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RESEARCH ARTICLE

Non-cooperative power control and spectrum allocation in cognitive radio networks: a game theoretic perspective

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ABSTRACT

The invention of cognitive radio (CR) concept aims to overcome the spectral scarcity issues of emerging radio systems by exploiting under-utilization of licensed spectrum. Determining how to allocate unused frequency bands among CR is one of the most important problems in CR networks. Because different CRs may have different quality-of-service requirements, they may have different objectives. In *voice communication*, high-speed transmission is the most important factor; hence, *voice radios* always try to maximize their transmission rate. However, in *data communication*, the most important factor is the bit error rate. The *data radios* always try to maximize their signal-to-interference-plus-noise ratio (SINR). In this paper, two non-cooperative games named interference minimization game and capacity maximization game, which reflect the target of *data radios* and *voice radios*, respectively, are proposed. From the simulations, after these games are applied, the average SINRs of all players at each channel are improved. The average SINR of players in each channel after applying the capacity maximization game is smaller than that after applying the interference minimization game. However, in comparison with that after applying the interference minimization game, the average capacity of players after applying capacity maximization approach is larger. Copyright © 2012 John Wiley & Sons, Ltd.

KEYWORDS

cognitive radio; game theory; potential game; player; Nash equilibrium

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1. INTRODUCTION

The concept of cognitive radio (CR) emerges as one promising solution to overcome the spectral scarcity issues of emerging radio systems by exploiting under-utilization of licensed spectrum. One of the most important problems in CR networks is the resource allocation problem, which provides the methods to schedule available resources including frequency bands, power, and modulation codes among radios. The resources should be allocated fairly among CRs in such a way that the CRs can meet their quality-of-service (QoS) requirements.

This problem has been studied broadly for various communication systems. One of the most commonly used approaches for the channel allocation problem is the graph coloring algorithm. In [1], the authors proposed a graph coloring algorithm for the channel/slot assignment problem in which all possible collisions were avoided for

a given network topology. In that paper, each channel/slot is mapped to a color, and each vertex corresponds to a user. The paper aims to minimize the color usage where each vertex is assigned with one color. In [2], a graph theoretical model to characterize the spectrum access problem under a number of different optimization functions and to devise rules for users to utilize available spectrum while avoiding interference with its neighbors was proposed. The available bandwidth is divided into a set of spectrum bands that are different from each other in bandwidth and transmission range. The user can utilize any number of spectrum band at a time. Unlike the traditional graph coloring proposed in [1], the approach in [2] tries to maximize the utility of the system such as max-sum-bandwidth, max-min-bandwidth, and max-proportional-fair. One of the limitations of the resource allocation approaches using graph coloring algorithm is its complexity. As we know, the optimal graph coloring is an NP-hard problem and has

difficulty in achieving an optimal solution. In addition, resource allocation approaches using graph coloring algorithm cannot reflect different users' objectives. An alternative to graph coloring for the resource allocation problem is the game theory, which provides a mathematical basis for modeling and analyzing interactive decision-making problems [3]. In game theory, the decision makers (i.e., players) have to make specific actions that may have mutual, possibly conflicting requirements and consequences. In general, a game is classified into cooperative and non-cooperative game. In non-cooperative games, the players make their decision independently to maximize their own benefit, whereas in cooperative games, coalitions of players are formed, and the players aim to maximize a common goal. For the context of this paper, only the non-cooperative games are discussed.

The applications of game theory in resource allocation problem in CR networks have been studied widely by various authors. In [4], a power control mechanism was proposed in order to maximize the total capacity of a CR network while allowing each CR user to contract or expand its bandwidth. In [5], a three-step cross-layer optimization for subcarrier, time slot, and power allocations for orthogonal frequency-division multiple access (OFDMA)-based CR system was studied. It was shown in [5] that the total capacity of OFDMA system can be maximized while the fairness among users was still guaranteed in a dynamic and time-varying communication environment. In [6], a near-optimal scheme for jointly allocating channels and power levels among CRs that competed for a channel access in a CR network was studied. This scheme aims to maximize the total sum capacity of the system while respecting total power constraints per user. In [7], a rate-maximization power-allocation game in frequency-selective Gaussian interference channel using iterative water-filling algorithm (IWFA) was proposed. In [8], the linear convergence of the distributed IWFA for arbitrary symmetric interference environment and for certain asymmetric channel conditions with any number of users was established. In [9], a sequential IWFA for the power allocation problem, in which, at each iteration, the players made their move one by one in order to maximize their own information rate was proposed. Owing to its sequential updating strategy, the sequential IWFA may suffer from slow convergence when the number of users is large. In [10,11], a simultaneous IWFA, in which users made their moves simultaneously, was proposed to overcome the slow convergence problem of the sequential IWFA.

