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Analytical Model for Compressive Arch Action in Horizontally Restrained Beam-Column Subassemblages

by Shao-Bo Kang and Kang Hai Tan

This paper presents an analytical model to predict the compressive arch action (CAA) of reinforced concrete beam-column subassemblages. In the model, a new method is proposed to quantify the strains of longitudinal reinforcement in the beam. Instead of equivalent rectangular concrete stress block, the constitutive model for concrete is employed to calculate the compression force sustained by concrete. Besides, tensile strength of engineered cementitious composites can also be considered. Comparisons between experimental and analytical results indicate that the model yields reasonable accuracy in predicting the CAA capacity and horizontal compression force of subassemblages. The effects of horizontal restraints on the CAA capacity of subassemblages are also investigated through the model. Furthermore, pseudo-static resistance of beam-column subassemblages is calculated based on the energy balance method. Finally, conclusions on the enhancement of CAA to structural resistance are drawn from analytical results.

Keywords: analytical model; beam-column subassemblage; compressive arch action; engineered cementitious composites; pseudo-static resistance.

INTRODUCTION

Compressive arch action (CAA) represents a mechanism through which the resistance of horizontally restrained beams is greatly enhanced due to the development of axial compression force in the beam. A relatively small vertical deflection of the bridging beam results in rotation and outward movement of the beam ends against stiff boundaries, which in turn generates compressive thrusts between the flexural compression zones at the middle and end joints.¹ These thrusts enhance the moment resistance via arching effect.²

Early researchers proposed rigid-plastic analytical models for predicting the peak capacity and corresponding midspan deflection of one-way slabs subjected to CAA.^{3,4} The term “rigid-plastic” refers to the assumption that plastic hinges have developed at specified points and the one-way slab typically remains rigid except for the elastic axial shortening and the inelastic rotation of plastic hinges.¹ These assumptions form the basis of following mechanical models for predicting the CAA. The models yield good predictions when used for calculating the CAA capacity of reinforced concrete beam-column subassemblages under column removal scenarios.^{5,6}

Nevertheless, Park’s model³ does not provide a clear determination of the stress state of steel bars in the compression zones, thereby hindering the precise prediction of the CAA capacity and horizontal compression force of reinforced concrete beams and slabs. In analyzing the response of reinforced concrete slabs, Guice et al.⁷ assumed that compressive concrete attained its ultimate strain and calculated the compressive steel strain by using the plane-section assumption. This method requires the predetermined peak capacity deflection, which leads to inaccuracy in predicting

the response of slabs. Yu and Tan⁸ proposed an analytical procedure to predict the CAA capacity of beam-column subassemblages under column removal scenarios and incorporated the rotational stiffness and connection gap of supports into the analytical model. It is capable of predicting the CAA capacity and peak axial compression of subassemblages with reasonable accuracy. However, in Yu and Tan’s model, it is assumed that tensile reinforcement has attained its yield strength and the strain of extreme compression fiber is constant at the crushing strain of concrete. This assumption results in a reduction in tensile strain of reinforcement, which violates the realistic variation of the tensile strain.⁹ Besides, application of the equivalent rectangular concrete block in the model leads to an overestimation of vertical load at the initial stage. Therefore, further improvement of the model is needed to predict the resistance of beam-column subassemblages at the CAA stage.

In this paper, a new analytical model is proposed for beam-column subassemblages in accordance with previous analytical studies, in which the actual stress state of compressive and tensile reinforcing bars is taken into account. It can also consider the tensile strength of engineered cementitious composites (ECC) used in structural topping. The model is calibrated with experimental results of conventional concrete and ECC beam-column subassemblages under column removal scenarios. Finally, a parametric study is conducted to investigate the effects of horizontal restraints on CAA of beam-column subassemblages.

RESEARCH SIGNIFICANCE

Previous analytical studies on CAA mainly provide reasonably accurate predictions of the CAA capacity and the peak horizontal compression force.^{7,8} However, the whole load-displacement curve, which is essential to analyze the energy-absorption capacity of structural members under pseudo-static loading conditions, cannot be obtained at the CAA stage. Besides, when ECC is used in beam-column subassemblages, its tensile strength cannot be considered in the models due to the assumption that tensile reinforcement has yielded and the ultimate strain of concrete has been attained at extreme compression concrete fiber. The current study offers an effective way to evaluate the quasi-static and pseudo-static resistances of reinforced concrete beam-column subassemblages.

