based on a maximum instantaneous received SIR threshold, which implies that a user connects to the BTS supplying it with the strongest instantaneous signal power. For distance-based path loss models, this is usually the nearest BTS, unless a deep fade is experienced on the transmission path. While connectivity based on instantaneous SIR is expected to increase coverage and rate in an HCN, the effect of regular handover ping-pongs has not been considered in this study. The authors provide tractable expressions for

coverage and rate, with focus on the case of small-scale Rayleigh fading. One of the interesting outcomes of this study is that coverage is not affected by the number of tiers in an HCN, or the relative density of BTSs across tiers.

The author in [63] presents a framework for accurately deriving the distribution of downlink SINR for multi-tiered HCNs based on the PPP, where Rayleigh fading is considered, and the SINR threshold can take values below 0 dB. This framework has been extended to [64], where a three-tier HCN comprising macrocells, and open and closed-access femtocells has been considered. It is shown here that the probability of a user camping on a femtocell or macrocell, and the coverage probability depends primarily on the relative transmit powers of the BTSs in the HCN, and the fraction of femtocell BTSs that operate under closed access. In [65], a similar model for multi-tier HCNs has been considered but incorporates user association bias. Applying the association bias implies that the user can connect to a subset of available BTSs only, and coverage has been analyzed for cases in which the serving BTS is either the nearest BTS, the BTS that supplies it with the strongest average signal, or the BTS that provides the maximum instantaneous SINR. Biased user association is also considered in [66]-[68], where the coverage probability of a two-tier PPP HCN is analyzed under different association bias factors for each tier. Here, the authors have proposed two biasing schemes based on maximum SINR, and maximum biased received power, and derive expressions for coverage probability for both cases. The authors suggest that the expressions allow for greater tractability and aid in generalizing the results for different cases of fading.

Quantifying user association in an HCN provides a measure of the effectiveness of small-cell BTSs in offloading users from their macrocell counterparts. In [68], the authors introduce the concept of stationary association cells, which quantify the region around BTSs in a PPP-based HCN that are expected to serve users. Two methods of quantifying association are used; first, association is based on maximum power of transmission, and secondly, association is based on maximum SINR at the user. In both cases, it is shown that association to femtocells decreases in less lossy environments but increases when the channel variance is greater due to fading. The maximum SINR connectivity model is relaxed in [69], to provide analysis for the association of users based on long-term maximum signal power. This relaxation results in making the analysis of PPP-based HCNs more tractable, and the resulting association of users is very flexible, i.e. they can connect to any BTS in open access.

Expressions for coverage, spectral efficiency and ergodic rate round of the analysis in this paper.

MIMO antennas have been known to enhance coverage and rate in cellular networks. The authors in [71] propose a framework based on PPP locations of MIMO antennas in an HCN and based on the channel models for MIMO antennas in [72], evaluate coverage probability for the HCN. This analysis is extended to the work in [73], where the authors demonstrate that HCN performance is better if several single antenna BTSs are spread across a space, as opposed to having few MIMO antennas, provided there are constraints on the total density of antennas that can be used. This analysis justifies the continued analysis of single antenna BTS models in an HCN, and techniques to optimize performance under such scenarios. Interference management is also critical to the performance of HCNs due to the increased aggregate interference caused by network densification with BTSs. In [74], the authors analyze the effect of various fractional frequency reuse techniques for both open access and closed access scenarios in PPP-based HCNs. This framework is extended to the case of multiple antennas in [75]. The authors in [76] and [77] propose a framework for joint resource partitioning over an HCN modelled by PPP BTSs and show that this is necessary to improve the overall rate distribution of users.

All the work discussed in this section thus far has utilized the PPP as the model for BTSs, and effectively considered uniformly distributed users through two-dimensional (2-d) spaces. The uniformly distributed users can also be modelled conveniently by a PPP, and a sufficiently large density of users yields what can be considered spatial performance results in terms of coverage, rate etc. The major drawback with the PPP from a practical standpoint, is that BTSs may be located excessively close to each other, which may be problematic in HCNs, especially if a small-cell BTS is overwhelmed by interference from a nearby macrocell BTS. This despite techniques such as range extension which exist for LTE small-cell BTSs [78]. Many other models in literature account for spatial dependencies between BTSs. In [79] and [80], the authors model BTS locations based on the Strauss point process (SPP) and Geyer saturation process, and fit curves to empirical models to justify their use in single-tier cellular networks. These models allow for the separation or clustering of BTSs based on saturation parameters. Another repulsive point process that tractably models single-tier macrocell networks with separation

between the BTSs is the Ginibre point process (GPP), which has been discussed in [81]. The authors validate the locations of GPP to model practical single-tier networks and derive expressions for coverage probability.

As HCNs are still an emerging technology, widespread empirical data about the nature of these deployments do not exist. Furthermore, while models such as the GPP and SPP are useful for modeling macrocell networks, their use has not been established for modeling HCNs. While the models are useful for the purposes of fitting, they do not give many insights about practical rules of deployment that can be applicable to an HCN. An important parameter that can be considered by cell planners when building HCNs is the distance between BTSs of the same tier and across different tiers. Fortunately, some useful models in the literature exist to model distance-based dependencies between BTSs.

The authors in [82] and [83] consider a distance-dependent BTS model based on the PCP, where it hypothesized that BTSs are usually clustered together in regions to serve similarly condensed user patterns. The authors in [84] consider the use of a Matern hardcore process (MHP) for a macrocell network, which can model separation between BTSs of the same tier. The MHP has two varieties, type-I and type-II [50]. The work in [85] models the interference from BTSs modelled under both types of the MHP and shows that surrounding interference can be approximated by an equivalent density PPP when the MHP type-II is considered. This conclusion has been used in [86] to model coverage and load balancing in an MHP type-II HCN, where BTSs from the same tier are separated from each other but are independent of BTSs across tiers. The drawback of this model is that some separation between macrocell BTSs and small-cell BTSs is required to allow for some interference reduction.

A good model for separation between BTSs of different tiers is the PHP. The most popular situation that has been analyzed is a two-tier HCN, where small-cell BTSs are forbidden to lie within a certain distance of their PPP macrocell counterparts. This effectively creates holes around macrocell BTSs in which there are no small-cell BTSs. Analysis of the PHP is harder than for a conventional PPP because of the absence of a PGFL for this model, but some good approximations exist in the literature. In [87] and [88], the authors present approximated expressions for an HCN that contains PPP macrocell BTSs and PHP picocell BTSs, where the approximation is based on modelling the latter as a PCP. The intuition behind this

technique is that if picocell BTSs do not exist in the holes around their macrocell counterparts, they generally tend to be clustered together in spaces outside the holes. The primary limitation of this model is that it permits only general user distributions. The authors in [89] present a model for analyzing HCNs where holes are created around users, and the coverage is approximated using the PGFL for an equivalent density PPP. While providing some intuition into the effect of prohibiting interfering BTSs to transmit within a certain radius around the user, the model is not completely accurate. A useful exposition on approximating Laplace transforms of interference is provided in [90], where the model derives accurate bounds for coverage to a typical user from PHP interferers in a wireless network. This framework is extended to a PHP-based HCN in [91], where open and closed access to small-cell BTSs is considered. Some work in our thesis is based on the intuition behind deriving the lower bound on the Laplace transforms.

Another aspect that requires serious consideration in HCNs is the correlation of users to BTSs and the general non-homogeneity of user patterns, which has been explored in some previous work. Inhomogeneous user distributions are modelled based on irregular allocations of density within Voronoi cells in [92] and using the inhomogeneous PPP in [93]. In each body of work, the authors test the validity of their models against peak-hour empirical user traffic data and prove the model to be a good fit. Another non-uniform user model is explored in [94], where conditional thinning of a PPP carrying potential BTSs yields a surrounding reduced density point process for a single-tier network of BTSs. This method biases users towards the interior of the serving BTSs Voronoi cell, resulting in a user pattern that is clustered around BTSs. One aspect of user distribution that could provide insights into the importance of HCNs are small user hotspots [9], [95], [96] that can be served by small cells. A non-homogeneous distribution of users was modeled by the log Gaussian Cox process (LGCP), where a basic k -means algorithm was used to select small-cell locations on top of a hexagonal grid of macrocell BTSs in [97]. Similarly, in [98], user hotspots were laid across an infinite macrocell hexagonal grid, and network performance was evaluated for three separate scenarios; each scenario was distinguished by distances from hotspots to small-cell, and macrocell BTSs. Related scenarios have been considered in [99], where users are correlated to Poisson distributed social attractors (SAs), and the deployments are conditioned on these

entities. The analysis in this study features arbitrary correlations between BTSs and users and cannot be generalized mathematically.

A model that considers non-uniform users that are clustered in condensed spaces is the PCP, where the BTS lies at the center of the user cluster. The authors in [100] show that this framework can be used for tractable analysis of association and coverage. One drawback of this proposed model is that it does not account for the possible distance offset between the user cluster and the deployed small-cell BTS that is supposed to serve them. Furthermore, possible separation between macrocell BTSs and small-cell BTSs has not been considered. The authors in [101] have proposed a framework that incorporate these separations that are imminent in an HCN deployment, and material from this paper is presented in the contributions of this thesis.

2.2 Stochastic Geometry in Wireless Networks

In this section, an introduction to important stochastic geometric models are presented in some detail as they are to be used in subsequent chapters as base models for the research carried out.

2.2.1 Point Processes

A collection of random countable points lying inside a Euclidean space $\mathbb{R}^d$, of d dimensions, can be defined as a point process [50]. Point processes can usually be visualized easily in 2 dimensions, where the locations of points can be concisely described by x and y coordinates. This can be easily extended to 3 dimensions by adding a z coordinate. Point processes can also be described as the union of several Borel sets $\mathcal{B}^d$, that in turn forms a Borel algebra (σ -algebra) on $\mathbb{R}^d$. The overall space, $\mathbb{R}^d$, can be characterized by the Lebesgue measure. Common examples of Lebesgue measures include area over two dimensions and volume over three dimensions¹ [50], [52].

A dichotomy in relation to point processes is how they are viewed as a collection of random points. If completely random, the point process can be viewed as a

¹ Volume can be referred to as the Lebesgue measure over an arbitrary number of dimensions but is used in the context of 3 dimensions here. In a different context, the volume of a ball in 2 dimensions is simply its area, for which generic formulae exist depending on the boundaries of the space.

random set that contains a discrete number of elements that describe the points within the space. Alternately, point processes can be viewed as random counting measures, characterized by the number of points, or density of points in the space. Both definitions can be used in conjunction with each other, forming a duality in definition, not dissimilar to the wave-particle duality for light [50]. Hence, we can define the number of points $N(B)$ falling on a region B as the counting measure for the point process, such that

$$N(B) \triangleq \#\{\Phi \cap B\}, \quad (2.1)$$

where Φ refers to the set of points in the point process, which can also be understood from the definition of $N(B)$ in (2.1). If we characterize the point locations in terms of the arbitrary coordinate vectors $\mathbf{x}$, Φ is mathematically represented as

$$\Phi \triangleq \{ \mathbf{x} \in \mathbb{R}^d : N(\{\mathbf{x}\}) = 1 \}. \quad (2.2)$$

Equation (2.1) is a description of random measure formalism while (2.2) refers to a random set formalism. A distinction needs to be made regarding which formalism can be used to describe a point process. If multiple points are found at the exact same location, then the point process is not a set of unique elements and therefore not completely random. Therefore, a random set formalism cannot be applied to it. This kind of point process is also called simple [50], [52], and this property is discussed further in Section 2.4. Random measure formalism for point processes is not restricted and can be used to describe simple point processes as well as point processes that incorporate some determinism; i.e. with co-located points [50].

By defining point processes as counting measures, we can define their density as the expected value of the number of points falling on a defined Euclidean space, e.g. a unit Lebesgue measure. Therefore, the density measure for a region B is defined as [50]

$$\Lambda(B) \triangleq \mathbb{E}(N(B)), \quad (2.3)$$

where $\mathbb{E}$ refers to the expectation operator in (2.3).

2.2.2 Distributions of Point Processes

Point process realizations on a Euclidean space are outcomes that are mapped from a probability space, and therefore can be described statistically. If we define an outcome event E for a point process Φ , we can describe the point process distribution [17] as

$$\Pr(E) \triangleq \mathbb{P}(N(\Phi) \in E), \quad \forall E \in \mathfrak{R}, \quad (2.4)$$

where $\mathfrak{R}$ is the set of all possible outcome spaces in (2.4). The counting measures $N(\Phi)$ are therefore random variables and can be measured in relation to different Borel sets B . If two Borel sets B_1 and B_2 are disjoint, then the counting measures for each $N(B_1)$ and $N(B_2)$ are independent. However, if the sets intersect, the counting measures are not independent.

Therefore, point process can be described by distributions that apply to random numerical variables such as the probability density function (pdf) and cumulative distribution function (CDF) [50], [52]. As such, by integrating over the space of simple sequences represented by point processes, we arrive at an alternate definition of its density, such that [50]

$$\Lambda(B) \triangleq \mathbb{E}(N(B)) = \int_{\mathbb{R}} \varphi(B) P(d\varphi). \quad (2.5)$$

The term P in (2.5) is the probability over the measure space, where $\mathbb{R} \rightarrow \mathfrak{R}$ is the domain over which the integral is measured. $\varphi(B)$ denotes the realization of point process Φ as a counting measure over Borel set B . To extend the definition of distributions of point processes, we define the finite dimensional (fidi) distributions of a point process as the joint probability distributions of the counting measures over different Borel sets, $(N(B_1), \dots, N(B_m))$ [50]. Using these definitions, we now proceed to outlining homogeneous point processes with a Poisson distribution.

2.2.3 Homogeneous Point Processes

A Poisson process is generally described by arrivals within a given time interval $[t_1, t_2)$, where the number of arrivals has a Poisson distribution, such that [50]

$$\mathbb{P}(N([t_1, t_2)) = k) = e^{-\lambda(t_2 - t_1)} \frac{(\lambda(t_2 - t_1))^k}{k!}. \quad (2.6)$$

In this context, λ is the mean number of arrivals within a unit time frame, i.e. the mean of the Poisson distribution. This definition is now extended to the general case of PPPs. A PPP that is realized across the finite space B on $\mathbb{R}^d$ with mean Λ ; where Λ admits a density λ is mathematically outlined as [50]

$$\mathbb{P}(N(B) = k) = \exp\left(-\int_{|B|} \lambda(x) dx\right) \frac{(\int \lambda(x) dx)^k}{k!}. \quad (2.7)$$

The domain of integration for $\lambda(x)$ is the Lebesgue measure of region B , also denoted as $|B|$. The second important characteristic that defines a PPP is the independence of disjoint sets (regions). If $B_1, \dots, B_m$ are disjoint, then the respective counting measures $N(B_1), \dots, N(B_m)$ are independent.

Note that the definition of a PPP in (2.7) describes the admitted density as a function $\lambda(x)$; for which the mean is Λ . This allows for various realizations of the PPP, with the density measure dependent on sections of the space defined by coordinates. Alternately, an arbitrary function can be used to define the density. In a situation where the density λ is simply equal to the mean Λ , the density of the PPP is homogeneous across the entire spatial region. Therefore, a homogeneous PPP is described by a homogeneous density. Following on, the number of points inside a finite space B can be characterized by the probability

$$\mathbb{P}(N(B) = k) = \frac{(\lambda|B|)^k e^{-\lambda|B|}}{k!}. \quad (2.8)$$

The homogeneous PPP is the most basic and ubiquitous spatial mathematical model found across various fields of study, and more recently, in wireless networks. We

now reiterate the definitions that summarize the characteristics of a homogeneous PPP [50]:

- The homogeneous PPP is characterized by a density of points λ . λ is the mean number of points expected in the multi-dimensional space $\mathbb{R}^d$. In the case of practical wireless networks, the number of dimensions considered is usually 2; therefore λ is the mean number of points expected in a unit area over $\mathbb{R}^2$.
- For every compact set B , the number of elements $N(B)$ has a Poisson distribution with mean intensity λ , as seen in (2.8).
- The points in the region B are identically and independently distributed (i.i.d).

These characteristics help define the process by which a PPP can be simulated to represent BTSs in a cellular, e.g. the layout of tier-1 macro-cell BTS location with intensity usually ranging from 1 to 2. To realize a PPP, these simple steps can be followed:

- Define the range of real values of each dimension; e.g. a range of 5 for x and y coordinates in a two-dimensional (2-d) space with unit area is 1 km², then the Borel set is a finite area of 25 km².
- Generate a Poisson random number with mean λ . This will represent the number of points falling within the finite space.
- For each point realized, generate a uniformly distributed (i.i.d) value that lies within the range specified for the coordinates of each dimension. The coordinate of each dimension can be realized separately and then combined to give the final vector coordinates of the point in a multi-dimensional space.

The PPP is a tractable because of its complete randomness and is a simple, yet powerful tool for the representation for BTS locations in both single-tier macrocell networks and multi-tier HCNs. However, there is some impracticality associated with it. For instance, there are chances of two BTS locations in a PPP realization lying very near each other without substantial spacing. The fidi distributions of a

homogeneous PPP for disjoint sets are simply the products of the probabilities of the counting measures of each set, due to the independence property of the PPP. Therefore,

$$\begin{aligned} \mathbb{P}(N(B_1) = k_1, \dots, N(B_m) = k_m) \\ = \frac{\lambda^{k_1 + \dots + k_m} |B_1|^{k_1} \dots |B_m|^{k_m}}{k_1! \dots k_m!} \exp\left(-\sum_{i=1}^m \lambda |B_i|\right). \end{aligned} \quad (2.9)$$

The formula in (2.9) implies that calculating joint probability distributions over a homogeneous PPP is straightforward. Across general dimensions, Borel sets B can be characterized as d -dimensional balls of radius r , in which case Equation (2.8) can be rewritten to incorporate a more specific Lebesgue measure $|B|$ [50], such that

$$\mathbb{P}(N(B) = k) = \frac{(\lambda c_d r^d)^k e^{-\lambda c_d r^d}}{k!} \quad (2.10)$$

The measure $c_d r^d$ refers to the volume of the ball in d dimensions, where the value of c_d can be calculated as follows:

$$c_d = \frac{\pi^{d/2}}{\Gamma(d/2 + 1)}, \quad (2.11)$$

where the gamma function is denoted by $\Gamma(n) = (n-1)!$. We now extend Equations (2.10) and (2.11) to the 2-d case as it applies to the research presented in this thesis. As such,

$$c_d r^d = \pi r^2 \quad (2.12 \text{ a})$$

$$\mathbb{P}(N(B) = k) = \frac{(\lambda \pi r^2)^k e^{-\lambda \pi r^2}}{k!} \quad (2.12 \text{ b})$$

2.2.4 Voronoi Tessellation

The Voronoi tessellation is an important spatial graph that can be applied to represent the cell regions in a wireless network. The basic definition of a Voronoi cell $V(\mathbf{y})$ of a point $\mathbf{y}$ that has been realized by a PPP Φ , is that it is the bounded set of locations whose Euclidean distances to $\mathbf{y}$ are less than or equal to their distances to other points within the spatial region, $\mathbf{z}$. A compact mathematical model for the Voronoi cell can be represented by [50]

$$V(\mathbf{y}) = \{\mathbf{x} \in \mathbb{R}^d: \|\mathbf{y} - \mathbf{x}\| \leq \|\mathbf{x} - \mathbf{z}\|, \forall \mathbf{z} \in \Phi\}. \quad (2.13)$$

A collection of points in a PPP can be surrounded by adjacent Voronoi cells that share boundaries with each other, thus realizing a Voronoi tessellation [50], [102]. Voronoi tessellations are useful in representing expected cell coverage areas in wireless cellular networks when the positioning of BTS locations is relatively random and not grid-based. Grids themselves are special cases of these tessellations, when the points within the spatial region are uniformly spaced. The kind of grids generated usually depend on the relative spacing between points, thus resulting in differently shaped cells; e.g. square or hexagonal. Figure 2.1 depicts the Voronoi tessellation in two dimensions of a single-tier macrocell network of density 2 km^{-2} , where the BTSs are represented by the black triangles, and the cell boundaries by the red lines.

Figure 2.1 Voronoi tessellation of a single-tier macrocell network of density $\lambda = 2 \text{ km}^{-2}$.

2.2.5 Properties of the PPP

Some properties that apply to the homogeneous PPP are useful in gaining an intuitive understanding of certain characteristics that are exploited when the PPP is used to model BTSs or forms the basis upon which another point process is constructed.

A. Simplicity

A point process is simple if there aren't multiple points at the same location, thus rendering such point processes as completely random. As the incidence of multiple co-located BTS sites – MIMO or sectorized antennas notwithstanding - with respect to a 2-d space is rare in cellular networks, the PPP and derivative models are suitable to model BTS locations. A similar case can be made for user locations as well.

B. Stationarity

A translated point process $\Phi_{\mathbf{x}}$ describes a point process that can be shifted by an arbitrary vector $\mathbf{x}$, such that

$$\Phi_{\mathbf{x}} \triangleq \{\mathbf{x}_1 + \mathbf{x}, \mathbf{x}_2 + \mathbf{x}, \dots\} \quad (2.14)$$

A point process whose distribution does not change under translation is called stationary or translation-invariant. Therefore, under stationarity the following condition holds mathematically in terms of outcome spaces E :

$$\mathbb{P}(\Phi \in E) = \mathbb{P}(\Phi_{\mathbf{x}} \in E) \quad (2.15)$$

An important feature of stationary point processes is that the density of the point process is constant. A homogeneous PPP and derivative models are an example of a stationary point process.

C. Isotropy

An isotropic point process is one whose distribution does not vary under an arbitrary rotation operation about the origin. This is described by the following condition related to the PPP Φ and event space E , such that

$$\mathbb{P}(\Phi \in E) = \mathbb{P}(\Phi_{\mathbf{r}} \in E) = \Pr(\mathbf{r}E) \quad (2.16)$$

$\mathbf{r}$ in (2.16) refers to the arbitrary rotation vector about the origin in $\mathbb{R}^d$ and $\Phi_{\mathbf{r}}$ is the consequent rotated point process. A homogeneous PPP is also isotropic. Combining the fact that a homogeneous PPP is both stationary and isotropic, it can be now called a motion-invariant point process. This feature renders the PPP and derivative models analytically tractable.

2.2.6 Distances in a PPP

Distance distributions are an important characteristic in random wireless networks, as it is important to characterize the positions of serving and interfering BTSs in a network. Due to the motion-invariant property of the PPP, it is possible to focus on any point in the space as the origin and define the distance distribution to neighbouring points in the PPP. The point focused on may be an element of the set defining the PPP, or a random point in the space outside it. Therefore, distance distributions in a PPP-based cellular network are the same from a BTS to its

neighbouring nodes, or from a user to its neighbouring BTSs. We consider the 2-d case for the purposes of this thesis. Given a random point $\mathbf{y} \in \mathbb{R}^2$, we consider r_n , the distance to the n^{th} neighbouring point of the PPP Φ , such that

$$r_n = \|\mathbf{y} - \mathbf{x}_n\|, \quad \mathbf{x}_n \in \Phi. \quad (2.17)$$

Consider the Borel set, $b(\mathbf{y}, r)$, ball of radius r centred at the said BTS location, $\mathbf{y}$. The volume of such a ball is πr^2 , as seen in (2.11). If the n^{th} nearest node is at a distance equal to or larger than r from the node in consideration, there are $n - 1$ nodes located inside this ball. As the CDF of the PPP approaches 1 as the number of nodes reaches an arbitrarily large value, we may define the distance CDF of distance to the n^{th} nearest node as

$$F_n(r) = 1 - \Pr(\#(b(\mathbf{y}, r) \cap \Phi) < n). \quad (2.18)$$

$\Pr(\#(b(\mathbf{y}, r) \cap \Phi) < n)$ is simply the sum of probabilities $\Pr(\#(b(\mathbf{y}, r) \cap \Phi) = k)$, where $k = 1, 2, \dots, n - 1$. Using this fact, and applying (2.10) to the 2-d case, $F_n(r)$ is rewritten as

$$F_n(r) = 1 - \exp(-\lambda\pi r^2) \sum_{k=0}^{n-1} \frac{(\lambda\pi r^2)^k}{k!}. \quad (2.19)$$

Note that $F_n(r)$ is only the CDF of the distance r to the n^{th} nearest neighbor. The PDF of this distance, $f_n(r)$, is obtained by deriving the CDF with respect to r , to obtain the generalized gamma distribution [50], [103]

$$f_n(r) = \frac{2}{\Gamma(n)} (\lambda\pi)^n r^{2n-1} e^{-\lambda\pi r^2}. \quad (2.20)$$

Of interest is the distance to the nearest node, as it accounts for the distance to the serving BTS from a PPP-based wireless network, if nearest-neighbour or strongest average received signal is used to model connectivity of users. Simply substituting $n = 1$ in (2.20) yields this as

$$f_1(r) = 2\lambda\pi r e^{-\lambda\pi r^2}. \quad (2.20)$$

Similarly, we can define the CDF $F_1(r)$, and complementary CDF (CCDF) $\bar{F}_1(r)$, as

$$F_1(r) = 1 - e^{-\lambda\pi r^2}, \quad (2.21)$$

$$\bar{F}_1(r) = e^{-\lambda\pi r^2}. \quad (2.22)$$

2.2.7 Association in PPP-Based Cellular Networks

We define the term association as the event in which a user is connected to a serving BTS to receive information signals. Association to a BTS in a PPP-based cellular network can be conditioned on its distance by simply using equations based on the CCDF defined in (2.21). We consider a connectivity model where a user connects to the BTS that supplies it with the strongest average signal power, discounting instantaneous gain from fading. For a single-tier macrocell network defined by the PPP Φ , assuming homogeneous path loss and transmission power levels, this BTS is simply the one nearest to the user. For across a BTS located at $\mathbf{x}$, at a distance $x = \|\mathbf{x}\|$ away from the user at the origin, the conditional association is given by

$$P_A(x) = \Pr(\#(b(\mathbf{o}, x) \cap \Phi_{\setminus \mathbf{x}}) = 0) = e^{-\lambda\pi x^2}, \quad (2.23)$$

where $\Phi_{\setminus \mathbf{x}}$ is the PPP excluding the point at $\mathbf{x}$. The intuition behind (2.23) is that no other BTSs can exist in the region $b(\mathbf{o}, x)$ if the user is to be associated with the BTS at $\mathbf{x}$. The association is more complex for an HCN, and we proceed to define formulae for a two-tier HCN, as this is the model used throughout the thesis. For brevity, we use abuse of language to term each BTS by its position vector.

A two-tier PPP HCN is defined by the sets Φ_1 and Φ_2 , which correspond to the tier-1 macrocell and tier-2 small-cell BTSs respectively. Each set of BTSs is characterized by their respective density, λ_1 and λ_2 . The corresponding transmission power levels of each tier are given by P_1 and P_2 . We consider a strictly

outdoor situation where path loss is distance-based and related to the path loss exponent α . For most outdoor scenarios, the value of α is within the range $2 < \alpha \leq 4$. The power received by a user from a BTS transmitting at power level P , and separated from it by distance x , is given by

$$P_{RX}(x; \alpha, P) = Px^{-\alpha}. \quad (2.24)$$

Typically, small-cell BTSs deployed outdoors are either microcells or picocells, and we choose the latter category for our research. While macrocell BTSs transmit at power levels of 40 W, picocell BTSs usually transmit within the range of 250 mW to 2W. Therefore, we can define the power ratio P_2/P_1 , where for a picocell-based two-tier HCN, $6.25 \times 10^{-3} \leq P_2/P_1 \leq 0.05$.

Based on these parameters, we transform the use to the origin, and first define the nearest BTS from the k^{th} tier as $\mathbf{x}_{k1}$, located at a distance $x_{k1} \equiv \|\mathbf{x}_{k1}\|$. From (2.21), we define PDF of distance to x_{k1} as $f_{X_{k1}} \equiv f_{X_{k1}}(x_{k1})$, such that

$$f_{X_{k1}} = 2\lambda_k \pi x_{k1} e^{-\lambda_k \pi x_{k1}^2}. \quad (2.25)$$

There are two possible association events under the defined connectivity model. If the user at the origin is associated with (or tags to) $\mathbf{x}_{k1}$, we define the event as $\mathbf{T}_{\mathbf{x}_{k1}}$. The probability of association to $\mathbf{x}_{k1}$ depends on the path loss exponent and the power ratio P_2/P_1 . Consider a user that receives equal signal power levels from both $\mathbf{x}_{11}$ and $\mathbf{x}_{21}$. If the distance x_{21} is known, the distance x_{11} can be computed using (2.24) as

$$x_{11} = x_{21}(P_1/P_2)^{-1/\alpha} = x_{21}\bar{P}_1, \quad (2.26)$$

where $\bar{P}_1 = (P_1/P_2)^{1/\alpha}$. Similarly, if x_{11} is known, $x_{21} = x_{11}\bar{P}_2$, where $\bar{P}_2 = \bar{P}_1^{-1}$. Using this result, the conditional probability of the association event $\mathbf{T}_{\mathbf{x}_{k1}}$ is given by

$$P_A(\mathbf{T}_{\mathbf{x}_{k1}} | X_{k1}) = \exp(-\lambda_{k'} |b(\mathbf{o}, x_{k1}\bar{P}_{k'})|) = e^{-\lambda_{k'} \pi (x_{k1}\bar{P}_{k'})^2}, \quad (2.27)$$

where $|b(\mathbf{o}, x_{k1} \bar{P}_{k'})|$ is the volume of the ball $b(\mathbf{o}, x_{k1} \bar{P}_{k'})$, and k' is the opposing tier. The intuition behind (2.27) is that no BTSs from the tier- k' must lie within the ball $b(\mathbf{o}, x_{k1} \bar{P}_{k'})$ if the user is to associate with $\mathbf{x}_{k1}$. This is equivalent to the CCDF of distance to the nearest BTS from tier-2, $\bar{F}_{X_{k'1}}(x_{k1} \bar{P}_{k'})$. By defining the joint association distribution, $f_A(\mathbf{T}_{\mathbf{x}_{k1}}, X_{k1})$ as

$$f_A(\mathbf{T}_{\mathbf{x}_{k1}}, X_{k1}) = f_{X_{k1}} P_A(\mathbf{T}_{\mathbf{x}_{k1}} | X_{k1}) = 2\lambda_k \pi x_{k1} e^{-(\lambda_k \pi x_{k1}^2 + \lambda_{k'} \pi (x_{k1} \bar{P}_{k'})^2)}, \quad (2.28)$$

we can define the overall association to either tier for an infinite space as

$$\begin{aligned} P_A(\mathbf{T}_{\mathbf{x}_{k1}}) &= \int_0^\infty f_A(\mathbf{T}_{\mathbf{x}_{k1}}, X_{k1}) dx_{k1} = \int_0^\infty \left[\frac{-\lambda_k e^{-\pi x_{k1}^2 (\lambda_k + \lambda_{k'} \bar{P}_{k'}^2)}}{(\lambda_k + \lambda_{k'} \bar{P}_{k'}^2)} \right] \\ &= \frac{\lambda_k}{\lambda_k + \lambda_{k'} \bar{P}_{k'}^2}. \end{aligned} \quad (2.29)$$

Figure 2.2 shows the association probability of a two-tier HCN depending on various parameters.

Figure 2.2 Association of users to tier-1 BTSs when the density of tier-1 BTSs is $\lambda_1 = 2 \text{ km}^{-2}$.

Figure 2.2 shows that users associate with tier-1 BTSs more under conditions of greater path loss, i.e. when the path loss exponent is higher, and when the power levels of the tier-2 BTSs are lower. We now proceed to look at coverage based on SIR-threshold for a PPP.

2.2.8 Downlink SIR-Based Coverage in PPP HCNs

The instantaneous SIR at a user connecting to tier- k of a two-tier HCN, under the stipulated connectivity scheme in subsection 2.2.7, is given by

$$SIR(X_{k1}) = \frac{P_k h_{\mathbf{x}_{k1}} X_{k1}^{-\alpha}}{\left(\sum_{j=1}^2 \sum_{\mathbf{x}_{ji} \in \Phi_j \setminus \mathbf{x}_{k1}} P_j h_{\mathbf{x}_{ji}} X_{ji}^{-\alpha} \right)}, \quad (2.30)$$

where $h_{\mathbf{x}_{ji}}$ is the instantaneous fading gain that affects the signal received at the user from the interference BTS at $\mathbf{x}_{ji}$, while $h_{\mathbf{x}_{k1}}$ affects the serving BTS's signal. For Rayleigh fading, this gain is well approximated as an exponential random variable, such that $h_{\mathbf{x}_{ji}} \equiv h \sim \exp(\mu)$, where μ is the mean gain [38], [48]. By substituting $\mu = 1$, and accounting for the total interference in the denominator of (2.30) by the term I , a tractable expression for the coverage, conditioned on distance to serving BTS, exists. A user connecting to tier- k is in coverage if its instantaneous SIR exceeds a specified threshold, β_k . As such, a user is in coverage if the condition $P_k h_{\mathbf{x}_{k1}} x_{k1}^{-\alpha} \geq \beta_k I$ holds. As such, we can rearrange terms in this condition and use the CCDF of an exponential distribution to formally define the conditional coverage probability for a PPP HCN. Denoting the coverage event as $\mathcal{C}_{\mathbf{x}_{k1}}$,

$$P_{c,k}^{\text{PPP}}(\mathcal{C}_{\mathbf{x}_{k1}} | X_{k1}, \mathbf{T}_{\mathbf{x}_{k1}}) = \Pr\left(h_{\mathbf{x}_{k1}} \geq \frac{\beta_k x_{k1}^\alpha I}{P_k}\right) = \mathbb{E}_I \exp\left(-\frac{\beta_k x_{k1}^\alpha I}{P_k}\right). \quad (2.31)$$

We consider a full-buffer traffic system where users are ready to transmit in all time slots and use up all subcarriers in an orthogonal system. As such, the system can be modelled as carrying one frequency. In order to obtain the expectation over the total interference I , we first split the interference per tier. As such, (2.31) is rewritten as

$$\begin{aligned}
P_{c,k}^{\text{PPP}}(\mathbf{C}_{\mathbf{x}_{k1}} | X_{k1}, \mathbf{T}_{\mathbf{x}_{k1}}) &= \mathbb{E}_I \exp \left(- \frac{\beta_k x_{k1}^\alpha}{P_k} \sum_{j=1}^2 P_j I_j \right) \\
&= \prod_{j=1}^2 \mathbb{E}_{I_j} \exp \left(- \frac{\beta_k x_{k1}^\alpha P_j I_j}{P_k} \right).
\end{aligned} \tag{2.32}$$

By characterizing the interfering signals as Rayleigh faded with gain g , and attenuated according to the path loss exponent, this becomes

$$\begin{aligned}
P_{c,k}^{\text{PPP}}(\mathbf{C}_{\mathbf{x}_{k1}} | X_{k1}, \mathbf{T}_{\mathbf{x}_{k1}}) &= \mathbb{E} \left[\prod_{j=1}^2 \left(\prod_{\mathbf{x}_{ji} \in \Phi_j \setminus (\tilde{A}_1 \cap \Phi_j)} \mathbb{E}_g \exp \left(- \frac{P_j \beta_k x_{k1}^\alpha g x_{ji}^{-\alpha}}{P_k} \right) \right) \right],
\end{aligned} \tag{2.33}$$

where $\tilde{A}_1 = b(\mathbf{o}, x_{k1} (P_j/P_k)^{1/\alpha})$, because no interfering BTS can exist closer to the user than the serving BTS. Solving the outside expectation in (2.33) requires use of the probability generating functional (PGFL) of the 2-d PPP characterized by density λ , which when taken in one dimension (to be a function of distance), is [50]

$$G(v) = \exp \left(-2\lambda\pi x \int_{\mathbb{R}^2} (1 - v(x)) dx \right), \tag{2.34}$$

for a function $v(x)$ applied over the PPP. Therefore, by applying the PGFL to the interior product in (2.33), and considering an infinite network, we rewrite (2.33) in terms of the product of Laplace transforms

$$P_{c,k}^{\text{PPP}}(\mathbf{C}_{\mathbf{x}_{k1}} | X_{k1}, \mathbf{T}_{\mathbf{x}_{k1}}) = \prod_{j=1}^2 \mathcal{L}_j^{x_{k1}}, \tag{2.35}$$

where

$$\mathcal{L}_j^{\mathbf{x}_{k1}} = \exp \left(-2\lambda_j \pi r \int_{x_{k1}(P_j/P_k)^{1/\alpha}}^{\infty} \left(1 - \mathbb{E}_g \exp \left(-\frac{P_j \beta_k x_{k1}^\alpha g r^{-\alpha}}{P_k} \right) \right) dr \right). \quad (2.36)$$

Taking expectation over g for the subtracted inside the integral in (2.36), we get

$$\mathcal{L}_j^{\mathbf{x}_{k1}} = \exp \left(-2\lambda_j \pi r \int_{x_{k1}(P_j/P_k)^{1/\alpha}}^{\infty} \frac{s_k P_j r^{1-\alpha}}{1 + s_k P_j r^{-\alpha}} dr \right), \quad (2.37)$$

where $s_k = \beta_k x_{k1}^\alpha / P_k$. Solving the integral in (2.37) yields

$$\mathcal{L}_j^{\mathbf{x}_{k1}} = \exp \left(-\lambda_j \pi \tau_j^{\mathbf{x}_{k1}} \left(x_{k1} (P_j/P_k)^{\frac{1}{\alpha}} \right) \right), \quad (2.38)$$

where $\tau_j^{\mathbf{x}_{k1}}(D)$ is a function of minimum distance to interferers and given by

$$\tau_j^{\mathbf{x}_{k1}}(D) = \frac{(s_k P_j)^{\frac{2}{\alpha}}}{\text{sinc}(2\pi/\alpha)} - D^2 {}_2F_1 \left(1, \frac{2}{\alpha}; \frac{\alpha+2}{\alpha}; -\frac{D^\alpha}{s_k P_j} \right), \quad (2.39)$$

where the subtracted term is the hypergeometric function of the parameters in the parenthesis. Finally, by substituting $D = x_{k1} (P_j/P_k)^{1/\alpha}$, we can rewrite (2.38) for the two-tier PPP HCN as

$$\begin{aligned} \mathcal{L}_j^{\mathbf{x}_{k1}} &= \exp \left(-\lambda_j \pi x_{k1}^2 \left(\frac{P_j}{P_k} \right)^{\frac{2}{\alpha}} \left(\frac{(\beta_k)^{\frac{2}{\alpha}}}{\text{sinc}(2\pi/\alpha)} - {}_2F_1 \left(1, \frac{2}{\alpha}; \frac{\alpha+2}{\alpha}; -\frac{1}{\beta_k} \right) \right) \right). \end{aligned} \quad (2.40)$$

Equations (2.31) to (2.40) characterize conditional coverage probability. In order to obtain the overall coverage of the network, we simply uncondition using the joint

association distribution in (2.28). As such the coverage probability of a user connecting to tier- k is given by

$$P_{c,k}^{\text{PPP}} = \int_0^\infty f_A(\mathbf{T}_{\mathbf{x}_{k1}}, X_{k1}) \left(\prod_{j=1}^2 \mathcal{L}_j^{\mathbf{x}_{k1}} \right) d\mathbf{x}_{k1}. \quad (2.41)$$

In a two-tier HCN, it is worth noting that $j = k$ or k' . Therefore, $(P_j/P_k)^{1/\alpha}$ is equal to 1 or $\bar{P}_k$ respectively. Substituting accordingly in (2.40), and solving the integral in (2.41) leads to

$$P_{c,k}^{\text{PPP}} = \frac{\lambda_k}{\lambda_k + \lambda_{k'} \bar{P}_k^2} \left(\frac{(\beta_k)^{\frac{2}{\alpha}}}{\text{sinc}(2\pi/\alpha)} - {}_2F_1\left(1, \frac{2}{\alpha}; \frac{\alpha+2}{\alpha}; -\frac{1}{\beta_k}\right) + 1 \right)^{-1}. \quad (2.42)$$

We do not present the detailed steps of solving the integral for the sake of maintaining brevity. The overall coverage of the two-tier HCN is simply the sum of $\sum_{k=1}^2 P_{c,k}^{\text{PPP}}$. Figures 2.3 to 2.5 shows the coverage probability of a two-tier HCN under different conditions.