In [12], a game theoretic framework for distributed adaptive channel allocation problem for CR networks was studied. The considered CR network consists of a set of N transmitting-receiving pairs of nodes (players) competing for a single channel among K available channels for transmission. The utility function for each player counts for the interference caused by the other players to this player and for the interference caused by it to other players if they share the common channel. The game was proved to be

an exact potential game, which will finally converge to a pure-strategy Nash equilibrium as proven in [13]. In [14], the authors showed that the interference minimization approach proposed in [12] can also be applied to the OFDMA subcarrier assignment, where N players compete for K subcarriers, with $K < N$. However, this game leads to a solution of a single subcarrier per user. In order to solve the dominated strategy problem, the authors restricted the strategy space such that each player is allowed to occupy a pre-defined number of subcarriers. In [15], a potential game framework was proposed to perform power allocation in an OFDM CR network, which was constituted by a primary licensed system and a secondary system. In particular, a DVB-SH-based satellite network was considered as the primary system. For the secondary system, an infrastructured wireless terrestrial network in which all secondary terminals communicated with a local base station was considered. Unlike in [12], the cognitive terminals (players) in [15] competed for K available subcarriers. Two constraints were imposed on each subcarrier. The first constraint was the minimum amount of power to allocate in order to respect the secondary system target bit error rate. The second constraint was the maximum amount of power on a certain subcarrier in order to protect the primary system. The utility function for each player was defined as the sum of capacity values acquired over K subcarriers subtracting the interference caused on the primary user. The potential games were also studied in [16,17] for the power control problem in code division multiple access system and in [18] to model an interference avoidance scheme. In addition, a potential game framework encompassing both power control and channel allocation in CR networks was proposed in [19].

The transmit power and spectral bands are the resources we have considered within the scope of the present paper. As we know, different radios may have different QoS requirements. In *voice communication*, high-speed transmission is the most important factor; hence, *voice radios* always try to maximize their transmission rate (i.e., maximize capacity). On the other hand, the most important factor for *data communication* is its bit error rate requirements. The *data radios*, hence, try to maximize the signal-to-interference-plus-noise ratio (SINR) (i.e., minimize the interference). In this paper, two different non-cooperative games named interference minimization game and capacity maximization game, which reflect the target of data radios and voice radios, respectively, are proposed. The proposed approach is different from [12] and [14] where only the cost is considered. For example, in the proposed interference minimization game, both the cost and the benefit for allocating power on channels are considered. The utility function of a player is determined as the benefit that the player earns minus its cost for choosing a specific strategy. The benefit of a player is defined as the total transmit power, whereas the cost is defined as the total interference that the other players generate on this player and the interference of this player to the others. This kind of utility function allows the player not to access the channel

when the benefit is smaller than the cost. In addition, in the proposed game, the players are able to access any number of channels at different transmit power. This is contrast to the games proposed in many of the existing literature; for example, in [12], the players are allowed to access a single channel at a fixed and pre-defined transmit power, and in [14], the players are allowed to access a fixed number of channels at a pre-estimated transmit power. The interference minimization game has been proved to be an exact potential game, which will finally converge to a pure-strategy Nash equilibrium solution by following the best response dynamics. In the capacity maximization game, the utility function of the game is determined as the total capacity that a radio acquires minus the interference that it generates to others. Different from that in the other works available in literature, in the proposed capacity maximization game, the player not only tries to maximize its capacity but also tries to reduce the interference that it generates to the others, in other words, to reduce the loss in the capacity of the others. As a result, it helps improve the performance of the whole system. Similar to those in interference minimization game, the players in capacity maximization game also follow the best response dynamics in order to achieve a steady state (i.e., the Nash equilibrium).

