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Robust Design of *LCL* Filters for Single-Current-Loop Controlled Grid-Connected Power Converters with Unit PCC Voltage Feedforward

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Abstract—The point of common coupling (PCC) voltage feedforward control is widely employed for *LCL*-type grid-connected converters to prevent large inrush current during startup, reduce steady-state tracking error and suppress disturbances of grid voltage. Generally, this feedforward loop is ignored in the system modeling and stability analysis for simplicity. However, the grid impedance would be introduced to the control system through the feedforward loop under practical power grids, leading to a different current loop model from the original one, then conventional *LCL* filter design for single-current-loop control may be inappropriate. In this paper, the detailed modeling of current control loop with unit PCC voltage feedforward is presented. Based on the established model, it is found that unstable open-loop poles may appear under grid impedance variation, which thus may cause system instability. Therefore, the robust design regions of *LCL* filters against grid impedance variation are proposed for single-current-loop control schemes. Moreover, another interesting finding is that the feedforward control can help maintain a high current control bandwidth under grid impedance variation. This feature is highly desirable if multiple resonant controllers are employed for harmonic compensation. Experimental results are finally presented to verify the theoretical analysis and robust design presented in this paper.

Index Terms—Grid impedance, *LCL* filter, PCC voltage feedforward, robust design region, single-current-loop, stability.

I. INTRODUCTION

Voltage-source-type grid-connected converters are widely used for AC/DC or DC/AC energy conversions, e.g., uninterrupted power supplies (UPS), photovoltaic (PV) and wind generation, and grid energy storage systems [1]. In order to attenuate output current harmonics at switching frequencies, passive power filters are normally required at the output terminal of converters. As third-order low-pass

filters, *LCL* filters have already been popularly used in grid-connected converters due to their high harmonic attenuation capability. However, the *LCL* resonance issue brings a stability challenge to the control system. Moreover, due to their direct interconnection with power grids, system stability performance may be adversely affected by uncertain grid conditions, e.g., grid impedance variation [2]–[5].

There are mainly two solutions to the unstable resonance issue, i.e., passive damping and active damping. Due to extra damping losses [6], the passive damping is gradually replaced by active damping in spite of its simplicity and high reliability. Typically, active damping methods can be classified into two categories, i.e., state-variable feedback and filter-based active damping. The filter-based active damping methods are realized by cascading a high-order filter with proportional-integral (PI) or proportional-resonant (PR) current controller, which are excessively dependent on the parameters of *LCL* filter [7]. The variation and uncertainty of filter parameters and grid impedance may result in damping failure. While for those based on state-variable feedback, e.g., filter capacitor current [8]–[11], extra sensors are required which would lead to a higher cost and failure rate. To avoid additional sensing, the filter capacitor voltage can be measured for grid synchronization and active damping [12]–[14]. Generally, the derivative of capacitor voltage has to be implemented for effective resonance damping, which would increase the complexity of current control. In order to avoid the above problems, [15] investigated the inherent damping characteristic of *LCL*-type grid-connected inverters, and the optimum damping can be achieved by solely using converter-side current control. In [16], the injected grid-side current was used as not only a feedback controlled variable but also a damping state-variable with a negative high-pass filter (HPF). Further, [17] also investigated the damping solution with controlled grid-side current by a HPF, and presented its equivalent physical meaning in the continuous *s*-domain and a systematic design for HPF in the discrete *z*-domain. Although these active damping methods do not need extra sensors, the design complexity of system is increased. More recently, it is found that the inevitable control delay caused by digital computation and pulse-width modulation (PWM) can help stabilize the undamped system [18]. Several literatures have investigated the stable design regions of *LCL* filter with single-loop current control schemes, and the conclusion is that $\omega_r < \omega_s/6$ is preferred for converter-side

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current feedback (CCF) [19], while $\omega_r > \omega_s/6$ should be adopted for grid-side current feedback (GCF) [18], [19], with ω_r and ω_s being resonant frequency and sampling frequency respectively. In [19], a thorough theoretical study was carried out to present the relationship between different control delays and stability of single-current-loop control with CCF or GCF, and a delay optimization method was proposed to improve system stability in [20]. [21] further explored the generalized design regions of *LCL* filter for single-current-loop control schemes, and even discussed the possibility to design resonant frequency beyond the Nyquist frequency [22]. Due to simple control design and no extra sensors costs, the control structure with single-current-loop is now attracting increasing research interests.

However, due to low-voltage distribution transformers and long distribution cables, etc., the grid impedance in modern distribution systems may be uncertain and vary in a wide range, which may challenge the stability and control performance of grid-connected converters. In [23], an optimal design for capacitor-current-based active damping was presented to improve its robustness against the wide range variation of grid impedance. Similarly, in order to avoid the instability caused by grid impedance, an improved capacitor-current-feedback method was proposed to widen the damping region for covering all the possible resonant frequencies in [11]. While for single-current-loop control, inductive grid impedance may shift the resonant frequency ω_r to unstable regions, e.g., ω_r becomes less than $\omega_s/6$ for GCF. A commonly adopted method is to design *LCL* filter with a sufficient stability margin, e.g., a much higher resonant frequency than $\omega_s/6$.

Practically, PCC voltage feedforward has been widely employed to prevent large inrush current during startup, reduce steady-state tracking error and suppress disturbances of grid voltage, such as dips and swells [24]-[27]. Moreover, PCC voltage feedforward can also suppress the effects of grid background harmonics on output current. In [28], a full feedforward scheme of PCC voltage was proposed, which involves a proportional (constant 1) and two derivative terms in the feedforward function. Due to noise amplification problem, the derivative terms are difficult to be implemented in digital control systems. Therefore, the unit PCC voltage feedforward is commonly performed. As the unit feedforward functions at the low-frequency region while the derivative terms function at the high-frequency region, the system with unit PCC voltage feedforward would have good rejection capability of low-order grid harmonics but poor rejection capability of high-order grid harmonics. In order to address this issue, an effective way is paralleling high-order harmonic controllers (HCs) with fundamental PR controller. However, in order to simplify the theoretical analysis, many stability or control studies for *LCL*-type current control ignored the effects of unit PCC voltage feedforward [11], [18]-[23]. This is obviously not accurate enough for the control design, especially under practical power grids, because the grid impedance would be

introduced to the control system through the feedforward loop, leading to a different current loop model from the original one. By designing a feedforward function of PCC voltage to shape the output impedance, strong robustness against grid impedance variation and high harmonic rejection capability were realized in [29]. Nevertheless, the feedforward function is complicated and difficult to be implemented in the digital controller due to multiple derivative terms and even a second-order derivative. [30] showed that an additional positive feedback of grid-side current is introduced by PCC voltage feedforward control. Furthermore, a brief and incomprehensive research about the effects of grid impedance on capacitor-current-based active damping and resonant controller or repetitive controller is presented. In order to improve system control performance under grid impedance variation, [31] then proposed an adaptive control by modifying the feedforward function of PCC voltage and adjusting the parameters of current controller online with an estimated grid impedance. However, it results in a complicated control structure and is sensitive to grid impedance variation. In the above studies, an additional damping loop, e.g., proportional capacitor-current feedback, has to be integrated into current control to stabilize the system. In [32], [33] and [34], it was found that either capacitor voltage feedforward or PCC voltage feedforward can be equivalent to a virtual impedance in parallel with the filter capacitor, thus the resonance characteristic of *LCL* filter can be changed. According to the above research, it may lead to incorrect *LCL* filter design and unstable current control for single-current-loop control schemes when unit PCC voltage feedforward is employed, which has not been reported in literature.