Figure 2.3 Coverage probability of a two-tier HCN vs. universal SIR threshold under various α , P_2 , and λ_2 in km^{-2} , where $\lambda_1 = 2 \text{ km}^{-2}$.

Figure 2.3 shows that coverage of a two-tier HCN only depends on path loss, and is better under conditions of greater path loss, i.e. when α is greater. Under a fixed value of α , the coverage probability of the HCN remains equal irrespective of the relative densities of BTSs across each tier, and their corresponding transmission levels. We now look at two cases where the SIR threshold is varied across one tier, while remaining fixed for the other one.

Figure 2.4: Coverage probability of a two-tier HCN vs. tier-2 SIR threshold under various α , P_2 , and λ_2 in km^{-2} , where $\lambda_1 = 2 \text{ km}^{-2}$ and $\beta_1 = 0 \text{ dB}$.

Figure 2.4 shows that for increased tier-2 SIR threshold, the HCN performs better under a reduced ratio of P_2/P_1 . This is a factor of association in the HCN, as more users tend to be served by tier-1 BTSs under such a setting. Conversely, we look at the case for which β_2 is fixed and β_1 is varied.

Figure 2.5: Coverage probability of a two-tier HCN vs. tier-1 SIR threshold under various α , P_2 , and λ_2 in km^{-2} , where $\lambda_1 = 2 \text{ km}^{-2}$ and $\beta_2 = 0 \text{ dB}$.

Figure 2.5 shows that the reverse occurs when tier-1 SIR threshold is increased, an increased ratio of P_2/P_1 works better for the HCN in terms of coverage. Overall, the HCN performs better under increased α , i.e. greater path loss conditions.

The probabilities presented are much lower than what cellular network operators would be satisfied with, but this is because of the assumptions of single-frequency network with a full buffer of user traffic. In reality, techniques such as ICIC and resource portioning can be performed to improve coverage because users do not need to receive information all the time. Furthermore, users tend to be correlated and located closer to BTSs, which naturally enhances the coverage of a system.

While the computation of coverage is straightforward in a PPP based HCN, and leads to a closed form mathematical formula, it is important to note that this is usually not possible when considering separation-based models such as the PHP. Instead, we borrow the intuition by which coverage probability is calculated, i.e. taking the product of distance distribution to serving BTS, conditional association and coverage, and then integrating over the range of possible distances that can be taken by users. This range is infinite when computing uniform users that are independent of the BTSs but can be limited when spatial coupling between the users and BTSs exists.

The conditional coverage can be used to compute the conditional Shannon rate of users, and consequently, the average Shannon rate for the entire network. For a received SIR value, the Shannon rate of a user in bits/s/Hz is given by

$$\gamma = \log_2(1 + \text{SIR}). \quad (2.43)$$

For the sake of brevity, we do not present the link between conditional coverage and rate in this section, but defer this to Chapter 4, where it is used to compute rate and ASE of the modelled HCN.

2.2.9 Poisson Cluster Processes

A Poisson cluster process (PCP) formed by a PPP of parent points, and a set of distance-conditioned Poisson number of daughter points for each parent point. If the density of the parent points is λ_p , and the density of daughter points is $\bar{c}$, the density of the PCP is $\lambda_p \bar{c}$. We discuss two main types of PCPs in this thesis and apply them to user models in Chapters 3 and 5.

A. Thomas Cluster Processes

A Thomas cluster process (TCP) has daughter points scattered symmetrically around the parent point according to a Gaussian distribution. Therefore, the distribution of the location of a daughter point, $\mathbf{x}_c$, is given by [50], [52], [100]

$$f_{\mathbf{x}_c}(\mathbf{x}_c) = \frac{1}{2\pi\sigma_c^2} \exp\left(-\frac{\|\mathbf{x}_c\|^2}{2\sigma_c^2}\right), \quad (2.44)$$

where $\mathbf{x}_c$ is the distance between the daughter point and the parent point, where the latter is assumed to be at the origin. The relative spread of daughter points in relation to the central parent is characterized by the variance σ_c^2 .

B. Matérn Cluster Processes

The Matérn Cluster process (MCP) is also composed of a homogeneous PPP parent process, with the daughters constrained within a radius, R_c , around each parent. As such, the distribution of $\mathbf{x}_c$ is given by [50], [52], [100]

$$f_{\mathbf{x}_c}(\mathbf{x}_c) = \begin{cases} 1/(\pi R_c^2) & \text{if } \|\mathbf{x}_c\| \leq R_c \\ 0 & \text{otherwise.} \end{cases} \quad (2.45)$$

2.2.10 Matérn Hardcore Process

Hardcore point processes contain points that are forbidden to lie within a specified minimum distance of each other. We look at the Matérn Hardcore Process (MHP), which is created from a base PPP. Two well-known variations of the MHP exist.

The MHP Type-I is modelled by drawing out a base PPP, Φ_b of density λ_b . All points that are within the specified minimum distance, δ , of each other, are removed to yield the set of MHP Type-I points, $\Phi_{\bar{m}}$, which is formally defined as [50]

$$\Phi_{\bar{m}} = \{\mathbf{x}_{\bar{m}} \in \Phi_b : \|\mathbf{x}_{\bar{m}} - \mathbf{y}_{\bar{m}}\| \geq \delta \forall \mathbf{y}_{\bar{m}} \in \Phi_b\}. \quad (2.46)$$

The probability of a point $\mathbf{x}_{\bar{m}} \in \Phi_b$ being retained in $\Phi_{\bar{m}}$ is equivalent to the probability that no points from the base PPP lie in the ball $b(\mathbf{x}_{\bar{m}}, \delta)$. This is equal to $e^{-\lambda_b \pi \delta^2}$, and therefore the resulting density of $\Phi_{\bar{m}}$ is $\lambda_{\bar{m}} = \lambda_b e^{-\lambda_b \pi \delta^2}$. Eliminating all points that violate the distance condition imposed by δ can be wasteful as a subset of the eliminated points can still be retained while the resulting point process adheres to the said condition. Fortunately, the MHP Type-II exists as a tractable model that yields a higher density of points.

In an MHP Type-II, each point from the base PPP, Φ_b , is assigned a uniformly random mark. Every point that does not lie within the distance δ of another carrying a lower (or higher) mark is retained to yield the MHP Type-II set, $\Phi_{\bar{m}}$. Formally [50],

$$\Phi_{\bar{m}} = \{\mathbf{x}_{\bar{m}} \in \Phi_b : m(\mathbf{x}_{\bar{m}}) < m(\mathbf{y}_{\bar{m}}) \forall \mathbf{y}_{\bar{m}} \in \Phi_b \cap b(\mathbf{x}_{\bar{m}}, \delta)\}. \quad (2.47)$$

The resulting density of $\Phi_{\bar{m}}$ is found by unconditioning over the uniform mark to yield

$$\lambda_{\bar{m}} = \frac{1 - e^{-\lambda_b \pi \delta^2}}{\pi \delta^2}. \quad (2.48)$$

The work in [85] shows that surrounding interference by BTSs in an MHP Type-II can be modelled as a PPP of equivalent density, $\lambda_{\bar{m}}$. For this reason, use this point

process to model separations between BTSs of the same tier in an HCN yields tractable analysis for coverage based on the PGFL of the PPP.

2.2.11 Poisson Hole Process

A Poisson hole process (PHP) is modelled by a base PPP, Φ_b , and a hole PPP, Φ_h . Points are retained in the PHP from Φ_b if they are separated from every point in the hole PPP. We denote this minimum separation by δ to maintain continuity, and formally define the PHP as

$$\Phi_r = \{\mathbf{x}_r \in \Phi_b: \|\mathbf{x}_r - \mathbf{x}_h\| \geq \delta \forall \mathbf{x}_h \in \Phi_h\}. \quad (2.49)$$

If the densities of Φ_b and Φ_h are λ_b and λ_h respectively, the density of Φ_r is $\lambda_r = \lambda_b e^{-\lambda_h \pi \delta^2}$, based on the fact that a point, $\mathbf{x}_r$, is retained in the PPP if $\#(b(\mathbf{x}_r, \delta) \cap \Phi_h) = 0$.

We now proceed to main technical sections of this thesis, where we combine some point process models, and use the intuition provided in this section to obtain important results for distance distributions, association and coverage. Related parameters such as rate and ASE are also discussed.

Chapter 3

Distance and Association in Separation-Based Two-Tier HCNs with Mixed User Distributions

3.1 Scope of Work

This chapter provides insights into a two-tier HCN model comprising tier-1 macrocell BTSs separated from tier-2 picocell BTSs, according to a cross-tier separation parameter (also known as the inter-tier separation in some literature). We examine a situation in which users are clustered around central points in interest termed social attractors (SAs) to form hotspots, that are modelled by PCPs. Each set of clustered users is potentially served by a tier-2 BTS intended for its SA. We introduce the concept of accounting for a naturally occurring random offset between the position of a potential tier-2 BTS deployment and its SA, as a novel modelling aspect. In related work presented in [100], the deployed tier-2 BTSs (for the two-tier case) are modelled at the centre of each hotspot, i.e. exactly at the SA. We contend that this may not always be practically possible due to installation constraints. For example, if a user hotspot that can be approximated by a PCP exists in a coffee shop, it may not always be possible to deploy the tier-2 BTS at the centre of the coffee shop due to impeding structures such as existing electrical wiring. Hence, some users may exist further away from the BTS than if a centralized deployment was possible. Therefore, it is worth studying what happens to the performance of an HCN when BTSs cannot be deployed perfectly at the centre of the approximated hotspot. Unlike the work in [96]-[99], the random offset between the tier-2 BTS and the SA can be generalized mathematically in our model and does not need to be arbitrarily fixed at a constant value. Subsequently, the model is completed by imposing the cross-tier BTS separation that limits the deployment (or

activation) of tier-2 BTSs. Unlike the HCN model in [100] that features clustered users, the proportion of hotspot users tagging to either tier-1 or tier-2 BTSs is influenced by the cross-tier separation parameter. The analysis in this chapter begins with equations for distance distributions to the serving BTS as well as nearest interferers across both tiers. Following this, we derive expressions for user association under scenarios in which the HCN is not resource constrained. Computation of these parameters requires the consideration of novel approximated topologies describing the relationships between clustered users to different serving BTSs in the network. Some of the challenges solved in this body of work including identifying key integration limits to compute distance PDFs and association probabilities. The work in this chapter is then rounded off with some numerical results that validate the analysis presented.

3.2 System Entities and Models

3.2.1 SAs and Clustered Users

This model considers users that are densely clustered around specific locations of significance, which are known as SAs, a phenomenon that is commonly witnessed in hotspots such as parks, office spaces and coffee shops [100]. The set of SAs, Φ_{SA} , is a PPP characterized by density λ_{SA} , and hence the clustered users may be modeled using an appropriate PCP, for which the SAs act as the centers of each cluster. The number of users per cluster is Poisson distributed with mean $\bar{c}_U$, and the probability of a clustered user existing at a location, $\mathbf{z}_U$, depends on its distance, $\|\mathbf{z}_U\|$, from the SA. The complete set of clustered users is described by the set Ψ_2 , of density of users $\rho_2 = \lambda_{SA}\bar{c}_U$. We follow the models outlined in subsection 2.2.9 and showcase two specific tractable cases of clustered user distribution.

A. Case 1 – Matérn Cluster Process (MCP)

Users are uniformly distributed within a fixed radius, R_U , around the SAs such that [50], [52], [100]

$$f_{\mathbf{z}_U}(\mathbf{z}_U) = \begin{cases} 1/(\pi R_U^2) & \text{if } \|\mathbf{z}_U\| \leq R_U \\ 0 & \text{otherwise.} \end{cases} \quad (3.1)$$

B. Case 2 – Truncated Thomas Cluster Process (TTCP)

Users are symmetrically scattered according to a Gaussian distribution, characterized by variance σ_U^2 , but limited by the radius R_U such that

$$f_{\mathbf{z}_U}(\mathbf{z}_U) = \begin{cases} \frac{e^{-\|\mathbf{z}_U\|^2/(2\sigma_U^2)}}{2\pi\sigma_U^2(1 - e^{-R_U^2/(2\sigma_U^2)})} & \text{if } \|\mathbf{z}_U\| \leq R_U \\ 0 & \text{otherwise.} \end{cases} \quad (3.2)$$

The limitation imposed by R_U in (3.2) is useful for practical reasons, as it forces dependency between the user cluster and its SA. It is useful to understand the range of distances that can be taken by R_U to model a practical hotspot. As a hotspot represents a condensed space with many users [100], we contend that the maximum distance, R_U , between an SA and a constituent user, must be much smaller than the distance between a conventional user and its nearest macrocell BTS; otherwise, a macrocell deployment would suffice to cover this hotspot. Therefore, we examine the PDF of distance from a typical user to its nearest macrocell BTS under different practical macrocell densities. The PDF follows the standard equation presented in (2.20), and is equal to $2\lambda_1\pi x_{11}e^{-\lambda_1\pi x_{11}^2}$, where λ_1 is the density of macrocell BTSs and x_{11} is the distance to the nearest macrocell BTS. Figure 3.1 showcases this distance PDF below.

Figure 3.1: PDF of distance from typical user to nearest macrocell BTS under different macrocell density settings, where the density is given by λ_1 .

The distance PDFs in Fig. 3.1 show that in a dense macrocell network, where the density of BTSs is $\lambda_1 = 2 \text{ km}^{-2}$, a user is separated from the nearest BTS with greatest probability by 150 to 250 m. As such, a practical hotspot would contain condensed users with a smaller separation from the SA, whose location is used as a reference point to potentially deploy a tier-2 BTS in this model, as described in subsection 3.2.2. Therefore, we contend that a suitable range for R_U is $20 \text{ m} \leq R_U \leq 100 \text{ m}$.

We now focus our attention on the TTCP in case 2, as described in (3.2), which is a modification of the standard TCP described in [50], [52] and [100]. To explain the use of R_U to restrict the distance of users from the SA in a TTCP, we look at the distribution of user locations following a standard Gaussian distribution according to the standard TCP, which is outlined in (2.44). The standard TCP does not consider the limitation imposed by R_U , and user locations that follow this model are distributed according to $f_{\mathbf{z}_U}(\mathbf{z}_U)$, where [50], [52], [100]

$$f_{\mathbf{z}_U}(\mathbf{z}_U) = \frac{e^{-\|\mathbf{z}_U\|^2/(2\sigma_U^2)}}{2\pi\sigma_U^2} \quad (3.3)$$

Examining (3.3), we understand that a user may exist at an unnaturally large distance from the SA with some probability, depending on the magnitude of σ_U^2 . It is hard to justify that such a user belongs to the hotspot, and therefore, we use the TTCP described in (3.2) to characterize the locations of users instead. In order to understand this better, we refer to Figures 3.2 and 3.3, which showcase the difference between the TTCP and TCP.

Figure 3.2: Users (green dots) surrounding an SA at the origin (red square) according to the TTCP distribution under case 2 described in equation (3.2). The TTCP is characterized by $\sigma_U = 50$ m and $R_{BTS} = 100$ m.

Figure 3.3: Users (green dots) surrounding an SA at the origin (red square) according to the TCP distribution described in equation (3.3). The TTCP is characterized by $\sigma_U = 50$ m.

Figure 3.2 shows that users modelled according to a TTCP characterized by $\sigma_U = 50$ m and $R_{BTS} = 100$ m are heavily clustered at distances near 50 m from the SA, and that no user exists beyond 100 m from the SA. Figure 3.3 shows that users modelled according to the TCP similarly show a lot of users clustered near 50 m from the SA, but that a few users from this model may lie far beyond 100 m from the SA. We contend that such users exist too far from the SA to be considered part of the hotspot. Therefore, we use the modified TTCP for our model.

It is also worth examining the distance from the SA to the typical user when the clustered users are modelled under case 1 and case 2. This gives us insights into how these models mirror a practical situation. Equation (3.1) shows us that all users are distributed uniformly around the SA within a radius of R_U under case 1, which is modelled by the MCP. We examine the PDF of distance from the user to the SA to understand where most users exist in the hotspot. In order to obtain the distance PDF,

we first transform the SA to the origin and define the random cartesian coordinates of a user located at $\mathbf{z}_U$ as (Q_1, Q_2) . We define the variable distance from the user to the SA as $Z_U = \|\mathbf{z}_U\|$. Therefore, by defining the variable angle Ω between the user and the SA, where $0 \leq \Omega \leq 2\pi$, we note that the cartesian coordinates can be written as

$$Q_1 = Z_U \cos(\Omega), \quad (3.4)$$

and

$$Q_2 = Z_U \sin(\Omega). \quad (3.5)$$

Using standard techniques, we may convert the cartesian distributions $f_{\mathbf{z}_U}(\mathbf{z}_U)$ given in (3.1) and (3.2) to the polar coordinate distribution $f_{Z_U, \Omega}(Z_U, \omega)$, such that

$$f_{Z_U, \Omega}(Z_U, \omega) = f_{Q_1, Q_2}(q_1, q_2) \left| \partial \left(\frac{q_1, q_2}{Z_U, \omega} \right) \right| = f_{\mathbf{z}_U}(\mathbf{z}_U) \left| \partial \left(\frac{q_1, q_2}{Z_U, \omega} \right) \right|, \quad (3.6)$$

where the determinant is given by

$$\left| \partial \left(\frac{q_1, q_2}{Z_U, \omega} \right) \right| = \left| \begin{bmatrix} \frac{\partial q_1}{\partial Z_U} & \frac{\partial q_1}{\partial \omega} \\ \frac{\partial q_2}{\partial Z_U} & \frac{\partial q_2}{\partial \omega} \end{bmatrix} \right|. \quad (3.7)$$

Solving for (3.7) by using the definition of Q_1 and Q_2 in (3.4) and (3.5), we get

$$\begin{aligned} \left| \partial \left(\frac{q_1, q_2}{Z_U, \omega} \right) \right| &= \cos(\omega) Z_U \cos(\omega) - (-\sin(\omega)) Z_U \sin(\omega) \\ &= Z_U (\cos^2(\omega) + \sin^2(\omega)) = Z_U. \end{aligned} \quad (3.8)$$

Substituting the result in (3.8) into (3.6), we get the joint PDF, $f_{Z_U, \Omega}(Z_U, \omega)$ of distance from the hotspot user to the SA. In order to get the marginal density

function $f_{Z_U}(z_U)$, we integrate with respect to ω over its domain. Therefore, the distance PDF $f_{Z_U}(z_U)$ is solved as

$$f_{Z_U}(z_U) = \int_0^{2\pi} z_U f_{\mathbf{Z}_U}(\mathbf{z}_U) d\omega = 2\pi z_U f_{\mathbf{Z}_U}(\mathbf{z}_U). \quad (3.9)$$

Accordingly, for case 1,

$$f_{Z_U}(z_U) = 2z_U/R_U^2, \text{ for } \|\mathbf{z}_U\| \leq R_U, \quad (3.10)$$

and for case 2,

$$f_{Z_U}(z_U) = \frac{z_U e^{-\|\mathbf{z}_U\|^2/(2\sigma_U^2)}}{\sigma_U^2(1 - e^{-R_U^2/(2\sigma_U^2)})}, \text{ for } \|\mathbf{z}_U\| \leq R_U. \quad (3.11)$$

Numerical analysis of the computed distance PDFs in (3.10) and (3.11) for either case of clustered users is useful in gaining insights into the characteristics of each distribution. For this purpose, we use Figure 3.4.

Figure 3.4: PDF of distance from a clustered user to its SA for different values of R_U under the MCP model described by case 1, and different values of σ_U when $R_U = 100$ m under the TTCP model described by case 2.

Figure 3.4 shows that for users distributed uniformly according to the MCP under case 1, probability of users existing at a distance $z_U \leq R_U$ increases linearly according to distance, therefore most users will exist near the edges of the hotspot in this model. However, it must be noted that users exist with equal density throughout the hotspot.

For the users distributed according to the TTCF, the standard deviation σ_U dictates where most of the users exist, with most of them existing at a distance which is near the value of σ_U . Therefore, when $\sigma_U = 25$ m while $R_U = 100$ m, most users will exist about a quarter of the distance to the edge of the hotspot, while when $\sigma_U = 50$ m, most users will exist at about half the distance to the edge of the hotspot. When $\sigma_U = R_U = 100$ m under case 2, most users exist near the circular edge of the hotspot. The distribution of users in this scenario is very close to the distribution of users under case 1, when $R_U = 100$ m.

Using these results, we contend that if users are distributed uniformly throughout a hotspot that can be modelled by a circular region, they may consequently be modelled by using case 1 with an appropriate radius, R_U , or by case 2, when $\sigma_U = R_U$. If it is expected that most users will congregate at a specific distance from the SA within the hotspot, case 2 can be used to model the users by setting an appropriate value of σ_U .

3.2.2 Two-Tier BTS Model

BTSs are categorized in two tiers, where tier-1 macrocell BTSs, and tier-2 picocell BTSs transmit at power levels denoted by P_1 and P_2 , respectively, where $P_1 > P_2$. The set of tier-1 BTSs, Φ_1 , is a PPP of density λ_1 , that is independent of Φ_{SA} . Primarily, tier-2 BTSs are planned to cater for users belonging to Ψ_2 , and therefore, *potential* tier-2 BTS points from the set $\Phi_{2,b}$ should exhibit some dependency on the location of the users. If each potential BTS is located exactly at its corresponding SA, it will be located exactly at the centre of the corresponding user cluster. However, we propose that in a realistic situation, this would not be possible due to practical installation constraints, and there would be some separation between the centre of the user cluster and the potential tier-2 BTS deployment. Furthermore, the distance between the SA and the potential tier-2 BTS is unlikely to

be constant, and instead is more likely a range of different values in a practical situation. This is because each installation site will have a different set of constraints. Therefore, in our model, we account for a distance offset between the SA and the potential tier-2 BTS that is random within a range of values.

The location of this potential tier-2 BTS by placing a *single randomly located point* within a stipulated radius, R_{BTS} around the SA. In this context, R_{BTS} is the maximum separation that is allowed between the SA and the potential tier-2 BTS. A potential tier-2 BTS, $\mathbf{x}_{2,b}$, may exist in the range $0 \leq \|\mathbf{z}_{BTS}\| \leq R_{BTS}$, where $\|\mathbf{z}_{BTS}\|$ is its distance to the SA. The potential tier-2 BTS locations, each denoted by $\mathbf{x}_{2,b}$, collectively form the set $\Phi_{2,b}$, which can be described mathematically by

$$\Phi_{2,b} := \{\mathbf{x}_{2,b} = \mathbf{x}_{SA} + \mathbf{z}_{BTS} \mid \forall \mathbf{x}_{SA} \in \Phi_{SA}\}, \quad (3.12)$$

where $\mathbf{z}_{BTS}$ is a random translation vector that displaces each realization of $\mathbf{x}_{2,b}$ from its corresponding SA, $\mathbf{x}_{SA}$, by an offset of $\|\mathbf{z}_{BTS}\|$. As a constant offset is unrealistic, we propose that the coordinates of $\mathbf{z}_{BTS}$ are randomly distributed according to the PDF $f_{\mathbf{z}_{BTS}}(\mathbf{z}_{BTS})$ that matches the distribution of the clustered users. Hence, for: (i) *Case 1*,

$$f_{\mathbf{z}_{BTS}}(\mathbf{z}_{BTS}) = \begin{cases} 1/(\pi R_{BTS}^2) & \text{if } \|\mathbf{z}_{BTS}\| \leq R_{BTS} \\ 0 & \text{otherwise,} \end{cases} \quad (3.13)$$

and (ii) *Case 2*,

$$f_{\mathbf{z}_{BTS}}(\mathbf{z}_{BTS}) = \begin{cases} \frac{e^{-\|\mathbf{z}_{BTS}\|^2/(2\sigma_{BTS}^2)}}{2\pi\sigma_{BTS}^2(1 - e^{-R_{BTS}^2/(2\sigma_{BTS}^2)})} & \text{if } \|\mathbf{z}_{BTS}\| \leq R_{BTS} \\ 0 & \text{otherwise.} \end{cases} \quad (3.14)$$

Instead of setting a range of values for the parameter R_{BTS} , we showcase the effect of different settings of R_{BTS} in the numerical results in subsections 3.7 and 4.4. This helps us understand what the maximum allowable offset between an SA and its tier-2 BTS must be in order to achieve performance thresholds in terms of coverage, or

to ensure that certain trends in probabilities of association are followed. The effect of R_{BTS} is described in greater detail in these subsections 3.7 and 4.4.

We may now define the distance between a single potential tier-2 BTS, $\mathbf{x}_{2,b}$, and its corresponding SA, as $Z_{BTS} = \|\mathbf{z}_{BTS}\|$. Accordingly, following the steps used to obtain (3.10) and (3.11) in the case of clustered users, we derive the general PDF of Z_{BTS} as

$$f_{Z_{BTS}}(z_{BTS}) = \int_0^{2\pi} z_{BTS} f_{\mathbf{z}_{BTS}}(\mathbf{z}_{BTS}) d\omega = 2\pi z_{BTS} f_{\mathbf{z}_{BTS}}(\mathbf{z}_{BTS}). \quad (3.15)$$

Accordingly, for case 1, $f_{Z_{BTS}}(z_{BTS})$ is given by

$$f_{Z_{BTS}}(z_{BTS}) = 2z_{BTS}/R_{BTS}^2, \text{ for } \|\mathbf{z}_{BTS}\| \leq R_{BTS}, \quad (3.16)$$

and for case 2, by

$$f_{Z_{BTS}}(z_{BTS}) = \frac{z_{BTS} e^{-\|\mathbf{z}_{BTS}\|^2/(2\sigma_{BTS}^2)}}{\sigma_{BTS}^2(1 - e^{-R_{BTS}^2/(2\sigma_{BTS}^2)})}, \text{ for } \|\mathbf{z}_{BTS}\| \leq R_{BTS}. \quad (3.17)$$

To understand the point process representing $\Phi_{2,b}$, the set of potential tier-2 BTSs that are offset from the SAs, we present Lemma 1, based on the stationarity property of the PPP.

Lemma 1. *The point process $\Phi_{2,b}$ is effectively equivalent to a PPP of density $\lambda_2 = \lambda_{SA}$.*

Proof: From (3.12), we note that realizations of $\mathbf{x}_{2,b}$ depend solely on their corresponding SAs, $\mathbf{x}_{SA} \in \Phi_{SA}$. It follows that all points $\mathbf{x}_{2,b} \in \Phi_{2,b}$ are independent of each other due to the inherent independence of the PPP SAs from each other. From the stationarity theorem that applies to PPPs [50], the resulting point process $\Phi_{2,b}$ is also a PPP of equal density. This is $\Phi_{2,b}$ is simply a random translation imposed on Φ_{SA} , irrespective of the distribution of the translation vector, $\mathbf{z}_{BTS}$. $\square$

We now consider the cross-tier separation that influences the deployment of tier-2 BTSs. $\|\mathbf{x}_{2,b} - \mathbf{x}_{11}\|$ is the distance from a potential location, $\mathbf{x}_{2,b}$, to the nearest tier-1 BTS. If the condition $\|\mathbf{x}_{2,b} - \mathbf{x}_{11}\| \geq \delta_{1,2}$, where $\delta_{1,2}$ is a stipulated distance parameter, is not satisfied for $\mathbf{x}_{2,b}$, and all $\mathbf{x}_{11} \in \Phi_1$, actual deployment does not occur at this location. The set of retained deployment points at which active tier-2 BTSs exist, denoted by Φ_2 , is given by

$$\Phi_2 = \{ \mathbf{x}_2 \in \Phi_{2,b}: \|\mathbf{x}_2 - \mathbf{x}_{11}\| \geq \delta_{1,2} \forall \mathbf{x}_{11} \in \Phi_1 \}. \quad (3.18)$$

From Lemma 1, it is clear that Φ_2 is accurately modelled by a PPP. Following the definition of a PHP in subsection 2.2.11, we note that Φ_2 is a PHP of density λ_2 , which is given by $\lambda_2 = \lambda_{2,b} \exp(-\lambda_1 \pi \delta_{1,2}^2)$. Therefore, based on selective deployment due to the cross-tier separation, only a portion of the SAs and their corresponding user clusters will be provisioned with a tier-2 BTS each.

Therefore, the set Ψ_2 can be split into two subsets, $\Psi_{2,A}$ and $\Psi_{2,B}$. The set of user clusters that are targeted with a tier-2 BTS is represented by $\Psi_{2,A}$, while the set of user clusters whose SA is not provisioned with a tier-2 BTS is represented by $\Psi_{2,B}$. As such, the respective densities of these users are given by $\rho_{2,A} = \lambda_2 \bar{c}_U$ and $\rho_{2,B} = (\lambda_{2,b} - \lambda_2) \bar{c}_U$. Figure 3.2 depicts the relationship between the tier-1 and tier-2 BTSs, as well as that between tier-2 BTSs and SAs. The black circles surrounding the macrocell BTSs showcase the region in which tier-2 BTSs (blue circles) cannot be deployed, while the red dotted circles around the SAs (red squares) depict the region around the SA within which the tier-2 BTS can take a random position. The diagram depicts the positions of the tier-2 BTSs as part of a PHP due to co-tier separation, as well as their offset positions from their respective SAs. We note that the SAs and tier-1 BTSs depicted here are modelled by PPPs.

Figure 3.5: Two-tier HCN model showcasing cross-tier separation between tier-1 BTSs (black triangles) and tier-2 BTSs (blue circles), where the latter has a random offset to its designated SA (red squares).

The analyses presented in subsections 3.5, 4.2, and 4.3.1 to 4.3.4 focus on spatial association and coverage probabilities for users in the two-tier HCN modelled in Chapters 3 and 4. For these sections of analysis, it is assumed that every BTS has sufficient spectral resources to accommodate all users connecting to them, under a full-buffer traffic system, where all users are ready to transmit in a given timeslot.

We may also consider the two-tier HCN where spectral resources are limited per BTS. Under a full-buffer traffic system, this means that although all users are ready to transmit in each given timeslot, each BTS has only a limited number of spectral resources to accommodate users. Under a 4G LTE system, these resources may be accounted for in terms of resource blocks (RBs), where each resource block contains a number of orthogonal subcarriers spaced 15 kHz to transmit in a 1 ms timeslot [4]. The number of resource blocks are limited depending on the bandwidth available to each BTS. For example, if the BTS transmits a downlink bandwidth of 10 MHz, 50 RBs are available for transmission. As the subcarriers are orthogonal, we can ignore intracell interference and only focus on interference when two frequency blocks overlap due to transmissions from interfering BTSs.

In subsection 3.6, we also consider the case where each BTS is afforded a limited number of RBs comprising orthogonal subcarriers, where the number of RBs per BTS is denoted by N_b . Numerical results from simulations pertaining to this scenario are found in subsection 3.7.3. The set of BTS RBs is denoted by $\mathbf{R}_b = \{b_1, b_2, \dots, b_{N_b}\}$.

3.2.3 Uniformly Distributed Independent Users SAs

In order to model a practical user scenario, we recognise that not all users will be part of hotspots, i.e. be present at specific locations due to the existence of SAs. Some users can be expected at locations that are independent of SAs [100], and more homogeneous in terms of their locations. An example of such users may be pedestrians in the streets. A uniform distribution of such users is appropriate due to the independent and homogeneous nature of their locations. As such, these users can be modelled by an independent PPP, Ψ_1 , of density ρ_1 , and some overlap may be present with clustered users. Such modelling of users has been used previously in the work presented in [38] and [100]. The combined set of users follow a mixed distribution - clustered and uniform - and is described by the set Ψ_o , of density $\rho_o = \rho_1 + \rho_2$. Figure 3.1 shows the distribution of mixed users, where clustered users are conditioned on the PPP SAs. Tier-1 macrocell BTSs have been included to show the locations of the users with respect to a PPP single-tier network.

Figure 3.6: Mixed user distributions for the user model specified in subsection 3.3.2, with clustered users (green) conditioned on SAs (red), and independent users (purples), surrounding macrocell BTSs (black triangles)

3.2.4 Connectivity Model

The HCN is assumed to be in open access in this work, and therefore, a user may freely associate with any BTS in the HCN. A typical user connects to the BTS with a free resource block that supplies it with the strongest received signal on average. We may transform the location of the origin to the user based on the stationarity of the BTSs surrounding it. Assuming a distance dependent path loss model that incorporates small-scale fading gain, the signal power received by a user located at the origin, from the i^{th} tier- k BTS, located at $\mathbf{x}_{ki}$ is given by

$$P_{RX}(\mathbf{x}_{ki}) = P_k h_{\mathbf{x}_{ki}} X_{ki}^{-\alpha}, \quad (3.19)$$

where $X_{ki} \equiv \|\mathbf{x}_{ki}\|$, is the random distance from the user to the BTS. α is the path loss exponent, and $h_{\mathbf{x}_{ki}}$ is the fading gain from a BTS located at $\mathbf{x}_{ki}$. Once again, we consider a Rayleigh fading scenario where $h_{\mathbf{x}_{ki}} \sim \exp(1)$, i.e., $h_{\mathbf{x}_{ki}}$ is exponentially distributed with parameter 1. From the path loss model defined in (3.19), the location of the k -tier BTS providing the strongest signal on average is also the nearest BTS from that tier. For the two-tier HCN considered in this model, the value of k in the

subscript of $\mathbf{x}_{ki}$ can take the values $k = 1, 2$. We now look at the range of values of i in the subscript of $\mathbf{x}_i$.

For notational simplicity, the value of i in the subscript of $\mathbf{x}_{ki}$ ranges from 1 to ∞ if $k = 1$, i.e., the nearest tier-1 BTS is located at $\mathbf{x}_{11}$, and subsequent tier-1 BTSs are found at $\mathbf{x}_{12}, \mathbf{x}_{13}$ etc. A distinction is made for a user belonging to $\Psi_{2,A}$, where a tier-2 BTS operates to serve its corresponding SA. This tier-2 BTS is located at $\mathbf{x}_{20}$. The nearest tier-2 BTS intended for a different cluster is located at $\mathbf{x}_{21}$, with the subscript i increasing in ascending order for each subsequently located tier-2 BTS. As each individual cluster is independently distributed according one of the PCP models presented, there is no distance dependency between their locations. Therefore, if a deployment intended for a different cluster is located close enough, $\|\mathbf{x}_{20}\|$ may be greater than $\|\mathbf{x}_{21}\|$ for a clustered user that belongs to $\Psi_{2,A}$.

For a user belonging to the sets $\Psi_{2,B}$ and Ψ_2 , $\mathbf{x}_{20}$ is non-existent as there is no tier-2 BTS conditioned on its location. Factoring out the average fading gain over several channel realizations as $\text{mean}(h_{\mathbf{x}_{ki}}) = 1$, the location of the k -tier BTS that provides the strongest signal is

$$\mathbf{x}_k^* = \arg \max_{\mathbf{x}_{ki} \in \Phi_k} P_k \|\mathbf{x}_{ki}\|^{-\alpha} \quad (3.20)$$

From (3.20), we note the magnitude of $P_k \|\mathbf{x}_{ki}\|^{-\alpha}$ is inversely proportional to $\|\mathbf{x}_{ki}\|$. Therefore, the strongest signal from either tier-1 is given by

$$\mathbf{x}_1^* \equiv \mathbf{x}_{11} \quad (3.21)$$

The strongest tier-2 BTS signal is supplied by $\mathbf{x}_2^*$, where

$$\mathbf{x}_2^* = \begin{cases} \mathbf{x}_{20}, & \text{if } \|\mathbf{x}_{20}\| \leq \|\mathbf{x}_{21}\| \\ \mathbf{x}_{21}, & \text{if } \|\mathbf{x}_{21}\| < \|\mathbf{x}_{20}\| \end{cases} \quad (3.22)$$

Following on, the user will connect to the BTS at location $\mathbf{x}^*$, which is selected from the two possible locations, $\mathbf{x}_k^*$, such that

$$\mathbf{x}^* = \arg \max_{\mathbf{x} \in \{\mathbf{x}_k^*\}} P_k \|\mathbf{x}\|^{-\alpha}, \quad (3.23)$$

where $\{\mathbf{x}_k^*\}$ is the set of coordinates $\mathbf{x}_k^*$, for $k = 1, 2$.

From the connectivity model, we may define the possible spatial association events $\mathbf{T}_{2,A}^{\mathbf{x}_{ki}}$, $\mathbf{T}_{2,B}^{\mathbf{x}_{ki}}$ and $\mathbf{T}_1^{\mathbf{x}_{ki}}$. These events describe the association to the BTS $\mathbf{x}_{ki}$, of the users $\mathbf{z}_{2,A} \in \Psi_{2,A}$, $\mathbf{z}_{2,B} \in \Psi_{2,B}$, and $\mathbf{z}_1 \in \Psi_1$ to the BTS respectively. Formally, by denoting $\bar{U} = 1$, $\bar{U} = 2, A$, or $\bar{U} = 2, B$, we may define

$$\mathbf{T}_{\bar{U}}^{\mathbf{x}_{ki}} = \mathbf{1} \left(\arg \max_{\mathbf{x} \in \{\mathbf{x}_k^*\}} P_k \|\mathbf{x}\|^{-\alpha} = \mathbf{x}_{ki} \right). \quad (3.24)$$

Under the case where RBs per BTS are constrained, we denote the set of free RBs for a user to connect to as $\mathbf{F}_{\mathbf{x}^*} \subset \mathbf{R}_b$. If $\mathbf{F}_{\mathbf{x}^*}$ is an empty set, i.e. $\mathbf{F}_{\mathbf{x}^*} = \emptyset$, will not associated to any BTS and therefore, is not in coverage. Therefore, the user will connect to the BTS at location $\mathbf{x}^*$, provided that that user has a free resource block for transmission. As such, under resource constraints,

$$\mathbf{x}^* = \arg \max_{\mathbf{x} \in \{\mathbf{x}_k^*\}} P_k \|\mathbf{x}\|^{-\alpha}, \text{ if } \mathbf{F}_{\mathbf{x}^*} \neq \emptyset. \quad (3.25)$$

The association event that guarantees transmission is given by $\bar{\mathbf{T}}_{\bar{U}}^{\mathbf{x}_{ki}}$, which occurs only if $\mathbf{F}_{\mathbf{x}^*} \neq \emptyset$, such that

$$\bar{\mathbf{T}}_{\bar{U}}^{\mathbf{x}_{ki}} = \mathbf{1} \left(\arg \max_{\mathbf{x} \in \{\mathbf{x}_k^*\}} P_k \|\mathbf{x}\|^{-\alpha} = \mathbf{x}_{ki} \right), \text{ if } \mathbf{F}_{\mathbf{x}_{ki}} \neq \emptyset \quad (3.26)$$

We now proceed to the discussion of important geometric notations of entities that exist when the intersection of two circles is considered. This is required to analyse the distance PDFs and probabilities of association that characterize this HCN in Section 3, and coverage analysis in Section 5. The definitions provided are also required in Section 5, for the analysis of the extended HCN BTS model that considers an additional co-tier separation.

3.3 Intersection of Circles and Geometric Entities

Section 2 contains the definition of the circle, which is the 2-d ball on $\mathbb{R}^2$. Consider a circle centred at the origin, $\mathbf{o}$, with radius R , defined by $b(\mathbf{o}, R)$. The volume of this ball is the area of the circle, given by $|b(\mathbf{o}, R)| = \pi R^2$. If we wish to obtain the volume of the ball across $\mathbb{R}^1$, in a 1-d space, we simply take the derivative with respect to R . This volume is the circumference of the circle, or the full arc of the circle subtended by an angle of 2π radians. Formally, we define this volume as $\cap(\mathbf{o}, R)$, where

$$|\cap(\mathbf{o}, R)| = \frac{d}{dR} |b(\mathbf{o}, R)| = 2\pi R. \quad (3.27)$$

It is now worth understanding the fractional areas of sectors and the length of the arcs of these sectors when two intersecting circles overlap. We first refer to the intersection of two circles in Figure 3.7.