The rest of this paper is organized as follows. Section 2 provides the system description. Section 3 provides the way to formulate the channel allocation problem as a non-cooperative game. The details of interference minimization game and capacity maximization game are also provided in this section. Section 4 provides the simulation results and the discussion. Finally, Section 5 provides the conclusion.

2. SYSTEM MODEL

Let us consider that a CR network consists of a set of N radios, each formed by a single transmitting–receiving pair of nodes, which are uniformly distributed in a geographical area. The nodes are assumed to be fixed or slowly moving so that the steady-state solution can be achieved. It is also assumed that there are K frequency channels available for data transmission, with $K < N$. The following assumptions are also made for the analysis:

- Multiple transmitter–receiver pairs are allowed to transmit at the same time over a shared channel.
- Access to all channels are equally likely for all transceivers.
- Complete channel information is available to all the transceivers.

Let p_{ik} be the transmit power of transmitter i over transmitting channel k . Let g_{ij}^k be the channel gain between transmitter i and receiver j when the transmission is made through a channel k . In general, for $i \neq j$, $g_{ij}^k \neq g_{ji}^k$. Under these assumptions, the SINR of the receiver i via transmitting channel k is given by

$$\text{SINR}(i, k) = \frac{p_{ik} g_{ii}^k}{\sum_{j=1, j \neq i}^N p_{jk} g_{ji}^k + \sigma_0} \quad (1)$$

where σ_0 is the average noise level and is assumed to be the same for all receivers. The total transmission capacity of transmitter–receiver pair i is defined as

$$C(i) = \sum_{k=1}^K \log \left(1 + \frac{p_{ik} g_{ii}^k}{\sum_{j=1, j \neq i}^N p_{jk} g_{ji}^k + \sigma_0} \right) \quad (2)$$

It is also assumed that the total transmit power of each transmitter is constrained by a maximum power constraint although different radios may have different power constraints. Let $P_{i,\max}$ be the power constraint of transmitter i ; the transmit power p_{ik} must satisfy the following condition:

$$\sum_{k=1}^K p_{ik} \leq P_{i,\max} \quad (3)$$

On the basis of these assumptions, the channel allocation problem can be modeled as the outcome of a non-cooperative game in which the players are the transmitter–receiver pairs of CRs. The details of the game formulation are presented in the next section.

3. A GAME THEORETIC FRAMEWORK

A game theoretic framework to allocate K channel to N transmitter–receiver pairs of CRs is proposed in this paper. For the non-trivial solution, only the case when $K < N$ is considered. The set of players, $\mathcal{N}$, consists of N transmitter–receiver pairs of CRs. For each player i , the strategy s_i is defined as a K -dimension vector representing the transmit power of player i over transmitting channels, represented as

$$s_i = \{p_{i1}, p_{i2}, \dots, p_{iK}\}$$

where p_{ik} is the transmit power of player i over transmitting channel k . In order to keep track of the channel assignment and the transmit power allocation on the assigned channel for all CRs, each player must maintain a channel allocation matrix and a power profile matrix defined as follows:

- The $N \times K$ dimension channel allocation matrix $A = \{a_{ik}\}$, in which element $a_{ik} = 1$ if player i communicates over transmitting channel k ($p_{ik} > 0$), and $a_{ik} = 0$ otherwise ($p_{ik} = 0$), for $k = 1, 2, \dots, K$.
- The $N \times K$ dimension power profile matrix $P = \{p_{ik}\}$, in which element p_{ik} is the transmit power of player i over channel k .

Let s_{-i} be the strategies chosen by all other players in the game, that is, $s_{-i} = \{s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_N\}$. The set

of strategies chosen by all players constitutes a strategy profile, which is denoted as $s = \{s_i, s_{-i}\}$. The last and also the most crucial task to formulate a game is to choose the utility function (payoff), u_i , which measures the outcome of the game for a player i given the particular strategy profile s . In a non-cooperative game, the players always make their decision in order to maximize their own utility function. In this paper, two utility functions, which respectively reflect the interference minimizing and capacity maximizing objectives, are proposed. The details of these games will be presented as follows.