In practical applications, the capacitor voltage can also be measured for grid synchronization and feedforward control. Generally, the derivative-based feedforward of capacitor voltage is used for resonance damping [13], [14], which is actually equivalent to the proportional feedforward of capacitor current [14] and thus also sensitive to grid impedance variation. In [35], a third-order *LCL*-filtered control system can be reduced to a first-order *L*-filtered one by feeding forward unit capacitor voltage, which exhibits a strong adaptability under weak power grids. However, one sampling period in advance needs to be implemented. Based on this thought, [36] further developed a unit capacitor voltage feedforward with full delay compensation for active damping, preventing large inrush current during startup and suppressing disturbances of grid voltage, which meanwhile exhibits the robustness against grid impedance variation. As an alternative sensing configuration, only unit PCC voltage feedforward is investigated in this paper.

In addition to the above stability issues caused by high-frequency *LCL* filter resonance, the dynamics of the phase-locked loop (PLL) would also bring a stability challenge to the control system under grid impedance variation [37]-[41]. According to the impedance stability analysis method [40], [42], the PLL introduces the negative damping in the low-

frequency range of the output impedance [37]-[41], [43], [44]. A large inductive grid impedance might intersect with the output impedance of converters at the negative damping region, thus causing system low-frequency oscillation [37], [38], [40], [41]. In order to avoid the instability, one method is to decrease the control bandwidth of PLL, which thus can reduce the negative damping region [38], [41]. Another solution is to reshape the output impedance by introducing a specified PCC feedforward at the current references. The magnitude of the negative damping then can be reduced [37]. As the stability issues caused by the PLL have been investigated in depth in many studies, it is not further discussed in this paper.

This paper investigates the stability of *LCL*-type single-current-loop controlled grid-connected converters. With unit PCC voltage feedforward control, the grid impedance would be introduced to the control system through the feedforward loop under practical power grids, leading to a different current loop model from the original one. The unstable open-loop poles may appear under grid impedance variation, which thus may cause system instability. Therefore, the robust design regions of *LCL* filters against grid impedance variation are proposed for single-current-loop control schemes. A conference paper of this idea with only grid-side current feedback control has been published in [45] by the authors, where preliminary thought and conclusion are presented. While in this paper, the detailed theoretical deviation and mathematical analysis of the proposed robust *LCL* design regions are illustrated, and more experimental results about system stability and control bandwidth under grid impedance variation are presented. Moreover, the system with converter-side current feedback control is also investigated. Note that one limitation of unit PCC voltage feedforward control is that system low-frequency instability or low-frequency current harmonics may be also induced when a large grid impedance appears, i.e., weak power grids [30]. An effective solution is to increase sampling frequency by adopting multi-sampling technique [46], which would be discussed later in the paper. In Section II, conventional mathematic models of an *LCL*-type single-phase grid-connected converter in the discrete *z*-domain are presented, and the stability design of *LCL* filters for single-current-loop control without PCC voltage feedforward is briefly reviewed. In Section III, the detailed modelling of unit PCC voltage feedforward control is presented, then the stability of single-loop current control schemes (GCF and CCF) is investigated under grid impedance variation, and the robust design regions of *LCL* filters are proposed. In Section IV, experimental results are provided to verify the theoretical analysis and robust design. Finally, Section V concludes this paper.

II. SYSTEM DESCRIPTION AND MATHEMATICAL MODEL

Fig. 1 illustrates an *LCL*-type single-phase grid-connected converter, where the single-current-loop control with GCF

Fig. 1. An *LCL*-type single-phase grid-connected converter with grid-side or converter-side current single-loop control.

or CCF is employed to regulate the output current. The *LCL* filter consists of a converter-side inductor L_1 , a grid-side inductor L_2 , and a filter capacitor C_f , where parasitic resistances of *LCL* filter have been ignored to assume the worst case. The power grid can be modeled by the Thevenin's theorem, consisting of an ideal voltage source e_g and an inductive grid impedance Z_g . Under the worst case assumption, only a pure inductive component L_g in Z_g is considered in this paper. v_{pcc} denotes PCC voltage, which is measured for synchronizing current control and suppressing disturbances of grid voltage when its unit feedforward is employed as shown in Fig. 1. i_1 and i_2 denote converter-side and grid-side current respectively. i_c denotes filter capacitor current. $v_{con}(s)$ denotes output voltage of converter. u_{dc} and i_{dc} denote dc-side voltage and current respectively.

For the basic control of grid-connected converters, sinusoidal pulse-width-modulation (SPWM) and software PLL are employed to obtain the duty cycles and the phase angle of grid voltage. $G_c(s)$ is a fundamental PR current controller, expressed as

$$G_c(s) = k_p + \frac{k_i s}{s^2 + \omega_1^2}. \quad (1)$$

where k_p and k_i are the proportional and integral gain respectively, and ω_1 is the fundamental angular frequency.

Applying the pre-warped Tustin *z*-transformation to (1), the discrete *z*-domain model can be derived as (2), where T_s is the sampling period.

$$G_c(z) = k_p + \frac{k_i \sin(\omega_1 T_s)}{2\omega_1} \cdot \frac{z^2 - 1}{z^2 - 2\cos(\omega_1 T_s)z + 1} \quad (2)$$

Fig. 2(a) shows the conventional single-current-loop control diagram with GCF or CCF in the mixed domain to represent the actual control system, where one sampling period delay z^{-1} is involved in the digital control system for execution of control algorithms. $G_{zoh}(s)$ is the transfer function of zero-order-hold (ZOH), given by

Fig. 2. Conventional single-current-loop control diagram with GCF or CCF. (a) In the mixed domain. (b) In the discrete z -domain.

$$G_{zoh}(s) = \frac{1 - e^{-T_s s}}{s}. \quad (3)$$

Corresponding to Fig. 2(a), Fig. 2(b) shows the discrete z -domain current control diagram, where $G_{i1}(z)$ is the transfer function from $v_{con}(z)$ to $i_1(z)$, and $G_{i2}(z)$ is the transfer function from $v_{con}(z)$ to $i_2(z)$. Define $L_T = L_2 + L_g$, and by ZOH z -transformation method, it can be derived as (4) and (5) shown at the bottom of this page, where

$$G_{i1}(s) = \frac{1}{L_1 L_T C_f} \cdot \frac{L_T C_f s^2 + 1}{s(s^2 + \omega_{rg}^2)}, \quad (6)$$

$$G_{i2}(s) = \frac{1}{L_1 L_T C_f} \cdot \frac{1}{s(s^2 + \omega_{rg}^2)}, \quad (7)$$

$$\omega_{rg} = \sqrt{\frac{L_1 + L_T}{L_1 L_T C_f}}. \quad (8)$$

ω_{rg} is defined as the actual resonant frequency when grid impedance L_g is considered. In order to make a distinction and facilitate further analysis, ω_r is defined as the original resonant frequency, expressed as

$$\omega_r = \sqrt{\frac{L_1 + L_2}{L_1 L_2 C_f}}. \quad (9)$$

From Fig. 2(b), the open-loop transfer functions with GCF and CCF can be derived as (10) and (11) respectively, shown at the bottom of this page.