Figure 3.7: The arcs $\cap(\mathbf{o}, R_1) \setminus b(\mathbf{x}, R_2)$ (blue dashed line), $\cap(\mathbf{o}, R_1) \cap b(\mathbf{x}, R_2)$ (black solid line), and circumference, $\cap(\mathbf{x}, R_2)$ (red solid curve), that result when two circles, $b(\mathbf{o}, R_1)$, and $b(\mathbf{x}, R_2)$ overlap. The angles $\theta_{R_2, R_1, X}$ and $\bar{\theta}_{R_2, R_1, X}$ can be used to compute $|\cap(\mathbf{o}, R_1) \cap b(\mathbf{x}, R_2)|$ and $|\cap(\mathbf{o}, R_1) \setminus b(\mathbf{x}, R_2)|$ respectively.

In Figure 3.7, we see two 2-d (or circles), $b(\mathbf{o}, R_1)$, centred at the origin, and $b(\mathbf{x}, R_2)$. The centre of the second circle is at a distance $\|\mathbf{x}\| = X$ away from the origin, and the circles overlap. In the 2-d space across $\mathbb{R}^2$, the volume of these balls is simply the area of the respective circles, given by πR_1^2 , and πR_2^2 . The circumference or full arc, $\cap(\mathbf{x}, R_2)$ of $b(\mathbf{x}, R_2)$ is depicted clearly with the red solid curve. Of interest are the respective magnitudes of the fractional arcs, $\cap(\mathbf{o}, R_1) \setminus b(\mathbf{x}, R_2)$, and $\cap(\mathbf{o}, R_1) \cap b(\mathbf{x}, R_2)$. We first formally define the arc $\cap(\mathbf{o}, R_1) \setminus b(\mathbf{x}, R_2)$, such that

$$\cap(\mathbf{o}, R_1) \setminus b(\mathbf{x}, R_2) = \cap(\mathbf{o}, R_1) - \cap(\mathbf{o}, R_1) \cap b(\mathbf{x}, R_2), \quad (3.28)$$

where $\cap(\mathbf{o}, R_1) \cap b(\mathbf{x}, R_2)$ is the part of the circumference, $\cap(\mathbf{o}, R_1)$, that intersects with the circle $b(\mathbf{x}, R_2)$.

From basic geometric theory [104], it can be observed that $\cap(\mathbf{o}, R_1) \cap b(\mathbf{x}, R_2)$ is the arc subtended by the angle $\theta_{R_2, R_1, X}$ in the circle $b(\mathbf{o}, R_1)$, where $\theta_{R_2, R_1, X}$ is a function of the distances R_1 , R_2 , and X in this context. $\theta_{R_2, R_1, X}$ is depicted clearly in Figure 3.7. Using basic geometry, $\theta_{R_2, R_1, X}$ is given by

$$\theta_{R_2, R_1, X} = \cos^{-1} \left(\frac{R_1^2 + X^2 - R_2^2}{2R_1X} \right). \quad (3.29)$$

Consequently, the magnitude of the arc $\cap(\mathbf{o}, R_1) \cap b(\mathbf{x}, R_2)$ is given by

$$|\cap(\mathbf{o}, R_1) \cap b(\mathbf{x}, R_2)| = 2\theta_{R_2, R_1, X}R_1. \quad (3.30)$$

Similarly, $\cap(\mathbf{o}, R_1) \setminus b(\mathbf{x}, R_2)$ is the arc subtended by the angle $\bar{\theta}_{R_2, R_1, X}$ in the circle $b(\mathbf{o}, R_1)$, where $\bar{\theta}_{R_2, R_1, X}$ is depicted clearly in Figure 3.7. $\bar{\theta}_{R_2, R_1, X}$ is given by

$$\bar{\theta}_{R_2, R_1, X} = \pi - \theta_{R_2, R_1, X}, \quad (3.31)$$

and consequently, the magnitude of the arc $\cap(\mathbf{o}, R_1) \setminus b(\mathbf{x}, R_2)$ is given by

$$|\cap(\mathbf{o}, R_1) \setminus b(\mathbf{x}, R_2)| = 2\bar{\theta}_{R_2, R_1, X}R_1. \quad (3.32)$$

We note that the radii R_1 and R_2 presented in this subsection can be substituted for distances such as the user cluster radius R_U or cross-tier separation $\delta_{1,2}$. Furthermore, the distance X can represent the distance between any two location points in the two-tier HCN, such as the distance between a user and an SA, or the distance between a user and the nearest BTS from a particular tier. As these parameters are variables, it is possible that under certain settings, two circles that may be used to model the analysis of important metrics either do not intersect, or fully intersect. We now analyse these situations for a formal standardized definition of the angles $\theta_{R_2, R_1, X}$ and $\bar{\theta}_{R_2, R_1, X}$, as well as the arcs $\cap(\mathbf{o}, R_1) \setminus b(\mathbf{x}, R_2)$ and $\cap(\mathbf{o}, R_1) \cap b(\mathbf{x}, R_2)$. Figure 3.8 showcases a situation in which the circles $b(\mathbf{o}, R_1)$ and $b(\mathbf{x}, R_2)$ do not intersect.

Figure 3.8: Geometric entities and their magnitudes that result when two circles, $b(\mathbf{o}, R_1)$, and $b(\mathbf{x}, R_2)$, do not intersect, i.e. when $X \geq R_1 + R_2$

Figure 3.8 shows that the two circles $b(\mathbf{o}, R_1)$, and $b(\mathbf{x}, R_2)$, will not intersect when the condition $X \geq R_1 + R_2$ is met. In this situation it is clear that the magnitude of the arc $\cap(\mathbf{o}, R_1) \setminus b(\mathbf{x}, R_2)$ is $|\cap(\mathbf{o}, R_1) \setminus b(\mathbf{x}, R_2)| = 2\pi R_1^2$. Working backwards from this magnitude, we understand from (3.31) and (3.30), that the functional angle $\bar{\theta}_{R_2, R_1, X}$ can be defined as equal to 2π . Similarly, $|\cap(\mathbf{o}, R_1) \cap b(\mathbf{x}, R_2)| = 0$, and $\theta_{R_2, R_1, X}$ can be defined as equal to 0.

We now look at the case where two circles fully intersect in Figure 3.9.

Figure 3.9: Geometric entities and their magnitudes that result when two circles, $b(\mathbf{o}, R_1)$, and $b(\mathbf{x}, R_2)$, fully intersect, i.e. when $X \geq R_2 - R_1$.

Figure 3.8 shows that the two circles $b(\mathbf{o}, R_1)$, and $b(\mathbf{x}, R_2)$, will fully intersect when the condition $X \geq R_2 - R_1$ is met. In this situation it is clear that the magnitude of the arc $\cap(\mathbf{o}, R_1) \cap b(\mathbf{x}, R_2)$ is $|\cap(\mathbf{o}, R_1) \cap b(\mathbf{x}, R_2)| = 2\pi R_1^2$. Working backwards from this magnitude, we understand from (3.31) and (3.30), that the functional angle $\theta_{R_2, R_1, X}$ can be defined as equal to 2π . Similarly, $|\cap(\mathbf{o}, R_1) \setminus b(\mathbf{x}, R_2)| = 0$, and $\bar{\theta}_{R_2, R_1, X}$ can be defined as equal to 0.

Following the definitions of the functional angles $\theta_{R_2, R_1, X}$ and $\bar{\theta}_{R_2, R_1, X}$ for the three separate cases in which the circles $b(\mathbf{o}, R_1)$ and $b(\mathbf{x}, R_2)$ partially intersect, do not intersect, and full intersect, we formalize the definition for $\theta_{R_2, R_1, X}$, such that

$$\begin{aligned} \theta_{R_2, R_1, X} &= \frac{1}{2} |\cap(\mathbf{o}, R_1) \cap b(\mathbf{x}, R_2)| \\ &= \begin{cases} \pi, & R_1 \leq R_2 - X \\ \cos^{-1} \left(\frac{R_1^2 + X^2 - R_2^2}{2R_1X} \right), & |R_2 - X| < R_1 < R_2 + X \\ 0, & \text{otherwise.} \end{cases} \end{aligned} \quad (3.33)$$

We also note that $\bar{\theta}_{R_2, R_1, X}$ can be defined formally as

$$\bar{\theta}_{R_2, R_1, X} = \pi - \theta_{R_2, R_1, X}. \quad (3.34)$$

The terms R_1 and R_2 can be simply interchanged in the formulae inside the curly bracket in (3.33) to obtain $\theta_{R_1, R_2, X} = \frac{1}{2} |\cap(\mathbf{x}, R_2) \cap b(\mathbf{o}, R_1)|$. Consequently, we note that $\bar{\theta}_{R_1, R_2, X} = \pi - \theta_{R_1, R_2, X}$.

It is also useful to understand the area of the portions of the circle $b(\mathbf{o}, R_1) \setminus b(\mathbf{x}, R_2)$, and $b(\mathbf{o}, R_1) \cap b(\mathbf{x}, R_2)$, as these measures are used with appropriate substitution of parameters to compute spatial probabilities of association that follow in Section 3, and are also seen in Section 5. The portion $b(\mathbf{o}, R_1) \setminus b(\mathbf{x}, R_2)$ of the circle $b(\mathbf{o}, R_1)$ is formally defined as

$$b(\mathbf{o}, R_1) \setminus b(\mathbf{x}, R_2) = b(\mathbf{o}, R_1) - b(\mathbf{o}, R_1) \cap b(\mathbf{x}, R_2), \quad (3.35)$$

where $b(\mathbf{o}, R_1) \cap b(\mathbf{x}, R_2)$ is the region of intersection between the circles $b(\mathbf{o}, R_1)$ and $b(\mathbf{x}, R_2)$. Therefore, $b(\mathbf{o}, R_1) \setminus b(\mathbf{x}, R_2)$ refers to the part of the circle that does not intersect with $b(\mathbf{x}, R_2)$ when an overlap between both circles is present. Under different conditions relating R_1 , R_2 , and X to one another, the magnitude of this portion is computed as

$$\begin{aligned} & |b(\mathbf{o}, R_1) \setminus b(\mathbf{x}, R_2)| \\ &= \begin{cases} 0, & R_1 \leq R_2 - X \\ \pi(R_1^2 - R_2^2), & R_1 \geq R_2 + X \\ \pi R_1^2 - |b(\mathbf{o}, R_1) \cap b(\mathbf{x}, R_2)|, & \text{otherwise if } R_1 + R_2 > X \end{cases} \end{aligned} \quad (3.36)$$

In the first scenario in (3.36) where $R_1 \leq R_2 - X$, the circles fully intersect, and therefore, $|b(\mathbf{o}, R_1) \cap b(\mathbf{x}, R_2)| = \pi R_1^2$. This scenario also describes a situation in which $R_1 \leq R_2$. In the second scenario under (3.36) where $R_1 \geq R_2 + X$, the circles also fully intersect, but the radii are related instead by $R_1 > R_2$. As such, $|b(\mathbf{o}, R_1) \cap b(\mathbf{x}, R_2)| = \pi R_2^2$.

The final scenario under (3.36) describes a situation in which the circles partially intersect. Such a situation may occur when $R_1 + R_2 > X$, irrespective of

the relative magnitudes of R_1 , R_2 and X . More specifically, this partial intersection may occur when $R_1 > R_2 - X$, if $R_2 > R_1$, or when $R_1 < R_2 + X$, if $R_1 \leq R_2$. In order to compute the magnitude $|b(\mathbf{o}, R_1) \cap b(\mathbf{x}, R_2)|$ exactly under this scenario, we use the angles $\theta_{R_2, R_1, X}$ and $\theta_{R_1, R_2, X}$ to get

$$\begin{aligned}
& |b(\mathbf{o}, R_1) \cap b(\mathbf{x}, R_2)| \\
&= R_1^2 \theta_{R_2, R_1, X} + R_2^2 \theta_{R_1, R_2, X} \\
&\quad - X \sqrt{R_2^2 - \left(\frac{R_1^2 - X^2 - R_2^2}{2X} \right)^2}.
\end{aligned} \tag{3.37}$$

Substituting (3.37) for $|b(\mathbf{o}, R_1) \cap b(\mathbf{x}, R_2)|$, we may compute the exact magnitude of $|b(\mathbf{o}, R_1) \setminus b(\mathbf{x}, R_2)|$ under the third scenario described in (3.36). In a situation where the circles $b(\mathbf{o}, R_1)$ and $b(\mathbf{x}, R_2)$ do not overlap, $|b(\mathbf{o}, R_1) \setminus b(\mathbf{x}, R_2)| = \pi R_1^2$, and $|b(\mathbf{o}, R_1) \cap b(\mathbf{x}, R_2)| = 0$.

As the clear definitions for important geometric entities, and the computations of their respective magnitudes have been successfully outlined for Sections 3.4 and 3.5 that follow henceforth in this chapter, we now proceed to the discussion of distance PDFs that characterize this HCN. We substitute the actual distances used in the computations that follow for R_1 , R_2 and X , when computing angles of the form $\theta_{R_2, R_1, X}$ and $\bar{\theta}_{R_2, R_1, X}$, or areas of the form $|b(\mathbf{o}, R_1) \setminus b(\mathbf{x}, R_2)|$ and $|b(\mathbf{o}, R_1) \cap b(\mathbf{x}, R_2)|$.

3.4 PDFs of Distances from Users to BTSs

We present expressions that describe the PDFs of distance from specific users to the nearest BTSs from each tier, which may correspondingly act as the serving BTSs for the user, or alternately act as the nearest interfering BTSs from either tier. We provide in detail the steps outlining the methods by which they can be derived.

3.4.1 PDF of Distance from Clustered Users to Potential Tier-2 BTS

The potential tier-2 BTS targeting a user cluster is located at $\bar{\mathbf{x}}_{20} \in \Phi_{2,b}$, where $\bar{\mathbf{x}}_{20}$ is randomly offset from the SA at $\mathbf{x}_{SA}$. We first define the distance, $\bar{X}_{20} = \bar{x}_{20}$, between $\bar{\mathbf{x}}_{20}$ and the user transformed to the origin, $\mathbf{z}_2 \in \Psi_2$, such that $\bar{X}_{20} \equiv \|\bar{\mathbf{x}}_{20} - \mathbf{z}_2\|$. From the definition of the random offset in (3.5) and (3.6), the distance between the SA of the cluster, and the potential BTS is $Z_{BTS} \equiv \|\mathbf{z}_{BTS}\| = \|\bar{\mathbf{x}}_{20} - \mathbf{x}_{SA}\|$. The angle formed at the vertex of the lines connecting $\mathbf{x}_{SA}$ to $\bar{\mathbf{x}}_{20}$, and $\bar{\mathbf{x}}_{20}$ to $\mathbf{z}_2$ is denoted by the random angle $\bar{\Omega} = \bar{\omega}$, where $\bar{\omega} \equiv \angle \mathbf{x}_{SA} \bar{\mathbf{x}}_{20} \mathbf{z}_2$. Figure 3.10 shows the relationship between these entities.

Figure 3.10: Relationship between SA, user from corresponding cluster, and potential tier-2 BTS.

We set $\bar{\mathbf{x}}_{20}$ at the origin, and then consider the random cartesian coordinates $\bar{Q}_1 = \bar{X}_{20} \cos(\bar{\Omega})$ and $\bar{Q}_2 = \bar{X}_{20} \sin(\bar{\Omega})$. The PDF $f_{\mathbf{z}_2}(\bar{q}_1, \bar{q}_2)$ is exactly equal to $f_{\mathbf{z}_2(\bar{\Omega})}(\mathbf{z}_2(\bar{\omega}))$, where the user's distance from the SA is $Z_2(\bar{\Omega}) = \|\mathbf{z}_2(\bar{\Omega})\|$. When $\bar{X}_{20} = \bar{x}_{20}$, $Z_{BTS} = z_{BTS}$, and $\bar{\Omega} = \bar{\omega}$, the distance $Z_2(\bar{\Omega}) = z_2(\bar{\omega})$ is given by

$$z_2(\bar{\omega}) = \sqrt{\bar{x}_{20}^2 + z_{BTS}^2 - 2\bar{x}_{20}z_{BTS}\cos(\bar{\omega})} \quad (3.38)$$

Conditioning on $Z_{BTS} = z_{BTS}$, we use standard techniques to convert $f_{\mathbf{Z}_2}(\bar{q}_1, \bar{q}_2 | z_{BTS})$ to the polar-coordinate distribution $f_{\bar{x}_{20}, \bar{\omega}}(\bar{x}_{20}, \bar{\omega} | z_{BTS})$. This is given by

$$\begin{aligned} f_{\bar{x}_{20}, \bar{\omega}}(\bar{x}_{20}, \bar{\omega} | z_{BTS}) &= f_{\mathbf{Z}_2}(\bar{q}_1, \bar{q}_2 | z_{BTS}) \left| \partial \left(\frac{\bar{q}_1, \bar{q}_2}{\bar{x}_{20}, \bar{\omega}} \right) \right| \\ &= f_{\mathbf{Z}_2}(\mathbf{z}_2(\bar{\omega}) | z_{BTS}) \left| \partial \left(\frac{\bar{q}_1, \bar{q}_2}{\bar{x}_{20}, \bar{\omega}} \right) \right|, \end{aligned} \quad (3.39)$$

where the determinant, $\left| \partial \left(\frac{\bar{q}_1, \bar{q}_2}{\bar{x}_{20}, \bar{\omega}} \right) \right|$ is given by

$$\left| \partial \left(\frac{\bar{q}_1, \bar{q}_2}{\bar{x}_{20}, \bar{\omega}} \right) \right| = \left\| \begin{bmatrix} \frac{\partial \bar{q}_1}{\partial \bar{x}_{20}} & \frac{\partial \bar{q}_1}{\partial \bar{\omega}} \\ \frac{\partial \bar{q}_2}{\partial \bar{x}_{20}} & \frac{\partial \bar{q}_2}{\partial \bar{\omega}} \end{bmatrix} \right\| \quad (3.40)$$

Solving for the determinant in (3.40), we get

$$\begin{aligned} \left| \partial \left(\frac{\bar{q}_1, \bar{q}_2}{\bar{x}_{20}, \bar{\omega}} \right) \right| &= \left\| \begin{bmatrix} \cos(\bar{\omega}) & -\bar{x}_{20}\sin(\bar{\omega}) \\ \sin(\bar{\omega}) & \bar{x}_{20}\cos(\bar{\omega}) \end{bmatrix} \right\| = \bar{x}_{20}(\cos^2(\bar{\omega}) + \sin^2(\bar{\omega})) \\ &= \bar{x}_{20}. \end{aligned} \quad (3.41)$$

Using (3.41), we may rewrite (3.39) to get

$$f_{\bar{x}_{20}, \bar{\omega}}(\bar{x}_{20}, \bar{\omega} | z_{BTS}) = \bar{x}_{20} f_{\mathbf{Z}_2}(\mathbf{z}_2(\bar{\omega}) | z_{BTS}). \quad (3.42)$$

In order to compute the PDF of $\bar{X}_{20}$ from this point, the expression in (3.42) needs to be unconditioned on z_{BTS} , and then the resulting joint distribution has to be marginalized to get $f_{\bar{x}_{20}} \equiv f_{\bar{x}_{20}}(\bar{x}_{20})$.

For the first step, we require the distribution $f_{Z_{BTS}}(z_{BTS})$, whose general form is presented in (3.15). The product of $f_{Z_{BTS}}(z_{BTS})$ in (3.15) and $f_{\bar{x}_{20}, \bar{\omega}}(\bar{x}_{20}, \bar{\omega} | z_{BTS})$

in (3.42) is the joint PDF $f_{\bar{x}_{20}, \bar{\omega}, z_{BTS}}(\bar{x}_{20}, \bar{\omega}, z_{BTS})$. Integrating this result over $\bar{\omega}$ and z_{BTS} gives the PDF $f_{\bar{x}_{20}} = f_{\bar{x}_{20}}(\bar{x}_{20})$, such that

$$f_{\bar{x}_{20}} = \int_0^{R_{BTS}} \int_0^{2\pi} 2\pi \bar{x}_{20} z_{BTS} f_{\mathbf{z}_2}(\mathbf{z}_2(\bar{\omega})|z_{BTS}) f_{\mathbf{z}_{BTS}}(\mathbf{z}_{BTS}) d\bar{\omega} dz_{BTS}, \quad (3.43)$$

for the domain of $\bar{x}_{20}$, which is $0 \leq \bar{x}_{20} \leq R_U + R_{BTS}$. Figure 3.10 shows how the maximum value of $\bar{x}_{20}$ is $R_U + R_{BTS}$.

The expression for $f_{\bar{x}_{20}}(\bar{x}_{20})$ is generalized for both cases of cluster distribution described in subsection 3.3.1 and the corresponding cases of potential BTS distribution in subsection 3.3.2. Further simplification is possible for case 1, where both the users and potential BTS are uniformly distributed around the SA within stipulated radii. We examine Fig 3.11 to understand this simplification.

We first fix the potential tier-2 BTS location, $\bar{\mathbf{x}}_{20}$ at the origin. Using simple geometry, it is observed that $f_{\mathbf{z}_2}(\mathbf{z}_2(\bar{\omega})|z_{BTS})$ is non-zero through the region $b(\mathbf{x}_{SA}, R_U) \cap b(\mathbf{o}, \bar{x}_{20})$. This implies that the range of the angle at the vertex $\mathbf{x}_{SA}\bar{\mathbf{x}}_{20}\mathbf{z}_2$ is $0 \leq \bar{\omega} \leq 2\theta_{R_U, \bar{x}_{20}, z_{BTS}}$, where $\theta_{R_U, \bar{x}_{20}, z_{BTS}}$ is a functional angle corresponding to $\theta_{R_2, R_1, X}$ in (3.33), such that R_U , $\bar{x}_{20}$, and z_{BTS} are substituted for R_1 , R_2 and X respectively.

Figure 3.11: The intersected region $b(\mathbf{x}_{SA}, R_U) \cap b(\mathbf{o}, \bar{x}_{20})$, where $\bar{\mathbf{x}}_{20} = \mathbf{o}$, and the functional angle $\theta_{R_U, \bar{x}_{20}, z_{BTS}}$.

By changing the limits to incorporate the functional angle $\theta_{R_U, \bar{x}_{20}, z_{BTS}}$, and observing the distribution of user coordinates under Case 1, we rewrite (3.41) to get

$$f_{\bar{x}_{20}} = \frac{2\bar{x}_{20}}{R_U^2} \int_0^{R_{BTS}} \int_0^{2\pi} z_{BTS} f_{\mathbf{z}_{BTS}}(\mathbf{z}_{BTS}) d\bar{\omega} dz_{BTS}. \quad (3.42)$$

Further observing that $f_{\mathbf{z}_{BTS}}(\mathbf{z}_{BTS}) = 1/(\pi R_{BTS}^2)$, and solving over the inside integral, we can simplify (3.42) further to get

$$f_{\bar{x}_{20}} = \frac{4\bar{x}_{20}}{(R_U R_{BTS})^2} \int_0^{R_{BTS}} z_{BTS} \theta_{R_U, \bar{x}_{20}, z_{BTS}} dz_{BTS}. \quad (3.43)$$

Unfortunately, the remaining integral in (3.43) cannot be presented in closed form as the integration of the functional $\theta_{R_U, \bar{x}_{20}, z_{BTS}}$ does not yield a standard result in closed form. However, computing the integral in (3.43) is a simple numerical procedure. For Case 2, the general formula is presented in (3.41), and further

simplifications are not possible. However, computing the double integral numerically is also a straightforward process.

The PDF $f_{\bar{x}_{20}}$ represents the distance between clustered users to the potential tier-2 BTS location that is conditioned on the location of the corresponding SA. The SA is provisioned with a tier-2 BTS if the potential location, $\bar{\mathbf{x}}_{20}$, is separated from its nearest corresponding tier-1 BTS by a distance greater than $\delta_{1,2}$. In this scenario, we can formalize the location $\bar{\mathbf{x}}_{20}$, as $\mathbf{x}_{20}$ instead, because an actual tier-2 BTS exists at this exact point. Consequently, the distance between a deployed tier-2 BTS and its users will follow the same distribution as that of a potential tier-2 BTS and the same users. Therefore, we may present the formalism that $f_{x_{20}} \equiv f_{\bar{x}_{20}}$, where $f_{x_{20}}$ is the PDF of distance between a user $\mathbf{z}_{2,A} \in \Psi_{2,A}$, when the cluster is provisioned a tier-2 BTS deployment at $\mathbf{x}_{20}$.

We now proceed to look at the PDF of distance to the nearest tier-1 BTS, which is important when computing parameters such as association and coverage due to the hole imposed by cross-tier separation.

3.4.2 PDF of Distance from Clustered Users to Nearest Tier-1 BTS

Another important measure required to compute association and coverage formulas is the PDF of distance from clustered users to their corresponding nearest tier-1 BTS, $\mathbf{x}_{11}$. As the cross-tier separation and the presence (or absence) of a tier-2 deployment at $\mathbf{x}_{20}$ determines the position of the nearest tier-1 interferer, we provide these PDFs as conditioned on the distance X_{20} , or $\bar{X}_{20}$.

A. Users from $\Psi_{2,A}$

For the user transformed to the origin, $\mathbf{z}_{2,A} \in \Psi_{2,A}$, the conditional PDF of distance to the nearest tier-1 BTS is denoted by $f_{X_{11}|X_{20}}^{z_{2,A}} \equiv f_{X_{11}}^{z_{2,A}}(x_{11}|x_{20})$, where the distance between the user and the BTS is denoted by X_{11} . The notation of the PDF contains the BTSs in the subscript and user in the superscript for brevity.

Transforming the origin to the user, we observe that the cross-tier separation forces the absence of tier-1 BTS can be found in the region $b(\mathbf{o}, r) \cap b(\mathbf{x}_{20}, \delta_{1,2})$, where r is an arbitrary radius. We note that tier-1 BTSs may be present instead in the region $b(\mathbf{o}, r) \setminus b(\mathbf{x}_{20}, \delta_{1,2})$. As distance is a measure in 1-d Euclidean space,

we convert to 1-d instead of 2-d to understand which portion of this space contains tier-1 BTSs. To perform this conversion to 1-d, we follow the steps outlined in (3.27), where the circumference $\cap(\mathbf{o}, r)$ is considered instead of the entire circle in 2-d, $b(\mathbf{o}, r)$.

We find that only a fraction of the circumference $\cap(\mathbf{o}, r)$ can contain tier-1 BTSs. Formally, this fractional arc is simply $\cap(\mathbf{o}, r) \setminus b(\mathbf{x}_{20}, \delta_{1,2})$, and is related to $b(\mathbf{o}, r) \setminus b(\mathbf{x}_{20}, \delta_{1,2})$ by

$$\cap(\mathbf{o}, r) \setminus b(\mathbf{x}_{20}, \delta_{1,2}) = \frac{d}{dr} b(\mathbf{o}, r) \setminus b(\mathbf{x}_{20}, \delta_{1,2}) \quad (3.44)$$

We now restate the PDF of distance to the nearest tier-1 BTS when $\delta_{1,2} = 0$. This is given by

$$f_{X_{11}|X_{20}}^{z_{2,A}} = 2\lambda_1 \pi x_{11} e^{-\lambda_1 \pi x_{11}^2}, \text{ if } \delta_{1,2} = 0 \quad (3.45)$$

We observe that the factor $2\lambda_1 \pi x_{11}$ represents the full arc, or circumference, $\cap(\mathbf{o}, x_{11})$, subtended by an angle 2π along which the nearest tier-1 BTS may exist with the probability defined by the PDF. When $\delta_{1,2} > 0$, the fractional arc that can contain the nearest tier-1 BTS is $\cap(\mathbf{o}, x_{11}) \setminus b(\mathbf{x}_{20}, \delta_{1,2})$. From the explanations leading to (3.32) in subsection 3.4, we understand that the magnitude of $\cap(\mathbf{o}, x_{11}) \setminus b(\mathbf{x}_{20}, \delta_{1,2})$ is $|\cap(\mathbf{o}, x_{11}) \setminus b(\mathbf{x}_{20}, \delta_{1,2})| = 2\bar{\theta}_{\delta_{1,2}, x_{11}, x_{20}} x_{11}$, where $\bar{\theta}_{\delta_{1,2}, x_{11}, x_{20}}$ is a functional angle corresponding to $\bar{\theta}_{R_2, R_1, X}$ in (3.34), such that $\delta_{1,2}$, x_{11} , and x_{20} are substituted for R_1 , R_2 and X respectively. Therefore, this fraction is factored instead of 2π in (3.45), yielding

$$f_{X_{11}|X_{20}}^{z_{2,A}} = 2\lambda_1 \bar{\theta}_{\delta_{1,2}, x_{11}, x_{20}} x_{11} e^{-\lambda_1 \pi x_{11}^2}. \quad (3.46)$$

Figure 3.12 shows how the angle $\bar{\theta}_{\delta_{1,2}, x_{11}, x_{20}}$ relates to $f_{X_{11}|X_{20}}^{z_{2,A}}$, and we note that the domain of X_{11} is given by $0 \leq X_{11} < \infty$.

Figure 3.12: The region $\cap(\mathbf{o}, x_{11}) \setminus b(\mathbf{x}_{20}, \delta_{1,2})$ along which the conditional PDF $f_{X_{11}|X_{20}}^{\mathbf{z}_{2,A}}$ is computed, where $|\cap(\mathbf{o}, x_{11}) \setminus b(\mathbf{x}_{20}, \delta_{1,2})| = 2\bar{\theta}_{\delta_{1,2}, x_{11}, x_{20}} x_{11}$ and the user $\mathbf{z}_{2,A}$ is found at the origin, $\mathbf{o}$.

Using standard probability theory, joint PDF $f_{X_{11}, X_{20}}^{\mathbf{z}_{2,A}}$ is now defined as

$$f_{X_{11}, X_{20}}^{\mathbf{z}_{2,A}} = f_{X_{20}} f_{X_{11}|X_{20}}^{\mathbf{z}_{2,A}} \quad (3.47)$$

B. Users from $\Psi_{2,B}$

Users from $\Psi_{2,B}$ are not afforded a tier-2 BTS conditioned on their location. This is because the candidate location for the tier-2 BTS, $\bar{\mathbf{x}}_{20}$ violates the cross-tier separation parameter, i.e. it is too close to a tier-1 BTS, $\mathbf{x}_{1i}$. For the sake of deriving the conditional PDF of distance from the user, $f_{X_{11}|\bar{X}_{20}}^{\mathbf{z}_{2,B}} \equiv f_{X_{11}|\bar{X}_{20}}^{\mathbf{z}_{2,B}}(x_{11}|\bar{x}_{20})$, we define two events, E_1 and E_2 .

E_1 describes the event in which $\mathbf{x}_{11}$ resides in the space $b(\bar{\mathbf{x}}_{20}, \delta_{1,2})$. Therefore, the BTS nearest to the user is also the one nearest to $\bar{\mathbf{x}}_{20}$. E_2 describes the event in which $\mathbf{x}_{11}$ resides outside the space $b(\bar{\mathbf{x}}_{20}, \delta_{1,2})$. In this scenario, $\mathbf{x}_{11}$ happens to be

the nearest tier-1 BTS to the user, but not the cause for voiding the candidate location, $\bar{\mathbf{x}}_{20}$. This situation implies that there are two very proximal tier-1 BTSs, which is rare due to the relative density of tier-1 BTSs. These events are disjoint to the independence property of the PPP describing tier-1 BTSs. Figure 3.13 showcases the regions along which $\mathbf{x}_{11}$ resides at a distance $X_{11} = x_{11}$ from the user at the origin, under both events, E_1 and E_2 .

Figure 3.13: The regions $\cap(\mathbf{o}, x_{11}) \cap b(\bar{\mathbf{x}}_{20}, \delta_{1,2})$ (event E_1) and $\cap(\mathbf{o}, x_{11}) \setminus b(\bar{\mathbf{x}}_{20}, \delta_{1,2})$ (event E_2), along which the tier-1 BTS, $\mathbf{x}_{11}$ may exist with respect to the user at the origin $\mathbf{o}$ (solid black circle) and the potential tier-2 BTS, $\bar{\mathbf{x}}_{20}$ (solid blue triangle), within a distance $\delta_{1,2} + \bar{x}_{20}$ (dotted magenta circle).

Figure 3.13 shows that the domain of distance between $\mathbf{x}_{11}$ and the user is given by $0 \leq X_{11} \leq \delta_{1,2} + \bar{x}_{20}$. We first focus on event E_1 . If the distance to $\mathbf{x}_{11}$ is $X_{11} = x_{11}$, then in 1-d, the nearest tier-1 BTS lies on the circumference of $b(\mathbf{o}, x_{11})$. From (3.27), the magnitude of this circumference is given by $|\cap(\mathbf{o}, x_{11})| = d/dx_{11}(|b(\mathbf{o}, x_{11})|)$. Furthermore, from the description of E_1 , $\mathbf{x}_{11}$ lies on the intersecting region $\cap(\mathbf{o}, x_{11}) \cap b(\bar{\mathbf{x}}_{20}, \delta_{1,2})$, as seen by the red-dotted arc in Figure 3.13. By examining (3.30), we know that $|\cap(\mathbf{o}, x_{11}) \cap b(\bar{\mathbf{x}}_{20}, \delta_{1,2})| = 2\theta_{\delta_{1,2}, x_{11}, x_{20}} x_{11}$, where $\theta_{\delta_{1,2}, x_{11}, x_{20}}$ is a functional angle corresponding to $\theta_{R_2, R_1, X}$ in (3.34), such that $\delta_{1,2}$, x_{11} , and x_{20} are substituted for R_1 , R_2 and X respectively. Ignoring the condition that the tier-1 BTS resides in the region $b(\bar{\mathbf{x}}_{20}, \delta_{1,2})$, we may factor this magnitude to obtain the distance PDF to $\mathbf{x}_{11}$ as $2\lambda_1 \theta_{\delta_{1,2}, x_{11}, x_{20}} x_{11} e^{-\lambda_1 \pi x_{11}^2}$, as tier-1 BTSs are located according to a standard PPP.

However, the condition $\#(\Phi_1 \cap b(\bar{\mathbf{x}}_{20}, \delta_{1,2}) > 0)$ needs to be accounted for. Noting that

$$\Pr\left(\#(\Phi_1 \cap b(\bar{\mathbf{x}}_{20}, \delta_{1,2}) > 0)\right) = 1 - e^{-\lambda_1 \pi \delta_{1,2}^2}, \quad (3.48)$$

we can write the conditional PDF of distance to $\mathbf{x}_{11}$ under the event E_1 by using Bayes theorem, such that

$$f_{X_{11}|\bar{X}_{20}, E_1}^{z_2, B} = \frac{2\lambda_1 \theta_{\delta_{1,2}, x_{11}, x_{20}} x_{11} e^{-\lambda_1 \pi x_{11}^2}}{1 - e^{-\lambda_1 \pi \delta_{1,2}^2}}. \quad (3.49)$$

We now turn our focus to event E_2 . If a tier-1 BTS is found at a distance x_{11} from the user, but outside the region $b(\bar{\mathbf{x}}_{20}, \delta_{1,2})$, then no tier-1 BTSs can be found in the region $b(\mathbf{o}, x_{11}) \cap b(\bar{\mathbf{x}}_{20}, \delta_{1,2})$. For ease in presenting the following equations, we formally define this region as

$$\tilde{A}_1 = b(\mathbf{o}, x_{11}) \cap b(\bar{\mathbf{x}}_{20}, \delta_{1,2}), \quad (3.50)$$

where the magnitude of $\tilde{A}_1$ can be computed using (3.37), by substituting $X_{11} = x_{11}$, $\delta_{1,2}$ and $\bar{X}_{20} = \bar{x}_{20}$ for R_1 , R_2 , and X respectively. However, we also know that

$b(\bar{\mathbf{x}}_{20}, \delta_{1,2})$ contains the tier-1 BTS responsible for voiding $\bar{\mathbf{x}}_{20}$. Therefore, a tier-1 BTS resides in the region

$$\tilde{A}_2 = b(\bar{\mathbf{x}}_{20}, \delta_{1,2}) \setminus b(\mathbf{o}, x_{11}), \quad (3.51)$$

where the magnitude of $\tilde{A}_2$ can be computed using (3.36) and (3.37), by substituting $\delta_{1,2}$, $X_{11} = x_{11}$ and $\bar{X}_{20} = \bar{x}_{20}$ for R_1 , R_2 , and X respectively. Therefore, E_2 is a joint event where the nearest tier-1 BTS is at a distance of x_{11} from the user, the number of points $\#(\Phi_1 \cap \tilde{A}_1 = 0)$, and $\#(\Phi_1 \cap \tilde{A}_2 > 0)$. The probabilities associated with the latter sub-events are obtained using the description of the PPP, such that

$$\Pr(\#(\Phi_1 \cap \tilde{A}_1 = 0)) = e^{-\lambda_1 |\tilde{A}_1|}, \quad (3.52)$$

and

$$\Pr(\#(\Phi_1 \cap \tilde{A}_2 > 0)) = 1 - e^{-\lambda_1 |\tilde{A}_2|}, \quad (3.53)$$

When a tier-1 BTS, $\mathbf{x}_{11}$, resides at a distance of x_{11} away from the user under event E_2 , we note that it exists outside the region $b(\mathbf{o}, x_{11}) \setminus b(\bar{\mathbf{x}}_{20}, \delta_{1,2})$ across a 2-d space. The magnitude of the space in 1-d that contains $\mathbf{x}_{11}$, is the magnitude of the fractional arc $\cap(\mathbf{o}, x_{11}) \setminus b(\bar{\mathbf{x}}_{20}, \delta_{1,2})$. From (3.32) and (3.27), the magnitude of $\cap(\mathbf{o}, x_{11}) \setminus b(\bar{\mathbf{x}}_{20}, \delta_{1,2})$ is given by $|\cap(\mathbf{o}, x_{11}) \setminus b(\bar{\mathbf{x}}_{20}, \delta_{1,2})| = d/dx_{11} (b(\mathbf{o}, x_{11}) \setminus b(\bar{\mathbf{x}}_{20}, \delta_{1,2})) = 2\bar{\theta}_{\delta_{1,2}, x_{11}, \bar{x}_{20}} x_{11}$.