3.1. Interference minimization game

As mentioned before, different radios may have different QoS requirements. As a result, they may have different targets. Whereas voice radios always try to maximize their transmission rate (i.e., maximize capacity), the data radios aim to maximize the SINR (i.e., minimize the interference). In this section, a game named interference minimization, which reflects the target of data radios, is proposed. A capacity maximization game, which reflects the target of voice radios, will be discussed in the next section. For the interference minimization game, the utility function for a player i given the particular strategy profile $s = \{s_i, s_{-i}\}$ can be defined as

$$u_i(s_i, s_{-i}) = \sum_{k=1}^K \left\{ p_{ik} g_{ii}^k - \sum_{j=1}^N \delta_{ji}^k p_{jk} g_{ji}^k - \sum_{j=1}^N \delta_{ij}^k p_{ik} g_{ij}^k \right\} \quad (4)$$

where $\delta_{ij}^k = 1$ if player i and player j ($i \neq j$) both transmit via channel k ($p_{ik} \times p_{jk} \neq 0$), and $\delta_{ij}^k = 0$ otherwise. The first component in the utility function represents the benefit that a player earns for allocating power on channels. The second and the third components represent the cost of a player: the interference caused by other players to this player and the interference caused by it to other players if they share the common channels. Similar to the game proposed in [12,14], the proposed game is an exact potential game. Potential game [13,20–22] is a special type of game that is characterized by a potential function whose change in value when any player deviates in their action is related to the change in the objective function of that player. There are various kinds of potential games. Among them, exact potential games are the most well-known games. A game is an exact potential game if there is an exact potential function that exactly reflects any unilateral change in the utility function of any player [23]. In the mathematical representation, a game is an exact potential game if there exists some exact potential function $\mathcal{P} : \mathcal{S} \rightarrow \mathcal{R}$ that satisfies

$$u_i(s_i, s_{-i}) - u_i(s'_i, s_{-i}) = \mathcal{P}(s_i, s_{-i}) - \mathcal{P}(s'_i, s_{-i}) \quad (5)$$

for $\forall i \in \mathcal{N}, \forall s \in \mathcal{S}$. The exact potential function of the proposed game is given as follows:

$$\mathcal{P}(s) = \frac{1}{2} \sum_{i=1}^N u_i(s) + \frac{1}{2} \sum_{i=1}^N \sum_{k=1}^K p_{ik} g_{ii}^k \quad (6)$$

Proving the proposed game as an exact potential game is similar to what was presented in [14]. As in [13], one important property of an exact potential game is that there is at least one pure-strategy Nash equilibrium. This equilibrium can be achieved by letting players repeatedly play in a myopic game [13]. In each play, the player will select the best response strategy, which maximizes its own utility, on the basis of the observed opponent strategies. This algorithm is known as best response dynamics [13]. Assuming that each player knows exactly the opponent strategies, determining the best response strategy of each player in each play is provided as follows.

From the utility function in Equation (4), for a player i , the utility (payoff) it receives by allocating power on a channel k is defined as

$$\begin{aligned} u_i^k(p_{ik}) &= p_{ik} g_{ii}^k - \sum_{j=1}^N \delta_{ji}^k p_{jk} g_{ji}^k - \sum_{j=1}^N \delta_{ij}^k p_{ik} g_{ij}^k \\ &= \alpha_i^k p_{ik} - \beta_i^k \end{aligned} \quad (7)$$

where $\alpha_i^k = g_{ii}^k - \sum_{j=1}^N \delta_{ji}^k g_{ji}^k$ and $\beta_i^k = \sum_{j=1}^N \delta_{ij}^k p_{ik} g_{ij}^k$. The values of α_i^k and β_i^k depend only on the strategies of other players and hence are known. If $\alpha_i^k \leq 0$, $u_i^k(p_{ik}) \leq 0$ for all $p_{ik} > 0$. That leads to Proposition 1, which is stated as follows.