Applying the Nyquist stability criterion to (10) and (11), the stability design of these two feedback control methods can be obtained as $\omega_r > \omega_s/6$ for GCF and $\omega_r < \omega_s/6$ for CCF respectively [18], [19]. However, inductive grid impedance would decrease the resonant frequency, and the system with GCF may be unstable in the case of $\omega_{rg} < \omega_s/6$ or low stability margin. Since $\omega_{rg} \approx \omega_{r0} = \sqrt{1/L_1 C_f}$ when $L_g \rightarrow +\infty$ [47], the robust design region of LCL filters for GCF against grid impedance variation is given as $\omega_{r0} > \omega_s/6$. Different from the control with GCF, inductive grid impedance will improve system stability margin for that with CCF due to $\omega_{rg} < \omega_r < \omega_s/6$ when $L_g > 0$. Therefore, the system can be stable simply by ensuring $\omega_r < \omega_s/6$.

III. STABILITY ANALYSIS AND ROBUST DESIGN REGIONS WITH UNIT PCC VOLTAGE FEEDFORWARD

From Section II, conventional stability design of LCL filters for single-current-loop control is $\omega_r > \omega_s/6$ for GCF while $\omega_r < \omega_s/6$ for CCF [18], [19]. Practically, unit PCC voltage feedforward is commonly integrated into current control loop to prevent large inrush current during startup and suppress disturbances of grid voltage. However, in

$$G_{i1}(z) = Z[G_{zoh}(s)G_{i1}(s)] = \frac{T_s}{(L_1 + L_T)(z-1)} + \frac{L_T \sin(\omega_{rg} T_s)}{L_1 (L_1 + L_T) \omega_{rg}} \cdot \frac{z-1}{z^2 - 2z \cos(\omega_{rg} T_s) + 1} \quad (4)$$

$$G_{i2}(z) = Z[G_{zoh}(s)G_{i2}(s)] = \frac{T_s}{(L_1 + L_T)(z-1)} - \frac{\sin(\omega_{rg} T_s)}{(L_1 + L_T) \omega_{rg}} \cdot \frac{z-1}{z^2 - 2z \cos(\omega_{rg} T_s) + 1} \quad (5)$$

$$\text{GCF: } G_{o2}(z) = \frac{G_c(z)}{(L_1 + L_T) \omega_{rg}} \cdot \frac{\omega_{rg} T_s [z^2 - 2z \cos(\omega_{rg} T_s) + 1] - \sin(\omega_{rg} T_s)(z-1)^2}{z(z-1)[z^2 - 2z \cos(\omega_{rg} T_s) + 1]} \quad (10)$$

$$\text{CCF: } G_{o1}(z) = \frac{G_c(z)}{L_1 (L_1 + L_T) \omega_{rg}} \cdot \frac{L_1 \omega_{rg} T_s (z^2 - 2z \cos \omega_{rg} T_s + 1) + L_T \sin \omega_{rg} T_s (z-1)^2}{z(z-1)(z^2 - 2z \cos \omega_{rg} T_s + 1)} \quad (11)$$

Fig. 3. Grid-side current single-loop control with unit PCC voltage feedforward and its equivalent forms. (a) Current control diagram. (b) Equivalent transformation. (c) Equivalent circuit.

order to simplify the analysis, most of existing stability studies for *LCL*-type current control have ignored its effects, as shown in Fig. 2. They are generally acceptable when grid-connected converters are interfaced with ideal power grids, but such stability analysis and *LCL* filter design may become unreasonable under practical power grids.

A. GCF with Unit PCC Voltage Feedforward

1) Physical Meaning

Fig. 3(a) shows the grid-side current single-loop control diagram with unit PCC voltage feedforward, where the feedforward path contains two parts, one coming from ideal fundamental grid voltage e_g , and another coming from grid-side current i_2 with grid impedance L_g . Further, its equivalent transformation and equivalent physical circuit are shown in Fig. 3(b) and Fig. 3(c) respectively, where fundamental grid voltage e_g is not considered. With this representation, it immediately informs that a virtual impedance $Z_{eq}(j\omega)$ in parallel with filter capacitor C_f is introduced by unit PCC voltage feedforward as derived in [32] and [34]. The same equivalent forms can also be derived when grid-side current or capacitor current is used as the damping variable [9], [11], [17]. It should be noted that the PCC voltage feedforward introduces a positive feedback of grid-side current [30], which is different from other variable-feedback-based active damping methods, and it then results in a negative expression of $Z_{eq}(j\omega)$, shown as follows

$$Z_{eq}(j\omega) = -\frac{j\omega T_s L_1 (L_2 + L_g) e^{j\omega T_s}}{L_g G_{zoh}(j\omega)}. \quad (12)$$

To clearly understand its physical meaning, a further expanded form of (12) can be derived as

$$Z_{eq}(j\omega) = R_{eq}(\omega) / jX_{eq}(\omega) = \frac{\omega^2 T_s L_1 (L_2 + L_g)}{2L_g \sin 0.5\omega T_s} (\sin 1.5\omega T_s - j \cos 1.5\omega T_s). \quad (13)$$

where $R_{eq}(\omega)$ and $X_{eq}(\omega)$ can be seen as a physical resistor and reactor in parallel with filter capacitor C_f , given as

$$\begin{cases} R_{eq}(\omega) = \frac{\omega^2 T_s L_1 (L_2 + L_g)}{2L_g \sin 0.5\omega T_s \sin 1.5\omega T_s} \\ X_{eq}(\omega) = -\frac{\omega^2 T_s L_1 (L_2 + L_g)}{2L_g \sin 0.5\omega T_s \cos 1.5\omega T_s} \end{cases}. \quad (14)$$

Similar to a real resistor, the equivalent resistor $R_{eq}(\omega)$ can damp the resonance peak of *LCL* filters. However, due to its frequency-dependent characteristic, a negative-value region may appear, and a pair of unstable open-loop poles will be generated when system resonant frequency is located in this range, thus leading to a non-minimum phase characteristic [17]. Likewise, the equivalent reactor $X_{eq}(\omega)$ can also simulate the behaviors of a real reactor, which decreases the resonance frequency when $X_{eq}(\omega) < 0$ and increases the resonance frequency when $X_{eq}(\omega) > 0$. It then results in actual system resonant frequency different from ω_{rg} shown in (8), and defined as ω'_{rg} here. While for conventional single-current-control without PCC voltage feedforward, its system resonant frequency is ω_{rg} with $L_g > 0$.

Considering the Nyquist frequency constraint $\omega < \omega_s/2$, letting $R_{eq}(\omega) > 0$ or $X_{eq}(\omega) > 0$ yields

$$\begin{cases} R_{eq}(\omega) > 0 \Rightarrow \omega < \omega_s/3 \\ X_{eq}(\omega) > 0 \Rightarrow \omega_s/6 < \omega < \omega_s/2 \end{cases}. \quad (15)$$

Fig. 4 shows the curves of $R_{eq}(\omega)$ and $X_{eq}(\omega)$ varying with frequency ω . It can be seen the frequency ranges for $R_{eq}(\omega) > 0$ and $X_{eq}(\omega) > 0$ are consistent with (15). From Section II, the resonant frequency ω_r of *LCL* filters should be designed to be greater than $\omega_s/6$ for GCF. In Fig. 4, it can be seen $X_{eq}(\omega) > 0$ in the range of $(\omega_s/6, \omega_s/2)$, which indicates that the PCC voltage feedforward control can mitigate the decrease of resonance frequency under inductive grid impedance. It can also be noted that the

Fig. 4. Curves of $R_{eq}(\omega)$ and $X_{eq}(\omega)$ varying with frequency ω .