Factoring this magnitude instead of 2π into the standard distance PDF to the nearest tier-1 BTS from the PPP describing, we then simply multiply the result with the probabilities in (3.52) and (3.53) (due to the independence property of the PPP representing tier-1) to yield the joint distance PDF defined by event E_2

$$f_{X_{11}|\bar{X}_{20}, E_2}^{z_{2,B}} = 2\lambda_1 \bar{\theta}_{\delta_{1,2}, x_{11}, \bar{x}_{20}} x_{11} e^{-\lambda_1 (\pi x_{11}^2 + |\tilde{A}_1|)} (1 - e^{-\lambda_1 |\tilde{A}_2|}) \quad (3.54)$$

Finally, adding the distance distributions corresponding to the disjoint events, E_1 and E_2 yields the conditional PDF $f_{X_{11}|\bar{X}_{20}}^{z_{2,B}}$, where

$$f_{X_{11}|\bar{X}_{20}}^{z_{2,B}} = \frac{2\lambda_1 x_{11} e^{-\lambda_1 \pi x_{11}^2}}{1 - e^{-\lambda_1 \pi \delta_{1,2}^2}} \tilde{\theta}_{X_{11}}^{2,B}, \quad (3.55)$$

and $\tilde{\theta}_{X_{11}}^{2,B}$ is given by

$$\theta_{\delta_{1,2}, x_{11}, \bar{x}_{20}} + \bar{\theta}_{\delta_{1,2}, x_{11}, \bar{x}_{20}} (e^{-\lambda_1 |\tilde{A}_1|} + e^{-\lambda_1 (|\tilde{A}_1| + |\tilde{A}_2|)}). \quad (3.56)$$

Once again, we may formalize the joint PDF $f_{X_{11}, \bar{X}_{20}}^{z_{2,B}} = f_{\bar{X}_{20}} f_{X_{11}|\bar{X}_{20}}^{z_{2,B}}$

3.4.3 PDF of Distance from Independent Uniform Users to Nearest Tier-1 and Tier-2 BTSs

The PDF of distance to $\mathbf{x}_{11} \in \Psi_1$ for the typical user $\mathbf{z}_1 \in \Psi_1$ is rather straightforward as the sets Ψ_1 and Φ_1 are independent PPPs. Transforming the origin to $\mathbf{z}_1$, and setting $X_{11} \equiv \|\mathbf{x}_{11}\|$, we follow the equation in (2.20) to get

$$f_{X_{11}}^{z_1} = 2\lambda_1 \pi x_{11} \exp(-\lambda_1 \pi x_{11}^2). \quad (3.57)$$

At a distance $x_{11}\bar{P}_2$ away from the user, a tier-2 BTS will transmit with the same average power level as that of the nearest tier-1 BTS at $\mathbf{x}_{11}$. Consequently, the nearest tier-2 BTS to the user at $\mathbf{x}_{21}$ must exist inside the region $b(\mathbf{o}, x_{11}\bar{P}_2)$. However, the presence of the hole created by $\mathbf{x}_{11}$ means that no tier-2 BTSs can exist inside $b(\mathbf{o}, x_{11}\bar{P}_2)$. Therefore, $\mathbf{x}_{21}$ resides inside $\tilde{A}_3$, where

$$\tilde{A}_3 = b(\mathbf{o}, x_{11}\bar{P}_2) \setminus b(\mathbf{x}_{11}, \delta_{1,2}), \quad (3.58)$$

the magnitude of which can be computed using (3.36) and (3.37), by substituting $X_{21} = x_{11}\bar{P}_2$, $\delta_{1,2}$, and $X_{11} = x_{11}$ for R_1 , R_2 , and X respectively. When a tier-2 BTS resides at a distance of $x_{21} \leq x_{11}\bar{P}_2$ away from the user, we note that it must exist

on the fractional arc $\cap(\mathbf{o}, x_{21}) \setminus b(\mathbf{x}_{11}, \delta_{1,2})$ in a 1-d Euclidean space, where from (3.27), $|\cap(\mathbf{o}, x_{21}) \setminus b(\mathbf{x}_{11}, \delta_{1,2})| = d/dx_{21}(|b(\mathbf{o}, x_{21}) \setminus b(\mathbf{x}_{11}, \delta_{1,2})|)$. In order to account for the density of tier-2 BTSs along this fractional arc, we note that tier-2 BTSs are reduced by the hole created by $\mathbf{x}_{11}$, and by other holes due to tier-1 BTSs lying further away from the user. However, we consider only the hole created by $\mathbf{x}_{11}$ to tractably account for the density of tier-2 BTSs around this hole. The logic behind considering only one hole is explained further in subsection 3.51.

The greatest possible density of tier-2 BTSs outside the nearest hole to the user is $\lambda_{2,b}$. As such, by approximating tier-2 BTSs outside the hole surrounding $\mathbf{x}_{11}$ as a PPP of density $\lambda_{2,b}$, and noting the magnitude of the fractional arc $\cap(\mathbf{o}, x_{21}) \setminus b(\mathbf{x}_{11}, \delta_{1,2})$ is $|\cap(\mathbf{o}, x_{21}) \setminus b(\mathbf{x}_{11}, \delta_{1,2})| = 2\bar{\theta}_{\delta_{1,2}, x_{21}, x_{11}} x_{21}$, we can obtain the conditional PDF $f_{X_{21}|X_{11}}^{z_1}$ as

$$f_{X_{21}|X_{11}}^{z_1} = 2\lambda_{2,b}\bar{\theta}_{\delta_{1,2}, x_{21}, x_{11}} x_{21} \exp(-\lambda_{2,b}\pi x_{21}^2). \quad (3.59)$$

We can now define the joint PDF $f_{X_{21}, X_{11}}^{z_1}$, such that

$$f_{X_{21}, X_{11}}^{z_1} = f_{X_{11}}^{z_1} f_{X_{21}|X_{11}}^{z_1}. \quad (3.60)$$

3.5 Spatial Association Probabilities of Users

For brevity in calculating conditional association probabilities, we restate the power ratios $\bar{P}_1 = (P_1/P_2)^{1/\alpha}$, and $\bar{P}_2 = 1/\bar{P}_1$. The users from $\Psi_{2,A}$ are referred to as provisioned clustered users because each cluster of users surrounding its corresponding SAs is provisioned with an active tier-2 BTS, provided the location of the BTS fulfils the condition imposed by the cross-tier separation parameter. The users from $\Psi_{2,B}$ are conversely referred to as the independent clustered users. The users from set Ψ_1 are referenced as independent uniform users.

3.5.1 Provisioned Clustered Users Associating with Conditioned Tier-2 BTS

We now turn our attention to the different spatial association events defined in subsection 3.2.3. The first association event to be considered is $\mathbf{T}_{2,A}^{\mathbf{x}_{20}}$, conditioned on the distances X_{11} and X_{20} , where the user $\mathbf{z}_{2,A} \in \Psi_{2,A}$ associates with the tier-2 BTS at $\mathbf{x}_{20}$, whose location is conditioned on the location of the corresponding SA. The conditional probability of this event is related to the density of surrounding tier-2 BTSs, which is further influenced by the total area of holes created by tier-1 BTSs in the vicinity. This is because tier-2 BTSs cannot exist in such holes. Calculating the total area of holes would require the consideration of all possible holes created by nearby tier-1 BTSs. As these holes are created around tier-1 BTSs from an independent PPP, it is difficult to compute an exact number of holes in the vicinity, and account for the multiple possible ways in which they overlap to yield the average area of holes. Therefore, a more tractable approximation is necessary to account for the absence of tier-2 BTSs other than $\mathbf{x}_{20}$ in the vicinity of the user. We use Fig. 3.14 to explain the approximated topology with which we compute the probability that $\mathbf{z}_{2,A}$ associates to the BTS at $\mathbf{x}_{20}$.

Figure 3.14: Approximated topology to compute the probability of the association event $\mathbf{T}_{2,A}^{\mathbf{x}_{20}}$, involving the user $\mathbf{z}_{2,A}$, transformed to the origin at $\mathbf{o}$.

We base our approximated topology on the work considered in [90], which was used when computing a lower bound on Laplace transform of interference in a PHP. Here, only the hole created by the nearest tier-1 BTS, $\mathbf{x}_{11}$ is considered, and no tier-2 BTSs can exist in this hole. The density of other tier-2 BTSs outside this whole is approximated by $\lambda_{2,b}$, which is the highest possible density of tier-2 BTSs outside a single hole. This can be explained intuitively by the fact that the density of tier-2 BTSs outside all holes in the HCN is equal to $\lambda_{2,b}$.

As such, we transform the user to the origin and observe that the probability of association to the BTS at $\mathbf{x}_{20}$, conditioned on the distances $X_{20} = x_{20}$ and $X_{11} = x_{11}$, is given by

$$P_A(\mathbf{T}_{2,A}^{\mathbf{x}_{20}} | X_{20}, X_{11}) = \begin{cases} \exp(-\lambda_{2,b} |\tilde{A}_4|), & x_{11} \geq x_{20} \bar{P}_1 \\ 0, & \text{otherwise,} \end{cases} \quad (3.61)$$

where, as in seen in Fig. 3.14,

$$\tilde{A}_4 = b(\mathbf{o}, x_{20}) \setminus b(\mathbf{x}_{11}, \delta_{1,2}), \quad (3.62)$$

and the magnitude of $\tilde{A}_4$ can be computed using (3.36) and (3.37) by substituting $X_{20} = x_{20}$, $\delta_{1,2}$, and $X_{11} = x_{11}$ for R_1 , R_2 and X respectively. As seen in Fig. 3.14, the condition, $x_{11} \geq x_{20}\bar{P}_1$ for the first case in (3.61) states the minimum distance, $X_{11} = x_{11}$ such that the user will associate to $\mathbf{x}_{20}$ instead of $\mathbf{x}_{11}$. To uncondition on X_{11} , we note from standard conditional probability theory that

$$\begin{aligned} P_A(\mathbf{T}_{2,A}^{\mathbf{x}_{20}} | X_{20}) &= \Pr(X_{11} > x_{20}\bar{P}_1 | X_{20} = x_{20}) \\ &\times \Pr(X_{21} > x_{20} | X_{20} = x_{20}, X_{11} > x_{20}\bar{P}_1), \end{aligned} \quad (3.63)$$

where the second conditional probability is given in the first case presented in (3.61). Furthermore, using the CCDF of distances in a PPP,

$$\Pr(X_{11} > \bar{P}x_{20} | X_{20} = x_{20}) = e^{-\lambda_1|\tilde{A}_5|}, \quad (3.64)$$

where

$$\tilde{A}_5 = b(\mathbf{o}, x_{20}\bar{P}_1) \setminus b(\mathbf{x}_{20}, \delta_{1,2}), \quad (3.65)$$

because no tier-1 BTS exists in $b(\mathbf{x}_{20}, \delta_{1,2})$ due to cross tier-separation, and the minimum distance to $\mathbf{x}_{11}$ is $x_{20}\bar{P}_1$. The magnitude of $\tilde{A}_5$ in (3.65) can be computed using (3.36) and (3.37) by substituting $X_{11} = x_{20}\bar{P}_1$, $\delta_{1,2}$ and $X_{20} = x_{20}$ for R_1 , R_2 , and X respectively.

As $\tilde{A}_4$ depends on the random distance X_{11} , we use the conditional PDF $f_{X_{11}|X_{20}}^{\mathbf{z}_{2,A}}$ in (3.45) to uncondition (3.61) on X_{11} , such that

$$P_A(\mathbf{T}_{2,A}^{\mathbf{x}_{20}} | X_{20}) = e^{-\lambda_1|\tilde{A}_5|} \int_{\mathbb{R}^2 \setminus \tilde{A}_6} f_{X_{11}|X_{20}}^{\mathbf{z}_{2,A}} e^{-\lambda_{2,b}|\tilde{A}_4|} d\mathbf{x}_{11}, \quad (3.66)$$

where the domain of integration, $\mathbb{R}^2 \setminus \tilde{A}_6$ discounts the region $\tilde{A}_5$ which does not contain tier-1 BTSs. $\tilde{A}_6$ is defined as

$$\tilde{A}_6 = b(\mathbf{o}, \bar{P}_1 x_{20}) \cup b(\mathbf{x}_{20}, \delta_{1,2}). \quad (3.67)$$

Finally, the probability that a user $\mathbf{z}_2 \in \Psi_{2,A}$ associates to $\mathbf{x}_{20}$, the BTS assigned for the cluster it belongs to is given by

$$P_A(\mathbf{T}_{2,A}^{\mathbf{x}_{20}}) = \int_0^{R_{BTS}+R_U} f_{X_{20}} P_A(\mathbf{T}_{2,A}^{\mathbf{x}_{20}} | X_{20}) dx_{20}. \quad (3.68)$$

We note that the integral in (3.68) is obtained by first computing the magnitude of the integrand $P_A(\mathbf{T}_{2,A}^{\mathbf{x}_{20}} | X_{20})$ in (3.66) for a given value of x_{20} . Computing the integrand requires the use of the functional angles $\theta_{\delta_{1,2}, x_{20}, x_{11}}$ and $\theta_{x_{20}, \delta_{1,2}, x_{11}}$ to compute $|\tilde{A}_4|$, and the use of the functional angles $\theta_{\delta_{1,2}, x_{20}, \bar{P}_1, x_{11}}$ and $\theta_{x_{20}, \bar{P}_1, \delta_{1,2}, x_{11}}$ to compute $|\tilde{A}_5|$, when the appropriate substitution for the formulae presented in (3.36) and (3.37) is carried out. Furthermore, the PDF $f_{X_{11}|X_{20}}^{\mathbf{z}_{2,A}}$ in the integrand of (3.66) is computed using the angle $\bar{\theta}_{\delta_{1,2}, x_{11}, x_{20}}$. Therefore, each integral leading to the computation of (3.68) requires integration of multiple inverse cosine functions for which closed form solutions are not possible. As such, a closed form representation of (3.68) does not exist. However, the integral can be solved numerically in a straightforward manner.

3.5.2 Provisioned Clustered Users Associating with Tier-1 BTS

When $X_{20} > \delta_{1,2}/(x_{20}\bar{P}_1 + 1)$, the user $\mathbf{z}_2 \in \Psi_{2,A}$ may associate with $\mathbf{x}_{11}$ instead of $\mathbf{x}_{20}$. We use Fig. 3.15 to describe this situation.

Figure 3.15: Approximated topology to compute the probability of the association event $\mathbf{T}_{2,A}^{\mathbf{x}_{11}}$, involving the user $\mathbf{z}_{2,A}$, transformed to the origin at $\mathbf{O}$.

Under the association event $\mathbf{T}_{2,A}^{\mathbf{x}_{11}}$ depicted in Fig. 3.15, the distance between the user and $\mathbf{x}_{11}$ is $x_{11} < x_{20} \bar{P}_1$. Conditioned on X_{11} ,

$$P_A(\mathbf{T}_{2,A}^{\mathbf{x}_{11}} | X_{20}, X_{11}) = \begin{cases} \exp(-\lambda_{2,b} |\tilde{A}_3|), & x_{11} < x_{20} \bar{P}_1 \\ 0, & \text{otherwise,} \end{cases} \quad (3.69)$$

Following the techniques used to obtain (3.66) we uncondition on X_{11} to get

$$P_A(\mathbf{T}_{2,A}^{\mathbf{x}_{11}} | X_{20}) = (1 - e^{-\lambda_1 |\tilde{A}_5|}) \int_{\tilde{A}_5}^{z_{2,A}} f_{X_{11}|X_{20}} e^{-\lambda_{2,b} |\tilde{A}_3|} dx_{11}. \quad (3.70)$$

Finally, unconditioning on X_{20} leads to $P_A(\mathbf{T}_{2,A}^{\mathbf{x}_{11}})$, the probability that a user, $\mathbf{z}_2 \in \Psi_{2,A}$ associates to its nearest tier-1 BTS $\mathbf{x}_{11}$, such that

$$P_A(\mathbf{T}_{2,A}^{\mathbf{x}_{11}}) = \int_0^{R_{BTS} + R_U} f_{X_{20}} P_A(\mathbf{T}_{2,A}^{\mathbf{x}_{11}} | X_{20}) dx_{20}. \quad (3.71)$$

Like in (3.68), computing the integrand in (3.71) requires the use of multiple functional angles, such that the integration of the inverse cosine function is necessary. As such, a closed form representation of (3.71) does not exist. However, solving the integral numerically is a straightforward process.

3.5.3 Provisioned Clustered Users Associating with Independent Tier-2 BTS and Related Distance PDF

Using the results presented in subsection 3.5.1 and 3.5.2, we may obtain the probability that the user tags to $\mathbf{x}_{21}$, which is the nearest tier-2 BTS deployment meant for a cluster surrounding a different SA. Conditioned on the distances $X_{20} = x_{20}$, and $X_{11} = x_{11}$, the probability that a user, $\mathbf{z}_{2,A} \in \Psi_{2,A}$ tags to the tier-2 BTS deployment $\mathbf{x}_{21}$, is given by

$$P_A(\mathbf{T}_{2,A}^{\mathbf{x}_{21}} | X_{20}, X_{11}) = \begin{cases} 1 - \exp(-\lambda_{2,b} |\tilde{A}_3|), & x_{11} < \bar{P}x_{20} \\ 1 - \exp(-\lambda_{2,b} |\tilde{A}_4|), & x_{11} \geq \bar{P}x_{20}, \\ 0, & \text{otherwise.} \end{cases} \quad (3.72)$$

Using the expressions in (3.72), we may define the conditional pdf of distance to $\mathbf{x}_{21}$ under the event $\mathbf{T}_{2,A}^{\mathbf{x}_{21}}$. We use approximation that nearby tier-2 BTSs outside the nearest hole created by $\mathbf{x}_{11}$ follow a PPP of density λ_2 . As such, the fractional arc $\cap(\mathbf{o}, r) \setminus b(\mathbf{x}_{11}, \delta_{1,2})$ is accounted for and factored into the standard PDF of distance to the nearest tier-2 BTS from the approximated PPP. Therefore,

$$f_{X_{21}|X_{20},X_{11}}^{\mathbf{z}_{2,A}} = 2\lambda_{2,b}x_{21}\theta_{\delta_{1,2},x_{11},x_{20}}e^{-\lambda_{2,b}\pi x_{21}^2}. \quad (3.73)$$

We may define the joint PDF $f_{X_{21},X_{20},X_{11}}^{\mathbf{z}_{2,A}} = f_{X_{21}|X_{20},X_{11}}^{\mathbf{z}_{2,A}} f_{X_{11},X_{20}}^{\mathbf{z}_{2,A}}$ from (3.73).

The expressions in (3.72) are useful when attempting to derive coverage probabilities. However, we also notice that there are three mutually exclusive user association events for users from the set $\Psi_{2,A}$. These are given by $\mathbf{T}_{2,A}^{\mathbf{x}_{20}}$, $\mathbf{T}_{2,A}^{\mathbf{x}_{11}}$, and $\mathbf{T}_{2,A}^{\mathbf{x}_{21}}$, which are the events in which they associate with their corresponding BTSs located at $\mathbf{x}_{20}$, $\mathbf{x}_{11}$ and $\mathbf{x}_{21}$ respectively. Therefore, we observe due to the mutual

exclusivity of these association events, that the conditional probability $P_A(\mathbf{T}_{2,A}^{\mathbf{x}_{21}}|X_{20})$ is simply the complement of the sum of the two conditional probabilities $P_A(\mathbf{T}_{2,A}^{\mathbf{x}_{20}}|X_{20})$ and $P_A(\mathbf{T}_{2,A}^{\mathbf{x}_{11}}|X_{20})$. Therefore, for the sake of brevity, we may also define $P_A(\mathbf{T}_{2,A}^{\mathbf{x}_{21}}|X_{20})$ in an alternate fashion, such that

$$P_A(\mathbf{T}_{2,A}^{\mathbf{x}_{21}}|X_{20}) = 1 - P_A(\mathbf{T}_{2,A}^{\mathbf{x}_{11}}|X_{20}) - P_A(\mathbf{T}_{2,A}^{\mathbf{x}_{20}}|X_{20}). \quad (3.74)$$

By the same rule, the overall association probability $P_A(\mathbf{T}_{1,A}^{\mathbf{x}_{21}})$ is the complement of the sum of the two probabilities $P_A(\mathbf{T}_{2,A}^{\mathbf{x}_{20}})$ and $P_A(\mathbf{T}_{2,A}^{\mathbf{x}_{11}})$. Therefore, the probability $P_A(\mathbf{T}_{1,A}^{\mathbf{x}_{21}})$, that a user $\mathbf{z}_2 \in \Psi_{2,A}$ associates with the tier-2 BTS at $\mathbf{x}_{21}$, can be defined as

$$P_A(\mathbf{T}_{1,A}^{\mathbf{x}_{21}}) = 1 - P_A(\mathbf{T}_{2,A}^{\mathbf{x}_{11}}) - P_A(\mathbf{T}_{2,A}^{\mathbf{x}_{20}}) \quad (3.75)$$

We note that the expression in (3.75) represents the probability that a user belonging to a cluster surrounding a specific SA will associate with a tier-2 BTS that has been deployed for a different proximal SA.

3.5.4 Independent Clustered Users Associating with Tier-1 and Tier-2 BTSs

Independent clustered users may tag to the nearest tier-1 or tier-2 BTS, i.e. $\mathbf{x}_{11}$ or $\mathbf{x}_{21}$. The distance to the nearest $\mathbf{x}_{11}$ is influenced by the location of the rejected potential tier-2 BTS at $\mathbf{x}_{20}$. We examine the association event $\mathbf{T}_{2,B}^{\mathbf{x}_{11}}$ in Fig. 3.16.

Figure 3.16: Approximated topology to compute the probability of the association event $\mathbf{T}_{2,B}^{\mathbf{x}_{11}}$, involving the user $\mathbf{z}_{2,B}$, transformed to the origin at $\mathbf{o}$.

Figure 3.16 shows that no tier-2 BTS should exist in the region $\tilde{A}_3$ if the user $\mathbf{z}_{2,B}$ associates to the nearest tier-1 BTS at $\mathbf{x}_{11}$. Therefore, conditioned on the distances $\bar{X}_{20}$ and X_{11} , the probability that a user, $\mathbf{z}_{2,B} \in \Psi_{2,B}$ tags to the tier-1 BTS deployment $\mathbf{x}_{11}$, is given by $P_A(\mathbf{T}_{2,B}^{\mathbf{x}_{11}} | \bar{X}_{20}, X_{11})$ is given by

$$P_A(\mathbf{T}_{2,B}^{\mathbf{x}_{11}} | \bar{X}_{20}, X_{11}) = \exp(-\lambda_{2,b} |\tilde{A}_3|). \quad (3.76)$$

To obtain (3.56), we reiterate the assumption that tier-2 BTSs are effectively a PPP outside the hole created by $\mathbf{x}_{11}$. Unconditioning on X_{11} , we get

$$P_A(\mathbf{T}_{2,B}^{\mathbf{x}_{11}} | \bar{X}_{20}) = \int_0^{\bar{X}_{20} + \delta_{1,2}} f_{X_{11}|X_{20}}^{\mathbf{z}_{2,B}} e^{-\lambda_{2,b} |\tilde{A}_3|} dx_{11}, \quad (3.77)$$

where the domain of integration accounts for the possible space in which $\mathbf{x}_{11}$ exists in relation to the user, and was explained in subsection 3.4.2 for users from $\Psi_{2,B}$. Consequently, we may uncondition on $\bar{X}_{20}$ using its PDF to obtain

$$P_A(\mathbf{T}_{2,B}^{\mathbf{x}_{11}}) = \int_0^{R_{BTS}+R_U} f_{\bar{X}_{20}} P_A(\mathbf{T}_{2,B}^{\mathbf{x}_{11}} | \bar{X}_{20}) dx_{20}. \quad (3.78)$$

Finally, we present the complementary probabilities, $P_A(\mathbf{T}_{2,B}^{\mathbf{x}_{21}} | \bar{X}_{20})$ and $P_A(\mathbf{T}_{2,B}^{\mathbf{x}_{11}})$, such that

$$P_A(\mathbf{T}_{2,B}^{\mathbf{x}_{21}} | \bar{X}_{20}) = 1 - P_A(\mathbf{T}_{2,B}^{\mathbf{x}_{11}} | \bar{X}_{20}), \quad (3.79)$$

and

$$P_A(\mathbf{T}_{2,B}^{\mathbf{x}_{21}}) = 1 - P_A(\mathbf{T}_{2,B}^{\mathbf{x}_{11}}). \quad (3.80)$$

Based on (3.51), the conditional distance PDF of X_{21} , in the event that the user is associated with $\mathbf{x}_{21}$ is derived by considering the fraction of the arc $\cap(\mathbf{o}, \bar{P}_2 x_{11}) \setminus b(\mathbf{x}_{11}, \delta_{1,2})$ that can contain $\mathbf{x}_{21}$. As such, we define the conditional PDF,

$$f_{X_{21} | \bar{X}_{20}, X_{11}}^{\mathbf{z}_{2,B}} = 2\lambda_{2,b} x_{21} \theta_{\delta_{1,2}, x_{11}, x_{21}} e^{-\lambda_{2,b} \pi x_{21}^2}. \quad (3.81)$$

Finally, we define the joint PDF $f_{X_{21}, \bar{X}_{20}, X_{11}}^{\mathbf{z}_{2,B}} = f_{X_{21} | \bar{X}_{20}, X_{11}}^{\mathbf{z}_{2,B}} f_{X_{11}, \bar{X}_{20}}^{\mathbf{z}_{2,B}}$ from (3.81).

3.5.5 Independent Uniform Users Associating with Tier-1 and Tier-2 BTSs

Based on the derivation of the PDFs in subsection 3.4.3, the conditional association of the user $\mathbf{z}_1 \in \Psi_1$ to a tier-2 BTS, $\mathbf{x}_{21}$, is straightforward and presented by

$$P_A(\mathbf{T}_1^{\mathbf{x}_{21}} | X_{11}) = 1 - \exp(-\lambda_{2,b} |\tilde{A}_3|). \quad (3.82)$$

The conditional probability of the user tagging to $\mathbf{x}_{11}$ instead is the complement of the probability in (3.82) and is given by

$$P_A(\mathbf{T}_1^{\mathbf{x}_{11}} | X_{11}) = \exp(-\lambda_{2,b} |\tilde{A}_3|). \quad (3.83)$$

As such, the overall spatial association probability to either BTS can be given by

$$P_A(\mathbf{T}_1^{\mathbf{x}_{k1}}) = \int_0^\infty f_{X_{11}}^{\mathbf{z}_1} P_A(\mathbf{T}_1^{\mathbf{x}_{k1}} | X_{11}) dx_{11}. \quad (3.84)$$

Like in the cases of the integrals in (3.68) and (3.71), computing the integrand in (3.84) requires the use of multiple functional angles, such that the integration of the inverse cosine function is necessary. As such, a closed form representation of (3.84) does not exist. However, solving the integral numerically is a straightforward process.

3.5.6 Overall Probability of Association with Tier-1 and Tier-2 BTSs

Based on the overall association probabilities for each set of users outlined in the previous sections, obtaining the overall association of users to tier-1 BTSs is straightforward. First, we recognize that there are three association events by which users may tag to a tier-1 BTS, defined previously as $\mathbf{T}_{2,A}^{\mathbf{x}_{11}}$, $\mathbf{T}_{2,B}^{\mathbf{x}_{11}}$, $\mathbf{T}_1^{\mathbf{x}_{11}}$ for the users belonging to the sets $\Psi_{2,A}$, $\Psi_{2,B}$, and Ψ_1 respectively. Using the respective densities of each user set, $\rho_{2,A}$, $\rho_{2,B}$, and ρ_1 , and by denoting $\bar{U} = 1$, $\bar{U} = 2, A$, or $\bar{U} = 2, B$, we may define the joint probability that a user located at $\mathbf{z}$ belongs to a set $\Psi_{\bar{U}}$ and tags to a tier-1 BTS as

$$\Pr(\mathbf{z} \in \Psi_{\bar{U}}, \mathbf{T}_{\bar{U}}^{\mathbf{x}_{11}}) = \frac{P_A(\mathbf{T}_{\bar{U}}^{\mathbf{x}_{11}}) \rho_{\bar{U}}}{\rho_o}, \text{ for } \bar{U} = 1, \bar{U} = 2, A, \text{ or } \bar{U} = 2, B \quad (3.85)$$

where $\rho_O = \rho_{2,A} + \rho_{2,B} + \rho_1$. Equation (3.85) is obtained by first observing that the probability of the event $\mathbf{z} \in \Psi_{\bar{U}}$ is simply $\rho_{\bar{U}}/\rho_O$, given that all user sets are independent and mutually exclusive. Subsequently, from the fact that the events, $\mathbf{z} \in \Psi_{\bar{U}}$ and $\mathbf{T}_{\bar{U}}^{\mathbf{x}_{11}}$, we may simply multiply their probabilities to get the joint probability in (3.85).

As each user set is independent of each other, it follows that the overall association probability to tier-1 can be obtained by simply summing the joint probabilities presented in (3.85). We define the overall association probability to tier-1 BTSs as $P_A(\mathbf{T}_{\bar{U}}^{\Phi_1})$, and expand the summation to get

$$P_A(\mathbf{T}_{\bar{U}}^{\Phi_1}) = \frac{P_A(\mathbf{T}_{2,A}^{\mathbf{x}_{11}})\rho_{2,A} + P_A(\mathbf{T}_{2,B}^{\mathbf{x}_{11}})\rho_{2,B} + P_A(\mathbf{T}_1^{\mathbf{x}_{11}})\rho_1}{\rho_O} \quad (3.86)$$

Finally, user association to tier-2 BTSs is mutually exclusive from user association to tier-1 BTSs. The probability of association of all users to tier-2 BTSs, is therefore the complementary probability $P_A(\mathbf{T}_{\bar{U}}^{\Phi_2})$, such that $P_A(\mathbf{T}_{\bar{U}}^{\Phi_2}) = 1 - P_A(\mathbf{T}_{\bar{U}}^{\Phi_1})$.

3.6 BTS Utilization Under Constrained Resources

We restate the definition of $\mathbf{R}_b$, which is the set of orthogonal RBs per BTS under a constrained environment, such that $\mathbf{R}_b = \{b_1, b_2, \dots, b_{N_b}\}$. The number of RBs per BTS is therefore given by N_b . If all N_b RBs are occupied by users ready for transmission based on union of association events $\bar{\mathbf{T}}_{\bar{U}}^{\mathbf{x}_{ki}}$ for a BTS $\mathbf{x}_{ki}$, any additional incumbent users are denied association to that BTS. As such, the probability that a user can connect to a free RB with probability $P_{f,\bar{U}}^{\mathbf{x}_{ki}}$. This factor is used in the next chapter when coverage is considered.

Therefore, by assuming that each user requires a single resource block per unit time and frequency, we define the instantaneous utilization of a BTS as

$$\xi_{\mathbf{x}_{ki}}(N_b; \rho_1, \rho_2) = \begin{cases} \frac{\sum N_{\bar{U}}(\mathbf{T}_{\bar{U}}^{\mathbf{x}_{ki}})}{N_b}, & \text{if } \sum N_{\bar{U}}(\mathbf{T}_{\bar{U}}^{\mathbf{x}_{ki}}) < N_b \\ 1, & \text{if } \sum N_{\bar{U}}(\mathbf{T}_{\bar{U}}^{\mathbf{x}_{ki}}) \geq N_b, \end{cases} \quad (3.87)$$

where $N_{\bar{U}}(\mathbf{T}_{\bar{U}}^{\mathbf{x}_{ki}})$ is the instantaneous number of users from user group Ψ_U that spatially associate with the user. We note that the subscript U may take the form $\bar{U} = 1, \bar{U} = 2, A$, or $\bar{U} = 2, B$.

The first case described in (3.87) represents a scenario where the BTS is undersubscribed. We assume that a full buffer traffic system is in place, where information has to be transmitted to all users across the network at the beginning of each timeslot. Furthermore, we assume here that each user requires one RB per unit timeslot for downlink transmission. This assumption can easily be adjusted to account for the average number of RBs required per user, but we do not take this into account for brevity. The second case in (3.87) describes a situation in which users are oversubscribed. In the numerical results section that follows in subsection 3.7, we compute the average utilization of RBs per BTS by simulation. Formally, this parameter is defined by $Y_{\mathbf{x}_{ki}}$, where

$$Y_{\mathbf{x}_{ki}} = \mathbb{E} \left(\xi_{\mathbf{x}_{ki}}(N_b; \rho_1, \rho_2) \right), \quad (3.88)$$

and $\mathbf{x}_{ki} = \mathbf{x}_{11}, \mathbf{x}_{20}$, or $\mathbf{x}_{21}$. We also compute the HCN's average utilization of RBs for each tier using simulation, such that this parameter is defined by Y_k , where

$$Y_k = \mathbb{E}_{\mathbf{x}_{ki}}(Y_{\mathbf{x}_{ki}}), \quad (3.89)$$

where the expectation is taken across all serving BTSs across tier- k that are designated by the subscript i . The average utilization of RBs across a tier signifies what percentage of RBs are used on average by BTSs of that tier.

3.7 Numerical Results and Discussion

3.7.1 Distance PDFs for Clustered Users

We first examine the PDFs of distance from clustered users belonging to the set Ψ_2 , under variations of the PCP describing the locations of users and BTSs in relation to SAs, under case 1 and case 2. Analysing the results that follow in Figs. 3.17, 3.18, and 3.19, we can then explain trends in association that follow in subsection 3.7.2. We first look at the numerical computation of the PDF, $f_{X_{20}} \equiv f_{\bar{X}_{20}}$, which is the PDF of the distance, X_{20} , between a typical clustered user and the BTS designated to serve it at $\mathbf{x}_{20}$. Initially, in Fig. 3.17, we examine $f_{X_{20}}$ under case 1, for $R_U = 50$ m, showcasing how the PDF varies in nature for different designations of R_{BTS} .

Figure 3.17: Numerical computation of the PDF of distance from the typical clustered user to the tier-2 BTS designated to serve it at $\mathbf{x}_{20}$, under case 1, where $R_U = 50$ m, for different values of R_{BTS} .

Figure 3.17 shows that when the BTS is deployed at the SA, i.e. at exactly the centre of the user cluster such that $R_{BTS} = 0$, most users are found near a distance of 50 m away from the BTS, mimicking the distance PDF result shown in Fig. 3.4. As the value of R_{BTS} increases to 25 m, such that the maximum distance between the SA and the deployed BTS is 25 m, a significant portion of users exist between 30 to 50 m from the user. However, there is a substantial portion of users that also exist at a distance greater than 50 m from the user, up to a distance of $R_{BTS} + R_U = 75$ m. For these users that exist further away, the average received signal strength is expected to be reduced, based on the path loss model described in (3.19). As the value of R_{BTS} is increased further, more users are found further away from the deployed BTS, which is the maximum distance to the user from the BTS also increases. Therefore, increasing R_{BTS} will result in more users receiving signals with diminished strength from the BTS at $\mathbf{x}_{20}$. As seen in subsection 3.7.2, this phenomenon also influences the association of users from a cluster to the BTS at $\mathbf{x}_{20}$. From a practical standpoint, this means that allowing the distance between the deployed tier-2 BTS to the SA to take a larger range of distances will result in a greater number of users receiving signals with diminished power from this BTS.

Case 1 also examines a situation where the location of the tier-2 BTS is uniformly random with respect to the SA, but contained within a maximum distance R_{BTS} .

We now examine how the PDF, $f_{X_{20}}$ behaves under case 2, where the user cluster is characterized by $\sigma_U = 25$ m and $R_U = 50$ m. Figure 3.18 shows how the PDF varies for different values of σ_{BTS} , when $R_{BTS} = 100$ m.

Figure 3.18: Numerical computation of the PDF of distance from the typical clustered user to the tier-2 BTS designated to serve it at $\mathbf{x}_{20}$, under case 2, where $\sigma_U = 25$ m, $R_U = 50$ m, for different values of σ_{BTS} , when $R_{BTS} = 100$ m.

Selecting the combination of $R_U = 50$ m and $R_{BTS} = 100$ m ensures that the maximum possible distance, X_{20} , between a user and the tier-2 BTS at $\mathbf{x}_{20}$, is $R_U + R_{BTS} = 150$ m, when σ_{BTS} is assigned a non-zero positive value. When the BTS is located at the SA, i.e. $\sigma_{BTS} = 0$, the maximum distance between the user and the tier-2 BTS is 50 m, while a significant portion of users will be located at approximately 25 m away from the BTS. As σ_{BTS} is increased, a larger portion of users will exist at greater distances from the tier-2 BTS at $\mathbf{x}_{20}$, and the average received signal strength across users from this BTS will diminish. This mimics what happens under case 1 when R_{BTS} is increased. However, we observe a notable difference between the trends of the PDF $f_{X_{20}}$ under case 1 and case 2 when

comparisons are made between Figs. 3.17 and 3.18. A designation of σ_{BTS} under case 2 when $R_{BTS} \gg \sigma_{BTS}$, will result in a greater portion of users existing further away from the tier-2 BTS, compared to a similar designation of R_{BTS} under case 1. We explore this further in Fig. 3.19, where we now set $R_{BTS} = 2\sigma_{BTS}$, while retaining the other settings used to obtain Fig. 3.18.

Figure 3.19: Numerical computation of the PDF of distance from the typical clustered user to the tier-2 BTS designated to serve it at $\mathbf{x}_{20}$, under case 2, where $\sigma_U = 25$ m, $R_U = 50$ m, for different values of σ_{BTS} , when $R_{BTS} = 2\sigma_{BTS}$.

Figure 3.19 shows that when $\sigma_{BTS} = 25$ m, the maximum distance between a user and the tier-2 BTS at $\mathbf{x}_{20}$ will be 100 m, as R_{BTS} is reduced to 50 m. Comparisons between Figs. 3.14 and 3.13 show that when $\sigma_{BTS} = 25$ m and $R_{BTS} = 50$ m, the distance PDF, $f_{X_{20}}$, follows trends very similar to when $\sigma_{BTS} = 25$ m and $R_{BTS} = 100$ m, except that a very small portion of users may be found beyond 100 m from the BTS in the latter case. However, for larger designations of σ_{BTS} and $R_{BTS} = 2\sigma_{BTS}$, a larger portion of users exist at even greater distances from the deployed BTS.

From a practical standpoint, when R_{BTS} is increased under case 2, the maximum distance allowed between the deployed tier-2 BTS is also increased. When σ_{BTS} is increased, most deployed BTSs are found near distances of σ_{BTS} from the SA. Increasing both values will result in a greater portion of users in the cluster receiving diminished signal strength.

We now proceed to look at how the parameters R_{BTS} and σ_{BTS} , together with the cross-tier separation $\delta_{1,2}$, can affect the association of users.

3.7.2 Validating Distance PDFs and Association Probabilities

This subsection examines the validity of the derived distance PDFs and association for clustered users in this chapter. We consider results for $\lambda_1 = 2/\text{km}^2$, $\lambda_{2,b} = \lambda_{SA} = 10/\text{km}^2$, $P_2 = 0.05P_1$, and $\alpha = 3.5$, which are standard parameters used in the literature. We assign other parameters of the considered offset parameters, R_{BTS} , and the cross-tier separation parameter, $\delta_{1,2}$, based on what is deemed a possible practical scenario. Under case 1, we obtain results for $R_U = 50$ m, and under case 2, $\sigma_U = 25$ m and $R_U = 50$ m. The proposed model was simulated over a square region of area 400 km^2 , and statistics were obtained for users within 7.5 km from the center of this region. The reason for centralizing the tested users was because the theoretical computations consider a user centred at the origin to which an infinite network of surrounding BTSs transmit. For each set of parameters, 100 variations of HCN BTSs and user locations were tested for fitting against the derived theoretical expressions. In each of the 100 simulations per set of parameters, a density of $\rho_2 = 100/\text{km}^2$ clustered users were designated. Therefore, the average number of users tested for each simulation per set of parameters within the stipulated 7.5 km radius was approximately 1.77×10^4 users. The approximate total number of users tested across the 100 simulations for computing association under each set of parameters was therefore 1.77×10^6 users. We note that according to the connectivity model described in subsection 3.3.4, we do not require Rayleigh fading channel realizations, as only the average power transmitted by BTSs determines which BTS a user may associate with. Numerical values from the derived theoretical expressions are plotted against simulated results in Figures 3.20 and 3.21.

We explain which specific equations were ultimately tested against simulations in each case.

Figure 3.20: Probability of association to the BTS at $\mathbf{x}_{20}$ vs. R_{BTS} or σ_{BTS} for cases 1 and 2 against various $\delta_{1,2}$ in m, where $R_{BTS} = 2\sigma_{BTS}$ for case 2 under the other HCN settings described in subsection 3.7.2.

In Figure 3.20, we test the probability of association of users from a cluster to its deployed BTS, for increasing R_{BTS} or σ_{BTS} , under case 1 and case 2, respectively. In case 2, the BTS distance from the SA is limited by $R_{BTS} = 2\sigma_{BTS}$. For case 1, the probability of associating to this BTS decreases quickly as R_{BTS} increases. This can be explained with the help of results in Fig. 3.17, from which we understand that increasing R_{BTS} will cause a greater number of users to exist at distances further away from the deployed tier-2 BTS. As such, the received signal power from the BTS at $\mathbf{x}_{20}$ is reduced for a larger portion of users. Consequently, the probability that these users could receive a signal with increased power from a neighbouring tier-1 or tier-2 BTS increases. From the connectivity model described in subsection 3.2.4, we note that this phenomenon will result in reduced association to the BTS at $\mathbf{x}_{20}$.