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Fig. 1—Plastic hinge mechanism of beam-column subassemblage.

(a) At free rotation stage of end support

(b) At CAA stage

Fig. 2—Compatibility condition of bridging beam.

DEVELOPMENT OF ANALYTICAL MODEL

With adequate horizontal restraints, the beam-column subassemblage is able to develop CAA between the compression zones of the bridging beam when the middle joint displacement is less than one beam depth. Based on rigid-plastic assumption, the system can be idealized as two rigid beam segments and four zero-length hinges concentrated at the faces of the middle joint and column stubs (refer to Fig. 1). The hinges at the interfaces sustain all flexural rotations, whereas the beam segments only experience axial shortening generated by net compression force in the beam.² However, in either the prototype structure or the test setup, horizontal restraints on the beam-column subassemblage are

far from fully fixed with infinite stiffness, and it is necessary to consider the actual horizontal movements and rotations of the supports in the model.

Compatibility condition

When the middle joint displacement is relatively small, the bridging beam is only subjected to flexural action and no axial compression force develops due to the connection gap between the beam and the horizontal support. The connection gap also allows free rotation of the end support. Figure 2(a) shows the deformed configuration of the beam at the free rotation stage of the end support. The beam only sustains sagging moment in the middle joint and axial compression force has not been mobilized in the beam. Thus, resultant force at Section 2 is zero. The initial connection gap is reduced with increasing vertical deflection of the beam, as calculated from Eq. (1). However, the compatibility condition of the beam cannot be invoked until the connection gap is closed.

$$t_1 = l(1 - \cos \phi) + t_0 - (0.5h - c_2) \tan \phi \quad (1)$$

$$\approx t_0 - (0.5h - c_2) \tan \phi$$

where l and h are the clear span and depth of the beam; t_0 is the connection gap prior to loading, measured in the experimental test from the measured horizontal force versus horizontal displacement relationship; ϕ is the rotation angle of the beam; and c_2 is the neutral axis depth at Section 2.

When free rotation angle Θ_0 of the end support is attained, axial compression force starts developing in the beam. Compatibility condition of the beam can be applied. Figure 2(b) shows the geometric condition of the beam. Due to inadequate stiffness of the rotational restraint, the support of the single-span beam experiences a rotation angle of Θ when the beam rotates at an angle of ϕ , and it compresses the top layer of the beam by $0.5h \tan \Theta$. Axial strain of the beam and the middle joint is assumed at a constant value of ϵ_b along the beam segments in Fig. 2(b). Thus, the middle joint is shortened by $0.5\epsilon_b(\beta - 2)l$. The beam compression force imposes a leftward movement of t to the end support.² In addition, connection gap t_0 between the beam end and the support is taken into consideration as well.⁸ Thus, horizontal distance l_{AB} between points A at the end support and B at the middle joint can be calculated as

$$(l_{AB})_1 = l + 0.5\epsilon_b(\beta - 2)l + t + t_0 - 0.5h \tan \Theta \quad (2)$$

where t and Θ are the horizontal translation and rotation of the end support, respectively; and ϵ_b is the axial compressive strain of the beam.

Support movement t and rotation Θ can be determined from Eq. (3) and (4), respectively. In calculating K_a and K_r from experimental results, the approach recommended by Yu and Tan⁸ is employed.

$$t = N/K_a \quad (3)$$

$$\Theta = \Theta_0 + M_1/K_r \quad (4)$$

where K_a and K_r are the respective stiffnesses of horizontal and rotational restraints and Θ_0 is the free rotation angle of the end support induced by connection gap.