Fig. 5. Grid-side current single-loop control diagram with unit PCC voltage feedforward in the discrete z -domain.

frequency $\omega_s/3$ for $R_{eq}(\omega)=\infty$ is the critical system resonant frequency as discussed in [9], [11], [17]. It means that there is no damping to attenuate the resonance peak when $\omega'_{rg}=\omega_s/3$. In other words, a pair of conjugate open-loop poles on the unit circle will be generated.

From the above analysis, the unit PCC voltage feedforward control would change system frequency characteristic, and thus conventional stability design of LCL filters may no longer be applicable to such systems. Therefore, how to design the LCL filter is the key to ensure system stability under grid impedance variation.

2) Discrete z -domain Model and Control Bandwidth

Corresponding to Fig. 3, Fig. 5 shows the discrete z -domain current control diagram, where $G_{pcc}(z)$ denotes the transfer function from $v_{con}(z)$ to $v_{pcc}(z)$, derived as follows

$$G_{pcc}(z) = Z[G_{zoh}(s)G_{pcc}(s)] = \frac{L_g(1 - \cos \omega_{rg} T_s)}{L_1 + L_T} \cdot \frac{z+1}{z^2 - 2z \cos \omega_{rg} T_s + 1}, \quad (16)$$

where,

$$G_{pcc}(s) = \frac{L_g}{L_1 L_T C_f} \cdot \frac{1}{s^2 + \omega_{rg}^2}. \quad (17)$$

From Fig. 5, the open-loop transfer function $G_{op2}(z)$ can be derived as

$$G_{op2}(z) = \frac{G_c(z)}{(L_1 + L_T)\omega_{rg}} \cdot \frac{\omega_{rg} T_s [z^2 - 2z \cos(\omega_{rg} T_s) + 1] - \sin(\omega_{rg} T_s)(z-1)^2}{(z-1)[z^3 - 2z^2 \cos(\omega_{rg} T_s) + (1-m)z - m]}, \quad (18)$$

where,

Fig. 6. Bode plots of open-loop transfer function $G_{op2}(z)$ and $G_{o2}(z)$.

$$m = \frac{L_g(1 - \cos \omega_{rg} T_s)}{L_1 + L_T}. \quad (19)$$

Comparing (10) with (18), it can be observed that the PCC voltage feedforward control introduces a damping term m to the denominator of $G_{op2}(z)$. It results in open-loop poles variation related to grid impedance L_g , the analysis of which will be given in the following subsection. Fig. 6 shows the Bode plots of open-loop transfer functions $G_{op2}(z)$ and $G_{o2}(z)$. As seen, with unit PCC voltage feedforward, the resonance peak is attenuated effectively when $L_g \neq 0$ as previously analyzed. Meanwhile, it can also be seen that $\omega_{rg} < \omega'_{rg} < \omega_r$, thus indicating that the PCC voltage feedforward control can mitigate the decrease of resonance frequency. In addition, an interesting finding is that the magnitude-frequency curve of $G_{op2}(z)$ keeps unchanged in the low-frequency range despite grid impedance variation. This indicates that grid impedance variation has negligible effects on the control bandwidth of current loop. In order to explain this, $G_{op2}(z)$ can be approximated as (20) in the low-frequency range by neglecting filter capacitor C_f .

$$G'_{op2}(z) = G_c(z)G'_{i2}(z) \frac{z^{-1}}{1 - z^{-1}G'_{pcc}(z)} = \frac{G_c(z)T_s}{(z-1)[(L_1 + L_T)z - L_g]}, \quad (20)$$

where,

$$G'_{i2}(z) = \frac{T_s}{(L_1 + L_T)(z-1)} \text{ and } G'_{pcc}(z) = \frac{L_g}{(L_1 + L_T)}. \quad (21)$$

Then the magnitude-frequency characteristic of $G'_{op2}(z)$ can be derived as (22) with $L_g \cos \omega T_s \approx L_g$ and $L_g \sin \omega T_s \approx 0$ in the low-frequency range, shown at the bottom of next page.

For comparison, the low-frequency characteristic of $G_{o2}(z)$ is also derived as

$$G'_{o2}(z) = z^{-1}G_c(z)G'_{i2}(z) = \frac{G_c(z)T_s}{(L_1 + L_r)z(z-1)}. \quad (23)$$

From (22), it is observed that $|G'_{op2}(z)|$ is basically independent of L_g , which indicates that the PCC voltage feedforward control can help maintain a high current control bandwidth under grid impedance variation. This feature is highly desirable if multiple HCs are employed for harmonic compensation, because the control bandwidth of current loop should be wide enough to cover all the resonant frequencies of HCs. If the resonant frequencies of HCs are close to or greater than the bandwidth of current loop, instead of compensating low-order current harmonics, the HCs will excite low-frequency instability [3], [36]. For that without PCC voltage feedforward, i.e., $G'_{o2}(z)$ in (23), inductive grid impedance L_g will decrease its magnitude and further challenge the control bandwidth of current loop, which might cause system instability.

3) Stability Analysis and Robust Design Regions against Grid Impedance Variation

From (18), the PCC voltage feedforward control would cause open-loop poles variation under grid impedance variation. According to the Nyquist stability criterion, the immediate task is to identify the locations of open-loop poles.

Define

$$D_1(z) = z^3 - 2z^2 \cos(\omega_r T_s) + (1-m)z - m. \quad (24)$$

Substituting (19) into (24) and applying w -transform and Routh's stability criterion [48], the constraint to avoid the unstable open-loop poles is derived in Appendix and shown as follows

$$f_1(L_g) = \cos(\omega_r T_s) > -\frac{L_1 + L_2}{2(L_1 + L_2) + 3L_g} = f_2(L_g) \quad (25)$$

From (25), it can be observed that $f_2(L_g)$ is negative. The inequality can always be valid by setting $\omega_r < \omega_s/4$ due to $\omega_{rg} < \omega_r$ and then $\cos(\omega_r T_s) > 0$. Nevertheless, it is over-conservative to design such an *LCL* filter, especially for symmetric regular sampling with $\omega_r < \omega_s/2$. In (25), if $L_g \rightarrow 0$, $\omega_{rg} = \omega_r$, and then it can be simplified as $\cos(\omega_r T_s) > -0.5$, i.e., $\omega_r < \omega_s/3$. Similarly, if $L_g \rightarrow +\infty$, it can be derived as $\cos(\omega_{r0} T_s) > 0$, i.e., $\omega_{r0} < \omega_s/4$, where ω_{r0} is the resonant frequency of converter-side inductor L_1 and filter capacitor C_f ,

$$\omega_{r0} = \sqrt{\frac{1}{L_1 C_f}}. \quad (26)$$

Therefore, if $\omega_r < \omega_s/3$ and $\omega_{r0} < \omega_s/4$, (25) is valid at the boundary values of the range $(0, +\infty)$ for L_g . Through further

(a)

(b)

Fig. 7. Curved-surface of $\Delta f = f_1 - f_2$ on q and ω_r/ω_s with $\omega_{r0} = \omega_s/4$. (a) 3D plots. (b) y-z view.

transformations, (25) can be expressed as

$$f_1(q) = \cos \sqrt{\frac{4\pi^2(1+q)}{\omega_s^2/\omega_r^2 + q\omega_s^2/\omega_{r0}^2}} > -\frac{1}{2+3q} = f_2(q). \quad (27)$$

where q represents the impedance ratio, given as $L_g/(L_1+L_2) \in (0, +\infty)$.