Under case 2, trends in association probability mimic the behaviour case 1, although the association probability decreases faster against increasing σ_{BTS} , when

$R_{BTS} = 2\sigma_{BTS}$. Observing Fig. 3.19, we understand that this is because due to the nature of the PDF of distance from the user to the deployed tier-2 BTS. The portion of users at a larger distance from the tier-2 BTS is greater for a stipulated value of σ_{BTS} under case 2 when $R_{BTS} = 2\sigma_{BTS}$, than for the same value of R_{BTS} under case 1.

In both cases, user association to the BTS at $\mathbf{x}_{20}$ tends to 0 for large values of R_{BTS} or σ_{BTS} . As either of these parameters is increased, the location of the deployed tier-2 BTS can be at increased distances from the SA. Consequently, the location of this BTS exhibits decreased correlation to the user cluster it was designated to serve. As a result, a typical user's received signal power from the BTS at $\mathbf{x}_{20}$ will decrease, and the probability of receiving a higher-powered signal from a different tier-1 or tier-2 BTS will increase.

Increased cross-tier separation, $\delta_{1,2}$, will effectively increase the distance from the nearest tier-1 BTS to the user, provided that the distance from the user to the tier-2 BTS is small enough. The absence of nearby tier-1 BTSs in these scenarios will increase the association to the BTS at $\mathbf{x}_{20}$. For example, we observe a higher probability of association to $\mathbf{x}_{20}$ when $R_{BTS} = 250$ m instead of $R_{BTS} = 100$ m under case 1. For larger values of $\delta_{1,2}$, it can be expected that most users that do not associate to the BTS at $\mathbf{x}_{20}$ will associate instead to a nearby tier-2 BTS at $\mathbf{x}_{21}$. As cross-tier separation is conditioned on the position of the deployed tier-2 BTS and not the SA or user cluster, the distance between the nearest tier-1 BTS and a user is influenced less when $\delta_{1,2} \ll R_{BTS}$ under case 1, or $\delta_{1,2} \ll R_{BTS}, \sigma_{BTS}$ under case 2. As such the disparity between probability of association reduces for different values of $\delta_{1,2}$ under such scenarios.

The results for association of clustered users to their corresponding BTS at $\mathbf{x}_{20}$ under large values of R_{BTS} and σ_{BTS} show that it would not make sense for the distance from an SA to a designated tier-2 BTS to be increased beyond a certain magnitude. This is particularly true when the radius of a user cluster is relatively limited, as can be seen in many hotspots in practical situations. If the distance between the BTS and the SA is allowed to take a much larger range than the radius of a typical user cluster, a good portion of the clustered users will not associate with the deployed BTS. This reduces the usefulness of the deployed tier-2 BTS as it does not serve one its primary purposes, to serve users from its designated cluster. As

such, we propose that the magnitudes R_{BTS} under case 1, and R_{BTS} and σ_{BTS} under case 2 are contained within reasonable limits. Specifically, we contend that in a practical situation, the R_{BTS} cannot take a much larger value than R_U in case 1, and R_{BTS} and σ_{BTS} cannot be much larger than R_U and σ_U in case 2.

The results in Fig. 3.20 feature a comparison with results that are expected when the cross-tier separation rule is not imposed for the deployment of tier-2 BTSs, i.e. all BTSs in the network can be modelled by a PPP, corresponding to the work presented for the general analysis of users in [39] and [40], and in [100] for the specific case of clustered users. When $R_{BTS} = 0$ under case 1, or $\sigma_{BTS} = 0$ under case 2, the probability of association to $\mathbf{x}_{20}$ is exactly the result expected for the models presented in [100] under the settings used to obtain Fig. 3.20. We observe that introducing a minimum distance between tier-1 and tier-2 BTSs by using a non-zero value of $\delta_{1,2}$ will result in better clustered user association to the designated BTS at $\mathbf{x}_{20}$. Furthermore, centralizing the BTS at the SA, as in the model in [100] ensures the best probability of association from a clustered user to its designated tier-2 BTS. However, we also understand that this might not be the most practical scenario due to installation constraints that may be found at potential tier-2 BTS deployment sites.

We now examine Fig. 3.21 to understand how the combination of the parameters $\delta_{1,2}$, R_{BTS} , and σ_{BTS} in case 2 can influence the overall association of clustered users from the set Ψ_2 , to either tier-1 or tier-2 BTSs in the HCN.

Figure 3.21: Probability of association tier-2 BTSs vs. $\delta_{1,2}$ for cases 1 and 2, respectively, against various R_{BTS} and σ_{BTS} in m, where $R_{BTS} = 100$ m for case 2 under the other HCN settings described in subsection 3.7.2.

Figure 3.21 examines the association of clustered users to tier-2 BTSs under increasing values of, $\delta_{1,2}$, for different R_{BTS} under case 1 and σ_{BTS} under case 2, where $R_{BTS} = 100$ m under case 2. We maintain the values of R_{BTS} and σ_{BTS} such that they do not exceed by a large amount, R_U under case 1, or R_U and σ_U under case 2. All user clusters will have one tier-2 BTS each designated to their corresponding SAs when $\delta_{1,2} = 0$. In this scenario, the density of tier-2 BTSs, λ_2 peaks at $10/\text{km}^2$ and is exactly equal to the density of potential tier-2 BTS sites, $\lambda_{2,b}$. The association to tier-2 BTSs in the HCN for clustered users peaks in this situation. This is because all clustered users will receive a strong signal from a nearby tier-2 BTS, preferably the one designated for its cluster.

As $\delta_{1,2}$ increases, potential tier-2 BTS sites are eliminated as candidates for deployment, and the density of active tier-2 BTSs in the HCN reduces in proportion to $\delta_{1,2}$. Therefore, as $\delta_{1,2}$ increases, the fraction of total clustered of users that belong to the set $\Psi_{2,B}$ increases as well. A typical user from this set is not served by a tier-2 BTS designated for its corresponding SA, but instead associates to an independent tier-1 or tier-2 BTS in the HCN, depending on which tier provides it

with the strongest signal. The simultaneous reduction of designated tier-2 BTSs to clustered users, and the overall reduction of tier-2 BTSs in the network means less users with associate to this tier. We examine these trends with reference to the two subsets of users, $\Psi_{2,A}$, and $\Psi_{2,B}$.

A typical user from $\Psi_{2,B}$ is very likely to associate to a tier-1 BTS under all values of $\delta_{1,2}$ because of the presence of the nearest tier-1 BTS at $\mathbf{x}_{11}$, due to which a designated tier-2 BTS is not provisioned to the serve the user. As $\delta_{1,2}$ increases, a larger number of such users may be found further away from their corresponding BTS at $\mathbf{x}_{11}$. As such, the probability of associating to a tier-1 BTS reduces slightly because some users from $\Psi_{2,B}$ may be located near the edge of the larger hole around $\mathbf{x}_{11}$ devoid of tier-2 BTSs. These users will receive slightly weaker signals from the BTS at $\mathbf{x}_{11}$, and may consequently associate to a nearby tier-2 BTS just outside the hole. However, the overall association probability of this group of users to tier-1 remains very close to 1 through all values of $\delta_{1,2}$ in Fig. 3.21.

Conversely, as $\delta_{1,2}$ increases, the portion of users belonging to $\Psi_{2,A}$ decreases, but the probability of these users associating to tier-2 BTSs increases and approaches 1. This is because the distance from the typical user to its nearest tier-1 BTS at $\mathbf{x}_{11}$ will increase as a result of cross-tier separation. Consequently, the strongest received signal is expected to be from a nearby tier-2 BTS. We also notice by using Fig. 3.20, that as $\delta_{1,2}$ increases, so does the probability that a typical user from $\Psi_{2,A}$ to its designated tier-2 BTS at $\mathbf{x}_{20}$. As such, it can be inferred that tier-2 BTSs are more likely to serve the clustered users they were meant to under increased $\delta_{1,2}$.

The overall association probability in Figure 3.21 shows that when $\delta_{1,2} \gg R_{BTS}$ or $\delta_{1,2} \gg \sigma_{BTS}$, the magnitudes of R_{BTS} , and σ_{BTS} have decreased effect on the association trends. Therefore, setting $\delta_{1,2}$ can be used as a rule of thumb in planning HCNs a for load balancing by means of forcing more users to associate to tier-1 BTSs. It can also be used to ensure tier-2 deployed tier-2 BTSs are more effectively utilized to serve the user clusters around their designated SAs.

The fitting of the theoretical and simulated curves showcases the validity of the distance PDFs contributing the derivation of $f_{X_{20}} \equiv f_{\bar{X}_{20}}$. They also show a tight fit

between theoretical and simulated values of the association probability to the conditioned tier-2 BTS at $\mathbf{x}_{20}$.

3.7.3 Utilization of RBs in Tier-1 and Tier-2 BTSs

We now examine trends in BTS utilization according to a fixed number of orthogonal RBs per BTS per timeslot, N_b , and different user densities. The BTS densities considered correspond to the ones in subsection 3.7.2. First, we examine the case where the spatial user densities for independent and clustered users are given by $\rho_1 = \rho_2 = 100/\text{km}^2$. Effectively, as $\lambda_{SA} = \lambda_{2,b} = 10/\text{km}^2$, this implies that there is an average of 10 users per cluster. Figures 3.17 and 3.18 showcase the separate utilization of tier-1 and tier-2 BTSs when $\alpha = 4$ and $R_U = 50$ m under the proposed connectivity model for constrained resources. We assume that each user in the HCN will be provided exactly one RB for downlink transmission per timeslot. The results were obtained by simulation, where 100 variants of the HCN were tested under each different combination of parameters in square a region of 400 km^2 . Statistics from users within 7.5 km from the centre of this region were collected, and the number of channel realizations simulated exceeded 10^6 in each case.

Figure 3.22: Utilization of tier-1 RBs vs $\delta_{1,2}$, for $\rho_1 = \rho_2 = 100/\text{km}^2$, $\alpha = 4$, and different values of R_{BTS} , under the density settings described in subsection 3.7.3.

Figure 3.23: Utilization of tier-2 RBs vs $\delta_{1,2}$, for $R_U = 50$ m, $\rho_1 = \rho_2 = 100/\text{km}^2$, $\alpha = 4$, and different values of R_{BTS} , under the density settings described in subsection 3.7.3.

Figure 3.22 shows that RBs from a tier-1 BTS are utilized heavily when $\delta_{1,2}$ is large, as most users will spatially associate to tier-1 BTSs due to the overall reduction in the density of proximal tier-2 BTSs. Furthermore, a larger portion of clustered users will belong the set $\Psi_{2,B}$, and these users spatially associate to tier-1 BTSs with very high probability, as was seen in subsection 3.7.2.

Similar trends are observed in Figure 3.23 for tier-2 BTSs, where the fraction of RBs utilized for a typical tier-2 BTS monotonically increases with $\delta_{1,2}$. This phenomenon occurs because clustered users from $\Psi_{2,A}$ are very likely to tag to their designated tier-2 BTS, as seen previously in Fig. 3.20, and are less likely to be poached by a nearby tier-1 BTS. Furthermore, Figs. 3.22 and 3.23 also show that deploying tier-2 BTSs closer to the SA tends to reduce tier-1 utilization, and conversely, increase tier-2 BTS utilization, because the average distance to a clustered user from a designated tier-2 BTS at $\mathbf{x}_{20}$ is reduced. Under conditions where path loss is greater, we note the supplied signal from $\mathbf{x}_{20}$ to a typical clustered user is likely to be greater than the signal from the nearest tier-1 BTS, $\mathbf{x}_{11}$. This means more users spatially associate to their designated BTS at $\mathbf{x}_{20}$. Consequently, the utilization of RBs tier-2 BTSs is greater under conditions where path loss is greater.

Under the settings for Figs. 3.22 and 3.23, the density of clustered and independent uniform users is equivalent, and the network is generally undersubscribed, i.e. more resources are available than users, if each user utilizes a single resource block. An interesting point to note is that the overall utilization of tier-2 BTS resources is much lower than that of its tier-1 counterparts under these circumstances, primarily because each BTS is designed for an individual cluster that has an average of 10 users when $\rho_1 = 100/\text{km}^2$. As $N_b = 50$, the resources for each tier-2 BTS would far exceed the number of users tagging to it, even if nearby independent users were considered. By contrast, tier-1 BTSs are likely to serve a greater portion of independent uniform users, while also poaching clustered users, especially when the cross-tier separation is larger.

We compare these results to those in Figs. 3.24 and 3.25, where different combinations of ρ_1 and ρ_2 are considered, when $R_{BTS} = 50$ m, and $\alpha = 4$.

Figure 3.24: Utilization of tier-1 RBs vs $\delta_{1,2}$, for $R_{BTS} = R_U = 50$ m, and $\alpha = 4$, for various denominations of ρ_1 and ρ_2 under the BTS density settings described in subsection 3.7.2.

Figure 3.25: Utilization of tier-2 RBs vs $\delta_{1,2}$, for $R_{BTS} = R_U = 50$ m, and $\alpha = 4$, for various denominations of ρ_1 and ρ_2 under the BTS density settings described in subsection 3.7.2.

Figure 3.24 shows that a larger density, ρ_1 , of independent uniform users, has a greater effect on increasing the utilization of tier-1 RBs when the value of $\delta_{1,2}$ is small, implying that these users consume most tier-1 RBs when user clusters are

densely accommodated by tier-2 BTS. An increased density of clustered users, ρ_2 , contributes to this effect to a smaller extent when $\delta_{1,2}$ is small. This is evident from the fact that the utilization of RBs from tier-1 is greatest at small $\delta_{1,2}$ when $\rho_1 = 300/\text{km}^2$, and $\rho_2 = 100/\text{km}^2$, even the overall density of users is the same for $\rho_1 = 200/\text{km}^2, \rho_2 = 200/\text{km}^2$ and $\rho_1 = 100/\text{km}^2, \rho_2 = 300/\text{km}^2$. Note that the last case renders the least utilization of tier-1 resources. For HCNs operating under larger cross-tier separation, however, the fraction of utilized tier-1 RBs approaches the same value across cases where the overall density of users, $\rho_0 = \rho_1 + \rho_2$ is equivalent. When $\delta_{1,2}$ is smaller, a much larger proportion of proximal clustered and uniform users are be offloaded to tier-2 BTSs. However, when $\delta_{1,2}$ is increased, tier-1 cells regions contain less tier-2 small-cells inside, and they serve a larger fraction of proximal clustered and independent users together in this situation. In particular, we note that increasing $\delta_{1,2}$ leads to the increase in the portion of clustered users from $\Psi_{2,B}$, and these users associate to the nearest tier-1 BTS with high probability. When the utilization of tier-1 BTSs reaches a figure of 1, the tier-1 BTSs are oversubscribed.

Figure 3.25 shows that a larger density of clustered users tends to increase the utilization of RBs by a greater fraction in tier-2 BTSs when $\delta_{1,2}$ is small. This is because tier-2 BTSs are designated to serve nearby clustered users around them, and any increase in density of these users would directly lead to the consumption of a larger fraction of RBs. When the cross-tier separation increases, the overall utilization of tier-2 RBs on average also increases, because of the absence of nearby tier-1 BTSs that poach a fraction of clustered and uniform users near the tier-2 BTSs. Figure 3.25 also shows that when $\delta_{1,2}$ is large, an increased density of either clustered or uniform users can have a similar effect on utilization of tier-2 RBs across different cases, provided the overall user density $\rho_0 = \rho_1 + \rho_2$ is equivalent.

Under-utilization of resources across the HCN does not necessarily detract from performance, as more available resources can be allocated to the users. If we assuming independent fading for each resource block, the chances of a user being in coverage are greater if multiple resource blocks are allocated to users. Furthermore, if users are in coverage in multiple channels, the overall user throughput and spectral efficiency of the HCN can be increased significantly. Figures 3.22 to 3.25 demonstrate that in a HCN densified with tier-2 BTSs and smaller cross-tier

separation, tier-2 BTSs are best placed to accommodate these additional resources, which is an incentive to populate the HCN with small-cells and keep cross-tier separation at a reasonably small value. A detailed study of how HCNs perform when users are afforded multiple resources is a good scope of extension for the model presented in this chapter.

3.8 Conclusion

This chapter presents a novel framework to model two-tier HCNs that incorporate separations between tier-1 and tier-2 BTSs, known as the cross-tier separation. We model users to form a mixed distribution where some users are clustered around social attractors while others are independent in nature. Each user cluster is either provisioned or denied a tier-2 BTS based on the cross-tier separation, and a random offset exists between deployed tier-2 BTSs and the centre of the user cluster. We derive distance PDFs and spatial association probabilities, showcasing the validity of the expressions through matching with simulated numerical results. Furthermore, we analyse using simulation the average utilization of RBs across BTSs in the HCN when resources are constrained per BTS, and show how cross-tier separation, as well as centralizing a tier-2 BTS to a user cluster affects the said utilization of RBs. The work in this chapter is extended to the analysis of coverage in Chapter 4.

Chapter 4

Coverage and ASE in Separation-Based Two-Tier HCNs with Mixed User Distributions

4.1 Scope of Work

In this section, we extend the work in Chapter 3 to provide expressions for the probability of coverage that apply to the proposed model. The theoretical analysis in this chapter focuses on spatial coverage probability for the expected user patterns outlined in Section 3.3, for which a number of topologies are introduced to approximate the Laplace transform of interference. In the analysis, we do not consider resource constraints for an HCN where there are only limited number of multiple orthogonal RBs per BTS. However, we describe how to compute using simulations, the coverage of the HCN when a limited number of RBs are available for transmission across each BTS in the HCN. Results are first validated numerically to support the analysis of spatial coverage, and we provide simulated results of ASE based on them. Furthermore, we showcase the probability of coverage and ASE when resource constraints are imposed. Trade-offs between coverage and ASE are presented in the numerical results, and they provide insights into how cross-tier separation can be used effectively to plan the locations or coordinate the activation of BTSs across the HCN.

4.2 Conditional Coverage Analysis

4.2.1 Definition of Coverage Based on SIR Threshold

A user associated with tier- k is in coverage if the SIR of its received signal exceeds the SIR threshold, β_k . The user will connect to the BTS located $\mathbf{x}^*$, defined in subsection 3.2.4. Transforming the origin to the user, where the distance to $\mathbf{x}^*$ is given by X^* , the SIR at the user is given by

$$SIR(X^*) = \frac{P_k h_{\mathbf{x}^*} (X^*)^{-\alpha}}{\left(\sum_{j=1}^2 \sum_{\mathbf{x}_{ji} \in \Phi_j \setminus \mathbf{x}^*} P_j h_{\mathbf{x}_{ji}} X_{ji}^{-\alpha} \right)}, \quad (4.1)$$

where α is the path loss exponent and h represents the fading. Conditioned on X^* and $\mathbf{T}_U^{\mathbf{x}^*}$, the probability of coverage under Rayleigh fading conditions, where the fading gain coefficients are $h \equiv g \sim \exp(1)$, the conditional probability of coverage follows the work presented in subsection 2.2.8. However, unlike in a PPP where BTSs are independent of each other, the association of users from all groups is dependent on a conditioned BTS. For example, the association of a user, $\mathbf{z}_{2,A} \in \Psi_{2,A}$, to its nearest tier-1 BTS, $\mathbf{x}_{11}$, is affected by the presence of the conditioned tier-2 BTS, $\mathbf{x}_{20}$. Similarly, the association of a user, $\mathbf{z}_1 \in \Psi_1$, or $\mathbf{z}_{2,B} \in \Psi_{2,B}$ to their respective nearest tier-2 BTS, $\mathbf{x}_{21}$ is affected by the presence of the nearest tier-1 BTS, $\mathbf{x}_{11}$, which creates a hole around it to deny deployed tier-2 BTSs. As such, the coverage probability is conditioned on the distance of interfering BTSs.

It is important to note that the SIR threshold, β_k , only influences whether a user is in coverage and able to successfully receive the downlink transmission from its tier- k serving BTS. Based on the connectivity model described in subsection 3.2.4, the user will associate to the BTS that provides it with the highest average received signal power, which according to (3.19), is not dependent on β_k .

We denote the set of conditioned interfering BTSs as $\{\mathbf{x}_I\} = \{\mathbf{x}_{I1}, \mathbf{x}_{I2}, \dots\}$, where each BTS is separated from the user by random distance X_{I1} . We restate the event where a user connecting to the BTS at $\mathbf{x}^*$ is in coverage, as $\mathcal{C}_U^{\mathbf{x}^*}$. Denoting the set of random distances as $\{X_{I1}\} = \{X_{I1}, X_{I1}, \dots\}$, we can modify the conditional coverage probability presented in subsection 2.2.8 to get

$$P_c(\mathbf{C}_U^{\mathbf{x}^*} | X^*, \{X_{I1}\}, \mathbf{T}_U^{\mathbf{x}^*}) = \prod_{j=1,2} \mathbb{E}_g(h > s_k g I_j) = \prod_{j=1,2} \mathcal{L}_U^j(\mathbf{x}^*), \quad (4.2)$$

where $s_k = (\beta_k(x^*)^\alpha / P_k)$, and I_j is the characteristic interference from tier- j . As different BTSs may serve a user from tier- k , we formally state that s_k is computed using the distance to serving BTS from tier- k in question. For example, if the serving BTS is $\mathbf{x}_{20}$, $s_2 = (\beta_2 x_{20}^\alpha / P_2)$. Conversely, if the serving BTS is $\mathbf{x}_{21}$, $s_2 = (\beta_2 x_{21}^\alpha / P_2)$. $\mathcal{L}_U^j(\mathbf{x}^*)$ is the Laplace transform of interference from tier- j to a user from set Ψ_U , that is tagged to $\mathbf{x}^*$. We expound on these transforms in the next subsection.

4.2.2 Laplace Transforms of Interference for Users Associating with Tier-1 BTSs

For a user tagged to $\mathbf{x}^*$, the Laplace transform of interference from PPP interferers of density λ_j is given by $\exp(-\lambda_j \pi \tau_j^{\mathbf{x}^*}(D))$, where we reproduce (2.39) to get

$$\tau_j^{\mathbf{x}^*}(D) = \left(\frac{(s_k P_j)^{\frac{2}{\alpha}}}{\text{sinc}(2\pi/\alpha)} - D^2 {}_2F_1\left(1, \frac{2}{\alpha}; \frac{\alpha+2}{\alpha}; -\frac{D^\alpha}{s_k P_j}\right) \right), \quad (4.3)$$

and $D = x^*(P_k/P_j)^{-1/\alpha}$. The function ${}_2F_1\left(1, \frac{2}{\alpha}; \frac{\alpha+2}{\alpha}; -\frac{D^\alpha}{s_k P_j}\right)$ in (4.3) refers to the hypergeometric function computed using the parameters inside the parenthesis. For all users tagged to $\mathbf{x}_{11}$, where $X_{11} = x_{11}$, we define the Laplace transform of interference from surrounding tier-1 BTSs as

$$\mathcal{L}_U^1(\mathbf{x}_{11}) \equiv \exp(-\lambda_1 \pi \tau_1^{\mathbf{x}_{11}}(x_{11})), \quad (4.4)$$

as the surrounding tier-1 interferers are a PPP, and no interfering BTS lies within a distance x_{11} from the user. We now proceed to look at the Laplace transform of interference from tier-2 BTSs for users tagged to $\mathbf{x}_{11}$.

The minimum distance from the user at which a tier-2 interferer exists is $x_{11}\bar{P}_2$. Without holes, total tier-2 interference would simply correspond to a PPP, except for the user $\mathbf{z}_{2,A} \in \Psi_{2,A}$, where the interference from the single planned tier-2 BTS, $\mathbf{x}_{20}$, must be considered. From the intuition in (4.2), it can be shown that the Laplace transform of interference from $\mathbf{x}_{20}$ only, can be presented as

$$\mathcal{L}_{\{2,A\}}^{\mathbf{x}_{20}}(\mathbf{x}_{11}) = \mathbb{E}_g(h > s_1 g P_2 x_{20}^{-\alpha}) = \mathbb{E}_g \exp(-s_1 g P_2 x_{20}^{-\alpha}). \quad (4.5)$$

Taking the expectation over the random fading variable $g \sim \exp(1)$, we get

$$\mathcal{L}_{\{2,A\}}^{\mathbf{x}_{20}}(\mathbf{x}_{11}) = \int_0^\infty \exp(-\bar{g}(1 + s_1 \bar{g} P_2 x_{20}^{-\alpha})) d\bar{g}. \quad (4.6)$$

Solving the integral in (4.6) gives us

$$\mathcal{L}_{\{2,A\}}^{\mathbf{x}_{20}}(\mathbf{x}_{11}) = \frac{1}{(1 + s_1 P_2 x_{20}^{-\alpha})} \quad (4.7)$$

We continue to provide the Laplace transform of interference from surrounding tier-2 BTSs to $\mathbf{z}_{2,A}$ when the user is associated with $\mathbf{x}_{11}$. From (4.3), and by noting that no independent tier-2 BTS is found within a distance of $x_{11}\bar{P}_2$, the Laplace transform of interference from surrounding tier-2 BTSs, if they were hypothetically distributed according to a PPP, is given by

$$\mathcal{L}_{\{2,A\}}^{2,PPP}(\mathbf{x}_{11}) = \exp\left(-\lambda_{2,b}\pi\left(\tau_2^{\mathbf{x}_{11}}(x_{11}\bar{P}_2)\right)\right) \quad (4.8)$$

However, in our proposed model, the surrounding interference from tier-2 BTS is reduced as compared to a PPP due to the holes created by tier-1 BTSs. We use Fig. 4.1 to depict the approximated topology that is used to compute the approximated reduced interference from surrounding tier-2 BTSs except for $\mathbf{x}_{20}$.

Figure 4.1: Approximated topology for the computation of reduced tier-2 BTS interference through the domain $x_{11} \bar{P}_2 \leq r \leq \delta_{1,2} + x_{11}$, when a user, $\mathbf{z}_{2,A}$, at the origin, associates with the nearest tier-1 BTS at $\mathbf{x}_{11}$.

Figure 4.1 showcases how we approximate this reduced interference by accounting for the hole of radius $\delta_{1,2}$, created by $\mathbf{x}_{11}$ only, following the methods used in [90]. The subtracted fraction of the Laplace transform of interference is therefore taken through the domain $x_{11} \bar{P}_2 \leq r \leq \delta_{1,2} + x_{11}$, where the limits are depicted in Fig. 4.1. Following the representation of the Laplace transform in terms of an integral in (2.37), we can write the subtracted fraction by integrating through the total region of reduced interference from tier-2 BTSs in 1-d, for which we use the angle $\theta_{\delta_{1,2}, r, x_{11}}$. To account for the subtracted fraction of interference, we observe that the density of tier-2 BTSs outside the nearest hole created by $\mathbf{x}_{11}$ is equal to $\lambda_{2,b}$. This subtracted fraction is now given by $\exp\left(-2\lambda_{2,b}v_{2,\{2,A\}}^{\mathbf{x}_{11}}\right)$, where

$$v_{2,\{2,A\}}^{\mathbf{x}_{11}} = \pi^{-1} \int_{x_{11}\bar{P}_2}^{\delta_{1,2}+x_{11}} \theta_{\delta_{1,2},r,x_{11}} \frac{s_1 P_2 r^{1-\alpha}}{1 + s_1 P_2 r^{-\alpha}} dr. \quad (4.9)$$

Therefore, the Laplace transform of interference from the surrounding PHP tier-2 BTSs in the proposed HCN model is approximated by dividing the transform in (4.8) by $\exp(-2\lambda_{2,b}v_{2,\{2,A\}}^{\mathbf{x}_{11}})$. Factoring in the interference by the conditioned BTS $\mathbf{x}_{20}$, we can write the overall Laplace transform of interference to the user as

$$\mathcal{L}_{\{2,A\}}^2(\mathbf{x}_{11}) = \frac{1}{(1 + s_1 P_2 x_{20}^{-\alpha})} \exp\left(-\lambda_{2,b}\pi\left(\tau_2^{\mathbf{x}_{11}}(x_{11}\bar{P}_2) - 2v_{2,\{2,A\}}^{\mathbf{x}_{11}}\right)\right). \quad (4.10)$$

We now examine the case where the user $\mathbf{z}_{2,B} \in \Psi_{2,B}$ tags to the tier-1 BTS, $\mathbf{x}_{11}$. We use Fig. 4.2 to depict the approximated topology that is used to compute the approximated reduced interference from surrounding tier-2 BTSs.

Figure 4.2: Approximated topology for the computation of reduced tier-2 BTS interference through the domain $x_{11}\bar{P}_2 \leq r \leq \delta_{1,2} + x_{11}$, when a user, $\mathbf{z}_{2,B} \in \Psi_{2,B}$, or a user, $\mathbf{z}_1 \in \Psi_1$ at the origin, associates with the nearest tier-1 BTS at $\mathbf{x}_{11}$.

Figure 4.2 showcases how to account for reduced interference inside the hole created by $\mathbf{x}_{11}$ only, by using the functional angle factor, $\theta_{\delta_{1,2}, r, x_{11}}$. We observe that a similar density of interfering tier-2 BTSs resides outside the hole created by $\mathbf{x}_{11}$, such that this density is equal to $\lambda_{2,b}$. By reusing the approximation of subtracted interference in the hole created by $\mathbf{x}_{11}$ only, we can follow the steps used to obtain (4.10) to describe the Laplace transform of interference from surrounding tier-2 BTSs. The user $\mathbf{z}_{2,B} \in \Psi_{2,B}$ is not targeted by a designated tier-2 BTS at $\mathbf{x}_{20}$. As such, we do not have to account for the Laplace transform of interference from this single conditioned tier-2 BTS. Hence, the Laplace transform, $\mathcal{L}_{\{2,B\}}^2(\mathbf{x}_{11})$, of interference from tier-2 BTSs to the user $\mathbf{z}_{2,B} \in \Psi_{2,B}$ that associates to the nearest tier-1 BTS at $\mathbf{x}_{11}$, is given by

$$\mathcal{L}_{\{2,B\}}^2(\mathbf{x}_{11}) = \exp\left(-\lambda_{2,b}\pi\left(\tau_2^{x_{11}}(x_{11}\bar{P}_2) - 2v_{2,\{2,B\}}^{x_{11}}\right)\right), \quad (4.11)$$

where $v_{2,\{2,B\}}^{\mathbf{x}_{11}} \equiv v_{2,\{2,A\}}^{\mathbf{x}_{11}}$, and $v_{2,\{2,A\}}^{\mathbf{x}_{11}}$ is described in (4.9).

The approximated interference pattern from tier-2 BTSs is the same for the clustered user $\mathbf{z}_{2,B} \in \Psi_{2,B}$ and the uniform users, $\mathbf{z}_1 \in \Psi_1$, because a conditioned BTS at $\mathbf{x}_{20}$ does not exist for both types of users. Therefore, we can also write the Laplace transform of interference from tier-2 BTSs to $\mathbf{z}_1$, when associating with $\mathbf{x}_{11}$, as

$$\mathcal{L}_{\{1\}}^2(\mathbf{x}_{11}) = \exp\left(-\lambda_{2,b}\pi\left(\tau_2^{\mathbf{x}_{11}}(x_{11}\bar{P}^{-1}) - 2v_{2,\{1\}}^{\mathbf{x}_{11}}\right)\right), \quad (4.12)$$

where $v_{2,\{1\}}^{\mathbf{x}_{11}} \equiv v_{2,\{2,A\}}^{\mathbf{x}_{11}}$, and $v_{2,\{2,A\}}^{\mathbf{x}_{11}}$ is described in (4.9). We note that the density of tier-2 BTSs outside the nearest hole is $\lambda_{2,b}$ to account for the density that is plugged into (4.12).

By assigning the subscript $\{\bar{U}\} = \{1\}, \{2, A\}, \{2, B\}$, we can definite the Laplace transform of interference from all BTSs to the user that tags to $\mathbf{x}_{11}$. This is given by the product of Laplace interference from tier-1 BTSs and tier-2 BTSs for each case, as seen in (4.2). As such the Laplace transform, $\mathcal{L}_{\{\bar{U}\}}(\mathbf{x}_{11})$, of total interference to the user that associates to the nearest tier-1 BTS at $\mathbf{x}_{11}$, is given by

$$\mathcal{L}_{\{\bar{U}\}}(\mathbf{x}_{11}) = \mathcal{L}_{\{\bar{U}\}}^2(\mathbf{x}_{11})\mathcal{L}_{\{\bar{U}\}}^1(\mathbf{x}_{11}). \quad (4.13)$$

From (4.2), we define the Laplace transform presented in (4.13) for all user categories to identify the set of conditioned interfering distances considered. For the users $\mathbf{z}_{2,A} \in \Psi_{2,A}$ and $\mathbf{z}_{2,B} \in \Psi_{2,B}$, the location of $\mathbf{x}_{11}$ is dependent on the conditioned tier-2 BTS, $\mathbf{x}_{20}$, and rejected candidate tier-2 BTS, $\bar{\mathbf{x}}_{20}$, respectively. Therefore, the conditional coverage probability pertaining to the event $\mathcal{C}_{2,A}^{\mathbf{x}_{11}}$ is conditioned on the distances X_{11} and X_{20} . From (4.13), we present the conditional probability of the event $\mathcal{C}_{2,A}^{\mathbf{x}_{11}}$ as

$$\mathrm{P}_c(\mathcal{C}_{2,A}^{\mathbf{x}_{11}} | X_{11}, X_{20}, \mathbf{T}_{2,A}^{\mathbf{x}_{11}}) \cong \mathcal{L}_{\{2,A\}}^2(\mathbf{x}_{11})\mathcal{L}_{\{2,A\}}^1(\mathbf{x}_{11}). \quad (4.14)$$

Similarly, the conditional coverage probability pertaining to the event $\mathcal{C}_{2,B}^{\mathbf{x}_{11}}$ is conditioned on the distances X_{11} and $\bar{X}_{20}$. From (4.13), we present the conditional probability of the event $\mathcal{C}_{2,B}^{\mathbf{x}_{11}}$ as

$$P_c(\mathcal{C}_{2,B}^{\mathbf{x}_{11}} | X_{11}, \bar{X}_{20}, \mathbf{T}_{2,B}^{\mathbf{x}_{11}}) \cong \mathcal{L}_{\{2,B\}}^2(\mathbf{x}_{11}) \mathcal{L}_{\{2,B\}}^1(\mathbf{x}_{11}). \quad (4.15)$$

For the user $\mathbf{z}_1 \in \Psi_1$, the event $\mathcal{C}_1^{\mathbf{x}_{11}}$ is conditioned on the location of the tier-1 BTS at $\mathbf{x}_{11}$ only. Therefore, conditional probability of the event $\mathcal{C}_1^{\mathbf{x}_{11}}$ is given by

$$P_c(\mathcal{C}_1^{\mathbf{x}_{11}} | X_{11}, \mathbf{T}_1^{\mathbf{x}_{11}}) \cong \mathcal{L}_{\{1\}}^2(\mathbf{x}_{11}) \mathcal{L}_{\{1\}}^1(\mathbf{x}_{11}). \quad (4.16)$$

4.2.3 Laplace Transforms of Interference for Provisioned Clustered Users Associating with Conditioned Tier-2 BTSs

We look at the tier-2 users, $\mathbf{z}_{2,A} \in \Psi_{2,A}$ if they associate with the tier-2 BTS conditioned on their cluster, $\mathbf{x}_{20}$. The cross-tier separation parameter dictates that there is a minimum separation, $\delta_{1,2}$, between $\mathbf{x}_{20}$ and any tier-1 BTS in the network. The reduction in interference from tier-1 BTSs can be modelled as holes around, $\mathbf{x}_{20}$, in which tier-1 interferers cannot exist. As such, the Laplace transform of interference from tier-1 BTSs to $\mathbf{x}_{20}$, when ignoring holes is given by

$$\mathcal{L}_{\{2,A\}}^{1,PPP}(\mathbf{x}_{20}) = \exp\left(-\lambda_{2,b}\pi\left(\tau_1^{\mathbf{x}_{20}}(x_{20}\bar{P}_1)\right)\right), \quad (4.17)$$

where the insertion of $\tau_1^{\mathbf{x}_{20}}(x_{20}\bar{P}_1)$ in (4.17) follows from the fact that no tier-1 BTS can exist in a radius, $x_{20}\bar{P}_1$, around $\mathbf{z}_{2,A}$ if the user associates to the BTS at $\mathbf{x}_{20}$. From a practical context, this means that the nearest tier-1 BTS exists at a distance greater than $x_{20}\bar{P}_1$ from the user, $\mathbf{z}_{2,A}$ if it is to associate to the BTS at $\mathbf{x}_{20}$. In this event, no tier-1 BTS transmits at a higher average power level to the user than the BTS at $\mathbf{x}_{20}$.

In order to explain how we account for the reduced interference from surrounding tier-1 BTSs when the user associates to $\mathbf{x}_{20}$, we use Fig. 4.3.

Figure 4.3: Approximated topology for the computation of reduced tier-1 BTS interference through the domain $x_{20}\bar{P}_1 \leq r \leq \delta_{1,2} + x_{20}$, when a user, $\mathbf{z}_{2,A} \in \Psi_{2,A}$, at the origin, associates with the conditioned tier-2 BTS at $\mathbf{x}_{20}$.

Figure 4.3 shows that interference from surrounding tier-2 BTSs is reduced through the domain $x_{20}\bar{P}_1 \leq r \leq \delta_{1,2} + x_{20}$, due to the hole around $\mathbf{x}_{20}$ in which tier-1 BTSs do not exist. It is important to note that this hole is a result of the actual hole that exists around the tier-1 BTSs due to cross-tier separation. If no tier-2 BTSs can exist within a distance of $\delta_{1,2}$ of a tier-1 BTS, then no tier-1 BTSs can exist within a distance of $\delta_{1,2}$ around a deployed tier-2 BTS. Following the method used to obtain (4.9), the reduced interference is given by $\exp\left(-2\lambda_1 v_{1,\{2,A\}}^{\mathbf{x}_{20}}\right)$, where $v_{1,\{2,A\}}^{\mathbf{x}_{20}}$ is computed using the function angle factor $\theta_{\delta_{1,2}, r, x_{20}}$ as seen in Fig. 4.3. However, in order to identify the limits pertaining to the integral used to compute $v_{1,\{2,A\}}^{\mathbf{x}_{20}}$, we notice that the nearest tier-1 BTS will exist at a distance $x_{11} \geq x_{20}\bar{P}_1$. As the distance $X_{11} = x_{11}$ is a variable distance distributed according to the distance PDF $f_{X_{11}|\mathbf{x}_{20}}^{\mathbf{z}_{2,A}}$, we rearrange the lower limit of the domain to be x_{11} , as no

tier-1 interferers can exist closer to the user than the BTS at $\mathbf{x}_{11}$. Hence, $v_{1,\{2,A\}}^{\mathbf{x}_{20}}$ is given by

$$v_{1,\{2,A\}}^{\mathbf{x}_{20}} = \pi^{-1} \int_{x_{11}}^{\delta_{1,2} + x_{20}} \theta_{\delta_{1,2}, r, x_{20}} \frac{s_2 P_1 r^{1-\alpha}}{1 + s_2 P_1 r^{-\alpha}} dr, \text{ if } \delta_{1,2} > x_{11} - x_{20}, \quad (4.18)$$

and $v_{1,\{2,A\}}^{\mathbf{x}_{20}} = 0$ if $\delta_{1,2} \leq x_{11} - x_{20}$.