From the geometric compatibility condition of the beam, spacing l_{AB} between points A and B can also be determined from the deformation of the beam, as expressed in Eq. (5). Accordingly, vertical displacement of the middle joint is calculated from Eq. (6)

$$(l_{AB})_2 = (l(1 - \epsilon_b) + (h - c_1) \tan(\varphi - \Theta) - c_2 \tan \varphi) \cos \varphi \quad (5)$$

$$\delta = (l + 0.5\epsilon_b(1 - 2\beta)l + t + t_0 - 0.5h \tan \Theta) \tan \varphi \quad (6)$$

where c_1 is the neutral axis depth at Section 1.

In addition to the whole beam, compatibility has also to be satisfied at Sections 1 and 2. It is assumed that the strain of beam top longitudinal reinforcement is zero at the inflection point of the beam and varies linearly between the inflection point and beam end sections, as shown in Fig. 2(b). Accordingly, at the centroid of reinforcement, total elongation of the top reinforcement between the inflection point and Section 1 is equal to the distance between the left end of the beam and the support, and the shortening of the bar between Section 2 and the inflection point is equal to the distance between the right end of the beam and the middle joint interface, as expressed in Eq. (7) and (8)

$$\Delta l_1 = (h - c_1 - a_{s1}) \tan(\varphi - \Theta) = 0.5\epsilon_{s1}l_1 \quad (7)$$

$$\Delta l_2 = (c_2 - a'_{s2}) \tan \varphi = 0.5\epsilon'_{s2}l_2 \quad (8)$$

where l_1 and l_2 are the respective distances from the inflection point of the beam to its left and right ends; ϵ_{s1} is the tensile strain of top reinforcement at Section 1; and ϵ'_{s2} is the compressive strain of top reinforcement at Section 2.

Due to different ratios of top and bottom longitudinal reinforcement in the beam, the inflection point is not located at the midspan of the beam between the end support and middle joint. It shifts with increasing middle joint displacement at the large deformation stage. The position of the inflection point is determined by the bending moments at Sections 1 and 2, as expressed in Eq. (9) and (10). The presence of axial compression force in the beam reduces the

length of beam segment between the inflection point and Section 1. However, in comparison with the maximum strain of tension reinforcement generated by bending moment, axial compressive strain of the beam is much smaller, and can be neglected in the model. Thus, the original beam length is used in place of its deformed length.

$$l_1 = \frac{lM_1}{M_1 + M_2} \quad (9)$$

$$l_2 = \frac{lM_2}{M_1 + M_2} \quad (10)$$

where M_1 is the hogging moment at Section 1; and M_2 is the sagging moment at Section 2. It is notable that M_1 is zero at free rotation stage of the end support.

With the neutral axis depth and strain at each section, strain profile of the section can be determined in accordance with the plane-section assumption, as expressed in Eq. (11) to (15)

$$\epsilon'_{s1} = \frac{c_1 - a'_{s1}}{h - c_1 - a_{s1}} \epsilon_{s1} \quad (11)$$

$$\epsilon_{s2} = \frac{h - c_2 - a_{s2}}{c_2 - a'_{s2}} \epsilon'_{s2} \quad (12)$$

$$\epsilon_{c1} = \frac{c_1}{c_1 - a_{s1}} \epsilon'_{s1} \quad (13)$$

$$\epsilon_{c2} = \frac{c_2}{c_2 - a_{s2}} \epsilon'_{s2} \quad (14)$$

$$\epsilon_{t1} = \frac{h - c_1}{h - c_1 - a_{s1}} \epsilon_{s1} \quad (15)$$

where ϵ_{s1}' is the compressive strain of bottom reinforcement at Section 1; ϵ_{s2} is the tensile strain of bottom reinforcement at Section 2; ϵ_{c1} and ϵ_{c2} are the strains of extreme compression fibers at Sections 1 and 2, respectively; ϵ_{t1} is the strain of extreme tension fiber; a_{s1} and a_{s2} are the distances from the centroid of tension reinforcement to the extreme tension fiber; and a_{s1}' and a_{s2}' are the distances from the centroid of compression reinforcement to the extreme compression fiber at Sections 1 and 2, respectively.