Fig. 7(a) shows the 3D plots of $\Delta f = f_1(q) - f_2(q)$ with $\omega_{r0} = \omega_s/4$, where x-axis and y-axis represent the values of impedance ratio q and frequency ratio ω_r/ω_s respectively, and z-axis represents the value of Δf . Fig. 7(b) shows the y-z

$$\begin{aligned} |G'_{op2}(z = e^{j\omega T_s})| &= \frac{|G_c(z = e^{j\omega T_s})|T_s}{\sqrt{2(1 - \cos \omega T_s)}|(L_1 + L_2)(\cos \omega T_s + j \sin \omega T_s) + L_g \cos \omega T_s - L_g + jL_g \sin \omega T_s|} \\ &\approx \frac{|G_c(z = e^{j\omega T_s})|T_s}{(L_1 + L_2)\sqrt{2(1 - \cos \omega T_s)}} \end{aligned} \quad (22)$$

TABLE I
PARAMETERS OF THE SYSTEM

Description	Symbol	Value
Grid voltage	e_g/V	80 (RMS)
Dc voltage	u_{dc}/V	160
Rated output power	P_o/kW	0.8
Fundamental frequency	f_i/Hz	50
Switching frequency	f_{sw}/kHz	10
Sampling frequency	f_s/kHz	10
<i>LCL</i> filter		
Converter-side inductance	L_1/mH	I 1.2 II 0.7 III 1.5 IV 1.2 V 1.2
Grid-side inductance	L_2/mH	1 1.5 0.7 1 1
Filter capacitance	$C/\mu F$	5 5 3 3 20
<i>LCL</i> resonance frequency	f_r/kHz	3.05 3.26 4.21 3.93 1.52
L_1C_f resonance frequency	f_{r0}/kHz	2.05 2.69 2.37 2.65 1.03

TABLE II
PERFORMANCE INDICATORS OF DIFFERENT *LCL* FILTER PARAMETERS AT THE RATED CONDITIONS

Description	Ripple ratio of i_1	Maximum switching current harmonic of i_2	Reactive power
I	11.79%	0.03%	1.26%
II	20.21%	0.04%	1.26%
III	9.43%	0.07%	0.75%
IV	11.79%	0.06%	0.75%
V	11.79%	0.01%	5%

view of Fig. 7(a). As seen, with $\omega_{r0}=\omega_s/4$, Δf is always greater than 0 with the increase of q when $\omega_r<\omega_s/3$, i.e., $f_1(q)>f_2(q)$, which indicates that the system always has no unstable open-loop poles under grid impedance variation with these two design constraints. Furthermore, according to (27), it can be derived as $f_1(q)|_{\omega_{r0}<\omega_s/4} > f_1(q)|_{\omega_{r0}=\omega_s/4}$ since f_1 is a monotone decreasing function on ω_{r0} . Therefore, for $\omega_{r0}<\omega_s/4$, it can also be obtained as $f_1(q)>f_2(q)$ regardless of q when $\omega_r<\omega_s/3$. From the above discussion, following the design of $\omega_r<\omega_s/3$ & $\omega_{r0}<\omega_s/4$ can ensure that the system always has no unstable open-loop poles under grid impedance variation.

As seen from Fig. 7, the incorrect *LCL* design may introduce unstable open-loop poles under grid impedance variation, which then may lead to unstable current control. For comparison, four cases are discussed with **Case I** ($\omega_r<\omega_s/3$, $\omega_{r0}<\omega_s/4$), **Case II** ($\omega_r<\omega_s/3$, $\omega_{r0}>\omega_s/4$), **Case III** ($\omega_r>\omega_s/3$, $\omega_{r0}<\omega_s/4$) and **Case IV** ($\omega_r>\omega_s/3$, $\omega_{r0}>\omega_s/4$). Fig. 8 shows the curves of $f_1(q)$ and $f_2(q)$ with the increase of q , where four different cases corresponding to *Parameter I, II, III* and *IV* listed in Table I are performed respectively. Note that *Parameter V* in Table I is for converter-side current feedback control which will be discussed in the following subsection. The performance indicators of these four sets of *LCL* filter parameters are given in Table II, all of which satisfy that the ripple ratio of converter-side current i_1 is less than 40%, the maximum switching current harmonic of grid-side current i_2 is less than 0.3% of the fundamental

Fig. 8. Curves of $f_1(q)$ and $f_2(q)$ with the increase of q for four different cases. (a) Case I. (b) Case II. (c) Case III. (d) Case IV.

Fig. 9. Bode plots of open-loop transfer function $G_{op2}(z)$ with the increase of q for four different cases. (a) *Case I*. (b) *Case II*. (c) *Case III*. (d) *Case IV*.

component, and the absorbed reactive power is not greater than 5% [22]. The shaded areas in Fig. 8 indicate the situations with unstable open-loop poles. As seen, $f_1(q)$ is always greater than $f_2(q)$ in Fig. 8(a), and always less than $f_2(q)$ in Fig. 8(d), which indicates that there is always no unstable open-loop pole for *Case I* from (27) and a pair of unstable open-loop poles for *Case IV*. Meanwhile, it can be seen that there is an intersection between $f_1(q)$ and $f_2(q)$ for *Case II* and *Case III*, shown in Fig. 8(b) and Fig. 8(c). Since $\omega_s/3$ is the critical system resonant frequency from Fig. 4, it can be yielded that $f_1(q)=f_2(q)$ by letting $D_1(z=e^{j\omega T_s/3})=0$. In other words, system resonant frequency ω'_{rg} is $\omega_s/3$ at these intersections with $q=q_c$. Similarly, according to (27), there is no unstable open-loop pole when $q < q_c$ and a pair of unstable open-loop poles

when $q > q_c$ for *Case II*. In contrast, there is no unstable open-loop pole when $q > q_c$ and a pair of unstable open-loop poles when $q < q_c$ for *Case III*. From these design examples, it can be seen that the system designed with *Case I* always has no unstable open-loop poles under grid impedance variation, which agrees with that derived above.

According to the Nyquist stability criterion [9], when the system exists a pair of unstable open-loop poles, a positive -180° crossing above 0 dB is required to ensure system stability, e.g., *Case II* when $q > q_c$, *Case III* when $q < q_c$, and *Case IV* when $q \neq 0$, which can be checked by Bode plots.