As the nearest source of interference from tier-1 BTSs depends on the position of $\mathbf{x}_{11}$, the expression in (4.17) should be modified to reflect this, such that the non-reduced interference from tier-1 PPP BTSs is $\exp\left(-\lambda_1 \pi \left(\tau_1^{\mathbf{x}_{20}}(x_{11})\right)\right)$ instead of $\exp\left(-\lambda_1 \pi \left(\tau_1^{\mathbf{x}_{20}}(x_{20} \bar{P}_1)\right)\right)$, $\tau_1^{\mathbf{x}_{20}}(x_{11})$. Subsequently, the Laplace transform of interference from tier-1 BTSs is given by

$$\mathcal{L}_{\{2,A\}}^1(\mathbf{x}_{20}) = \exp\left(-\lambda_1 \pi \left(\tau_1^{\mathbf{x}_{20}}(x_{11}) - 2v_{1,\{2,A\}}^{\mathbf{x}_{20}}\right)\right). \quad (4.19)$$

The interference from tier-2 BTSs to the user $\mathbf{z}_{2,A}$ associated to the BTS at $\mathbf{x}_{20}$ is conditioned on the nearest hole of radius $\delta_{1,2}$ created by $\mathbf{x}_{11}$, as no tier-2 BTSs can exist within this hole. We refer to Fig. 4.4 to understand how to compute the reduced interference factor in the Laplace transform.

Figure 4.4: Approximated topology for the computation of reduced tier-2 BTS interference through the domain $x_{20} \leq r \leq \delta_{1,2} + x_{11}$, when a user, $\mathbf{z}_{2,A} \in \Psi_{2,A}$, at the origin, associates with the conditioned tier-2 BTS at $\mathbf{x}_{20}$.

From Fig. 4.4, we note that the minimum distance to the nearest interfering tier-2 BTS is x_{20} as it cannot be closer to the user than $\mathbf{x}_{20}$. Following the steps used in (4.17) to (4.19), we note that the reduced interference factor is dependent on the functional angle factor, $\theta_{\delta_{1,2}, r, x_{11}}$. As such, the Laplace transform of interference from tier-2 BTSs, $\mathcal{L}_{\{2,A\}}^2(\mathbf{x}_{20})$, when the user $\mathbf{z}_{2,A} \in \Psi_{2,A}$ associates with the conditioned tier-2 BTS at $\mathbf{x}_{20}$ is given by

$$\mathcal{L}_{\{2,A\}}^2(\mathbf{x}_{20}) = \exp\left(-\lambda_{2,b}\pi\left(\tau_2^{\mathbf{x}_{20}}(x_{20}) - 2v_{2,\{2,A\}}^{\mathbf{x}_{20}}\right)\right), \quad (4.20)$$

where

$$v_{2,\{2,A\}}^{\mathbf{x}_{20}} = \pi^{-1} \int_{x_{20}}^{\delta_{1,2} + x_{11}} \theta_{\delta_{1,2}, r, x_{11}} \frac{s_2 P_2 r^{1-\alpha}}{1 + s_2 P_2 r^{-\alpha}} dr. \quad (4.21)$$

We note again the interference is conditioned on the locations of $\mathbf{x}_{20}$ and $\mathbf{x}_{11}$, and hence the distances X_{20} and X_{11} . Therefore, the conditional probability of the coverage event $\mathcal{C}_{2,A}^{\mathbf{x}_{20}}$ is given by

$$P_c(\mathcal{C}_{2,A}^{\mathbf{x}_{20}} | X_{20}, X_{11}, \mathbf{T}_{2,A}^{\mathbf{x}_{20}}) \cong \mathcal{L}_{\{2,A\}}^2(\mathbf{x}_{20}) \mathcal{L}_{\{2,A\}}^1(\mathbf{x}_{20}). \quad (4.22)$$

4.2.4 Laplace Transforms of Interference for Provisioned Clustered Users Associating with Independent Tier-2 BTSs

The user $\mathbf{z}_{2,A} \in \Psi_{2,A}$ may associate with the nearest independent tier-2 BTS at $\mathbf{x}_{21}$. As the conditioned tier-2 BTS at $\mathbf{x}_{20}$ acts as an interferer in this case, we define the Laplace transform due to this BTS as

$$\mathcal{L}_{\{2,A\}}^{\mathbf{x}_{20}}(\mathbf{x}_{21}) = \frac{1}{(1 + s_2 P_2 x_{20}^{-\alpha})} \quad (4.23)$$

The interference from the remaining tier-2 BTSs is reduced by the hole around the nearest tier-1 BTS at $\mathbf{x}_{11}$. We refer to Fig. 4.5 to showcase how the reduced interference factor can be accounted for in the Laplace transform of interference from surrounding tier-2 BTSs.

Figure 4.5: Approximated topology for the computation of reduced tier-2 BTS interference through the domain $x_{21} \leq r \leq \delta_{1,2} + x_{11}$, when a user, $\mathbf{z}_{2,A} \in \Psi_{2,A}$, at the origin, associates with the independent tier-2 BTS at $\mathbf{x}_{21}$.

Figure 4.5 shows that tier-BTS interference is reduced through the domain $x_{21} \leq r \leq \delta_{1,2} + x_{11}$. We may use the functional angle factor $\theta_{\delta_{1,2}, r, x_{11}}$ to account for the reduction in tier-2 interference when computing its Laplace transform. By recognizing that no interfering tier-2 BTS exists at a distance closer than x_{21} , conditioning on the presence of the BTS $\mathbf{x}_{20}$, and accounting for reduced interference, we may compute the Laplace transform $\mathcal{L}_{\{2,A\}}^2(\mathbf{x}_{21})$, as

$$\mathcal{L}_{\{2,A\}}^2(\mathbf{x}_{21}) = \frac{1}{(1 + s_2 P_2 x_{20}^{-\alpha})} \exp\left(-\lambda_{2,b} \pi \left(\tau_2^{\mathbf{x}_{21}}(x_{21}) - 2v_{2,\{2,A\}}^{\mathbf{x}_{21}}\right)\right), \quad (4.24)$$

where

$$v_{2,\{2,A\}}^{\mathbf{x}_{21}} = \pi^{-1} \int_{x_{21}}^{\delta_{1,2} + x_{11}} \theta_{\delta_{1,2}, r, x_{11}} \frac{s_2 P_2 r^{1-\alpha}}{1 + s_2 P_2 r^{-\alpha}} dr. \quad (4.25)$$

A few assumptions are made to tractably compute $v_{2,\{2,A\}}^{x_{21}}$ in (4.25), and these are observed in Fig. 4.5. It is assumed that $\mathbf{x}_{21}$ and $\mathbf{x}_{20}$ share the same tier-1 neighbour, $\mathbf{x}_{11}$. Secondly, we assume that $\mathbf{x}_{20}$ is closer to this BTS than $\mathbf{x}_{21}$, which yields the upper limit on the integral.

We now proceed to the Laplace transform of interference from tier-1 BTSs to the user when it tags to $\mathbf{x}_{21}$. Two holes are created around $\mathbf{x}_{20}$ and $\mathbf{x}_{21}$, and tier-1 interference must be subtracted from these regions. We analyse the subtracted fraction of the Laplace transform, given by $\exp(-2\lambda_1 v_{1,\{2,A\}}^{x_{21}})$, and focus on the term $v_{1,\{2,A\}}^{x_{21}}$. Considering the overlaps between the holes, we present the situation where the distance between $\mathbf{x}_{20}$ and $\mathbf{x}_{21}$ is at a minimum, i.e. $\|\mathbf{x}_{20} - \mathbf{x}_{21}\| = x_{20} - x_{21}$. We examine this scenario in Fig. 4.6.

Figure 4.6: Approximated topology for the computation of reduced tier-1 BTS interference through $x_{11} \leq r \leq \delta_{1,2} + x_{20}$, when a user, $\mathbf{z}_{2,A} \in \Psi_{2,A}$, at the origin, associates to the independent tier-2 BTS at $\mathbf{x}_{21}$, when $\|\mathbf{x}_{20} - \mathbf{x}_{21}\| = x_{20} - x_{21}$.

In this scenario depicted in Fig. 4.6, we notice that the functional angle used to compute the reduced interference is now $\max(\theta_{\delta_{1,2},r,x_{20}} + \theta_{\delta_{1,2},r,x_{21}})$. In the specific depiction in Fig. 4.6, $\max(\theta_{\delta_{1,2},r,x_{20}} + \theta_{\delta_{1,2},r,x_{21}}) = \theta_{\delta_{1,2},r,x_{20}}$. We note that an overlap between the holes around $\mathbf{x}_{21}$ and $\mathbf{x}_{20}$ can only occur if $\delta_{1,2} \geq x_{11} - x_{20}$. Hence the modified parameter $\bar{v}_{1,\{2,A\}}^{\mathbf{x}_{21}}$ is given by

$$\bar{v}_{1,\{2,A\}}^{\mathbf{x}_{21}} = \pi^{-1} \int_{x_{11}}^{\delta_{1,2} + x_{20}} \max(\theta_{\delta_{1,2},r,x_{20}} + \theta_{\delta_{1,2},r,x_{21}}) \frac{s_2 P_2 r^{1-\alpha}}{1 + s_2 P_2 r^{-\alpha}} dr, \quad (4.26)$$

if $\delta_{1,2} \geq x_{11} - x_{20}$, and $\bar{v}_{1,\{2,A\}}^{\mathbf{x}_{21}} = 0$ otherwise.

Similarly, we may account for the overlap between the holes under $\max(\|\mathbf{x}_{20} - \mathbf{x}_{21}\|) = x_{20} + x_{21}$. This scenario is depicted in Fig. 4.7 below.

Figure 4.7: Approximated topology for the computation of reduced tier-1 BTS interference through $x_{11} \leq r \leq \delta_{1,2} + x_{20}$, when a user, $\mathbf{z}_{2,A} \in \Psi_{2,A}$, at the origin, associates to the independent tier-2 BTS at $\mathbf{x}_{21}$, when $\|\mathbf{x}_{20} - \mathbf{x}_{21}\| = x_{20} + x_{21}$.

In the scenario depicted in Fig. 4.7, we note that the functional angle used to compute the reduced interference is now $\theta_{\delta_{1,2},r,x_{20}} + \theta_{\delta_{1,2},r,x_{21}}$. Therefore, we present the modified parameter $\bar{v}_{1,\{2,A\}}^{\mathbf{x}_{21}}$, such that, if $\delta_{1,2} \geq x_{11} - x_{20}$,

$$\bar{v}_{1,\{2,A\}}^{\mathbf{x}_{21}} = \pi^{-1} \int_{x_{11}}^{\delta_{1,2}+x_{20}} (\theta_{\delta_{1,2},r,x_{20}} + \theta_{\delta_{1,2},r,x_{21}}) \frac{s_2 P_2 r^{1-\alpha}}{1 + s_2 P_2 r^{-\alpha}} dr, \quad (4.27)$$

if $\theta_{\delta_{1,2},r,x_{20}} + \theta_{\delta_{1,2},r,x_{21}} < \pi$, or

$$\bar{v}_{1,\{2,A\}}^{\mathbf{x}_{21}} = \int_{x_{11}}^{\delta_{1,2}+x_{20}} \frac{s_2 P_2 r^{1-\alpha}}{1 + s_2 P_2 r^{-\alpha}} dr, \quad (4.28)$$

if $\theta_{\delta_{1,2},r,x_{20}} + \theta_{\delta_{1,2},r,x_{21}} \geq \pi$. If $\delta_{1,2} < x_{11} - x_{20}$, $\bar{v}_{1,\{2,A\}}^{\mathbf{x}_{21}} = 0$. The situations depicted to account for the reduction in interference by tier-1 BTSs due to holes surrounding both $\mathbf{x}_{20}$ and $\mathbf{x}_{21}$ present two extreme situations for the distance between $\mathbf{x}_{20}$ and $\mathbf{x}_{21}$. In reality, the distance between these two BTSs would take a range of values between $x_{20} - x_{21}$ and $x_{20} + x_{21}$. However, computing all these increments of interference reduction to get the average reduction in interference by tier-1 BTSs is an intractable process. Hence, we average over the two situations presented, to account for the parameter $v_{1,\{2,A\}}^{\mathbf{x}_{21}}$ that is used to compute the reduction in tier-1 interference, such that

$$v_{1,\{2,A\}}^{\mathbf{x}_{21}} = \frac{\bar{v}_{1,\{2,A\}}^{\mathbf{x}_{21}} + \bar{v}_{1,\{2,A\}}^{\mathbf{x}_{21}}}{2}. \quad (4.29)$$

Following on, the Laplace transform of tier-1 interference to the user $\mathbf{z}_{2,A} \in \Psi_{2,A}$, when associating with $\mathbf{x}_{21}$ is given by

$$\mathcal{L}_{\{2,A\}}^1(\mathbf{x}_{21}) = \exp\left(-\lambda_1\left(\tau_1^{\mathbf{x}_{21}}(x_{11}) - 2v_{1,\{2,A\}}^{\mathbf{x}_{21}}\right)\right). \quad (4.30)$$

By taking the product of the two transforms, and accounting for the distances on which they are conditioned, we get

$$P_c(\mathbf{C}_{2,A}^{\mathbf{x}_{21}} | X_{21}, X_{20}, X_{11}, \mathbf{T}_{2,A}^{\mathbf{x}_{21}}) \cong \mathcal{L}_{\{2,A\}}^2(\mathbf{x}_{21}) \mathcal{L}_{\{2,A\}}^1(\mathbf{x}_{21}). \quad (4.31)$$

4.2.5 Laplace Transforms of Interference for Independent Clustered Users Associating with Tier-2 BTSs

The user $\mathbf{z}_{2,B} \in \Psi_{2,B}$ may associate with the nearest tier-2 BTS at $\mathbf{x}_{21}$, that exists outside the hole of radius $\delta_{1,2}$ created by the conditioned BTS, $\mathbf{x}_{11}$. The conditioned interference from this BTS creates the Laplace transform

$$\mathcal{L}_{\{2,B\}}^{\mathbf{x}_{11}}(\mathbf{x}_{21}) = \frac{1}{(1 + s_2 P_1 x_{11}^{-\alpha})} \quad (4.32)$$

Using the techniques presented thus far, the transform of interference from tier-1 is simply

$$\mathcal{L}_{\{2,B\}}^1(\mathbf{x}_{21}) = \frac{1}{(1 + s_2 P_1 x_{11}^{-\alpha})} \exp\left(-\lambda_1 \pi \left(\tau_1^{\mathbf{x}_{21}}(x_{11}) - 2v_{1,\{2,B\}}^{\mathbf{x}_{21}}\right)\right), \quad (4.33)$$

where

$$v_{1,\{2,B\}}^{\mathbf{x}_{21}} = \pi^{-1} \int_{x_{11}}^{\delta_{1,2} + x_{21}} \theta_{\delta_{1,2}, r, x_{21}} \frac{s_2 P_1 r^{1-\alpha}}{1 + s_2 P_1 r^{-\alpha}} dr, \quad \text{if } \delta_{1,2} \geq x_{11} - x_{20}, \quad (4.34)$$

and $v_{1,\{2,B\}}^{\mathbf{x}_{21}} = 0$ if $\delta_{1,2} < x_{11} - x_{20}$.

Similarly, we present the Laplace transform of interference from tier-2 BTSs to the user as

$$\mathcal{L}_{\{2,B\}}^2(\mathbf{x}_{21}) = \exp\left(-\lambda_{2,b} \pi \left(\tau_2^{\mathbf{x}_{21}}(x_{21}) - 2v_{2,\{2,B\}}^{\mathbf{x}_{21}}\right)\right), \quad (4.35)$$

where

$$v_{2,\{2,B\}}^{\mathbf{x}_{21}} = \pi^{-1} \int_{x_{21}}^{\delta_{1,2} + x_{21}} \theta_{\delta_{1,2}, r, x_{21}} \frac{s_2 P_2 r^{1-\alpha}}{1 + s_2 P_2 r^{-\alpha}} dr. \quad (4.36)$$

The conditional probability of coverage relating to this association case is dependent on the distances X_{21} , X_{11} and $\bar{X}_{20}$. Thus, we may write

$$P_c(\mathbf{C}_{2,B}^{\mathbf{x}_{21}} | X_{21}, X_{11}, \bar{X}_{20}, \mathbf{T}_{2,B}^{\mathbf{x}_{21}}) \cong \mathcal{L}_{\{2,B\}}^2(\mathbf{x}_{21}) \mathcal{L}_{\{2,B\}}^1(\mathbf{x}_{21}). \quad (4.37)$$

4.2.6 Laplace Transforms of Interference for Independent Uniform Users Associating with Tier-2 BTSs

The intuition behind deriving the transforms of interference to the user $\mathbf{z}_1 \in \Psi_1$ is identical to that behind doing the same for the user $\mathbf{z}_{2,B} \in \Psi_{2,B}$. As such, we simply outline the Laplace transform of interference from tier-1, $\mathcal{L}_{\{1\}}^1(\mathbf{x}_{21})$ as

$$\mathcal{L}_{\{1\}}^1(\mathbf{x}_{21}) = \frac{1}{(1 + s_2 P_1 x_{11}^{-\alpha})} \exp\left(-\lambda_1 \pi \left(\tau_1^{\mathbf{x}_{21}}(x_{11}) - 2v_{1,\{1\}}^{\mathbf{x}_{21}}\right)\right), \quad (4.38)$$

where $v_{1,\{1\}}^{\mathbf{x}_{21}} \equiv v_{1,\{2,B\}}^{\mathbf{x}_{21}}$. Similarly, the Laplace transform of interference from tier-2 is

$$\mathcal{L}_{\{1\}}^2(\mathbf{x}_{21}) = \exp\left(-\lambda_{2,b} \pi \left(\tau_2^{\mathbf{x}_{21}}(x_{21}) - 2v_{2,\{1\}}^{\mathbf{x}_{21}}\right)\right), \quad (4.39)$$

Where $v_{2,\{1\}}^{\mathbf{x}_{21}} \equiv v_{2,\{2,B\}}^{\mathbf{x}_{21}}$.

The conditional probability of coverage relating to this association case is dependent on the distances X_{21} and X_{11} only. Thus, we may write

$$P_c(\mathbf{C}_1^{\mathbf{x}_{21}} | X_{21}, X_{11}, \mathbf{T}_1^{\mathbf{x}_{21}}) \cong \mathcal{L}_{\{1\}}^2(\mathbf{x}_{21}) \mathcal{L}_{\{1\}}^1(\mathbf{x}_{21}). \quad (4.40)$$

We now move on to the section that discusses the overall coverage probabilities and provides formulae that represent rate, ASE, and BTS utilization.

4.3 Total Coverage and ASE

We first define $\mathcal{L}_{\{U\}}(\mathbf{x}_{ko}) = \mathcal{L}_{\{U\}}^2(\mathbf{x}_{ko})\mathcal{L}_{\{U\}}^1(\mathbf{x}_{ko})$, where the term U indexes the subscripts 1, 2, A , and $2, B$ to represent the user groups, and the serving BTS is denoted by $\mathbf{x}_{ko}$. We now look at coverage of specific users.

4.3.1 Overall Coverage of Provisioned Clustered Users

Using the joint distance PDFs and conditional association probabilities derived in Chapter 3, we first uncondition on distance and association to get the joint coverage and association probability of a user when it follows the association event $\mathbf{T}_{2,A}^{\mathbf{x}_{20}}$, such that

$$\begin{aligned} & \Pr(\mathbf{T}_{2,A}^{\mathbf{x}_{20}}, \mathbf{C}_{2,A}^{\mathbf{x}_{20}}) \\ &= \int_{X_{20}} \int_{\bar{P}_1 X_{20}}^{\infty} f_{X_{11}, X_{20}}^{\mathbf{z}_{2,A}} P_A(\mathbf{T}_{2,A}^{\mathbf{x}_{20}} | X_{20}, X_{11}) \mathcal{L}_{\{2,A\}}(\mathbf{x}_{20}) dx_{11} dx_{20}, \end{aligned} \quad (4.41)$$

where the domain of X_{20} is $0 \leq X_{20} \leq R_{BTS} + R_U$. We follow the same procedure for the association event $\mathbf{T}_{2,A}^{\mathbf{x}_{11}}$ to obtain

$$\begin{aligned} & \Pr(\mathbf{T}_{2,A}^{\mathbf{x}_{11}}, \mathbf{C}_{2,A}^{\mathbf{x}_{11}}) \\ &= \int_{X_{20}} \int_0^{\bar{P}_1 X_{20}} f_{X_{11}, X_{20}}^{\mathbf{z}_{2,A}} P_A(\mathbf{T}_{2,A}^{\mathbf{x}_{11}} | X_{20}, X_{11}) \mathcal{L}_{\{2,A\}}(\mathbf{x}_{11}) dx_{11} dx_{20} \end{aligned} \quad (4.42)$$

Likewise, for the association event $\mathbf{T}_{2,A}^{\mathbf{x}_{21}}$,

$$\begin{aligned} & \Pr(\mathbf{T}_{2,A}^{\mathbf{x}_{21}}, \mathbf{C}_{2,A}^{\mathbf{x}_{21}}) \\ &= \int_{X_{20}} \int_0^{\infty} \int_0^{X_{20}} f_{X_{21}, X_{11}, X_{20}}^{\mathbf{z}_{2,A}} P_A(\mathbf{T}_{2,A}^{\mathbf{x}_{21}} | X_{21}, X_{20}, X_{11}) \mathcal{L}_{\{2,A\}}(\mathbf{x}_{21}) dx_{21} dx_{11} dx_{20} \end{aligned} \quad (4.43)$$

The overall coverage probability of the user $\mathbf{z}_{2,A} \in \Psi_{2,A}$ is obtained by adding these joint probabilities across all mutually exclusive association events, such that

$$P_c(\mathbf{C}_{2,A}) = \Pr(\mathbf{T}_{2,A}^{\mathbf{x}_{20}}, \mathbf{C}_{2,A}^{\mathbf{x}_{20}}) + \Pr(\mathbf{T}_{2,A}^{\mathbf{x}_{11}}, \mathbf{C}_{2,A}^{\mathbf{x}_{11}}) + \Pr(\mathbf{T}_{2,A}^{\mathbf{x}_{21}}, \mathbf{C}_{2,A}^{\mathbf{x}_{21}}). \quad (4.44)$$

4.3.2 Overall Coverage of Independent Clustered Users

Following the same methods used in the previous subsection, we look at the joint coverage and association probabilities of the user $\mathbf{z}_{2,B} \in \Psi_{2,B}$. For the association event $\mathbf{T}_{2,B}^{\mathbf{x}_{11}}$, we get

$$\begin{aligned} & \Pr(\mathbf{T}_{2,B}^{\mathbf{x}_{11}}, \mathbf{C}_{2,B}^{\mathbf{x}_{11}}) \\ &= \int_{\bar{X}_{20}} \int_0^{\bar{X}_{20} + \delta_{1,2}} f_{X_{11}, \bar{X}_{20}}^{\mathbf{z}_{2,B}} P_A(\mathbf{T}_{2,B}^{\mathbf{x}_{11}} | \bar{X}_{20}, X_{11}) \mathcal{L}_{\{2,B\}}(\mathbf{x}_{11}) dx_{11} d\bar{x}_{20}, \end{aligned} \quad (4.45)$$

where the domain of $\bar{X}_{20}$ is $0 \leq \bar{X}_{20} \leq R_{BTS} + R_U$. Similarly, for the association event $\mathbf{T}_{2,B}^{\mathbf{x}_{21}}$, it is observed that

$$\begin{aligned} & \Pr(\mathbf{T}_{2,B}^{\mathbf{x}_{21}}, \mathbf{C}_{2,B}^{\mathbf{x}_{21}}) \\ &= \int_{\bar{X}_{20}} \int_{X_{11}} \int_0^{\bar{X}_{20}} f_{X_{21}, X_{11}, \bar{X}_{20}}^{\mathbf{z}_{2,B}} P_A(\mathbf{T}_{2,B}^{\mathbf{x}_{21}} | X_{21}, X_{20}, X_{11}) \mathcal{L}_{\{2,B\}}(\mathbf{x}_{21}) dx_{21} dx_{11} d\bar{x}_{20}, \end{aligned} \quad (4.46)$$

where the domain of X_{11} is given by $0 \leq X_{11} \leq \bar{x}_{20} + \delta_{1,2}$. Following on,

$$P_c(\mathbf{C}_{2,B}) = \Pr(\mathbf{T}_{2,B}^{\mathbf{x}_{11}}, \mathbf{C}_{2,B}^{\mathbf{x}_{11}}) + \Pr(\mathbf{T}_{2,B}^{\mathbf{x}_{21}}, \mathbf{C}_{2,B}^{\mathbf{x}_{21}}). \quad (4.47)$$

4.3.3 Overall Coverage of Independent Uniform Users

Continuing with the same unconditioning methods used in previous sections, the joint association and coverage probabilities for the user $\mathbf{z}_1 \in \Psi_1$ are given by

$$\Pr(\mathbf{T}_1^{\mathbf{x}_{11}}, \mathbf{C}_1^{\mathbf{x}_{11}}) = \int_0^\infty f_{X_{11}}^{\mathbf{z}_1} P_A(\mathbf{T}_1^{\mathbf{x}_{11}} | X_{11}) \mathcal{L}_{\{1\}}(\mathbf{x}_{11}) dx_{11}. \quad (4.48)$$

and

$$\begin{aligned} & \Pr(\mathbf{T}_1^{\mathbf{x}_{11}}, \mathbf{C}_1^{\mathbf{x}_{11}}) \\ &= \int_0^\infty \int_0^{\bar{P}_2 x_{11}} f_{X_{11}, X_{20}}^{\mathbf{z}_{2,A}} P_A(\mathbf{T}_{2,A}^{\mathbf{x}_{11}} | X_{20}, X_{11}) \mathcal{L}_{\{1\}}(\mathbf{x}_{11}) dx_{21} dx_{11}. \end{aligned} \quad (4.49)$$

Following on,

$$P_c(\mathbf{C}_1) = \Pr(\mathbf{T}_1^{\mathbf{x}_{11}}, \mathbf{C}_1^{\mathbf{x}_{11}}) + \Pr(\mathbf{T}_1^{\mathbf{x}_{21}}, \mathbf{C}_1^{\mathbf{x}_{21}}). \quad (4.50)$$

4.3.4 Overall Coverage of Users Connecting to Each Tier

The overall coverage of users across the two-tier HCN that associate to the k^{th} tier is given by P_c^k , where $k = 1, 2$. The probability P_c^k is dependent on the density of users. Given that all user groups are mutually exclusive, and association to tier-1 or tier-2 BTSs are mutually exclusive events, the joint association and coverage probability of a user tagging to tier-1 is given by P_c^1 , where

$$P_c^1 = \frac{\Pr(\mathbf{T}_{2,A}^{\mathbf{x}_{11}}, \mathbf{C}_{2,A}^{\mathbf{x}_{11}}) \rho_{2,A} + \Pr(\mathbf{T}_{2,B}^{\mathbf{x}_{11}}, \mathbf{C}_{2,B}^{\mathbf{x}_{11}}) \rho_{2,B} + \Pr(\mathbf{T}_1^{\mathbf{x}_{11}}, \mathbf{C}_1^{\mathbf{x}_{11}}) \rho_1}{\rho_o}. \quad (4.51)$$

The summed probabilities are taken across all possible mutually exclusive association events. Similarly, the joint association and coverage probability of a user tagging to tier-2 is given by P_c^2 , where

$$\begin{aligned} & P_c^2 \\ &= \frac{(\Pr(\mathbf{T}_{2,A}^{\mathbf{x}_{20}}, \mathbf{C}_{2,A}^{\mathbf{x}_{20}}) + \Pr(\mathbf{T}_{2,A}^{\mathbf{x}_{21}}, \mathbf{C}_{2,A}^{\mathbf{x}_{21}})) \rho_{2,A} + \Pr(\mathbf{T}_{2,B}^{\mathbf{x}_{21}}, \mathbf{C}_{2,B}^{\mathbf{x}_{21}}) \rho_{2,B} + \Pr(\mathbf{T}_1^{\mathbf{x}_{21}}, \mathbf{C}_{2,B}^{\mathbf{x}_{21}}) \rho_1}{\rho_o}. \end{aligned} \quad (4.52)$$

As association to the two-tiers is mutually exclusive, the overall coverage probability of the HCN is defined by P_c^{HCN} , where

$$P_c^{\text{HCN}} = P_c^1 + P_c^2 \quad (4.53)$$

4.3.5 Coverage Under Constrained Resources

The coverage under constrained resources is related to the probability of tagging to a free block, which was defined in Section 3.6 as $P_{f,U}^{x_{ki}}$. Although we don't present analysis for this parameter, we may simply plug it in to the derived coverage probabilities in (4.42) and (4.43) to understand the coverage under constrained resources. This can be done since the parameter depends solely on the density of users and the available resources per BTS. Defined the coverage event as $\bar{\mathcal{C}}_{\bar{U}}^{x_{ko}}$ to go with the association event $\bar{\mathcal{T}}_{\bar{U}}^{x_{ko}}$, we define the coverage probability of users associating with tier- k

$$\Pr(\bar{\mathcal{T}}_{\bar{U}}^{x_{ko}}, \bar{\mathcal{C}}_{\bar{U}}^{x_{ko}}) = P_{f,U}^{x_{ki}} \Pr(\mathcal{T}_{\bar{U}}^{x_{ko}}, \mathcal{C}_{\bar{U}}^{x_{ko}}). \quad (4.54)$$

Similarly, from (4.51), (4.52) and (4.53), we get the probability of coverage for users associating to tier-1 and tier-2, when BTS resources are constrained, as

$$\bar{P}_c^1 = \frac{\Pr(\bar{\mathcal{T}}_{2,A}^{x_{11}}, \bar{\mathcal{C}}_{2,A}^{x_{11}})\rho_{2,A} + \Pr(\bar{\mathcal{T}}_{2,B}^{x_{11}}, \bar{\mathcal{C}}_{2,B}^{x_{11}})\rho_{2,B} + \Pr(\bar{\mathcal{T}}_1^{x_{11}}, \bar{\mathcal{C}}_1^{x_{11}})\rho_1}{\rho_{2,A} + \rho_{2,B} + \rho_1} \quad (4.55)$$

and

$$\begin{aligned} \bar{P}_c^2 &= \frac{(\Pr(\bar{\mathcal{T}}_{2,A}^{x_{20}}, \bar{\mathcal{C}}_{2,A}^{x_{20}}) + \Pr(\bar{\mathcal{T}}_{2,A}^{x_{21}}, \bar{\mathcal{C}}_{2,A}^{x_{21}}))\rho_{2,A} + \Pr(\bar{\mathcal{T}}_{2,B}^{x_{21}}, \bar{\mathcal{C}}_{2,B}^{x_{21}})\rho_{2,B} + \Pr(\bar{\mathcal{T}}_1^{x_{21}}, \bar{\mathcal{C}}_{2,B}^{x_{21}})\rho_1}{\rho_{2,A} + \rho_{2,B} + \rho_1}. \end{aligned} \quad (4.56)$$

The overall probability of coverage of the HCN under constrained resources is given by $\bar{P}_c^{\text{HCN}} = \bar{P}_c^1 + \bar{P}_c^2$.

4.3.6 Rate and ASE

The instantaneous Shannon-theoretic rate of a user tagged to a tier- k BTS when in coverage is given by

$$\zeta_{x_{ki}} = \log_2(1 + \text{SIR}(X_{ki})), \text{SIR}(X_{ki}) \geq \beta_k \quad (4.57)$$

where $\mathbf{x}_{ki}$ refers to the serving BTS in this case, and the rate is taken in bits/s/Hz. For clustered users, $\mathbf{x}_{ki}$ may refer to $\mathbf{x}_{20}$, $\mathbf{x}_{11}$ or $\mathbf{x}_{21}$, while for independent uniform users, the serving BTSs are denoted by $\mathbf{x}_{11}$ or $\mathbf{x}_{20}$. When the instantaneous rate is spatially averaged for this BTS, we define the average Shannon rate when the user is in coverage, $\check{\zeta}_{\mathbf{x}_{ki}}$, as

$$\check{\zeta}_{\mathbf{x}_{ki}} = \mathbb{E}(\zeta_{\mathbf{x}_{ki}}). \quad (4.58)$$

We do not present proofs for deriving the average rate based on coverage in this thesis, but instead produce simulated results related to this parameter. The average rate per tier is given by

$$\zeta_k = \mathbb{E}_{\mathbf{x}_{ki}}(\zeta_{\mathbf{x}_{ki}}), \quad (4.59)$$

where the expectation is taken across all serving BTSs across tier- k that are designated by the subscript i . The average rate per tier in (4.48) is directly related to the ASE of the system from the equation

$$ASE = \lambda_k P_c(k) \zeta_k. \quad (4.60)$$

4.4 Numerical Results and Discussion

We present different types of numerical results in this section for discussion. These include results that validate the expressions describing the analysis of probability of coverage, explore the effects of increased cross-tier separation in the HCN design, and look at how coverage is affected when there are limited RBs for transmission across each BTS in the network. For all scenarios we consider a two-tier outdoor HCN comprising tier-2 BTSs transmitting at power levels corresponding to picocell BTSs. A Rayleigh fading environment is considered.

4.4.1 Validation of Spatial Coverage Probability

In this subsection, we validate our expressions for coverage probability in Figs. 4.8 and 4.9. We use set of values of $\lambda_1 = 2/\text{km}^2$, $\lambda_{2,b} = 12/\text{km}^2$, $P_2 = 0.05P_1$, and $\alpha = 4$. The SIR threshold for tier-1 is fixed as $\beta_1 = 0$ dB. Different combinations of $\delta_{1,2}$, R_{BTS} , and σ_{BTS} are tested. For ease in presentation, we define these combinations in terms of distance sets. Under case 1, where each set corresponds to the values $\{R_{BTS}, \delta_{1,2}\}$, Set 1 = {50 m, 100 m}, Set 2 = {50 m, 250 m}, and Set 3 = {50 m, 350 m}. Under case 2, we define Set 4 in terms of $\{\sigma_{BTS}, R_{BTS}, \delta_{1,2}\}$, where Set 4 = {50 m, 100 m, 100 m}. The radius of the user cluster is fixed at $R_U = 50$ m under case 1, while $\sigma_U = 25$ m and $R_U = 50$ m under case 2.

Figure 4.8: Spatial coverage probability of users from Ψ_2 vs tier-2 SIR threshold, for $\beta_1 = 0$ dB, and different combinations of $\delta_{1,2}$ and R_{BTS}/σ_{BTS} under cases 1 and 2, under the BTS density and path loss settings in subsection 4.4.1.

Figure 4.9: Spatial coverage probability of users from Ψ_1 vs tier-2 SIR threshold, for $\beta_1 = 0$ dB, and different combinations of $\delta_{1,2}$ and R_{BTS} under case 1, under the BTS density and path loss settings in subsection 4.4.1.

Figure 4.8 shows that the expressions for spatial coverage probability against varying tier-2 SIR threshold are validated as tight lower bounds. The curves in Fig. 4.8 indicate that the coverage of clustered users declines when there is an increase in the offset between the BTS and the centre of the user cluster. The reason for this was described in detail in Chapter 3 but is reiterated now. When the parameter R_{BTS} or σ_{BTS} under case 1 or case 2, the distance between a designated tier-2 BTS and a clustered user will increase on average. Therefore, more users will receive signals with diminished power levels. If the user associates to this BTS at $\mathbf{x}_{20}$, then the received SIR is also lower. Therefore, centralizing the deployed BTS inside the expected user cluster benefits coverage. If user patterns in hotspots can be approximated by circular regions corresponding to the user models presented, deploying BTSs close to the centre of this region could be an important task for cell planners. The decline in coverage is greater for case 2 than case 1, if $\sigma_{BTS} = R_{BTS}$, due to the difference in user distribution. Coverage is increased by increasing $\delta_{1,2}$ up to a limit, primarily due to the reduction in nearby tier-1 BTS interference, which then increases the received SIR of users associating to tier-2 BTSs. The reverse occurs as well; any user associating to a tier-1 BTS will experience decreased interference from tier-2 BTSs when $\delta_{1,2}$ is increased up to an extent.

Figure 4.9 demonstrates that this trend also exists for independent uniform users from Ψ_1 , where the increase in $\delta_{1,2}$ improves coverage. However, centralizing the deployed tier-2 BTS to SAs does not affect these users, as their locations are independent of these deployments.

We notice that similar trends in coverage probability persist even when the tier-1 SIR threshold, β_1 is increased. We examine the coverage probability of both groups of users in Figs. 4.10 and 4.11, when $\beta_1 = 3$ dB.

Figure 4.10: Spatial coverage probability of users from Ψ_2 vs tier-2 SIR threshold, for $\beta_1 = 3$ dB, and different combinations of $\delta_{1,2}$ and R_{BTS}/σ_{BTS} under cases 1 and 2, for the BTS density and path loss settings in subsection 4.4.1.

Figure 4.11: Spatial coverage probability of users from Ψ_1 vs tier-2 SIR threshold, for $\beta_1 = 3$ dB, and different combinations of $\delta_{1,2}$ and R_{BTS} under case 1, for the BTS density and path loss settings in subsection 4.4.1.

Figures 4.10 and 4.11 validate the derived expressions for overall coverage probability when $\beta_1 = 3$ dB. Both these graphs show that there is a very slight drop in coverage probability for each set of parameters when $\beta_1 = 3$ dB as opposed to when it is 0 dB. This is to be expected as a smaller proportion of users associated to tier-1 BTSs will receive signals with a higher SIR threshold. As the fraction of users associating with tier-1 BTSs is much lower than those associating to tier-2 BTSs, the reduction in coverage probability is quite small. This ratio of association to users was made clear in numerical results describing the association of users in Chapter 3. On the other hand, we notice that if β_2 is increased significantly, coverage probability drops much faster in the two-tier HCN. An alternate way of looking at the probability of coverage under increased SIR thresholds β_1 and β_2 , is that it tells us how many users receive signals at higher SIR thresholds, and by extension, are transmitted to with a higher average rate.

The trade-offs in performance against increasing $\delta_{1,2}$ are explored in the next subsection.

4.4.2 Effect of Increasing Cross-Tier Separation on Coverage

We explore the effect of increasing $\delta_{1,2}$ for case 1, where $R_U = 50$ m, and $\beta_1 = \beta_2 = 0$ dB. We set the density of the users, $\rho_1 = \rho_2 = 100/\text{km}^2$, and retain the settings for tier-1 BTS and SA density. For Figs. 4.12 to 4.14, we combine different values of power ratio P_1/P_2 , and path loss α , where the sets are defined as $\{P_1/P_2, \alpha\}$. Thereby, Set 1 = Set 2 = $\{20, 4\}$, Set 3 = $\{100, 4\}$, and Set 4 = $\{20, 3\}$. To understand the effect of cross-tier separation, we compare the results with an equivalent PPP of deployed tier-2 BTSs under each set of parameters.

Figure 4.12: Spatial coverage probability of users from Ψ_2 vs $\delta_{1,2}$, for $\beta_1 = \beta_2 = 0$ dB, and different values of R_{BTS} , under the BTS density and path loss settings described in subsection 4.4.2.

Figure 4.12 reiterates the fact that coverage of clustered users from Ψ_2 improves when the BTS is located closer to the SA. Furthermore, overall coverage of clustered users increases in proportion to $\delta_{1,2}$ up to a point, and then declines. This is because small increments of $\delta_{1,2}$ eliminate proximal interference to a user from the opposite tier. However, increasing $\delta_{1,2}$ beyond this point results in many distant users tagging to tier-1 BTSs, while suffering interference from tier-2 BTSs near the cell edge. Therefore, there is an optimal figure of cross-tier separation that aids the improvement of coverage if all users follow spatial distributions similar to the

clusters modelled by Ψ_2 . For the network parameters described in this subsection, this distance is approximately 150 m.

In order to prove that the improvements in coverage is due to the cross-tier separation, and not simply by reducing the density of the tier-2 BTSs, we examine the coverage of the proposed planning technique against PPPs of equivalent density, i.e. where the equivalent PPP carries equal density to the tier-2 BTSs from our model. Figure 4.12 shows that the equivalent PPP exhibits depleted coverage in comparison to an HCN that considers cross-tier separation. This is because of undesired interference to users associating with tier-2 BTSs from overly proximal tier-1 BTSs, and vice-versa. Therefore, planning tier-2 BTSs with minimum separation presents a method by which the spatial coverage of an HCN can be boosted naturally if clustered user patterns are expected. We now look at how the HCN behaves when uniform users only are considered.

Figure 4.13: Spatial coverage probability of users from Ψ_1 vs $\delta_{1,2}$, for $\beta_1 = \beta_2 = 0$ dB, and different values of R_{BTS} , under the BTS density and path loss settings described in subsection 4.4.2.