Equilibrium condition

Due to symmetry of geometry and loading, only the single-span beam is extracted from the plastic hinge mechanism, as shown in Fig. 3. Through equilibrium,⁸ the correlation between the applied vertical force on the middle joint and corresponding vertical deflection can be established as

$$P = \frac{2(M_1 + M_2 - N\delta - q l^2/2)}{l} \quad (16)$$

where N is the beam axial compression force; q is the self-weight of the affected beam; P is the concentrated point force on the middle beam-column joint; and δ is the deflection of the middle joint.

To calculate the vertical load on the middle joint, axial compression force N and bending moments M_1 and M_2 have to be quantified based on the deformed geometry shown in Fig. 3. Therefore, cross-sectional analysis at Sections 1 and 2 are necessary. Figure 4 shows the strain profile and internal force equilibrium at Beam Sections 1 and 2.

When ECC is used in the structural topping of beam-column subassemblages, its tensile strength has to be considered at Section 1 (within the depth of h_t) subjected to hogging moment, as shown in Fig. 4(a). Compared to conventional concrete subassemblages, an additional term is added to the equilibrium condition of internal forces at Section 1 of ECC subassemblages, as expressed in Eq. (17)

$$T_{t1} = b \int_{h-c_1-h_t}^{h-c_1} \sigma_{t1} dx \quad (17)$$

where T_{t1} is the tension force sustained by the ECC topping; b is the width of the beam section; h_t is the thickness of the ECC topping; and σ_{t1} is the tensile stress of the ECC topping.

In determining the compression force sustained by concrete, compressive stresses are integrated across the compression zone at Section 1, as expressed in Eq. (18)

$$C_{c1} = b \int_0^{c_1} \sigma_{c1} dx \quad (18)$$

$$C_{c2} = b \int_0^{c_2} \sigma_{c2} dx \quad (19)$$

where C_{c1} and C_{c2} are the compression forces in concrete; and σ_{c1} and σ_{c2} are the respective compressive stresses of concrete.

A similar procedure can be used to calculate the internal forces at Section 2. It is notable that at Section 2 only the beam longitudinal reinforcement sustains tension force, whereas ECC topping is subjected to compression (refer to Fig. 4(b)). Thus, Eq. (19) represents the compression force sustained by ECC topping.

In the analytical model, axial compression force in the beam is assumed to be constant along the beam length.^{2,8} From the internal force equilibrium illustrated in Fig. 4, axial compression force at Section 1 is expressed in Eq. (20). Bending moment at Section 1 can also be calculated. For ECC subassemblages, total hogging moment is a summation of four terms—namely, contributions of compressive concrete, compressive reinforcement, tensile reinforcement, and tensile ECC, as expressed in Eq. (21). Sagging moment at Section 2 is only contributed by three terms, as expressed in Eq. (22). Contribution of tensile ECC is eliminated, as the ECC topping is in compression at Section 2.

$$N = C_{c1} + C_{s1} - T_{s1} - T_{t1} = C_{c2} + C_{s2} - T_{s2} \quad (20)$$

$$M_1 = b \int_0^{c_1} (0.5h - c_1 + x) \sigma_{c1} dx + C_{s1} (0.5h - a'_{s1}) + T_{s1} (0.5h - a_{s1}) + b \int_{h-c_1-h_t}^{h-c_1} (x + c_1 - 0.5h) \sigma_{c1} dx \quad (21)$$

Fig. 3—Free-body diagram of single-span beam.

$$M_2 = b \int_0^{c_2} (0.5h - c_2 + x) \sigma_{c2} dx + C_{s2} (0.5h - a'_{s2}) + T_{s2} (0.5h - a_{s2}) \quad (22)$$

in which C_{s1} and C_{s2} are the compression forces in the compressive reinforcement; and T_{s1} and T_{s2} are the tension forces sustained by reinforcement at Sections 1 and 2, respectively.

The single-span beam is assumed to sustain elastic compressive strain under CAA due to the axial compression force. The compression force in the beam can be quantified by its compressive strain,⁹ as expressed in Eq. (23)

$$N = b h E_c \epsilon_b \quad (23)$$

in which E_c is the equivalent modulus of elasticity of the section, approximately equal to $4700\sqrt{f'_c}$ (in MPa).¹⁰ For sections with several concrete layers of different strengths, the elastic modulus can be calculated by $E_c = \Sigma(E_{ci} b_i h_i) / b h$.

