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2016

Wu, Q., Gao, F., Song, Z., Lin, X., Zhang, Y., Chen, H., et al. (2016). Experimental verification of free-space singular boundary conditions in an invisibility cloak. *Journal of Optics*, 18(4), 044008-.

<https://hdl.handle.net/10356/84534>

<https://doi.org/10.1088/2040-8978/18/4/044008>

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[<http://dx.doi.org/10.1088/2040-8978/18/4/044008>].

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Experimental verification of free-space singular boundary conditions in an invisibility cloak

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Abstract: A major issue in invisibility cloaking, which caused intense mathematical discussions in the past few years but still remains physically elusive, is the plausible singular boundary conditions associated with the singular metamaterials at the inner boundary of an invisibility cloak. The perfect cloaking phenomenon, as originally proposed by Pendry et al. for electromagnetic waves, cannot be treated as physical before a realistic inner boundary of a cloak is demonstrated. Although a recent demonstration has been done in a waveguide environment, the exotic singular boundary conditions should apply to a general environment as in free space. Here we fabricate a metamaterial surface that exhibits the singular boundary conditions and demonstrate its performance in free space. Particularly, the phase information of waves reflected from this metamaterial surface is explicitly measured, confirming the singular responses of boundary conditions for an invisibility cloak.

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Introduction

Invisibility cloaking of electromagnetic waves has caused intense mathematical discussions in the past few years for its exotic boundary conditions at the inner boundary of a perfect invisibility cloak. As originally proposed by Pendry et al. [1], this inner boundary of a cloak arises from a singular coordinate transformation, perfectly separating the electromagnetic domain into the inner, or “hidden,” region and those outside. However, the original derivation in Ref. [1] did not discuss the singularity in the coordinate transformation, which corresponds to the inner boundary of the cloak. On the other hand, subsequent mathematical analysis on this inner boundary using regular boundary conditions encountered difficulty. For example, Ref. [2] analyzed the case of active sources inside the inner region of a cloak, and concluded that the “finite energy solutions” to Maxwell’s equations do not exist. It is Ref. [3] that provided a solution to Maxwell’s equations and ascribed the exotic responses of the inner boundary, which behaves as a perfect electric conductor (PEC) for transverse- electric (TE) waves and as a perfect magnetic conductor (PMC) for transverse-magnetic (TM) waves, to its “surface voltage” effect—the electromagnetic fields in the normal direction blows up like a delta function. Although many experimental efforts [4] have been made to build a cloak in practice, they all have avoided implementing this singular inner boundary of the perfect cloak and thus are only approximately valid. In that sense, the theoretical proposal in Ref. [1] still remains unphysical before the inner boundary of the cloak can be demonstrated physically.

The analysis in Ref. [3] also implied a practical model for a set of boundary conditions (called “DB” boundary conditions [5]) for which, when the normal components of the D and B fields on a boundary are zero, the electromagnetic fields inside the domain enclosed by the boundary are uniquely determined [5,6,7]. This set of boundary conditions can be categorized into a more general mathematical form of boundary conditions in the sense of the Schwartz distribution theory [8,9]. Yet how to physically implement the boundary conditions is the focus of this research. A recent demonstration has been done in a waveguide environment where the mode spectrum is discretized [10]. Whether this set of DB boundary conditions still holds in a general environment as in free space is unverified. Some other works are mainly focused on numerical analysis of singularities of invisibility cloaking [16]. In this paper, we design a metamaterial whose permittivity and permeability approach to zero in the normal direction (as in the DB boundary) and demonstrate the exotic responses of the metamaterial surface in free space as a verification of previous analysis on the boundary conditions for invisibility cloaking. This metamaterial surface, serving as the inner boundary of the perfect invisibility cloak, is a key step in realizing metamaterial-based transformation invisibility cloaking.