Figure 4.13 shows that when user patterns are uniformly distributed and uncorrelated with BTS locations, even greater cross-tier separation is required to attain the maximum coverage. This is because unlike with the clustered users, these users do not have a conditioned tier-2 BTS allocated to serve them. As such,

independent uniform users associating to tier-2 BTSs will connect to them from much larger distances, and consequently, experience lower received signal power levels. As such, a greater cross-tier separation is required to mitigate interference from tier-1 BTSs to these users. In all cases presented, the optimal separation is approximately between 400 to 450 m, beyond which the coverage probability plateaus and then declines. Comparison with equivalent PPP BTSs also reiterates the benefit of reducing the tier-2 BTSs in the network using cross-tier separation instead of random selection. The density of tier-2 BTSs in a PPP has no effect on coverage, confirming the discussion in subsection 2.2.8. Intuitively, if the density of PPP tier-2 BTSs is reduced, more users will be poached by tier-1 BTSs at further distances such that their received signal power is lower. However, the overall interference across such an HCN is also reduced due to the reduction in tier-2 BTSs, and therefore the average received SIR is maintained. As such coverage probability will remain equivalent in a two-tier HCN modelled by a PPP, irrespective of what the relative density of tier-2 BTSs is.

The general order of coverage probability is also significantly reduced due to the absence of correlation between users and BTSs. Centralizing BTSs to social attractors has no effect on this category of users as they are independent from SAs. The number of BTSs in the HCN that are active under larger cross-tier separation is reduced drastically, and therefore, the expected ASE is reduced significantly as well. However, if all BTSs are provisioned with sufficient bandwidth to match the density of users, having a smaller number of tier-2 BTSs operating is beneficial in order to save operating costs and energy. Therefore, if the user pattern is described by Ψ_1 in a situation, e.g. during off peak hour traffic, the cross-tier separation can be used as an activation mechanism, such that tier-2 BTSs that are too close to tier-1 BTSs are switched off. This will depend on the relative density of users however, and therefore some form of BTS cooperation is needed to carry this activation mechanism out effectively. We now look at a situation where both user categories are combined to form a mixed user distribution.

Figure 4.14: Spatial coverage probability of mixed users vs $\delta_{1,2}$, for $\beta_1 = \beta_2 = 0$ dB, and different values of R_{BTS} , under the BTS density and path loss settings in subsection 4.4.2.

Figure 4.14 shows the effect of combining the user sets Ψ_2 and Ψ_1 when $\rho_2 = \rho_1$. The curves appear to follow the trends in Fig. 4.12, as a higher portion of users are in coverage from Ψ_2 . However, the optimal cross-tier separation is skewed to a larger value due to presence of uniform users as well, by approximately 50 m. The presence of users from Ψ_1 also tends to reduce the overall coverage probability of the network. In any case, having operating tier-2 BTSs at a minimum distance of their counterpart tier-1 BTSs is beneficial to the spatial coverage of an HCN. If each BTS is provisioned with enough resources to support the users tagging to it, it makes sense to consider this parameter when planning or activating tier-2 BTSs in a network. The situation described is therefore applicable to when the HCN is uniformly undersubscribed.

Additionally, Figures 4.12 to 4.14 show that coverage declines when path loss is less, i.e. under Set 4, when $\alpha = 3$. Under Set 3, when $P_1/P_2 = 100$, as opposed to $P_1/P_2 = 20$ in Sets 1 and 2, coverage is reduced at small to medium values of $\delta_{1,2}$ because tier-1 BTS interference to tier-2 users is greater. However, the coverage under Set 3 exceeds that of Set 1 and Set 2 when $\delta_{1,2}$ is large. In this situation, tier-1 users form the majority, and total interference to those located at the cell-edge from tier-2 BTSs is lower due to smaller P_2 . We now examine the effect of increasing cross-tier separation on ASE and present some trade-offs.

4.4.3 Effect of Increasing Cross-Tier Separation on ASE

From the equation in (4.60), ASE depends on coverage, average rate of users in coverage, and the total density of BTSs in the HCN. Figures 4.12 to 4.14 show clearly that coverage increases when cross-tier separation is increased for all types of user distributions considered. As the coverage probability increases, the average rate of users in coverage also increases in conjunction with it as the expected number of users receiving higher SIR levels is greater. However, we notice that the density of tier-2 BTSs, and hence the density of BTSs across the HCN decreases with increased cross-tier separation in the network. As such, increasing ASE may work off a trade-off between density and coverage. We examine this trade-off for different user groups in Figures 4.15 to 4.17 that follow, while maintaining the same parameters used in the discussion in subsection 4.4.2.

Figure 4.15: ASE of users from Ψ_2 vs $\delta_{1,2}$, for $\beta_1 = \beta_2 = 0$ dB, and different values of R_{BTS} , under the BTS density and path loss settings described in subsection 4.4.2

Figure 4.15 demonstrates that some amount of cross-tier separation aids the increase of ASE of the HCN when only clustered users from the user group Ψ_2 are considered. This means that the boost in coverage overrides the reduction of tier-2 BTSs upto a point where $\delta_{1,2} \cong 100$ m. Centralizing user hotspots is the most

effective way of planning correlated BTSs to increase ASE by boosting coverage as well. As most users are served by tier-2 BTSs when $\delta_{1,2}$ is small, reduction in the tier-2 BTS power level leads to reduced coverage and ASE, as seen by the curve corresponding to Set 3 in Figure 4.15. Similarly, reduced path loss levels increase inter-cell interference and cause reduction in coverage and ASE.

At large values of $\delta_{1,2}$, the ASE of the HCN declines because of the reduction in the density of tier-2 BTSs, which implies that the reuse factor in the HCN is reduced.

The decline in ASE at larger values of $\delta_{1,2}$ presents a trade-off between ASE and coverage that needs to be considered when designating a magnitude of cross-tier separation in order to plan the HCN. One way of tackling this trade-off is to set the cross-tier separation to match the maximum coverage probability for clustered users, also this results in a less than optimal ASE. Conversely, ASE may be prioritized by reducing $\delta_{1,2}$ further. An alternate way of looking at this is to specify achievable thresholds for both coverage probability and ASE and then find a value of $\delta_{1,2}$ that satisfies both these thresholds.

We proceed to examine the ASE for uniformly distributed users that are independent of tier-2 BTSs in Figure 4.16.

Figure 4.16: ASE of users from Ψ_1 vs $\delta_{1,2}$, for $\beta_1 = \beta_2 = 0$ dB, and different values of R_{BTS} , under the BTS density and path loss settings described in subsection 4.4.2.

Figure 4.16 shows that ASE declines rapidly against $\delta_{1,2}$ for the system when only uniform independent users from Ψ_1 exist in the HCN. This is because the increase in coverage probability and average rate of users is insufficient to offset the decline in tier-2 BTS density. As such, the reduced tier-2 BTS density is the most important factor that affects the ASE of the network when only independent uniform users are considered. As the optimal ASE is found at very small values of cross-tier separation, the trade-off between coverage and ASE for these users presents a starker contrast. Cell operators may wish to target a threshold ASE for such users when planning the HCN, or a minimum coverage threshold, and decide on the cross-tier separation accordingly. We now proceed to examine the trade-offs between ASE and coverage under a mixed user distribution where the densities of user groups are equivalent.

Figure 4.17: ASE of mixed users vs $\delta_{1,2}$, for $\beta_1 = \beta_2 = 0$ dB, and different values of R_{BTS} , under the BTS density and path loss settings in subsection 4.4.2.

Figure 4.17 shows that when user groups are combined, the ASE is optimal at $\delta_{1,2} < 50$ m. As such, cell operators may once again select the optimal cross-tier separation for planning or activation on BTSs by considering thresholds for ASE or coverage for this user pattern. If the clustered user density is much larger than the uniform user density, some freedom is allowed in selecting a larger value of $\delta_{1,2}$; however, this diminishes as the densities become more equal. Based on the

fact that being in coverage is usually a more important factor than the average rate a user can receive, it is possible that coverage probability may be prioritized when considering this trade-off.

4.4.4 Effect of Increasing Cross-Tier Separation Under Constrained Resources

We examine the coverage probability of the HCN when orthogonal RBs per BTS are constrained by using simulations to obtain relevant results in Figs. 4.18 to 4.20. We first consider Figure 4.18, where different values are assigned to N_b , and a few combinations of ρ_1 and ρ_2 are considered for the user cluster radius is $R_U = 50$ m under case 1 of clustered users. The density of BTSs follows subsection 4.4.2. We consider a full buffer traffic system where each BTS is always ready to transmit to users. Each user will be afforded a single orthogonal RB for transmission in each timeslot. For example, if 50 users are ready to be served by a BTS that has 50 RBs for transmission, the BTS will use all its 50 RB. If 40 users are ready to be served instead, the BTS will allocate 40 of its available RBs for transmission, while the remainder of RBs will be idle. If more than 50 users are ready to receive signals from the serving BTS, a fraction of the users will not be served during that timeslot. Therefore, during that timeslot, this fraction of users is not in coverage. Figure 4.18 describes the probability of coverage of users during a single timeslot.

Figure 4.18: Coverage probability of mixed users vs $\delta_{1,2}$, for $P_2 = 0.05P_1$, $\beta_1 = \beta_2 = 0$ dB and $R_U = 50$ m under case 1 of clustered users for the density, resource allocation and path loss settings in subsection 4.4.4.

From Figure 4.18, where $\rho_1 = \rho_2 = 100/\text{km}^2$, and $N_b = 50$, i.e. N_b is reasonably high, there is a slight improvement in coverage per timeslot for the overall HCN when $\delta_{1,2} \leq 100 \text{ m} = 2R_U$. This is because most of the covered users belong to Ψ_2 , and the density of users per cluster is approximately 10. As tier-2 BTS density is not reduced by much, a large number of resources still exist for users that do not have conditioned tier-2 BTSs. The coverage boost received at low values of $\delta_{1,2}$ therefore exceeds the loss the loss of coverage due to depletion of available resources.

However, increasing $\delta_{1,2}$ further reduces available resource blocks, and in turn, coverage probability for the transmission timeslot. This situation will be compounded by an overall reduction in ASE. Therefore, when resources are constrained such that they do not outnumber users by a large margin, the cross-tier separation must be limited. Cell operators may devise this separation for planning or activation of BTSs based on instantaneous user densities or expected user patterns that have been observed over a period of time. One way to aid the increase of coverage probability under resource constraints is to increase available system bandwidth. For instance, the bandwidth per BTS on the downlink for 50 resource blocks is usually 10 MHz as per the LTE standard. Increasing this to 20 MHz

affords each BTS 100 resource blocks and the expected coverage is boosted, with a larger value of cross-tier separation permitted.

We now examine what happens when varying levels of transmission power, P_2 , are used to by tier-2 BTSs. The coverage probability under these scenarios is depicted for $R_U = 50$ m, $R_{BTS} = 50$ m, and $\alpha = 4$ under case 1 of clustered users. Figure 4.19 describes the trends in coverage observed for different levels of P_2 .

Figure 4.19: Coverage probability of mixed users vs $\delta_{1,2}$, for varying ratios of P_2/P_1 , where $\beta_1 = \beta_2 = 0$ dB, $\alpha = 4$, and $R_U = R_{BTS} = 50$ m under case 1 of clustered users, for the density, resource allocation and path loss settings in subsection 4.4.4.

Figure 4.19 shows that an increase value of P_2 , such that $P_2 = 0.1P_1$ increase overall coverage of the HCN. This is because the stronger signal levels transmitted by tier-2 BTSs will ensure that more clustered users are poached by them, and the overall received SIR of such users is increased. When more clustered users are offloaded to tier-2 BTSs, we also note that the number of RBs that are occupied by tier-1 BTSs reduces. Hence, users previously not served by tier-1 BTSs due to the lack of RBs to serve them can now be covered during a single timeslot. These factors combine to raise the overall probability of coverage of the HCN when P_2 is increased. However, the reverse occurs when P_2 is reduced. Less clustered users

associate to tier-2 BTSs, and those that do experience a worse received SIR. A larger portion of users will now attempt to associate to the limited tier-1 BTSs in the network and will encounter more instances of not being in coverage due to the lack of tier-1 RBs to serve them. As such, the overall coverage of the network will decline.

We now examine the coverage probability under varying user group densities in Figure 4.20, for the case where $R_{BTS} = 50$ m, and $\alpha = 4$.

Figure 4.20 Coverage probability of mixed users vs $\delta_{1,2}$, for $P_2 = 0.05P_1$, $\beta_1 = \beta_2 = 0$ dB, $\alpha = 4$, and $R_U = R_{BTS} = 50$ m under case 1 of clustered users, for the density, resource allocation and path loss settings in subsection 4.4.4.

Figure 4.20 shows that the overall coverage of the system is greater when $\rho_1 = 50/\text{km}^2$, i.e. there are more users from Ψ_2 , and the trend of increased coverage for small $\delta_{1,2}$ holds. However, when $\rho_1 = 150/\text{km}^2$, overall coverage of the system declines, and the system performs best with no cross-tier separation. This is because the number of available resources with strong signals from serving BTSs reduces in proportion to the overall user population. To understand this effect, we look at equation (3.84) in Chapter 3, which shows that the distances through which independent users from Ψ_1 associate to either tier is much larger than for users from Ψ_2 . As such, an increased density of users Ψ_1 will result in more users associating to BTSs from larger distances, and consequently, receive diminished signal power.

A similar reduction in coverage probability is observed when the number of resources per BTS is reduced, i.e. when $N_b = 30$. In this situation, the probability that a user will have a free RB for transmission reduces, thereby reducing the overall coverage probability. Therefore, the benefits of cross-tier separation are amplified in two situations: *a)* when clustered users dominate the overall user pattern, and *b)* when the system bandwidth is sufficient such that enough orthogonal resources are available per BTS. Cell operators could use this information for an activation mechanism for the HCN, e.g. more BTSs are active when more users exist, and less cross-tier separation is employed when independent uniformly distributed users dominate the environment.

4.5 Conclusion

This chapter extends the work in Chapter 3 to provide expressions that describe coverage and ASE in an HCN that is planned according to the proposed model in Chapter 3. We then present numerical results that validate the expressions for coverage probability and examine what parameters influence coverage and ASE. Firstly, the results show that the coverage of the two-tier HCN can be increased by deploying tier-2 BTSs near the centre of clustered user hotspots. By extension, this will also increase the average rate of covered users, as well as the overall ASE of the HCN. Secondly, increasing cross-tier separation enhances coverage when the HCN is undersubscribed, across both clustered and independent user groups. A trade-off exists between reduced ASE and increased coverage when cross-tier separation is increased. This is because the increased coverage in the HCN comes at the cost of reduced tier-2 BTS density, and this in turn will lead to a reduction in ASE. We show that optimal ASE is generally obtained at smaller values of cross-tier separation, while optimal coverage is obtained at larger values of coverage when the number of BTS resources are enough to accommodate all users in the network. As such, cross-tier separation may be selected by assigning thresholds to each performance parameter, or by prioritizing one over the other.

However, when the number of resources per BTS are constrained, it is best to avoid imposing large inter- tier separation. The work is expected to provide some insight into realistic planning scenarios for an HCN according to specific user patterns environments that are possible.

Chapter 5

Two-Tier HCNs with Co-Tier and Cross-Tier Separation Dependencies

5.1 Scope of Work

In this chapter, we extend the BTS model proposed in Chapter 3 and 4, that considers only cross-tier separation between a tier-1 and tier-2 BTS. We contend that there are two types of separation possible in a two-tier HCN, that comprise tier-1 macrocell BTSs and tier-2 picocell BTSs. The new separation parameter that is introduced between BTSs of the same tier is defined as *co-tier* separation. In [86], the authors have explored a form of co-tier separation, where the BTSs from each tier are separated from other BTSs from the same tier only. The work treats tiers as independent and shows that there are benefits to increasing co-tier separation, as it increases coverage of an HCN. However, ignoring the separation that could exist between tier-1 and tier-2 BTSs could lead to a significant amount of undesired interference when BTSs are too close to each other. In this chapter, we explore the performance of an HCN when cross-tier separation between tier-1 and tier-2 BTSs exists together with co-tier separation between tier-2 BTSs only. The intuition behind imposing the latter separation category for small-cell BTSs is that their higher density increases the chances of two overly proximal BTSs. Maintaining tier-1 as a PPP allows to maintain tractability in deriving expressions.

We derive analytical expressions to approximate the tier-2 BTS density, user association, the probability of coverage, and spectral efficiency of the HCN under different parameters. The discussion showcases separation limits and examines trade-offs between coverage and spectral efficiency for increased separation, with additional insights into packet performance in the network. These findings could

be useful to cell planners in optimizing the deployment of a two-tier HCN, to meet enhanced user and network requirements.

5.2 Modelling of BTSs and Connectivity

5.2.1 BTSs in Two-Tier HCN Model

Tier-1 macro-cell BTSs are modelled by an independent PPP set, Φ_1 , of density λ_1 , and transmit at power level P_1 . We consider a two-tier HCN where tier-2 pico-cell BTSs transmitting at power P_2 are overlaid on the existing macro-cell network. The set $\Phi_{2,b}$ contains the potential location points at which tier-2 BTSs may be deployed and is modelled by a point process of density $\lambda_{2,b}$, where $\lambda_{2,b} \gg \lambda_1$. Due to the relative densities considered, the probability that a pair of tier-2 BTSs lie arbitrarily close to one another is much larger than for a pair of tier-1 BTSs. Figure 5.1 showcases the CCDF of distance in PPPs of different density.

Figure 5.1: CCDF of distance to nearest interfering BTS in different PPPs characterized by density λ in km^{-2} .

Figure 5.1 shows that the CCDF of distance drops very sharply, where in most cases for $\lambda \geq 8 \text{ km}^{-2}$, there is a neighbouring interferer within 200 m of a BTS. The higher densities in Figure 5.1 are likely to represent small-cell BTSs in an

HCN, and therefore, we contend that co-tier separation is more important across tier-2.

Many different means of deploying separated tier-2 BTSs are possible. For example, separated tier-2 BTSs can be selected in regions that are exclusively outside cross-tier separation radii around tier-1 BTSs. However, analysis of an HCN based on such a model proves to be rather intractable. Therefore, we propose a model in which that points in $\Phi_{2,b}$ are already minimally separated by co-tier separations parameter, δ_2 , and then deployed based on cross-tier separation. The set $\Phi_{2,b}$ is represented by

$$\Phi_{2,b} = \{\mathbf{x}_{2,b} : \|\mathbf{x}_{2,b} - \mathbf{y}_{2,b}\| \geq \delta_2, \forall \mathbf{y}_{2,b} \in \Phi_{2,b}\}. \quad (5.1)$$

The Matérn hard-core process (MHP) type-II [50], [52], [85]-[86] is selected as a tractable model to describe $\Phi_{2,b}$ when $\delta_2 > 0$. This point process is selected because it retains a relatively high density of tier-2 BTSs after a fraction of points are eliminated for being closer than δ_2 to each other. Furthermore, the interference of surrounding tier-2 BTSs can be approximated as the interference from a PPP of equivalent density, as seen in [85] and [86]. The density of locations, $\lambda_{2,b}$ is limited to $\lambda_{2,b,\max}$, which is given by

$$\lambda_{2,b,\max} = 1/(\pi\delta_2^2). \quad (5.2)$$

Conversely, for a fixed value of $\lambda_{2,b}$, we may define the maximum co-tier separation possible, $\delta_{2,\max}$, such that

$$\delta_{2,\max} = 1/(\pi\lambda_{2,b})^{1/2} \quad (5.3)$$

The final set of deployed tier-2 BTSs are conditioned on a minimum separation from all tier-1 BTSs, which is the cross-tier separation parameter, $\delta_{1,2}$, also described in Chapters 3 and 4. If any potential tier-2 location is separated from a tier-1 BTS by a distance less than $\delta_{1,2}$, it will not contain a deployment. The set of deployed tier-2 BTSs is described by the set Φ_2 , where

$$\Phi_2 = \{\mathbf{x}_2 \in \Phi_{2,b} : \|\mathbf{x}_2 - \mathbf{x}_1\| \geq \delta_{1,2}, \forall \mathbf{x}_1 \in \Phi_1\} \quad (5.4)$$

Φ_2 is the set of points retained from $\Phi_{2,b}$ that are outside holes surrounding tier-1 BTSs from the PPP, Φ_1 . Therefore, Φ_2 can be modelled by a modified Poisson hole process (PHP). Unlike in a conventional PHP, where the base process that is subject to reduction in density, is a PPP, the base process here is an MHP type-II. The resulting density of the tier-2 BTSs is bounded by the standard PHP density, such that

$$\lambda_2 \geq \lambda_{2,b} \exp(-\lambda_1 \pi \delta_{1,2}^2). \quad (5.5)$$

We examine (5.5) in more detail. When $\delta_2 = 0$, the density of the resulting tier-2 BTSs is equivalent to the PHP density, which is given on the R. H. S. of (5.5). The holes of radius $\delta_{1,2}$ can prohibit any number of tier-2 BTSs in this situation. However, the number of prohibited tier-2 BTSs is limited when $\delta_2 > 0$. To understand this, we look at a situation where a single tier-2 BTS is prohibited inside the hole of radius $\delta_{1,2}$. As another hole of radius δ_2 exists around the tier-2 BTS, additional tier-2 BTSs can be prohibited by the same tier-1 hole in the region $b(\mathbf{x}_1, \delta_{1,2}) \setminus b(\mathbf{x}_2, \delta_2)$, as opposed to the full hole $b(\mathbf{x}_1, \delta_{1,2})$. If $\delta_2 > 0$, the relative magnitude of each region is related by $b(\mathbf{x}_1, \delta_{1,2}) \setminus b(\mathbf{x}_2, \delta_2) < b(\mathbf{x}_1, \delta_{1,2})$. As such, less tier-2 BTSs are removed in this situation, and the density of resulting tier-2 BTSs by cross-tier separation increases in proportion to δ_2 .

The resultant density in (5.5) cannot be accounted for exactly in a tractable manner because measuring the increase in tier-2 density due to δ_2 would require computing through multiple different possible instances of $b(\mathbf{x}_1, \delta_{1,2}) \setminus b(\mathbf{x}_2, \delta_2)$ where the region for prohibition of tier-2 BTSs is reduced. This is an intractable process, and therefore, we present the bound on λ_2 only. The density of tier-2 BTSs outside the holes can still be modelled as $\lambda_{2,b}$, and therefore, when a single-hole assumption is made to model distance PDFs, association and coverage according to [90], we can still apply this density in the analysis. The number of eventually deployed tier-2 BTSs does affect ASE however, and we apply the simulated density to reflect this.

The equation in (5.5) presents the bound on the density of a general case of the model, where λ_2 depends on both the cross-tier and co-tier separation parameters. Ignoring one or both types of separation will result in the set of tier-2 BTSs following a classic point process. This is examined in Remark 1.

Remark 1. Consider a situation where $\delta_2 = 0$, while $\delta_{1,2} > 0$. As the potential tier-2 locations are now modelled by a PPP, Φ_2 can be modelled by an exact PHP, where holes created by Φ_1 reduces the number of deployments according to probability of retention $\exp(-\lambda_1 \pi \delta_{1,2}^2)$.

Alternately, if $\delta_{1,2} = 0$, while $\delta_2 > 0$ means that $\exp(-\lambda_1 \pi \delta_{1,2}^2) = 1$. Therefore, $\Phi_2 \equiv \Phi_{2,b}$, and $\lambda_2 = \lambda_{2,b}$, because all potential tier-2 points are retained for deployment. This implies that the retained set of BTSs is exactly an MHP type-II. If $\delta_{1,2} = \delta_2 = 0$, all BTSs can lie arbitrarily close to each other, and both tier-1 and tier-2 BTSs are then effectively modelled by PPPs.

Figure 5.2 presents the BTS model proposed in this chapter, where the cross-tier separation is depicted by the black solid line circular holes around the microcell BTSs, depicted by black triangles. The tier-2 BTSs are depicted by the red squares and have co-tier separation circular holes drawn by red dashed lines.

Figure 5.2: Two-tier HCN model showcasing cross-tier separation between tier-1 BTSs (black triangles) and tier-2 BTSs (red squares), and co-tier separation between two tier-2 BTSs.

5.2.2 User Model

For tractable analysis, we simplify the user model in Chapters 3 and 4 and reduce all users as uniformly distributed throughout the space. As such, the users can be modelled by an independent PPP. The association and coverage will thus reflect the spatial performance of the HCN. Planning the HCN according to the separation categories is therefore assessed in terms of how effectively coverage holes are reduced in the HCN. Reduction of coverage holes will intuitively lead to the increase in coverage for BTS-conditioned users as well, so the performance improvements presented here are likely to have a similar effect even such user distributions.

5.2.3 Connectivity Model

We assume an open access HCN, where a user connects to the BTS supplying it with the greatest long-term average received signal power level. If a connected user migrates to a region where a different BTS supplies the strongest signal, handover between BTSs will occur. Focusing on the user at the origin, we assume distance

dependent path loss with small-scale Rayleigh fading. The BTS locations around the user are denoted by $\mathbf{x}_{ki}$, where k is the BTS tier, while i refers to the position of the BTS in terms of distance from the user. For example, the nearest tier-1 BTS is located at $\mathbf{x}_{11}$ and subsequent tier-1 BTSs are found at $\mathbf{x}_{12}, \mathbf{x}_{13}$ etc. If the distance between the user and the BTS at $\mathbf{x}_{ki}$ is $X_{ki} \equiv \|\mathbf{x}_{ki}\|$, the instantaneous received signal power from this BTS is

$$P_{RX}(\mathbf{x}_{ki}) = P_k h_{\mathbf{x}_{ki}} X_{ki}^{-\alpha} \quad (5.6)$$

where α is the path loss exponent, and $h_{\mathbf{x}_{ki}}$ is the Rayleigh fading gain, where both parameters follow the stipulation in Chapters 2 to 4. Thus, the general domain of α is given by $2 < \alpha \leq 4$ and $h_{\mathbf{x}_{ki}} \equiv h \sim \exp(1)$, i.e., $h_{\mathbf{x}_{ki}}$ is independent and exponentially distributed with mean 1. As the long-term average fading gain $\mathbb{E}\{h\} = 1$, we define the location of the serving BTS as

$$\mathbf{x}^* = \arg \max_{\mathbf{x} \in \{\mathbf{x}_k^*\}} P_k \|\mathbf{x}\|^{-\alpha}. \quad (5.7)$$

In (5.7), based on the function $P_k \|\mathbf{x}\|^{-\alpha}$, $\{\mathbf{x}_k^*\} \equiv \{\mathbf{x}: \mathbf{x}_{11}, \mathbf{x}_{21}\}$, corresponding to the nearest BTS from each tier. We consider a single frequency system that describes the worst-case interference scenario that also applies to LTE-based frequency reuse but does not affect distance PDFs or association as these metrics are solely dependent on the relative distance between BTSs.

5.3 Distance PDFs and Association

5.3.1 PDF of Distance to Nearest Tier-1 BTS

With the user at the origin, the distance to the nearest tier-1 BTS, is $\|\mathbf{x}_{11}\| \equiv X_{11}$. As Φ_1 is modelled by a PPP, the probability density function (PDF) of X_{11} is given by $f_{X_{11}} \equiv f_{X_{11}}(x_{11})$, where

$$f_{X_{11}} = 2\lambda_1 \pi x_{11} \exp(-\lambda_1 \pi x_{11}^2). \quad (5.8)$$

5.3.2 Conditional Association Probabilities

From the connectivity model described in (5.6) and (5.7), a user connects to one of the BTSs at $\mathbf{x}_{11}$ and $\mathbf{x}_{21}$. We give these spatial association events the notations $\mathbf{T}_{\mathbf{x}_{11}}$ and $\mathbf{T}_{\mathbf{x}_{21}}$, respectively. We first look at the association of a typical user to a tier-1 BTS. Transforming the origin, $\mathbf{o}$, to the user, we note that the user will tag to the BTS at $\mathbf{x}_{11}$ if the region $b(\mathbf{o}, \mathbf{x}_{11} \bar{P}^{-1/\alpha}) \setminus \tilde{A}_0$ is free of tier-2 BTSs, where $\tilde{A}_0$ is given by

$$\tilde{A}_0 = \bigcup_{\mathbf{x}_{1i} \in \Phi_1} b(\mathbf{x}_{1i}, \delta_{1,2}). \quad (5.9)$$

$\tilde{A}_0$ is the union of all holes created by tier-1 BTSs due to cross-tier separation, in which tier-2 BTSs are absent. Accounting for this exact area is intractable due to overlaps among holes. Therefore, following the intuition used in Chapters 3 and 4 from [90], we consider the hole created by the nearest tier-1 BTS, $\mathbf{x}_{11}$, to approximate for the absence of tier-2 BTSs.

To compute the probability of the event $\mathbf{T}_{\mathbf{x}_{11}}$, we would need to obtain the CCDF of this area outside the hole with respect to the BTS set Φ_2 . The effective approximated density of tier-2 BTSs outside the nearest hole is $\lambda_{2,b}$. As we are conditioning on the position of the $\mathbf{x}_{11}$, we note that the positions of tier-2 BTSs that are well outside the hole are generally independent of $\mathbf{x}_{11}$. This has been confirmed in the work carried out in [85]. However, the position of tier-2 BTSs near the hole created by $\mathbf{x}_{11}$ is influenced by the voided candidate tier-2 locations inside the hole, because each candidate has a co-tier separation of δ_2 around it. As it is intractable to account for all the holes considered, we consider a single prohibited tier-2 BTS located at $\tilde{\mathbf{x}}_2$.

Given that the radius of the hole around $\mathbf{x}_{11}$ is $\delta_{1,2}$, the possible distance between $\tilde{\mathbf{x}}_2$ and $\mathbf{x}_{11}$ is within the domain $0 \leq \|\tilde{\mathbf{x}}_2 - \mathbf{x}_{11}\| \leq \delta_{1,2}$. The PDF of this distance is simply the characteristic PDF that applies to the PPP base set $\Phi_{2,b}$. We present this PDF as $f_{\|\tilde{\mathbf{x}}_2 - \mathbf{x}_{11}\|}$, such that

$$f_{\|\tilde{\mathbf{x}}_2 - \mathbf{x}_{11}\|} = 2\lambda_{2,b}\pi\|\tilde{\mathbf{x}}_2 - \mathbf{x}_{11}\|\exp(-\lambda_{2,b}\pi(\|\tilde{\mathbf{x}}_2 - \mathbf{x}_{11}\|)^2) \quad (5.10)$$

Therefore, conditioning on the existence of the location $\tilde{\mathbf{x}}_2$, we note that no tier-2 BTSs can exist in the region $\tilde{A}_1$, which is given by

$$\tilde{A}_1 = b(\mathbf{x}_{11}, \delta_{1,2}) \cup b(\tilde{\mathbf{x}}_2, \delta_2) \quad (5.11)$$

Note that both the cross-tier separation around $\mathbf{x}_{11}$, and the co-tier separation around $\tilde{\mathbf{x}}_2$ have been considered in $\tilde{A}_1$. Following on, we note that a user will tag to the nearest tier-2 BTS if its distance is $x_{11}\bar{P}_2$, where $\bar{P}_2 = (P_2/P_1)^{1/\alpha}$. Therefore, approximating the positions of the tier-2 BTSs outside this region, we define the association to tier-1 BTSs from the equivalent CCDF of a PPP, such that

$$P_A(\mathbf{T}_{\mathbf{x}_{11}} | X_{11}, \tilde{X}_2) \cong \exp(-\lambda_{2,b} |b(\mathbf{o}, x_{11}\bar{P}_2) \setminus \tilde{A}_1|), \quad (5.12)$$

where the user has been transformed to the origin. Unconditioning on $\tilde{X}_2$ requires the PDF of distance from the user to $\tilde{\mathbf{x}}_2$. Conditioning on the two holes, we transform the user to the origin and consider its distance, $\tilde{X}_2$ to the prohibited candidate, $\tilde{\mathbf{x}}_2$. As $0 \leq \|\tilde{\mathbf{x}}_2 - \mathbf{x}_{11}\| \leq \delta_{1,2}$, it follows that the domain of $\tilde{X}_2$ is $|\delta_{1,2} - X_{11}| \leq \tilde{X}_2 \leq X_{11} + \delta_{1,2}$. The PDF of this distance, $f_{\tilde{X}_2} \equiv f_{\tilde{X}_2}(\tilde{x}_2)$ follows a PPP such that

$$f_{\tilde{X}_2} = 2\lambda_{2,b}\pi\tilde{x}_2 \exp(-\lambda_{2,b}\pi\tilde{x}_2^2) \quad (5.13)$$

Unconditioning on $\tilde{X}_2$, we get

$$P_A(\mathbf{T}_{\mathbf{x}_{11}} | X_{11}) \cong \int_{|\delta_{1,2} - X_{11}|}^{X_{11} + \delta_{1,2}} f_{\tilde{X}_2} \exp(-\lambda_{2,b} |b(\mathbf{o}, x_{11}\bar{P}_2) \setminus \tilde{A}_1|) d\tilde{x}_2, \quad (5.14)$$

From (5.12), the conditional association to the nearest tier-2 BTS, $\mathbf{x}_{21}$, is simply the complement of $P_A(\mathbf{T}_{\mathbf{x}_{11}} | X_{11})$, and is given by

$$P_A(\mathbf{T}_{\mathbf{x}_{21}}|X_{11}) \cong \int_{|\delta_{1,2}-X_{11}|}^{X_{11}+\delta_{1,2}} f_{\tilde{X}_2} \left(1 - \exp(-\lambda_{2,b}|b(\mathbf{o}, x_{11}\bar{P}_2)\backslash\tilde{A}_1|) \right) d\tilde{x}_2. \quad (5.15)$$

We look at the domain of X_{11} through which the user may associate to the tier-2 BTS. Noting that the minimum distance from the user to the nearest possible tier-2 BTS is given by $|\delta_{1,2} - x_{11}|$, the user will associate with this BTS only if the condition

$$\delta_{1,2} - x_{11} = x_{11}\bar{P}_2 \quad (5.16)$$

holds. Therefore, rearranging these terms, we get

$$\min(X_{11}|\mathbf{T}_{\mathbf{x}_{21}}) = \frac{\delta_{1,2}}{(1 + \bar{P}_2)} \quad (5.17)$$

We proceed to derive the PDF of distance to $\mathbf{x}_{21}$.

5.3.3 PDF of Distance to Nearest Tier-2 BTS

Following on, we note that the nearest tier 2 BTS may lie along the fractional arc $\cap(\mathbf{o}, x_{21})\backslash\tilde{A}_1$. Factoring in this fraction to the standard PDF of distance the approximated PPP of density $\lambda_{2,b}$ outside the hole $b(\mathbf{x}_{11}, \delta_{1,2})$, we get the conditional PDF of distance to $\mathbf{x}_{21}$, $f_{X_{21}|X_{11}} \equiv f_{X_{21}|X_{11}}(x_{21}|x_{11})$, such that

$$f_{X_{21}|X_{11}} \cong 2\lambda_{2,b}\pi x_{21} e^{-\lambda_{2,b}\pi x_{21}^2} \int_{|\delta_{1,2}-X_{11}|}^{X_{11}} f_{\tilde{X}_2} \min(\bar{\theta}_{\delta_{1,2},x_{21},x_{11}}, \bar{\theta}_{\delta_{1,2},x_{21},\tilde{x}_2}) d\tilde{x}_2, \quad (5.18)$$

where $\bar{\theta}_{\delta_{1,2},x_{21},x_{11}}$, and $\bar{\theta}_{\delta_{1,2},x_{21},\tilde{x}_2}$ conform to the standard definition of $\theta_{R_2,R_1,X}$ in Appendix A.

5.3.4 Minimum Radius of Association around Tier-2 BTSs

For an arbitrary BTS located at $\mathbf{x}_2$ at distance $X_2 = x_2$ from the user at the origin, association to this BTS occurs if the nearest tier-1 BTSs is not within a distance $X_{11} = x_2 \bar{P}_2$. Jointly, the nearest tier-2 BTSs cannot exist within a distance of $X_{21} = x_2$. The combination of co-tier and cross-tier separations around the tier-2 BTS implies that no tier-1 BTSs exists inside $b(\mathbf{x}_2, \delta_{1,2})$, and no tier-2 BTSs exists inside $b(\mathbf{x}_2, \delta_2)$. If the user exists outside either of these regions, the association event cannot be guaranteed as another BTS can lie arbitrarily close to it.

Therefore, the minimum distance from the user to the boundary of the hole $b(\mathbf{x}_2, \delta_{1,2})$ is simply given by $\delta_{1,2} - x_2$. This is also the distance to the nearest possible tier-1 BTS. If the user is to associate with the BTS at $\mathbf{x}_2$, the condition stipulated below in (5.16) must hold, such that

$$\delta_{1,2} - x_2 \leq x_2 \bar{P}_1. \quad (5.19)$$

Furthermore, the minimum distance from the user to the boundary of the hole $b(\mathbf{x}_2, \delta_2)$ is $\delta_2 - x_2$, and the condition

$$\delta_2 - x_2 \leq x_2. \quad (5.20)$$

must hold if the user is to avoid associating to another tier-2 BTS. Defining the maximum distance from the BTS, $\mathbf{x}_2$, to the user as η_2 , we rearrange the terms in (5.16) and (5.17) to obtain

$$\eta_2 = \min\left(\frac{\delta_{1,2}}{(1 + \bar{P}_1)}, \frac{\delta_2}{2}\right). \quad (5.21)$$

Therefore, η_2 defines the minimum radius around a tier-2 BTS, within which a user is guaranteed association to it, if the long-term average signal connectivity model is used. Hence by setting an appropriate co-tier and cross-tier separation, we may model a form of spatial biasing of users to tier-2 BTSs. This may be particularly useful if clustered users are found around the tier-2 BTSs and might be a useful extension of this work.

5.4 Analysis of Coverage and ASE

5.4.1 Conditional Coverage Probability

Following the definitions provided in previous chapters, the SIR of a typical user at the origin, who is connected to the k -tier serving BTS at $\mathbf{x}_{k1}$, is

$$SIR(X_{k1}) = \frac{P_k h_{\mathbf{x}_{k1}} X_{k1}^{-\alpha}}{\left(\sum_{j=1}^2 \sum_{\mathbf{x}_{ji} \in \Phi_j \setminus \mathbf{x}_{k1}} P_j h_{\mathbf{x}_{ji}} X_{ji}^{-\alpha} \right)}. \quad k = 1, 2 \quad (5.22)$$

The user is in coverage if the received SIR is at least equal to the tier- k threshold, β_k , and this event is given by $\mathcal{C}_U^{\mathbf{x}_{k1}}$. Under Rayleigh fading conditions, where $h \sim \exp(1)$, the coverage probability of the user, conditioned on the distance $X_{k1} = x_{k1}$, and the association event $\mathbf{T}_{\mathbf{x}_{k1}}$, is presented by modifying (4.2) such that

$$P_c(\mathcal{C}_U^{\mathbf{x}_{k1}} | X_{k1}, \mathbf{T}_{\mathbf{x}_{k1}}) = \prod_{j=1,2} \mathbb{E}_g(h > s_k g I_j) = \prod_{j=1,2} \mathcal{L}^j(\mathbf{x}_{k1}), \quad (5.23)$$

where $s_k = (\beta_k x_{k1}^\alpha / P_k)$, and I_j is the characteristic interference from tier- j . $\mathcal{L}^j(\mathbf{x}_{k1})$ is the Laplace transform of interference from tier- j to the user that is associated to $\mathbf{x}_{k1}$. We restate $\mathcal{L}^j(\mathbf{x}_{k1})$ for a PPP-based HCN as

$$\mathcal{L}^j(\mathbf{x}_{k1}) = \exp\left(-\lambda_k \left(\tau_j^{\mathbf{x}_{k1}} \left(x_{k1} (P_j / P_k)^{1/\alpha}\right)\right)\right), \quad (5.24)$$

where the parameter $\tau_j^{\mathbf{x}_{k1}}(D)$ is given by

$$\tau_j^{\mathbf{x}_{k1}}(D) = \left(\frac{(s_k P_j)^{\frac{2}{\alpha}}}{\text{sinc}(2\pi/\alpha)} - D^2 {}_2F_1\left(1, \frac{2}{\alpha}; \frac{\alpha+2}{\alpha}; -\frac{D^\alpha}{s_k P_j}\right) \right), \quad (5.25)$$

As no PGFL exists to account for imposing cross-tier and/or co-tier separation on base PPP sets, the effect of separations is approximated instead. We do this by subtracting nearby interference from the result in (5.24).