Through internal force equilibrium of the single-span beam, the relationship between the axial compression force and bending moment at the end section is established. To calculate the internal forces from strains at Sections 1 and 2, constitutive models for steel reinforcement, concrete, and ECC need to be defined in tension and compression.

Constitutive models

Steel reinforcement—Experimental tests on beam-column subassemblages demonstrate that strains of beam bottom longitudinal reinforcement at the middle joint face and top reinforcement at the end column stub face keep increasing under column removal scenarios, whereas top steel bars at the middle joint and bottom bars at the end column stub are subjected to compressive strains at the initial stage and gradually shifted to tension with increasing middle joint displacement.¹¹ Therefore, the constitutive law of reinforcing bars is defined in tension and compression. Figure 5 shows the elastic-perfectly-plastic stress-strain relationship of steel bars, with yield strains of ϵ_y in tension and ϵ_y' in compression. As the strain of compressive steel reinforcement is reduced at large deformations, a linear unloading phase is defined in the compressive branch of the stress-strain curve, with its stiffness equal to the elastic modulus of steel. Correspondingly, the stress-strain relationship of steel reinforcement in tension and compression is expressed in Eq. (24) and (25)

$$\begin{aligned} \sigma_s &= E_s \epsilon_s, \text{ if } \epsilon_s < \epsilon_y \\ \sigma_s &= f_y, \text{ if } \epsilon_s \geq \epsilon_y \end{aligned} \quad (24)$$

$$\sigma'_s = E_s \epsilon'_s, \text{ if } \epsilon'_s < \epsilon'_y \text{ and } \epsilon'_m < \epsilon'_y$$

Fig. 4—Force equilibrium of beam sections.

$$\begin{aligned} \sigma'_s &= f'_y, \text{ if } \epsilon'_s \geq \epsilon'_y \text{ and } \epsilon'_m \leq \epsilon'_s \\ \sigma'_s &= f'_y - (\epsilon'_m - \epsilon'_s) E_s, \text{ if } \epsilon'_m \geq \epsilon'_y \text{ and } \epsilon'_m > \epsilon'_s \end{aligned} \quad (25)$$

where σ_s and σ'_s are the tensile and compressive stresses of reinforcement, respectively; E_s is the modulus of elasticity of steel bars; ϵ_s and ϵ'_s are the tensile and compressive strains of reinforcement; ϵ_y and ϵ'_y are the yield strains of steel in tension and compression, respectively; f_y and f'_y are the tensile and compressive yield strengths of steel reinforcement, respectively; and ϵ'_m is the maximum compressive strain that reinforcement has attained.

Concrete—In previous analytical studies on horizontally restrained reinforced concrete slabs, equivalent rectangular concrete stress block was used to determine the load-displacement curves of slabs.^{2,12} A similar approach was used for beam-column subassemblages under column removal scenarios.⁸ However, vertical load on the middle joint is substantially overestimated at the initial stage. Besides, severe crushing of concrete occurs in the compression zone of bridging beams indicates that the rectangular concrete stress block is not valid under large deformations. Thus, realistic stress-strain relationship of concrete has to be used to calculate the compressive stress of concrete. In deriving the analytical model, the constitutive model for unconfined concrete in uniaxial compression proposed by Mander et al.¹³ is employed to determine the compression force sustained by concrete and corresponding bending moment.

Engineered cementitious composites—Different from conventional concrete with limited ductility in tension, ECC exhibits tensile strain-hardening behavior and superior strain capacity.¹⁴⁻¹⁶ To consider the tensile strength of ECC in the analytical model, the stress-strain relationship of ECC in uniaxial tension can be simplified as a bilinear

Fig. 5—Stress-strain relationship of steel bars.