Results

The metamaterial consists of two kinds of resonators: split ring resonator (SRR)[11] and electric inductive capacitive (ELC) resonator [12]. The former can excite magnetic resonance and the latter can excite electric resonance. The patterned structures are shown in Fig.1 (a) and (b), respectively. Here we combine the SRR and ELC as a unit cell as in Fig. 1(c) such that the magnetic resonance and the electric resonance can be aligned along the same z axis. Both SRR and the ELC were printed on the FR4 substrate ($\epsilon_r = 4.4 + 0.01i$) and arrayed periodically in the

x-y plane and along z axis to form a metamaterial. (Later we will see the thickness along z axis is a secondary factor.) We set the geometrical parameters of SRR as follows: $l = 3.7\text{mm}$, $ll = 2.7\text{mm}$, $w = 0.33\text{mm}$ and $g = 0.1\text{mm}$. The periodicity in the x-y plane is set as $a = 5\text{mm}$, smaller than $1/5\lambda$. The geometric parameters of ELC are defined as $h = 3.5\text{mm}$, $hh = 1.7\text{mm}$, $v = 0.2\text{mm}$ and $c = 0.2\text{mm}$. The thickness of the SRR and ELC made of copper wires is $t = 0.026\text{mm}$. The thickness of the FR4 substrate is $d = 0.4\text{mm}$.

We then numerically retrieve the effective parameters based on the unit cell with standard retrieval procedure [13]. The dispersion of ε_z and μ_z are plotted in red and blue, respectively, in Fig.1(d)

from 8 GHz to 11 GHz. The frequency point at which $\varepsilon_z \rightarrow 0$, $\mu_z \rightarrow 0$ can be found at 10 GHz.

Note that this retrieval was done with incidence wave vector in the y direction. The slight spatial dispersion in this unit cell will affect the dispersion profile, but does not change the frequency point where $\varepsilon_z \rightarrow 0$, $\mu_z \rightarrow 0$. Another retrieval with incidence wave vector in the x direction has confirmed this point.

Figure 1: Design of the singular metamaterial. (a) The pattern of Split Ring Resonator(SRR) printed on a FR4 substrate. $l = 3.7\text{mm}$, $ll = 2.7\text{mm}$, $w = 0.33\text{mm}$, and $g = 0.1\text{mm}$. (b) The pattern of Electric Inductive Capacitive (ELC) resonator printed on the FR4 substrate. $h = 3.5\text{mm}$, $hh = 1.7\text{mm}$, $v = 0.2\text{mm}$, and $c = 0.2\text{mm}$. In both (a) and (b), the thickness of FR4 substrate is $d = 0.4\text{mm}$, and the thickness of copper wires is $t = 0.026\text{mm}$. (c) The unit cell that combines the SRR and ELC. The period in the x-y plane is $a = 5\text{mm}$. (d) the retrieved effective parameters in the z direction from 8 GHz to 11 GHz. The blue solid curve and dashed curve represent the real and imaginary parts of effective permittivity, respectively. The real and imaginary parts of effective permeability are denoted as the red solid and dashed curves.

According to previous analysis, if the anisotropic metamaterial meets the condition $\varepsilon_z \rightarrow 0$, $\mu_z \rightarrow 0$,

its surface can reflect all waves back because of the “surface voltage” effect. In the following we simulate the response of the metamaterial with the commercial software CST MicroWave Studio as shown in Fig. 2. To show that the response is indeed a surface effect, we use only two layers along the z axis in the simulation. Both TE and TM polarized waves are incident on the two-layer sample obliquely with 45° incidence angle. We can see that in both cases, waves can be fully reflected. Particularly, the normal component of magnetic (electric) field reaches a high value and concentrate on the surface for incidence of TE (TM) waves. This is exactly the “surface voltage” effect as revealed in Ref. [3], as the underlying mechanism for the boundary to behave as a PEC for TE waves and a PMC for TM waves. Note that when the boundary behaves as a PEC for TE waves, it is not the electric current but the singular magnetic field in the normal direction that supports the discontinuity of tangential magnetic field across the boundary. Similarly, when the boundary behaves as a PMC for TM waves, it is the singular electric field in the normal direction, not the magnetic current, that supports the discontinuity of tangential electric field across the boundary. Because of this “surface voltage” effect, the thickness of this metamaterial in z direction is unimportant. It should be emphasized that the “surface voltage” effect fails for the normal incidence of an ideal plane wave. Yet an ideal plane wave does not exist. One needs to put a localized source in front of the metamaterial surface that consists of a spectrum of plane waves. The normal incidence at exact 90 degree occupies zero energy in the spectrum.