In order to observe the frequency characteristic of *LCL* plant with unit PCC voltage feedforward, the effect of current controller $G_c(z)$ is ignored and set to 1. Fig. 9 shows the Bode plots of open-loop transfer function $G_{op2}(z)$ with the increase of q for four different cases, which correspond

Fig. 10. Closed-loop poles of grid-side current control with the increase of q for four different cases. (a) *Case I*. (b) *Case II*. (c) *Case III*. (d) *Case IV*.

to the four plots in Fig. 8 respectively. As seen, there exists a positive -180° crossing located at ω'_{rg} for **Case II** when $q > q_c$, **Case III** when $q < q_c$, and **Case IV** when $q \neq 0$. Normally, a resonance peak far greater than 0 dB exists at ω'_{rg} , and thus the system can be stable. However, the equivalent resistor $R_{eq}(\omega)$ may damp the resonance peak below 0 dB as shown in Fig. 9, then the system will be unstable. A possible solution is to adopt a large enough proportional gain k_p and make the magnitude of resonance greater than 0 dB. However, this may inevitably increase the system crossover frequency and significantly reduce its phase margin, or even lead to instability. Moreover, it is well-known that k_p must be limited to avoid the interference of high frequency noises.

Therefore, **Case I** ($\omega_r < \omega_s/3$, $\omega_{r0} < \omega_s/4$) is the only option that can guarantee a stable closed-loop control regardless of grid impedance variation as long as the proportional gain k_p is properly designed. The other three cases do not exhibit

such robustness due to the unstable open-loop poles introduced by unit PCC voltage feedforward control. Moreover, it should also be noted that the control with unit PCC voltage feedforward can be equivalent to that without PCC voltage feedforward in the case of stiff grids. From Section II, the robust design region of *LCL* filters for the control without PCC voltage feedforward is $\omega_{r0} > \omega_s/6$ [47]. In view of this, combining **Case I** ($\omega_r < \omega_s/3$, $\omega_{r0} < \omega_s/4$) with $\omega_{r0} > \omega_s/6$ yields the proposed robust design region of *LCL* filters, i.e., $\omega_r < \omega_s/3$ & $\omega_s/6 < \omega_{r0} < \omega_s/4$, for the control with unit PCC voltage feedforward against grid impedance variation.

To further prove the conclusions derived above, Fig. 10 shows the closed-loop poles of grid-side current control for these four different cases, where the proportional gain k_p is set as a constant to observe the variation of close-loop poles with the increase of q . As seen, there exist two pairs of closed-loop poles, one is related to *LCL* resonance located at

the left-side of the unit circle, and another one is related to control bandwidth located at the right-side of the unit circle. From Fig. 10(a), the poles related to *LCL* resonance gradually move inside the unit circle with the increase of q , while for Fig. 10(b), Fig. 10(c) and Fig. 10(d), the poles may be outside the unit circle under grid impedance variation. These are consistent with the analysis presented above, i.e. the positive -180° crossing above 0 dB is not satisfied for **Case II** ($\omega_r < \omega_s/3$, $\omega_{r0} > \omega_s/4$) with $q > q_c$, **Case III** ($\omega_r > \omega_s/3$, $\omega_{r0} < \omega_s/4$) with $q < q_c$ and **Case IV** ($\omega_r > \omega_s/3$, $\omega_{r0} > \omega_s/4$) with $q \neq 0$.

Actually, in addition to the above high-frequency instability related to *LCL* resonance, the low-frequency instability issues may also appear under a large grid impedance when implementing unit PCC voltage feedforward, which has been discussed in [30] in detail. The fundamental reason is that the unit PCC voltage feedforward causes phase lag of the open-loop transfer function under grid impedance variation, as shown in Fig. 9(a). To tackle the low-frequency instability issues and meanwhile implement the unit PCC voltage feedforward control, one method is to reserve enough stability margin with maximum possible grid impedance when designing the control system. Another method is to increase sampling frequency f_s by adopting multi-sampling technique [46]. With this method, the effects of grid impedance L_g on system low-frequency phase can be mitigated, which can be derived similar as (22).

B. CCF with Unit PCC Voltage Feedforward

Fig. 11 shows the converter-side current single-loop control diagram with unit PCC voltage feedforward in the discrete z -domain. From Fig. 11, the open-loop transfer function $G_{op1}(z)$ can be derived as

$$G_{op1}(z) = \frac{G_c(z)}{L_1(L_1 + L_T)\omega_{rg}} \cdot \frac{L_1\omega_{rg}T_s[z^2 - 2z\cos(\omega_{rg}T_s) + 1] + L_T\sin(\omega_{rg}T_s)(z-1)^2}{(z-1)[z^3 - 2z^2\cos(\omega_{rg}T_s) + (1-m)z - m]} \quad (28)$$

By comparing (28) with (18), it can be seen $G_{op1}(z)$ has the same poles with $G_{op2}(z)$, which indicates that for the control with CCF, unit PCC voltage feedforward control can also damp the *LCL* resonance and shift its resonance frequency as that with GCF under grid impedance variation. Moreover, since $G_{op1}(z)$ has the same low-frequency characteristic with $G_{op2}(z)$ derived by neglecting the filter capacitor C_f , the conclusion that unit PCC voltage feedforward control can help maintain a high current control bandwidth under grid impedance variation is also applicable to CCF.

Since $G_{op1}(z)$ has the same poles with $G_{op2}(z)$, the above constraints for the unstable poles of $G_{op2}(z)$ can also be applied to $G_{op1}(z)$. In order to avoid the unstable open-loop poles under grid impedance variation, only **Case I** ($\omega_r < \omega_s/3$, $\omega_{r0} < \omega_s/4$) is investigated for CCF. From Section II, the stability design of *LCL* filters for converter-side current

Fig. 11. Converter-side current single-loop control diagram with unit PCC voltage feedforward in the discrete z -domain.

Fig. 12. Bode plots of open-loop transfer function $G_{op1}(z)$ with the increase of q . (a) $\omega_r < \omega_s/6$. (b) $\omega_s/6 < \omega_r < \omega_s/3$ & $\omega_{r0} < \omega_s/4$.

single-loop control is $\omega_r < \omega_s/6$. Considering the case of stiff grids, **Case I** is divided into two regions, i.e. $\omega_s/6 < \omega_r < \omega_s/3$ & $\omega_{r0} < \omega_s/4$ and $\omega_r < \omega_s/6$. It should be noted that when $\omega_r < \omega_s/6$, ω_{r0} is always less than $\omega_s/4$ due to $\omega_{r0} < \omega_r$.

Fig. 12 shows the Bode plots of open-loop transfer function $G_{op1}(z)$ with the increase of q for the above two different cases, where the parameters of *LCL* filter correspond to *Parameter V* and *I* listed in Table I

(a)

(b)

Fig. 13. Closed-loop poles of converter-side current control with the increase of q . (a) $\omega_r < \omega_s/6$. (b) $\omega_s/6 < \omega_r < \omega_s/3$ & $\omega_{r0} < \omega_s/4$.

respectively. It can be seen from Fig. 12 that the resonance peak is well damped with the increase of q . Since there is no unstable open-loop pole, the system is stable as long as there is no -180° crossing above 0 dB according to the Nyquist stability criterion. Clearly, the system shown in Fig. 12(a) can always meet this stability condition if the proportional gain k_p is properly designed, whereas the one shown in Fig. 12(b) may lose stability when q or L_g is zero or very small due to the infinite resonant peak at the second negative -180° crossing.