curve with hardening behavior, as shown in Fig. 6(a). Prior to cracking, ECC remains in the elastic stage and its tensile stress increases linearly with strain. After the first cracking, a hardening stage develops with multi-cracking along the tensile coupon length. Equation (26) expresses the tensile stress-strain relationship of ECC.¹⁷

$$\begin{aligned} \sigma &= \frac{\sigma_{tc}}{\epsilon_{tc}} \epsilon, \text{ if } \epsilon \leq \epsilon_{tc} \\ \sigma &= \sigma_{tc} + (\sigma_{tu} - \sigma_{tc}) \frac{\epsilon - \epsilon_{tc}}{\epsilon_{tu} - \epsilon_{tc}}, \text{ if } \epsilon_{tc} < \epsilon \leq \epsilon_{tu} \end{aligned} \quad (26)$$

where σ_{tc} and ϵ_{tc} are the first cracking strength and strain of ECC in tension; σ_{tu} is the ultimate tensile strength; and ϵ_{tu} is the associated tensile strain at σ_{tu} .

(a) In uniaxial tension

(b) In compression

Fig. 6—Stress-strain relationship of ECC.

Under compression, a bilinear stress-strain relationship of ECC is assumed in the ascending branch of compressive stress,¹⁸ as shown in Fig. 6(b). After attaining the compressive strength, the stress linearly drops to 50% of the compressive strength, and then decreases slowly with increasing middle joint displacement. Thus, a constant compressive stress is assumed with increasing compressive strain. Equation (27) expresses the compressive stress-strain relationship of ECC.¹⁷

$$\sigma = 2 \frac{\sigma_{c0}}{\epsilon_{c0}} \epsilon, \text{ if } 0 \leq \epsilon \leq \frac{1}{3} \epsilon_{c0}$$

$$\sigma = \frac{2}{3} \sigma_{c0} + \frac{\sigma_{c0}}{2\epsilon_{c0}} (\epsilon - \frac{1}{3} \epsilon_{c0}), \text{ if } \frac{1}{3} \epsilon_{c0} \leq \epsilon \leq \epsilon_{c0} \quad (27)$$

$$\sigma = \sigma_{c0} + (\sigma_{cu} - \sigma_{c0}) \frac{\epsilon - \epsilon_{c0}}{\epsilon_{cu} - \epsilon_{c0}}, \text{ if } \epsilon_{c0} \leq \epsilon \leq \epsilon_{cu}$$

$$\sigma = \sigma_{cu}, \text{ if } \epsilon_{cu} \leq \epsilon$$

where σ_{c0} is the compressive strength of ECC; ϵ_{c0} is the compressive strain at σ_{c0} ; σ_{cu} is the ultimate compressive strength in the post-peak branch; and ϵ_{cu} is the ultimate compressive strain. It is assumed that $\sigma_{cu} = 0.5\sigma_{c0}$ and $\epsilon_{cu} = 1.5\epsilon_{c0}$.

Solution procedure

Prior to the onset of CAA in the beam, end support experiences free rotation due to the presence of connection gaps in horizontal restraints. At this stage, only sagging moment is mobilized at Section 2 and axial force in the beam is zero, as shown in Fig. 2(a). Thus, only equilibrium at Section 2 has to be satisfied. Once connection gap is closed up, axial compression force develops in the beam, indicating the commencement of CAA. At the CAA stage, both equilibrium and compatibility need to be satisfied. Figure 7 shows the solution procedure for CAA stage of the beam. A sample calculation is detailed in the Appendix.*

Fig. 7—Solution procedure for analytical model.

VALIDATION OF ANALYTICAL MODEL

The proposed analytical model is validated by experimental results of beam-column subassemblages.^{8,19-21} Each subassemblage comprises a middle column, two single-span beams, and two enlarged end column stubs (refer to Fig. 8). Geometry and reinforcement details of the beams are summarized in Table 1. Figure 8 shows the setup for subassemblages tested by Yu.⁹ Roller support was placed underneath each

*The Appendix is available at www.concrete.org/publications in PDF format, appended to the online version of the published paper. It is also available in hard copy from ACI headquarters for a fee equal to the cost of reproduction plus handling at the time of the request.

Fig. 8—Test setup for beam-column subassemblages.