Proposition 1. *The strategies by which a player i allocates power on a channel k , where $\alpha_i^k \leq 0$, are always dominated by strategies by which the player does not occupy that channel.*

Proposition 2. *The strategies by which a player allocates power on a group of channels are always dominated by the strategies by which the player allocates all of its power available for transmission (maximum power constraint) on a single channel.*

Proof. Assuming that s_i is a strategy by which a player i allocates power on a group of channels, $\mathcal{K}_1 \subset \{1, 2, \dots, K\}$. The utility earned by that player for choosing strategy s_i is provided as

$$\begin{aligned} u_i(s_i, s_{-i}) &= \sum_{k \in \mathcal{K}_1} u_i^k(p_{ik}) \\ &= \sum_{k \in \mathcal{K}_1} \{\alpha_i^k p_{ik}\} - \sum_{k \in \mathcal{K}_1} \{\beta_i^k\} \end{aligned} \quad (8)$$

Let $m = \arg \max_{k \in \mathcal{K}_1} \{\alpha_i^k\}$ and $\alpha_{i,\max} = \alpha_i^m$. Because the overall transmit power available for player i is

constrained by a maximum power constraint as in Equation (3), the following inequality is given:

$$u_i(s_i, s_{-i}) < \alpha_{i,\max} P_{i,\max} - \sum_{k \in \mathcal{K}_1} \beta_i^k < \alpha_i^m P_{i,\max} - \beta_i^m$$

Let s'_i be the strategy by which the player i allocates transmit power of $P_{i,\max}$ on the channel m :

$$u_i(s'_i, s_{-i}) = \alpha_i^m P_{i,\max} - \beta_i^m$$

Consequently, $u_i(s_i, s_{-i}) < u_i(s'_i, s_{-i})$, and strategy s_i is said to be dominated by strategy s'_i . $\square$

Corollary 1. *The best response of a player should be a strategy by which either only one channel (single-channel strategy) or none at all is accessed. In the case where one channel is accessed (i.e., channel k), the following conditions must be satisfied:*

- $\alpha_i^k > 0$;
- the transmit power $p_{ik} = P_{i,\max}$;
- the utility function $u_i > 0$.

As a result, the best response strategy for a player in each play can be determined as a strategy that generates the largest utility function among strategies satisfying conditions stated in Corollary 1. If there is no single-channel strategy generating a positive utility, the best response strategy is not to access any channel.

Next, the details of the best response dynamics, which consists of two states, the initialized state and repeated state are given as follows.

Initialized state:

- Each player is allowed to access K channels. For a player i , the transmit power allocation on a k channel is $p_{ik} = P_{(i,\max)}/K$.

Repeated state: this state is repeated until the game converges.

- Each player plays in a pre-defined order in each iteration.
 - On the basis of the observed opponent strategies (in the previous iteration), player finds the best response strategy among K single-channel strategies that satisfy Corollary 1 in order to maximize the utility function.
 - Player plays the best response strategy and updates the power profile matrix and channel allocation matrix.

Because there are, at most, K single-channel strategies satisfying conditions stated in Corollary 1 and because for each single-channel strategy, computing the utility function requires $O(N)$ addition operations, the complexity to find the best response for each player is $O(KN)$. There are N players, and as a result, the complexity of this game is $O(KN^2)$.

3.2. Capacity maximization game

For the capacity maximization target, the utility function for each player is defined as

$$u_i(s_i, s_{-i}) = \sum_{k=1}^K \log \left(1 + \frac{p_{ik} g_{ii}^k}{\sum_{j=1, j \neq i}^N p_{jk} g_{ji}^k + \sigma_0} \right) - c_i \left(\sum_{k=1}^K \sum_{j=1, j \neq i}^N p_{ik} g_{ij}^k a_{jk} \right) \quad (9)$$

where the first item is the player's benefit, which is determined as the total transmission capacity as defined in Equation (2). The second item is the cost of the player and is determined as the product of the interference that the player generates on other players and the cost-per-interference constant c_i . Similar to that in interference minimization game, in capacity maximization game, players also follow the best response dynamics in order to achieve the steady state (i.e., Nash equilibrium). By following the best response dynamics, players repeatedly play the game until a steady state is reached. In each play, each player selects the best response strategy, which maximizes its own utility, on the basis of the observed opponent strategies. Assuming that each player knows exactly the opponent strategies, finding the best response strategy for a player i can be formulated as an optimization problem as follows.