Fig. 13 further plots the closed-loop poles of converter-side current control with the increase of q . It can be seen the poles related to LCL resonance gradually move inside the unit circle with the increase of q , which helps stabilize the system. However, the system stability for the case $\omega_s/6 < \omega_r < \omega_s/3$ & $\omega_{r0} < \omega_s/4$ shown in Fig. 13(b) depends on q or grid impedance L_g . For a stiff grid where the grid impedance is negligible, the system would be unstable.

TABLE III
COMPARISON OF DESIGN REGIONS OF LCL FILTERS

		GCF	CCF
W/o PCC voltage feedforward	Stable design region with $L_g=0$	$\omega_r > \omega_s/6$	$\omega_r < \omega_s/6$
	Robust design region with $L_g \neq 0$	$\omega_{r0} > \omega_s/6$	$\omega_r < \omega_s/6$
With unit PCC voltage feedforward	Stable design region with $L_g=0$	$\omega_r > \omega_s/6$	$\omega_r < \omega_s/6$
	Robust design region with $L_g \neq 0$	$\omega_r < \omega_s/3$ and $\omega_s/6 < \omega_{r0} < \omega_s/4$	$\omega_r < \omega_s/6$

From the above analysis, the robust case against grid impedance variation is shown in Fig. 12(a) and Fig. 13(a) with the design constraint $\omega_r < \omega_s/6$, which is the same as that without PCC voltage feedforward.

C. Summary and Discussion

Table III finally presents the comparison of design regions of LCL filters under different scenarios. It shows that the conventional design with $\omega_{r0} > \omega_s/6$ to improve the robustness of grid-side current single-loop control against grid impedance variation is not applicable to the case with unit PCC voltage feedforward.

In this paper, only 1.5 sampling periods delay consisting of one sampling period of calculation delay and equivalent half sampling period of ZOH delay is considered [9], [36]. However, the voltage sampling or any other filters in voltage feedforward loop for specified applications may also introduce a time delay. With this additional time delay involved in the control loop, it would lead to a different robust design region. The resultant design region depends on the used filter, and can still be derived with the same analysis method given in this paper.

For the voltage sampling, a low-pass filter is commonly used to attenuate the switching or sampling noise, which generally has a very high bandwidth. Under this scenario, the time delay is very small and can be neglected, thus the feedforward can be seen as a unit feedforward. Actually, the similar situations can be found in many studies [27], [29], [30], [32], [34] and [36], where the effects of the low-pass filter are ignored. While for the filters for positive-negative sequences decomposition in unbalanced three-phase systems, the additional time delay is serious and cannot be neglected, which would lead to a different robust design region. Actually, when the three-phase voltages are unbalanced, the sampled PCC voltage can also be fed forward directly to the control system before positive-negative sequences decomposition in the stationary $\alpha\beta$ frame, thus the conclusion presented in this paper can still hold.

IV. EXPERIMENTAL RESULTS

A single-phase experimental prototype has been constructed for verification of the theoretical analysis and robust design. The experimental circuit topology and control

(a)

(b)

Fig. 14. Transient experimental results of grid-side current single-loop control with unit PCC voltage feedforward when $q=0$ or $L_g=0$. (a) Experimental waveforms. (b) Grid-side current spectra.

diagram are shown in Fig. 1, and the system parameters are shown in Table I. A Chroma 61502 programmable ac power supply and a dc load were connected to the ac-side and dc-side of the prototype respectively.

Grid-side current single-loop control with unit PCC voltage feedforward is first investigated. Fig. 14(a) shows the transient experimental results of grid-side current and PCC voltage, where *Parameter I* with $\omega_r > \omega_s/6$ in Table I is employed. Since the grid impedance is zero, the system has the same stability performance with that without PCC voltage feedforward. As seen, the output current is always stable despite of a step change of dc load. Fig. 14(b) further presents the grid-side current spectra. The total THD is 2.47% ($< 5\%$). Moreover, as the unipolar SPWM is adopted, the equivalent switching frequency is $2f_s$. It can be seen that the switching current harmonics around $2f_s$ is obvious compared to those at other switching frequencies, which satisfies the

Fig. 15. Bode plots of open-loop transfer function $G_{op2}(z)$ with selected grid impedance L_g for four different cases.

requirement of less than 0.3% of the fundamental component according to the IEEE std. 1547-2003.

Considering grid impedance L_g , four groups of experiments with four sets of *LCL* filter parameters listed in Table I (*Parameter I, II, III, IV*) are performed respectively. Note that these four sets of parameters correspond to the four cases shown in Fig. 8 and Fig. 9. From Fig. 8, there will be a pair of unstable open-loop poles when $q > q_c$ for *Parameter II* and $q < q_c$ for *Parameter III*. The values of q_c are 0.17 and 2 respectively, and the corresponding grid impedance L_g are 0.37 mH and 4.4 mH respectively. In contrast, there is always no unstable open-loop pole for *Parameter I* and a pair of unstable open-loop unstable poles for *Parameter IV*. In order to investigate the unstable situations, grid impedance L_g are selected to be 2 mH, 2mH, 0.7 mH and 0.7mH respectively for these four cases with resonant frequencies ω_{rg} being 2.43 kHz, 2.95 kHz, 3.41 kHz and 3.46 kHz, and thus there exists a pair of unstable open-loop poles for *Parameter II, III* and *IV*. According to the Nyquist stability criterion, a positive -180° crossing above 0 dB by $G_{op2}(z)$ is required for *Parameter II, III* and *IV* to ensure system stability. With proportional gain $k_p=4$ and the selected grid impedance L_g , Fig. 15 shows the Bode plots of open-loop transfer function $G_{op2}(z)$. As seen, the magnitude of $G_{op2}(z)$ is less than 0 dB at the positive -180° crossing for *Parameter II, III* and *IV*, and thus the system will be unstable. By plotting the zero-pole maps, the frequencies of unstable close-loop poles are 3.45 kHz, 3.71 kHz and 3.76 kHz respectively. While for *Parameter I*, the system is stable due to no unstable open-loop pole and no -180° crossing above 0 dB.

Fig. 16 shows the experimental results for these four cases. Here, for the sake of safety, the output power for the unstable cases is reduced. As analyzed above, the output current is unstable for *Parameter II, III* and *IV* which correspond to **Case II**, **Case III** and **Case IV** respectively,

Fig. 16. Experimental results of grid-side current single-loop control with unit PCC voltage feedforward when $q \neq 0$ or $L_g \neq 0$. (a) Case I. (b) Case II. (c) Case III. (d) Case IV.

while is stable for *Parameter I* which corresponds to **Case I**. It is noticeable that the unstable situation of **Case II** shown in Fig. 16(b) is different from those of **Case III** and **Case IV** where the output current tends to be divergent. This is because **Case II** is close to the critical stable state which can be seen from Fig. 15. In addition, it can also be seen that the PCC voltage contains a large amount of harmonics, which are caused by the current harmonics through grid impedance L_g . By importing the data of grid-side current waveform acquired by a Lecory Wavesurfer 3024 digital oscilloscope to Matlab, the current harmonics are analyzed by FFT function, shown in Fig. 17. From Fig. 17(b), Fig. 17(c) and Fig. 17(d), it can be seen the frequencies of unstable current harmonics are 3.20 kHz, 3.55 kHz and 3.60 kHz, respectively, all of which are close to the theoretical values. The tolerance of filter parameters, equivalent series resistance (ESR) and sampling errors, etc., can cause frequency shifting. For **Case I** shown in Fig. 17(a), the total THD is 2.46%. Moreover, compared to Fig. 14, the

switching current harmonics around $2f_s$ are further attenuated due to the presence of inductive grid impedance.