5.4.2 Laplace Transforms of Interference for Users Associating with Tier-1 BTSs

A user that tags to $\mathbf{x}_{11}$ under the association event $T_{\mathbf{x}_{11}}$ is surrounded by tier-1 interferers minimally separated by the distance to $\mathbf{x}_{11}$, $X_{11} = x_{11}$. As the surrounding tier-1 interferers are simply a PPP, the Laplace transform of interference from tier-1 to the user is given by

$$\mathcal{L}^1(\mathbf{x}_{11}) = \exp\left(-\lambda_1 \pi \tau_1^{\mathbf{x}_{11}}(x_{11})\right) \quad (5.26)$$

We now look at the Laplace transform of interference from tier-2 for this association event. Noting that tier-2 BTS interference reduced by the hole of radius $\delta_{1,2}$ around $\mathbf{x}_{11}$, we need to incorporate this factor into the Laplace transform. However, from the derivation of conditional association in this chapter, we notice that prohibited tier-2 BTSs lying inside the hole may push the actual tier-2 deployment further away from interferers.

First, the Laplace transform of interference from tier-2 BTSs, ignoring interference reduction is given by approximating the MHP as an equivalent PPP, such that

$$\mathcal{L}^{2,\text{MHP}}(\mathbf{x}_{11}) = \exp\left(-\lambda_{2,b} \pi \tau_2^{\mathbf{x}_{11}}(x_{11} \bar{P}_2)\right). \quad (5.27)$$

This follows the fact that MHP interference can be characterized as an equivalent PPP [85], and that no tier-2 BTS exists within $x_{11} \bar{P}_2$ of the user for the association event. The reduced interference is given by $\exp(-2\lambda_{2,b} \pi v_2^{\mathbf{x}_{11}})$. Unconditioning on the position of the modelled prohibited tier-2 BTS, $\tilde{\mathbf{x}}_2$, the factor $v_2^{\mathbf{x}_{11}}$ is shown to be

$$v_2^{\mathbf{x}_{11}} = \pi^{-1} \int_{x_{11}\bar{P}_2}^{\delta_{1,2}+x_{11}} \int_{|\delta_{1,2}-x_{11}|}^{\delta_{1,2}+x_{11}} \max(\theta_{\delta_{1,2},r,x_{11}}, \theta_{\delta_{2,r},\tilde{x}_2}) \frac{s_1 P_2 r^{1-\alpha}}{1 + s_1 P_2 r^{-\alpha}} dr. \quad (5.28)$$

Following on, the Laplace transform of tier-2 interference to the user is approximated as

$$\mathcal{L}^2(\mathbf{x}_{11}) \cong \exp\left(-\lambda_{2,b}\pi(\tau_2^{\mathbf{x}_{11}}(x_{11}\bar{P}_2) - 2v_2^{\mathbf{x}_{11}})\right). \quad (5.29)$$

The conditional coverage probability of the user associating with $\mathbf{x}_{11}$ is therefore given as

$$P_c(\mathcal{C}_U^{\mathbf{x}_{11}} | X_{11}, \mathbf{T}_{\mathbf{x}_{11}}) \cong \mathcal{L}^2(\mathbf{x}_{11}) \mathcal{L}^1(\mathbf{x}_{11}). \quad (5.30)$$

5.4.3 Laplace Transforms of Interference for Users Associating with Tier-2 BTSs

The Laplace transforms of interference to the user tagging to the nearest tier-2 BTS, $\mathbf{x}_{21}$, are conditioned on the existence of the nearest tier-1 BTS, $\mathbf{x}_{11}$. We first examine the Laplace transform of interference from $\mathbf{x}_{11}$. Following the work presented in Chapter 4, we present this figure as

$$\mathcal{L}^{\mathbf{x}_{11}}(\mathbf{x}_{21}) = \frac{1}{1 + s_2 P_1 x_{11}^{-\alpha}} \quad (5.31)$$

As no tier-1 BTS is closer to the user than $\mathbf{x}_{11}$, the total transform of surrounding PPP tier-1 BTS interference, ignoring reduction by holes, is given by

$$\mathcal{L}^{1,\text{PPP}}(\mathbf{x}_{21}) = \exp\left(-\lambda_1 \pi \tau_1^{\mathbf{x}_{21}}(x_{11})\right) \quad (5.32)$$

However, tier-1 interference is reduced by the hole around $\mathbf{x}_{21}$, and the transform of this interference is given by $\exp(-2\lambda_1 \pi v_1^{\mathbf{x}_{21}})$, where

$$v_1^{\mathbf{x}_{21}} = \pi^{-1} \int_{x_{11}}^{x_{21} + \delta_{1,2}} \theta_{\delta_{1,2}, r, x_{21}} \frac{s_2 P_1 r^{1-\alpha}}{1 + s_2 P_1 r^{-\alpha}}, \text{ if } \delta_{1,2} > x_{11} - x_{21} \quad (5.33)$$

and $v_1^{\mathbf{x}_{21}} = 0$ if $\delta_{1,2} \leq x_{11} - x_{21}$. Therefore, the total Laplace transform of interference from tier-1 BTSs to the user associating with $\mathbf{x}_{21}$ is obtained by multiplying the three transforms described in (5.31), (5.32) and using the factor in (5.33), such that

$$\mathcal{L}^1(\mathbf{x}_{21}) = \frac{1}{1 + s_2 P_1 x_{11}^{-\alpha}} \exp\left(-\lambda_1 \pi (\tau_1^{\mathbf{x}_{21}}(x_{11}) - 2v_1^{\mathbf{x}_{21}})\right). \quad (5.34)$$

We now turn our attention to the interference due to surrounding tier-2 BTSs. Once again, by approximating the MHP interference using a PPP, the Laplace transform of interference from tier-2 prior to interference reduction is given by

$$\mathcal{L}^{2, \text{MHP}}(\mathbf{x}_{21}) = \exp\left(-\lambda_{2,b} \pi \tau_2^{\mathbf{x}_{21}}(x_{21})\right). \quad (5.35)$$

For the reduction in interference, we consider that interfering tier-2 BTSs are absent in the region $b(\mathbf{x}_{21}, \delta_2) \cup b(\mathbf{x}_{11}, \delta_{1,2})$, while ignoring the possible hole of radius δ_2 that may be created by a prohibited tier-2 BTS inside $b(\mathbf{x}_{11}, \delta_{1,2})$ to maintain tractability.

For the distance r from the user through which tier-2 interference is reduced, we look at the fractions of the arc $\cap(\mathbf{o}, r)$ surrounding the user that intersect the region $b(\mathbf{x}_{21}, \delta_2) \cup b(\mathbf{x}_{11}, \delta_{1,2})$. As such, the reduced interference is given by $\exp(-2\lambda_{2,b} \pi v_2^{\mathbf{x}_{21}})$, where $v_2^{\mathbf{x}_{21}}$ is derived using the intuition applied to deriving $v_{1,\{2,A\}}^{\mathbf{x}_{21}}$ in (4.29). We first account for when the separation between $\mathbf{x}_{11}$ and $\mathbf{x}_{21}$ is maximum, i.e. $\|\mathbf{x}_{11} - \mathbf{x}_{21}\| = x_{11} + x_{21}$. Here, the modified factor, $\bar{v}_2^{\mathbf{x}_{21}}$ is given by

$$\bar{v}_2^{\mathbf{x}_{21}} = \pi^{-1} \int_{x_{21}}^{\max(\delta_2 + x_{21}, \delta_{1,2} + x_{11})} \max(\theta_{\delta_{1,2}, r, x_{11}} + \theta_{\delta_2, r, x_{21}}) \frac{s_2 P_2 r^{1-\alpha}}{1 + s_2 P_2 r^{-\alpha}} dr. \quad (5.36)$$

The second situation we consider computing a modified factor, $\bar{\bar{v}}_2^{\mathbf{x}_{21}}$ is when $\|\mathbf{x}_{11} - \mathbf{x}_{21}\|$ is minimum, i.e. $\|\mathbf{x}_{11} - \mathbf{x}_{21}\| = x_{11} - x_{21}$. As such, $\bar{\bar{v}}_2^{\mathbf{x}_{21}}$ is given by

$$\bar{\bar{v}}_2^{\mathbf{x}_{21}} = \pi^{-1} \int_{x_{21}}^{\max(\delta_2 + x_{21}, \delta_{1,2} + x_{11})} (\theta_{\delta_{1,2}, r, x_{11}} + \theta_{\delta_2, r, x_{21}}) \frac{s_2 P_2 r^{1-\alpha}}{1 + s_2 P_2 r^{-\alpha}} dr, \quad (5.37)$$

if $\theta_{\delta_{1,2}, r, x_{11}} + \theta_{\delta_2, r, x_{21}} < \pi$, and

$$\bar{\bar{v}}_2^{\mathbf{x}_{21}} = \int_{x_{21}}^{\max(\delta_2 + x_{21}, \delta_{1,2} + x_{11})} \frac{s_2 P_2 r^{1-\alpha}}{1 + s_2 P_2 r^{-\alpha}} dr, \quad (5.38)$$

if $\theta_{\delta_{1,2}, r, x_{11}} + \theta_{\delta_2, r, x_{21}} \geq \pi$. By averaging over the two situations, present the factor $v_2^{\mathbf{x}_{21}}$ as

$$v_2^{\mathbf{x}_{21}} = \frac{\bar{\bar{v}}_2^{\mathbf{x}_{21}} + \bar{v}_2^{\mathbf{x}_{21}}}{2}. \quad (5.39)$$

Finally, the Laplace of transform of total interference from tier-2 BTSs to the user tagged to $\mathbf{x}_{21}$ is given by

$$\mathcal{L}^2(\mathbf{x}_{21}) = \exp\left(-\lambda_{2,b} \pi (\tau_2^{\mathbf{x}_{21}}(x_{21}) - 2v_2^{\mathbf{x}_{21}})\right). \quad (5.40)$$

The conditional coverage probability of the user associating with $\mathbf{x}_{11}$ is therefore given as

$$P_c(\mathcal{C}_U^{\mathbf{x}_{21}} | X_{11}, X_{21}, \mathbf{T}_{\mathbf{x}_{21}}) \cong \mathcal{L}^2(\mathbf{x}_{21}) \mathcal{L}^1(\mathbf{x}_{21}). \quad (5.41)$$

5.4.4 Overall Coverage Probability

The joint association and coverage probability under the association event $\mathcal{C}_U^{\mathbf{x}_{11}}$ is given by

$$\Pr(\mathbf{C}_U^{\mathbf{x}_{11}}, \mathbf{T}_{\mathbf{x}_{11}}) = \int_0^\infty f_{X_{11}} P_A(\mathbf{T}_{\mathbf{x}_{11}} | X_{11}) \mathcal{L}^2(\mathbf{x}_{11}) \mathcal{L}^1(\mathbf{x}_{11}) dx_{11} \quad (5.42)$$

In order to derive $\Pr(\mathbf{C}_U^{\mathbf{x}_{21}}, \mathbf{T}_{\mathbf{x}_{21}})$, we note that the conditional association probability $P_A(\mathbf{T}_{\mathbf{x}_{21}} | X_{11}, X_{21})$ is given by

$$P_A(\mathbf{T}_{\mathbf{x}_{21}} | X_{11}, X_{21}) = \begin{cases} 1, & \text{if } x_{21} < x_{11} \bar{P}_2 \\ 0, & \text{otherwise.} \end{cases} \quad (5.43)$$

As such, the joint coverage and association probability is obtained as

$$\begin{aligned} & \Pr(\mathbf{C}_U^{\mathbf{x}_{21}}, \mathbf{T}_{\mathbf{x}_{21}}) \\ &= \int_0^\infty \int_0^{x_{11} \bar{P}_2} f_{X_{11}} f_{X_{21} | X_{11}} P_A(\mathbf{T}_{\mathbf{x}_{21}} | X_{11}, X_{21}) \mathcal{L}^2(\mathbf{x}_{21}) \mathcal{L}^1(\mathbf{x}_{21}) dx_{21} dx_{11}. \end{aligned} \quad (5.44)$$

As the users considered are uniform, the probability of coverage in the HCN, p_c^{HCN} is given by

$$p_c^{\text{HCN}} = \Pr(\mathbf{C}_U^{\mathbf{x}_{21}}, \mathbf{T}_{\mathbf{x}_{21}}) + \Pr(\mathbf{C}_U^{\mathbf{x}_{11}}, \mathbf{T}_{\mathbf{x}_{11}}). \quad (5.45)$$

5.4.5 Rate and ASE

We restate the definition of rate and ASE for this HCN model, by following the work presented in subsection 4.4.6. The instantaneous Shannon-theoretic rate of a user tagged to a tier- k BTS when in coverage is given by

$$\zeta_{\mathbf{x}_{k1}} = \log_2(1 + \text{SIR}(X_{k1})), \text{SIR}(X_{k1}) \geq \beta_k \quad (5.46)$$

where k indexes the tier, and the rate is taken in bits/s/Hz. The average rate per tier is given by

$$\zeta_k = \mathbb{E}(\zeta_{\mathbf{x}_{k1}}). \quad (5.47)$$

where the expectation is taken across all serving BTSs across tier- k that are designated by $\mathbf{x}_{k1}$. The average rate per tier in (5.47) is directly related to the ASE of the system from the equation

$$ASE = \lambda_k \Pr(\mathbf{C}_U^{\mathbf{x}_{k1}}, \mathbf{T}_{\mathbf{x}_{k1}}) \zeta_k. \quad (5.48)$$

5.5 Numerical Results and Discussion

We present different types of numerical results in this section for discussion. These include results that validate the expressions describing the analysis of probability of coverage, explore the effects of increased cross-tier and co-tier separations in the HCN design, analyse practical voice packet performance in the HCN, and showcase the minimum radius of association guaranteed for tier-2 BTSs. For all scenarios we consider a two-tier outdoor HCN comprising tier-2 BTSs transmitting at power levels corresponding to picocell BTSs. A Rayleigh fading environment is considered.

5.5.1 Validation of Coverage Probability

For the sake of validating the bounds presented in the analysis, we look at two cases, where $\lambda_{2,b} = 12 \text{ km}^{-2}$ for a relatively high density of potential tier-2 BTS locations, and $\lambda_{2,b} = 8 \text{ km}^{-2}$ for a sparser density of picocell BTSs. For all cases, we choose $\alpha = 3.5$, and $\beta_1 = 0 \text{ dB}$, and test the results against varying β_2 . The different combinations of co-tier and cross-tier separations are designated in terms of sets, where each set is defined as $\{\delta_2, \delta_{1,2}\}$. As such, Set 1 = {50 m, 0}, Set 2 = {50 m, 150 m}, Set 3 = {100 m, 150 m}, and Set 4 = {100 m, 250 m}. The final densities of picocell BTSs are influenced by the cross-tier separation. We present the validated results in Figures 5.3 and 5.4.

The approximated theoretical results are validated as lower bounds across different SIR thresholds, as seen in Figs. 5.3 and 5.4 on the next page. As expected, increasing the tier-2 SIR threshold reduces overall coverage. More importantly, planning the HCN with increased separations between BTSs tends to enhance coverage because of reduced interference levels. As described in the subsequent

section, there is a limit to which both separation parameters can be increased to achieve enhanced coverage, i.e. excessively separating BTSs from each other does not yield improvement in coverage. We proceed to examine these trade-offs in the next subsection, while considering the ASE of the system as well.

Figure 5.3: Probability of coverage for various separation sets for $\lambda_{2,b} = 12 \text{ km}^{-2}$, $\lambda_1 = 2 \text{ km}^{-2}$, and the other parameters defined in subsection 5.5.1.

Figure 5.4: Probability of coverage for various separation sets for $\lambda_{2,b} = 8 \text{ km}^{-2}$, $\lambda_1 = 2 \text{ km}^{-2}$, and the other parameters defined in subsection 5.5.1.

5.5.2 Effects of Increasing Co-Tier and Cross-Tier Separation

In this subsection, we assess the improvement in coverage that is made possible by increasing the co-tier separation for tier-2 BTSs as well increasing cross-tier separation. We find that there are limits to which both separations need to be contained within, and we discuss the reasoning behind them. We also present trade-offs between coverage and ASE for increased magnitudes of δ_2 and $\delta_{1,2}$.

Figure 5.5: Probability of Coverage vs. $\delta_{1,2}$ for $\lambda_{2,b} = 12 \text{ km}^{-2}$, except where noted, $\lambda_1 = 2 \text{ km}^{-2}$, $\beta_1 = \beta_2 = 0 \text{ dB}$, and varying δ_2 , when $\alpha = 3.5$. Comparisons are made with an equivalent density PPP HCN (Eq. PPP in the graphs).

Figure 5.5 depicts the effect that co-tier and cross-tier separations. We consider a system where the SIR thresholds that guarantee coverage are $\beta_1 = 0 \text{ dB}$, and $\beta_2 = 0 \text{ dB}$. The densities of the BTSs are given by $\lambda_1 = 2 \text{ km}^{-2}$ and $\lambda_{2,b} = 12 \text{ km}^{-2}$, except where noted. The path loss exponent considered is $\alpha = 3.5$.

Figure 5.5 shows that increasing $\delta_{1,2}$ enhances the coverage of the HCN until it peaks at a certain value, for all different fixed values of δ_2 . This result mirrors the one presented in Figure 4.4 from the previous chapter for uniform users. Therefore, by deploying tier-2 BTSs that are sufficiently separated from tier-1 BTSs, operators can reduce holes in coverage by eliminating overwhelming interference from tier-1 BTSs to tier-2 users or vice-versa. However, beyond a certain limit of $\delta_{1,2}$, the

coverage of the HCN declines again. While there are less holes near the tier-1 BTSs, more users associate to tier-1 BTSs from larger distances. The reduced signal power levels and increased interference from tier-2 BTSs lying just outside the tier-1 cell edge contribute to larger holes at this increased distance. Therefore, excessive separation of tier-2 BTSs from their tier-1 counterparts does not enhance network performance.

Figure 5.5 also shows that increasing co-tier separation for tier-2 BTSs will enhance coverage by eliminating coverage holes due to proximal tier-2 interferers. This affect can be achieved consistently through increased δ_2 without affecting the density of deployed tier-2 BTSs, up to the limit $\delta_{2,\max}$, which is given in (5.3). In the example provided in Figure 5.5, the maximum distance possible between a pair of tier-2 BTSs is 162 m, if the density of potential locations is limited to 12/km². When this co-tier separation is increased to 178 m, the maximum achievable density of potential locations is now 10 km⁻². At this setting, coverage of the HCN is slightly less due to the introduction of more holes in regions that are further away from tier-2 BTSs. This implies that the magnitude of δ_2 must be limited if optimal coverage probability in the HCN is desired.

If optimized coverage is the main objective pursued by cellular operators, the HCN may be planned with a sensible limit imposed on $\delta_{1,2}$. This limit may be approximated by identifying the point beyond which coverage probability declines for settings of $\lambda_{2,b}$ and λ_1 . Therefore, cell planners may avoid increasing $\delta_{1,2}$ arbitrarily and limit the reduction tier-2 BTSs. For example, consider the settings used to obtain Figure 5.5. The maximum coverage probability is achieved if $\delta_{1,2}$ is approximately 300 m. As this applies to a specific path loss environment, cell planners may condition this separation parameter on the expected path loss inside the HCN.

The limit on the co-tier separation, δ_2 is slightly more complicated to identify in terms of practicality. Figure 5.5 shows that maximizing δ_2 for a target base density of potential tier-2 BTS locations provides the best probability of coverage in the network. This probability of coverage can be enhanced further by using techniques such as beamforming or power control in cases where user patterns are clearly identifiable. In particular, beamforming will increase the received signal strength

for users from serving BTSs, and also reduce the overall interference from surrounding BTSs. This will result in a higher probability of coverage.

Furthermore, increasing this co-tier separation to reduce tier-2 BTS density further does not provide a distinct advantage. However, one important practical consideration that may limit the value of δ_2 is the homogeneity of distances between tier-2 BTSs when δ_2 increases. For instance, it may not be practically possible to select potential tier-2 sites with a density of $\lambda_{2,b} = 12 \text{ km}^{-2}$ if co-tier separation is set to the limit of $\delta_2 = 162 \text{ m}$.

We now look at the trends in coverage against increased co-tier and cross-tier separation when larger SIR threshold values are used, in Figs. 5.6 and 5.7.

Figure 5.6: Probability of Coverage vs. $\delta_{1,2}$ for $\lambda_{2,b} = 12 \text{ km}^{-2}$, except where noted, $\lambda_1 = 2 \text{ km}^{-2}$, $\beta_1 = 0 \text{ dB}$, $\beta_2 = 3 \text{ dB}$ and varying δ_2 , when $\alpha = 3.5$. Comparisons are made with an equivalent density PPP HCN (Eq. PPP in the graphs).

Figure 5.6 considers the case where $\beta_1 = 0 \text{ dB}$ and $\beta_2 = 3 \text{ dB}$. The overall coverage under all cases declines because the higher SIR threshold for users associated to tier-2 BTSs means that a smaller fraction of them will be in coverage. It is interesting to note that in the case of the equivalent PPP, the coverage is no

longer equal across all densities of tier-2 BTSs. This is because when the density of tier-2 BTSs reduces, less users associate to them, and therefore less users require a higher SIR threshold to be in coverage. As such, the coverage of the equivalent PPP will increase with an increase in $\delta_{1,2}$.

Similarly, we notice in Fig. 5.6 that the optimal cross-tier separation to maximize coverage is now found at a higher value of $\delta_{1,2}$. This is because increasing $\delta_{1,2}$ will reduce the interference to tier-2 users from tier-1 BTSs, which now require a higher SIR threshold to connect. Overall, however, the probability of coverage when β_2 is increased is less than when it is equal to 0 dB.

Figure 5.7: Probability of Coverage vs. $\delta_{1,2}$ for $\lambda_{2,b} = 12 \text{ km}^{-2}$, except where noted, $\lambda_1 = 2 \text{ km}^{-2}$, $\beta_1 = \beta_2 = 3 \text{ dB}$, and varying δ_2 , when $\alpha = 3.5$. Comparisons are made with an equivalent density PPP HCN (Eq. PPP in the graphs).

Figure 5.7 considers the case where $\beta_1 = 3 \text{ dB}$ and $\beta_2 = 3 \text{ dB}$. The overall coverage under all cases declines because the higher SIR threshold for users associated to all BTSs in the network means that a smaller fraction of them will be in coverage. The trends for optimal cross-tier separation are exactly the same as those observed in Fig. 5.5 when $\beta_1 = \beta_2 = 0 \text{ dB}$, and the equivalent PPP exhibits the same probability of coverage across all densities of tier-2 BTSs.

We now proceed to expound on the ASE as a factor in deciding how the distances between BTSs can be set in a two-tier HCN. Figure 5.6 presents the ASE of the network under the settings used to compute results in Figure 5.5.

Figure 5.8: ASE vs. $\delta_{1,2}$ for $\lambda_{2,b} = 12 \text{ km}^{-2}$ except where noted, $\lambda_1 = 2 \text{ km}^{-2}$, $\beta_1 = \beta_2 = 0 \text{ dB}$, and varying δ_2 , when $\alpha = 3.5$. Comparisons are made with an equivalent density PPP HCN (Eq. PPP in the graphs).

Figure 5.8 shows that the ASE of the network generally declines sharply with increased cross-tier separation, $\delta_{1,2}$. This reflects the assessment of ASE for uniformly independent users in Chapter 4 and shows that the increased coverage is not sufficient to compensate for reduction in tier-2 BTS density that influences ASE. As tier-2 BTS density can be maintained beyond the bound presented in subsection 5.2.1, increasing co-tier separation aids the increase in ASE, up to the maximum limit. However, increasing δ_2 beyond this limit reduces the BTS density even further and subsequently causes a reduction in ASE. This is showcase by the values of ASE across different $\delta_{1,2}$, for $\delta_2 = 162 \text{ m}$ vs $\delta_2 = 178 \text{ m}$. In the former case, the maximum achievable density of potential tier-2 BTSs is $12/\text{km}^{-2}$. A planned HCN with this density exhibits a higher ASE than the latter case, where the corresponding tier-2 density of potential tier-2 BTSs is $10/\text{km}^{-2}$. In order to

understand the effect that final tier-2 BTS density has on the ASE, we examine Fig. 5.9.

Figure 5.9: ASE vs. $\delta_{1,2}$ for $\lambda_1 = 2 \text{ km}^{-2}$, $\beta_1 = \beta_2 = 0 \text{ dB}$, and varying δ_2 and $\lambda_{2,b}$, when $\alpha = 3.5$.

Figure 5.9 shows that when $\lambda_{2,b}$ is at its highest value, the ASE of the network is at also maximized because of the availability of great spectral resources due to a higher density of tier-2 BTSs. When the co-tier separation is set at its maximum value, e.g. $\delta_2 = 145 \text{ m}$ when $\lambda_{2,b} = 15/\text{km}^2$ or $\delta_2 = 162 \text{ m}$ when $\lambda_{2,b} = 12/\text{km}^2$, the ASE is maximized for a particular base density. This is because the probability of coverage, which factors into the ASE is also maximized when δ_2 is set tot its highest possible value for a corresponding density.

The trade-offs between ASE and coverage imply that the right setting for cross-tier separation must be selected carefully. Consider a situation where a spatial coverage probability threshold of 0.55 is desired. The maximum coverage probability is achieved if $\delta_{1,2}$ is approximately 300 m. However, if δ_2 is 150 m, the maximum area spectral efficiency is achieved when $\delta_{1,2}$ is between 150 and 200 m, while achieving the coverage probability threshold of 0.55.

Alternately, cell planners may wish to limit tier-2 BTSs to reduce the cost and energy needed of operation per unit area and optimize separations to achieve this density. For example, from Figure 5.5, a final tier-2 BTS density of $\lambda_2 = 10 \text{ km}^{-2}$ can be achieved with $\lambda_{2,b} = 12 \text{ km}^{-2}$, $\delta_2 \geq 0$, and $\delta_{1,2} \cong 170 \text{ m}$. Examining Figure 5.5 and Figure 5.8 shows that the equivalent density of tier-2 BTSs can be achieved $\lambda_{2,b} = 10 \text{ km}^{-2}$, $\delta_2 \geq 0$, and $\delta_{1,2} = 0$. However, a better probability of coverage and ASE is achieved when $\delta_2 \geq 150 \text{ m}$, and $\delta_{1,2} \cong 170 \text{ m}$, to attain $\lambda_2 = 10 \text{ km}^{-2}$, when compared to $\delta_2 \geq 178 \text{ m}$, and $\delta_{1,2} = 0$. This shows that both co-tier and cross-tier separations can be used wisely in conjunction with each other to plan the locations of tier-2 BTSs in the HCN yields the best performance. Using cross-tier separation in isolation, as modelled by the PHP [19]-[21], or co-tier separation in isolation, as modelled by the MHP [17], results in reduced coverage and ASE as compared to the proposed technique.

To justify the proposed planning model, we compare coverage and spectral efficiency of the model against equivalent PPPs, where no separation between BTSs is required for deployment. For the equivalent PPP, the density of tier-1 BTSs considered is equal, and the density of tier-2 BTSs is equal to the final density of the picocell BTSs after cross-tier separation. Figure 5.5 shows improved coverage probability when both cross-tier and co-tier separations are used simultaneously when compared to an equivalent PPP. This is because of the reduction of interference from very proximal tier-1 and tier-2 BTSs, as compared to deployments that do not consider separation rules, i.e. the PPP. Under a pure PPP where deployment of BTSs is completely random, the reduction in density has no effect on coverage confirming the hypothesis presented in subsection 2.2.8. Consequently, improved coverage under the proposed model leads to better ASE as compared to the equivalent PPP HCN, as seen in Figure 5.8.

The coverage of an equivalent PPP can also be viewed as the performance that would be expected by a single-tier macrocell network, as this measure is independent of the number of tiers deployed. Therefore, the results in Figure 5.8 shows that tier-2 BTSs can be deployed by operators to increase ASE without considering separation rules, by reusing the frequency across more BTSs. However, the optimizing co-tier and cross-tier separations can help operators use tier-2 BTSs to achieve a better ASE while fill coverage holes more effectively.

From these results, we propose that cell planners can deploy an HCN by: a) maximizing co-tier separation between potential tier-2 locations according to a target density, b) optimizing cross-tier separation according to thresholds in coverage, ASE, and the desired final density of tier-2 BTSs.

5.5.3 Practical Packet Performance under the Proposed Model.

In order to expound on the benefits of using our proposed cell planning technique, we examine the performance of packets under a practical scenario, by mapping coverage probability to understand this. In Figures 5.7 and 5.8, we map the coverage and rate described in Fig. 5.5 and Fig. 5.6 to the average throughput and delay of voice packets over a single BTS's downlink channel. We consider a long-term evolution (LTE) network that carries voice-over-LTE (VoLTE) packets of a fixed size of 300 bits that need to be transmitted every 20 ms [3]. Each BTS has 100 resource blocks (RBs), each occupying a timeslot of 1 ms, and approximately 128 kHz bandwidth. The total downlink bandwidth per cell is therefore 20 MHz channel. We present the performance when different modulation schemes are used.

Figure 5.10: VoLTE average packet throughput and delay per BTS vs. $\delta_{1,2}$ for $\lambda_{2,b} = 12 \text{ km}^{-2}$, $\lambda_1 = 2 \text{ km}^{-2}$, and varying δ_2 in km, when $\alpha = 3.5$. Each BTS carries 20MHz downlink bandwidth with 100 RBs per cell with non-adaptive 16-QAM modulation for 100 users per BTS.

In Figure 5.10, we consider a non-adaptive 16 QAM modulation scheme, which can support 150 bits per RB [29], and 2 RBs per voice packet. An SIR threshold of 1 dB can support the minimum rate of 1.18 bits/s/Hz for this. As such, only one timeslot is required to transmit the packet successfully if the target SIR threshold is met. However, if the target SIR threshold is not met, a retransmission is required under the HARQ scheme used in LTE. Thus, the average number of transmissions required is found to be equivalent to the reciprocal of coverage probability. The delay is then computed by multiplying the average number of transmissions required into the time occupied by a unit timeslot.

The result demonstrates that increasing co-tier separation and optimizing the cross-tier separation enhances voice packet throughput and reduces the average delay. Similarly, in Figure 5.11, we consider a non-adaptive QPSK modulation scheme, which needs about 16 RBs per voice packet [29] and is supported by an SINR threshold of 0 dB for a maximum of 50 users. Similar trends are observed for increased separations although the overall packet throughput is lower, and delay higher, because the modulation scheme supports less bits per second.

Figure 5.11: VoLTE average packet throughput and delay per BTS vs. $\delta_{1,2}$ for $\lambda_{2,b} = 12 \text{ km}^{-2}$, $\lambda_1 = 2 \text{ km}^{-2}$, and varying δ_2 in km, when $\alpha = 3.5$. Each BTS carries 20MHz downlink bandwidth with 100 RBs per cell with non-adaptive QPSK modulation for 100 users per BTS.

This advantage can be enhanced for other types of variably sized packets, if adaptive modulation schemes are used to transmit more bits per RB when the channel quality permits this. For adaptive modulation schemes, we look at the maximum achievable throughput, assuming all RBs are used in full buffer traffic system. For reference, the average throughput in terms of bits/s under the 16-QAM modulation scheme for the settings in Figure 5.10 is given in Figure 5.12.

Figure 5.12: VoLTE throughput in bits/s vs. $\delta_{1,2}$ for $\lambda_{2,b} = 12 \text{ km}^{-2}$, $\lambda_1 = 2 \text{ km}^{-2}$, and varying δ_2 in km, when $\alpha = 3.5$. Each BTS carries 20MHz downlink bandwidth with 100 RBs per cell with non-adaptive 16-QAM modulation for 100 users per BTS.

Under an appropriate adaptive modulation scheme, e.g. that could also consider 64-QAM for high channel quality, and variable packet sizes, this throughput could be increased by a large amount. In fact, by considering the theoretical average rate of users in coverage used in computing the ASE, the Shannon throughput per BTS under the same settings, is much higher, as seen in Figure 5.13.

Figure 5.13: Average Achievable Shannon throughput in bits/s vs. $\delta_{1,2}$ for $\lambda_{2,b} = 12 \text{ km}^{-2}$, $\lambda_1 = 2 \text{ km}^{-2}$, and varying δ_2 in km, when $\alpha = 3.5$. Each BTS carries 20MHz downlink bandwidth with 100 RBs per cell and 100 users per BTS.

5.5.4 Minimum Radius of Guaranteed Association for Tier-2 BTSs

We examine the minimum radius of association guaranteed for a tier-2 BTS according to the separation settings. The results in Figure 5.10 demonstrate that either $\delta_{1,2}$ or δ_2 may dictate the minimum radius of association. Once again, we consider settings where $\lambda_{2,b} = 12 \text{ km}^{-2}$ and $\lambda_1 = 2 \text{ km}^{-2}$, and $\alpha = 3.5$. Denoting a combined designation of settings by the set $\{\delta\}_i = \{\delta_{1,2}, \delta_2\}$, where $i = 1, 2, 3, 4$, we look at the 4 sets separately such that $\{\delta\}_1 = \{0.175 \text{ km}, 0.1 \text{ km}\}$, $\{\delta\}_2 = \{0.175 \text{ km}, 0.2 \text{ km}\}$, $\{\delta\}_3 = \{0.3 \text{ km}, 0.1 \text{ km}\}$, and $\{\delta\}_4 = \{0.3 \text{ km}, 0.2 \text{ km}\}$.

The minimum radius η_2 , is equal to 0.05 km, 0.052 km, 0.05 km, and 0.089 km for these respective sets. Analysing these values tells us that for $\{\delta\}_1$ and $\{\delta\}_3$, the value of η_2 is dictated by δ_2 . Conversely, for $\{\delta\}_2$ and $\{\delta\}_4$, the value of η_2 is dictated by $\delta_{1,2}$. Assigning cross-tier and co-tier separations together for tier-2 BTS planning can guarantee that a tier-2 BTS serves a decent radius around it, thus increasing the average utilization of a tier-2 BTS compared to random deployment.

Figure 5.14: Association probability to arbitrary tier-2 BTS vs distance to said BTS for various joint co-tier and cross-tier separation settings, $\{\delta\}_i$, with highlighted also is the minimum radius of association, η_2 .

5.6 Conclusion

This chapter analyses the performance of a two-tier HCN in terms of coverage, ASE, and practical performance in terms of packet throughput and delay, when cross-tier separation between tier-1 and tier-2 BTSs is combined with co-tier separation between tier-2 BTSs to decide the locations of active tier-2 deployments. Using theoretical approximations that were validated by simulation, it was shown that the coverage of proximal users may be enhanced by designing an HCN where the cross-tier and co-tier separations are significant in terms of magnitude up to a certain limit. Beyond a certain magnitude of cross-tier separation, the coverage of the HCN will decline below its peak, and therefore the limit to this type of separation must be identified when planning an HCN according to the proposed model. The results also show that co-tier separation, i.e. the spacing between a pair of tier-2 BTSs must be increased to its maximum possible value for a base density of tier-2 BTSs in order to enable the best performance of the HCN in terms of coverage. Beyond a certain limit of co-tier separation, the density of deployed tier-2 BTSs falls without providing any significant gain in coverage. Therefore, a limit to co-tier separation can be identified accordingly.

Furthermore, gains in probability coverage come at the cost of ASE as tier-2 BTS density reduces due to cross-tier separation. This means that the best

combination of co-tier and cross-tier separation values needs to be decided by considering the trade-off between coverage and ASE. This decision may be taken by assigning thresholds to either of these performance parameters or prioritising one of them over the other. Accordingly, we have defined a set of rules that could be used by cell planners to decide how cross-tier and co-tier separations can be set to improve these practical cell performance parameters when deploying an HCN. Another useful design aspect related to this model is that a minimum radius of association may be guaranteed for a tier-2 base station, based on the values of co-tier and cross-tier separation parameters.

Chapter 6

Conclusion and Recommendations

6.1 Conclusions

The work in this thesis provides two novel frameworks for the modelling and analysis of two-tier HCNs in practical scenarios. The primary contribution is the modelling of two-tier HCNs with mixed user distributions that comprise uniform random users, as well as clustered users that are conditioned on the locations of outdoor small-cell BTSs. We introduce a framework that incorporates the possible separation between macrocell and picocell BTSs in the HCN, and stipulate that there is a random offset between the centres of user clusters and deployed picocell BTSs. Following a thorough analysis of distance PDFs, association probabilities and coverage probabilities, we present numerical results that validate the derived mathematical expression representing these parameters. Furthermore, a discussion of the HCN's performance in terms of coverage, spectral efficiency, and the utilization of resources per BTS is presented. The discussion reflects some rules that can be used by cell planners in planning the locations of picocell BTSs when they are overlaid on an existing macrocell network. These rules may also be considered a mechanism by which tier-2 BTSs are activated in the network instead.

In particular, we hypothesize that it is important for cellular network operators to identify expected user patterns in the network that correspond to hotspots and deploy tier-2 BTSs that are centrally located within these hotspots. Furthermore, depending on expected user patterns, the separation between tier-1 and tier-2 BTSs must be decided. A larger separation allows for the simultaneous improvement in coverage probability and spectral efficiency for the HCN, provided that the BTSs are sufficiently provisioned to capture users in the network. However, as resources are constrained, the separation that can be allowed is reduced. In such situations,

provision each BTS with more resources can allow for further separation and improved coverage and spectral efficiency.

This primary HCN model is then extended to consider an additional separation category, which stipulates a minimum distance between two picocell BTSs. Expressions for distance PDFs, association probability and coverage probability are derived for a uniformly random user situation. As such, statistics yielded from the analysis showcase spatial coverage and spectral efficiency in the network. The numerical results are first used to validate the derived expressions, and then examined for trade-offs in coverage and spectral efficiency. It is shown that cell planning or activation based on the optimal selection of cross-tier separation and co-tier separation allows for optimal coverage and ASE statistics for the HCN. These are shown to influence practical packet performance as well. Another interesting aspect of planning HCNs with these two separation categories is that picocell BTSs can be generally guaranteed a radius of association.

The author hopes that the models presented serve as useful baseline models for further research.

6.2 Recommendations for Further Research

There are several areas that can be researched in relation to the HCN models presented in this thesis. Firstly, we may contend that expressions may be derived to account for theoretical BTS utilization, rate, and spectral efficiency for both models presented in this paper. The second model incorporating co-tier and cross-tier separation can be extended by using the clustered user model as well, where it can be hypothesized that BTS offset from the user cluster is linked to the co-tier separation. Furthermore, imposing resource constraints on this model may provide a different perspective on how optimal separation between BTS is planned.

Other recommendations include analysing the network for indoor scenarios, particularly for clustered users, where the power ratio between BTSs can vary if even lower powered BTSs are used. Indoor fading models may also be different to the standard Rayleigh fading model used throughout this thesis due to the availability of line of sight links. We propose that these studies may be good extensions of the models presented for future research.

Author's Publications

1. M. Sinnen, K. H. Li, K. C. Teh, "Study of distance distributions and clustered user association in separation-based two-tier HCNs," in *Proc. IEEE WCNC*, April 2018.
2. M. Sinnen, K. H. Li, K. C. Teh, "Modelling and analysis of two-tier HCNs with co-tier and cross-tier separation dependencies", accepted for publication in *IET Communications*, 2019, available at DOI: 10.1049/iet-com.2018.5195.

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