$$\max_{s_i} u_i, \forall i \in \mathcal{N}$$

s.t.

$$\begin{cases} p_{ik} \geq 0, \forall i, k \\ \sum_{k=1}^K p_{ik} \leq P_{i,\max} \end{cases}$$

The Lagrangian $\mathcal{L}$ associated with the aforementioned optimization problem is defined as follows:

$$\begin{aligned} \mathcal{L}(s_i) &= (-u_i) + \lambda \left\{ \sum_{k=1}^K p_{ik} - P_{i,\max} \right\} + \sum_{k=1}^K \mu_k (-p_{ik}) \\ \mathcal{L}(s_i) &= - \sum_{k=1}^K \log \left(1 + \frac{p_{ik} g_{ii}^k}{\sum_{j=1, j \neq i}^N p_{jk} g_{ji}^k + \sigma_0} \right) \\ &\quad - c_i \left\{ \sum_{k=1}^K \sum_{j=1, j \neq i}^N p_{ik} g_{ij}^k a_{jk} \right\} \\ &\quad + \lambda \left\{ \sum_{k=1}^K p_{ik} - P_{i,\max} \right\} + \sum_{k=1}^K \mu_k (-p_{ik}) \end{aligned} \quad (10)$$

where λ is a non-negative constant and μ_k are constant for $k = 1, 2, \dots, K$. The partial derivative of $\mathcal{L}(s_i)$ is given as

$$\frac{\partial \mathcal{L}(s_i)}{\partial p_{ik}} = -\frac{1}{\ln(2)} \left(\frac{\frac{g_{ii}^k}{\sum_{j=1, j \neq i}^N p_{jk} g_{ji}^k + \sigma_0}}{1 + \frac{p_{ik} g_{ii}^k}{\sum_{j=1, j \neq i}^N p_{jk} g_{ji}^k + \sigma_0}} \right) + c_i \left\{ \sum_{j=1, j \neq i}^N g_{ij}^k a_{jk} \right\} + \lambda - \mu_k$$

for $k = 1, 2, \dots, K$. From the Karush–Kuhn–Tucker conditions [24], the following equations must be satisfied:

$$\frac{\partial \mathcal{L}(s_i)}{\partial p_{ik}} = 0 \quad (11)$$

$$\lambda \left\{ \sum_{k=1}^K p_{ik} - P_{i,\max} \right\} = 0 \quad (12)$$

$$\sum_{k=1}^K \mu_k (-p_{ik}) = 0 \quad (13)$$

Assume that $p_{ik} > 0, \forall k \in \{1, 2, \dots, K\}$. From Equation (13), $\mu_k = 0, \forall k \in \{1, 2, \dots, K\}$. From Equation (11), we have

$$-\frac{1}{\ln(2)} \left(\frac{\frac{g_{ii}^k}{\sum_{j=1, j \neq i}^N p_{jk} g_{ji}^k + \sigma_0}}{1 + \frac{p_{ik} g_{ii}^k}{\sum_{j=1, j \neq i}^N p_{jk} g_{ji}^k + \sigma_0}} \right) + c_i \left\{ \sum_{j=1, j \neq i}^N g_{ij}^k a_{jk} \right\} + \lambda - \mu_k = 0, \forall k \in \{1, 2, \dots, K\}$$

Let $I_{ik} = \sum_{j=1, j \neq i}^N p_{jk} g_{ji}^k + \sigma_0$ and $C_{ik} = \sum_{j=1, j \neq i}^N g_{ij}^k a_{jk}$; the previous equation is re-written as

$$\frac{1}{\ln(2)} \left\{ \frac{g_{ii}^k}{p_{ik} g_{ii}^k + I_{ik}} \right\} = c_i C_{ik} + \lambda$$

As a result,

$$p_{ik} = \frac{1}{\ln(2) (c_i C_{ik} + \lambda)} - \frac{I_{ik}}{g_{ii}^k} \quad (14)$$