The robustness of converter-side current single-loop control with unit PCC voltage feedforward against grid impedance variation is also investigated by experimental tests. With the stability requirements presented in Section II, $\omega_r < \omega_s/6$ needs to be satisfied when $q=0$ or $L_g=0$. The transient experimental results are shown in Fig. 18(a), where *Parameter V* in Table I is employed. It can be seen the output current is always stable despite a step change of dc load. While when $q \neq 0$ or $L_g \neq 0$, due to no unstable open-loop pole, the system can be stable when a proper proportional gain k_p is set to ensure no -180° crossing above 0 dB as shown in Fig. 12(a). Fig. 18(b) shows the transient experimental results when $L_g=2$ mH, which is stable as the theoretical analysis. Fig. 19 shows the grid-side current spectra. As seen, the switching current harmonics are well attenuated for both two cases, and the total THDs of grid-side current are 2.88% and 2.29% respectively.

Fig. 17. Grid-side current spectra with grid-side current single-loop control. (a) Case I. (b) Case II. (c) Case III. (d) Case IV.

Fig. 18. Transient experimental results of converter-side current single-loop control with unit PCC voltage feedforward. (a) $L_g = 0$. (b) $L_g = 2 \text{ mH}$.

(a)

(b)

Fig. 19. Grid-side current spectra with converter-side current single-loop control. (a) $L_g=0$. (b) $L_g=2$ mH.

Through the above experimental tests, the robust design regions of *LCL* filters against grid impedance variation, i.e. $\omega_r < \omega_s/3$, $\omega_s/6 < \omega_{r0} < \omega_s/4$ for GCF and $\omega_r < \omega_s/6$ for CCF, are verified.

Finally, the effect of grid impedance variation on the control bandwidth of current loop is investigated. Only the control with GCF is performed and *Parameter I* listed in Table I is employed. In order to test the dynamic response, a step change (from 150 V to 180 V) of dc voltage reference is carried out, thus leading to a 6 A set-up change on the current reference as the proportional gain of dc voltage controller is 0.2 ($0.2 \times 30 = 6$ A). Fig. 20 shows the transient experimental results of grid-side current. Note that current waveform ① also indicates the control without PCC voltage feedforward as $L_g = 0$ mH from Fig. 3(a). From Fig. 20(a), it can be seen that current waveform ① tracks the current reference well and a 6 A step change is observed. For current waveform ②, only a 4 A step change is observed as the grid impedance L_g decreases the control

(a)

(b)

Fig. 20. Transient experimental results of grid-side current. (a) Comparison of ① and ②. (b) Comparison of ① and ③. (①: with unit PCC voltage feedforward and $L_g = 0$ mH. ②: without PCC voltage feedforward and $L_g = 2$ mH. ③: with unit PCC voltage feedforward and $L_g = 2$ mH.)

bandwidth of current loop and results in a slower dynamic response. From Fig. 20(b), current waveform ③ has a similar dynamic response with current waveform ①. This proves that grid impedance variation has negligible effects on the control bandwidth of current loop when unit PCC voltage feedforward control is performed.

V. CONCLUSION

This paper has investigated the stability of *LCL*-type single-current-loop controlled grid-connected converters and robust design regions of *LCL* filters against grid impedance variation. With unit PCC voltage feedforward implemented to prevent large inrush current during startup and suppress disturbances of grid voltage, a virtual resistor and a virtual reactor are equivalently paralleled with filter capacitor, which can damp the resonance peak and shift resonant frequency. However, it is not sufficient to ensure

system stability as this feedforward control may lead to unstable open-loop poles. Then the robust design regions of *LCL* filters against grid impedance variation are derived for single-loop current control schemes, i.e. $\omega_r < \omega_s/3$ & $\omega_s/6 < \omega_{r0} < \omega_s/4$ for GCF and $\omega_r < \omega_s/6$ for CCF. The proposed design can avoid unstable open-loop poles under grid impedance variation and thus improve the system robustness. Moreover, one interesting finding is that the unit PCC voltage feedforward can help maintain a high current control bandwidth under grid impedance variation. This feature is highly desirable if multiple resonant controllers are employed for harmonic compensation. Experimental results are finally presented to verify the theoretical analysis and robust design.

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APPENDIX

Generally, w transformation $z = (w+1)/(w-1)$ is used for mapping the inside of the unit circle in the z -plane into the left half of the w -plane, thus the Routh's stability criterion can be adopted to analyze the stability of discrete control systems [48].

In the complex plane, w can be defined as $w = \sigma + j\omega$. With $z = (w+1)/(w-1)$ and $|z| < 1$, it can be derived as

$$|z| = \left| \frac{w+1}{w-1} \right| = \left| \frac{\sigma + j\omega + 1}{\sigma + j\omega - 1} \right| < 1. \quad (\text{A1.1})$$

According to the further derivation, it can be yielded as

$$(\sigma+1)^2 + \omega^2 < (\sigma-1)^2 + \omega^2 \Rightarrow \sigma < 0. \quad (\text{A1.2})$$

Thus, the inside of the unit circle in the z -plane ($|z| < 1$) corresponds to the left half of the w -plane. And also, the unit circle in the z -plane is mapped into the imaginary axis in the w -plane, and the outside of the unit circle in the z -plane is mapped into the right half of the w -plane [48]. Therefore, the Routh's stability criterion could be adopted to analyze system stability.

Applying w transformation $z = (w+1)/(w-1)$ to $D_1(z)$ in (24), it can be derived as

$$D_1(w) = D_1(z) \Big|_{z=\frac{w+1}{w-1}} = \frac{2(a_3 w^3 + a_2 w^2 + a_1 w + a_0)}{(w-1)^3}, \quad (\text{A1.3})$$

where

$$\begin{cases} a_3 = 1 - \cos(\omega_r T_s) - m \\ a_2 = 1 - \cos(\omega_r T_s) + 2m \\ a_1 = 1 + \cos(\omega_r T_s) - m \\ a_0 = 1 + \cos(\omega_r T_s) \end{cases} \quad (\text{A1.4})$$

Then according to the Routh's stability criterion, the Routh table of $a_3 w^3 + a_2 w^2 + a_1 w + a_0 = 0$ is given as

$$\begin{array}{ccc} w^3 & a_3 & a_1 \\ \hline w^2 & a_2 & a_0 \\ \hline w^1 & (a_1 a_2 - a_0 a_3)/a_2 & 0 \\ \hline w^0 & a_0 & \end{array} \quad (\text{A1.5})$$

With $m = L_g(1 - \cos \omega_r T_s)/(L_1 + L_T)$ in (19), $a_3 > 0$, $a_2 > 0$ and $a_1 > 0$. Hence, to avoid the unstable open-loop poles, the constraint condition can be derived as

$$\frac{a_1 a_2 - a_0 a_3}{a_2} > 0 \Rightarrow \cos(\omega_r T_s) > -\frac{L_1 + L_2}{2(L_1 + L_2) + 3L_g}. \quad (\text{A1.6})$$

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