Figure 2: The field pattern when the wave at 10GHz is incident onto the two-layer sample with 45 degree incidence angle. (a) Magnetic field (Hz) for TE wave. (b) Electric field pattern (Ez) for TM wave.

We then experimentally demonstrate the responses of the metamaterial surface. As shown in Fig. 2, this “surface voltage” effect arises only in the first layer while the second layer can be actually removed. We thus only used a single-layer sample in experiment. We fabricated the one-layer sample with dimensions of $50cm \times 50cm$, whose unit cell is as in Fig. 1(c). Two horn antennas, connected with Agilent vector network analyzer, were then used to launch the incident wave and measure the reflected wave, respectively. We first tuned the incident waves to be obliquely incident on the sample with incidence angles varying from 20° to 80° in the x - z plane. The measured results of reflectivity from 7 to 12 GHz are shown in Fig. 3(c-d), being consistent with simulation results in Fig. 3(a-b). We can see that around 10 GHz, at which both the permittivity and permeability goes to zero in the z direction, very high reflectivity can be achieved for both TE and TM waves in a broad range of incidence angles. Incidence angle variation in the y - z plane was also performed with similar results, as shown in Fig. 3(g-h), being consistent with simulation in Fig. 3(e-f). These results confirm that around 10 GHz, the metamaterial surface can achieve

omni-directional high reflectivity.

To show that this metamaterial surface around 10 GHz behaves like a PEC for TE waves and a PMC for TM waves, we also measured the relative phase of the reflected waves compared to the waves reflected from a PEC in the frequency range from 9.9 to 10.1 GHz. For simplicity we only show results at incidence angle of 45° . Fig. 4(a-b) show the phase measurement results in the x-z plane for TE (Fig. 4(a)) and TM (Fig. 4(b)) waves. Fig. 4(c-d) show the phase measurement results in the y-z plane for TE (Fig. 4(c)) and TM (Fig. 4(d)) waves. In Fig. 4(a) and (c), the phase of reflected waves overlaps with that from the PEC, meaning that the metamaterial surface behaves as a PEC for TE waves omnidirectionally. In Fig. 4(b) and (d), the phase of reflected waves has a 180 degree shift compared to that from the PEC, meaning that the metamaterial surface behaves as a PMC for TM waves omnidirectionally. It should be noted that the phase information does not depend on the incidence plane, which is different from previous metamaterial surfaces that have “geometrical phase” involved [14,15].

Figure 3: Reflectivity of the singular metamaterial surface. (a-b) Simulation of reflection for TE waves (a) and TM waves (b) from 7 GHz to 12 GHz with incidence in the x-z plane. (c-d) Measured reflection for TE waves (c) and TM waves (d) from 7 GHz to 12 GHz with incidence in the x-z plane. (e-f) Simulation of reflection for TE waves (e) and TM waves (f) from 7 GHz to 12 GHz with incidence in the y-z plane. (g-h) Measured reflection for TE waves (g) and TM waves (h) from 7 GHz to 12 GHz with incidence in the y-z plane.

Figure 4: Phase measurement of reflected waves compared to that from a PEC in the frequency range from 9.9 GHz to 10.1 GHz. (a-b) The incidence is in x-z plane with the incident angle $\theta_i = 45^\circ$ for TE (a) and TM (b) waves. (c-d) The incidence is in y-z plane with the incident angle $\theta_i = 45^\circ$ for TE (c) and TM (d) waves.

Conclusion

In conclusion, we demonstrate a metamaterial surface that exhibits the same boundary conditions as the inner boundary of an invisibility cloak as proposed by Pendry et al. This metamaterial is constructed with SRR and ELC to achieve normal components of permeability and permittivity approaching zero at the same frequency point. The metamaterial surface behaves like a PEC for TE waves and PMC for TM waves omnidirectionally, as a result of previously revealed “surface voltage” effect. Phase information is explicitly measured to confirm the boundary conditions.

Acknowledgement

This work was sponsored by the China Scholarship Council under Grant No. 201306920012, Nanyang Technological University under Nanyang Assistant Professorship Grant, and Singapore Ministry of Education under Grants Tier 1 RG27/12 and MOE2011-T3-1-005.

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