From Equations (12) and (14),

$$\lambda \left\{ \sum_{k=1}^K \left(\frac{1}{\ln(2) (c_i C_{ik} + \lambda)} - \frac{I_{ik}}{g_{ii}^k} \right) - P_{i,\max} \right\} = 0 \quad (15)$$

The value of λ can be determined by solving Equation (15). After that, the value of p_{ik} can be calculated by replacing the value of λ into Equation (14). It is important to note that if the value of p_{il} found in Equation (14) is smaller than 0, for $l \in \{1, 2, \dots, K\}$, p_{il} is set to 0, that is, $p_{il} = 0$. After that, we need to re-calculate the value of λ on the basis of the following equation and replace the value of λ into Equation (14) to find the value of p_{ik} , for $k \in \{1, 2, \dots, K\}, k \neq l$:

$$\lambda \left\{ \sum_{k=1, k \neq l}^K p_{ik} - P_{i,\max} \right\} = 0 \quad (16)$$

As in the previous case, the best response dynamics consists of two states, the initialized state and repeated state. The details of this algorithm are given as follows.

Initialized state:

- Each player is allowed to access K channels. For a player i , the transmit power allocation on a k channel is $p_{ik} = P_{(i,\max)}/K$

Repeated state: this state is repeated until the game converges.

- Each player plays in a pre-defined order in each iteration.
 - On the basis of the observed opponent strategies (in the previous iteration), player finds the best response strategy, which is determined as in Equation (14).
 - Player plays the best response strategy and updates the power profile matrix and channel allocation matrix.

It is important to note that, in order to find the value of λ , we need to solve a K -degree equation, that is, Equation (15). Thus, capacity maximization game is more complex than interference minimization game. Assume that computing I_{ik} and C_{ik} is processed sequentially. Consequently, it requires $O(KN)$ time to calculate these values. Assume that it requires θ time to solve the K -degree equation, the total time to compute the best response strategy for a player is $\theta + O(KN)$. There are N players, and as a result, the complexity of this game is $N\theta + O(KN^2)$.

4. SIMULATION RESULTS

In this section, the performances of the proposed game are provided and discussed. The simulations consider 20

transmitter–receiver CR pairs (players) and 5 channels. The transmitters and receivers are randomly located in a square region of dimension $L \times L$, where L is 1 km as in Figure 1. The power constraints of the players are identical and are equal to 40 mW. The channel gains are determined on the basis of the path loss model. The path loss exponent is set to 3. It is assumed that each player has complete knowledge about channel gains between any transmitter and any receiver, that is, g_{ij}^k is known, $\forall i, j \in \mathcal{N}$, and $k \in \{1, 2, \dots, K\}$. For the initialization, channels are assigned randomly to N players such that the maximum power constraint for each player is satisfied. After that, the players repeatedly play the game until the Nash equilibrium is reached. In each play, the players make their moves in a pre-defined sequence, and the best response strategy, on the basis of the observed opponent strategies, is chosen.

4.1. Simulation results of interference minimization game

Firstly, the convergence character of the player's strategy is studied. Let the channel allocation vector of player i at the iteration l be $a_i^l = \{a_{ik} | k = 1, \dots, K\}^l$. The channel allocation strategy is defined as the decimal conversion of the channel allocation vector. For example, at the first iteration, if the channel allocation vector of player 1 is $a_1^1 = \{10, 000\}$, the corresponding channel allocation strategy is 16. Because there are five channels, the total number of channel allocation strategies is 32. The channel allocation strategies that players take in each iteration are shown in Figure 2. From the figure, it can be seen that, starting from the initial state, the game takes a maximum of five iterations to reach the steady state, which is the Nash equilibrium. The convergence characteristic of the proposed game is also shown in Figure 3 where the values of the potential function of the proposed game after each step (on each step, only one player will take the action) are plotted. As seen from the figure, after 80 steps, the potential function is stable and reaches its maximum value.

Figure 1. An example of transmitter–receiver allocation.

Figure 2. Convergence character of player's strategy of interference minimization game.

Figure 3. Potential function of interference minimization game.

The SINR of players before and after applying the channel allocation is also investigated. Figure 4(a) shows the average SINR of players in different channels at the initial state (before applying the channel allocation game), and Figure 4(b) shows the average SINR of players in different channels after applying the interference minimization game. It can be seen that the average SINR of players at each channel increases significantly after applying the proposed game. In particular, at initial state, the SINR of some of the players have the negative values. After the proposed game is applied, the SINR of all players at each channel are improved because of the reduction in interference.

4.2. Simulation results of capacity maximization game

In the simulations studied in this section for the capacity maximization game, the cost-per-interference constants of players are the same and are set to 1, that is, $c_i = 1$ for $i \in \{1, 2, \dots, N\}$. The convergence character of player's strategy for the capacity maximization game is shown in Figure 5. As seen from the figure, similar to that in the

Figure 4. (a) Initial allocation of average signal-to-interference-plus-noise ratio (SINR) of players in different channels and (b) final allocation of average SINR of players in different channels after applying interference minimization game.

interference minimization game, starting from the initial state, the capacity maximization game takes a maximum of five iterations to reach the steady state, which is a Nash equilibrium. The convergence of the capacity maximization game is also shown in Figure 6, which plots the total capacity per unit bandwidth of players of capacity maximization game after each step. As seen from the figure, it takes a maximum of 80 steps for the total capacity per unit bandwidth of the whole system to be stable. The performance of the capacity maximization game is also studied in terms of SINR. The average SINR of players in different channels after applying the capacity maximization game is plotted in Figure 7. The average SINR of players in different channels at the initial state is the same as in the previous game (interference minimization) and is given in Figure 4(a). As seen from these figures, the capacity maximization game helps increase the average SINR of players in each channel significantly.

Figure 5. Convergence character of player's strategy of capacity maximization game.

Figure 6. Total capacity per unit bandwidth of players of capacity maximization game after each step.

Figure 7. Final allocation of average signal-to-interference-plus-noise ratio (SINR) of players in different channels after applying capacity maximization game.

Figure 8. Average capacity per unit bandwidth of players after applying the proposed approaches.

Next, the performances of interference minimization game and capacity maximization game are compared. As shown in Figure 4(b) and in Figure 7, the average SINR of players in each channel after applying the interference minimization game is greater than that after applying the capacity maximization game. In addition, the average capacities of players after applying the proposed approaches are also studied, and the results are given in Figure 8. As seen from Figure 8, in comparison with that after applying the interference minimization game, the average capacity of players after applying the capacity maximization approach is larger. The total capacity per unit bandwidth that the whole system (20 players) can obtain by using capacity maximization is 225.56 bits/Hz. By applying interference minimization game, the total capacity per unit bandwidth of the whole system is only 144.49 bits/Hz. It can be easily realized that the performance of the proposed approaches reflects exactly the designing intention. In the interference minimization game, the players aim to minimize the interference and hence maximize the SINR, whereas in the capacity maximization game, the players aim to maximize their capacity. It can be also inferred from Figure 8 that, after applying the proposed approaches, the players have more or less the same average capacity per unit bandwidth values. Thus, the radio resources have been shared fairly among CRs by applying the proposed approaches.

5. CONCLUSIONS

In this paper, two different non-cooperative games to jointly allocate power and spectrum among CRs in CR networks are proposed. For the interference minimization game, the utility function of a player is determined as the benefit that the player earns minus its cost for choosing a specific strategy. The benefit of a player is defined as the total transmit power, whereas the cost is defined as the total interference that the other players generate on

this player and the interference of this player to the others. The proposed game is proved to be a potential game, which will finally converge to a Nash equilibrium by following the best response dynamics. In addition, determining the best responses strategy for each player on the basis of the observed opponent strategies is also provided. For the capacity maximization game, the utility function of the game is determined as the benefit (i.e. total capacity) that a player acquires minus its cost that is defined as the product of the interference that the player generates on other players and the cost-per-interference constant. From the simulations, after the interference minimization game was applied, the SINR of all players at each channel were improved because of the reduction in interference, whereas after the capacity maximization game was applied, the players acquired very high average capacities (higher than those after applying interference minimization game).

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