Table 1—Comparisons of experimental and analytical results

Specimen		Cross section, mm (in.)	Clear span, mm (in.)	Reinforcement at beam end*		Boundary conditions				Capacity of CAA			Maximum horizontal compression								
				Top	Bottom	Axial stiffness, kN/m (kip/ft)	Connection gap, mm (in.)	Rotational stiffness, kN-m/rad (kip-ft/rad)	Free rotation, rad	Test results, kN (kip)	Analytical results, kN (kip)		Test results, kN (kip)	Analytical results, kN (kip)							
Yu and Tan ⁸	S1	150 × 250 (5.9 × 9.8)	2750 (108.3)	2T10 + T13	2T10	1.06×10^5 (7.3 × 10 ⁴)	0.5 (0.11)	1.0×10^4 (7.4 × 10 ³)	0	41.6 (9.4)	41.9 (9.4)	1.01	177.9 (40.0)	186.7 (42.0)	1.05						
	3T10			2T10	1.2 (0.27)		38.4 (8.6)			36.8 (8.3)	0.96	155.9 (35.0)	169.2 (38.0)	1.09							
	3T13			2T10	4.29×10^5 (2.94 × 10 ⁵)	1.2 (0.27)	3.0×10^4 (2.21 × 10 ⁴)	54.5 (12.3)		51.8 (11.6)	0.95	221.0 (49.7)	235.3 (52.9)	1.06							
	3T13			2T13		1.0 (0.22)		63.2 (14.2)		59.1 (13.3)	0.94	212.7 (47.8)	251.8 (56.6)	1.18							
	3T13			3T13		0.8 (0.18)		70.3 (15.8)		68.4 (15.4)	0.97	238.4 (53.6)	267.5 (60.1)	1.12							
	3T16			2T13		0.8 (0.18)		70.3 (15.8)		71.0 (16.0)	1.01	218.1 (49.0)	221.7 (49.8)	1.02							
	S7		2150 (84.6)	3T13	2T13	1.2 (0.27)	82.8 (18.6)	82.9 (18.6)		1.00	233.1 (52.4)	272.0 (61.1)	1.17								
	S8		1550 (61.0)	3T13	2T13	0.8 (0.18)	121.3 (27.3)	129.5 (29.1)		1.07	272.5 (61.3)	293.8 (66.0)	1.08								
FarhangVesali et al. ¹⁹	F1	180 × 180 (7.1 × 7.1)	2200 (86.6)	2N10	2N10	1.00×10^6 (6.9 × 10 ⁴)	0	0	1.45 × 10 ⁴ (1.07 × 10 ⁴)	40.5 (9.1)	37.5 (8.4)	0.93	—	211.1 (52.4)	—						
	F2			2N10	2N10				1.35 × 10 ⁴ (1.00 × 10 ⁴)							35.7 (8.0)	35.7 (8.0)	1.00	—	193.0 (43.4)	—
	F3			2N10	2N10				1.85 × 10 ⁴ (1.36 × 10 ⁴)							41.4 (9.3)	37.5 (8.4)	0.91	—	210.6 (47.3)	—
	F4			3N10	2N10				1.80 × 10 ⁴ (1.33 × 10 ⁴)							40.1 (9.0)	39.3 (8.8)	0.98	—	177.9 (40.0)	—
	F5			3N10	2N10				1.60 × 10 ⁴ (1.18 × 10 ⁴)							41.6 (9.4)	39.3 (8.8)	0.94	—	189.3 (42.6)	—
	F6			3N10	2N10				1.45 × 10 ⁴ (1.07 × 10 ⁴)							39.4 (8.9)	39.5 (8.9)	1.00	—	191.7 (43.1)	—

Table 1 (cont.)—Comparisons of experimental and analytical results

Specimen		Cross section, mm (in.)	Clear span, mm (in.)	Reinforcement at beam end ^a		Boundary conditions				Capacity of CAA			Maximum horizontal compression		
				Top	Bottom	Axial stiffness, kN/m (kip/ft)	Connection gap, mm (in.)	Rotational stiffness, kN-m/rad (kip-ft/rad)	Free rotation, rad	Test results, kN (kip)	Analytical results, kN (kip)		Test results, kN (kip)	Analytical results, kN (kip)	
Kang and Tan ^{20,21}	MJ-B-0.52/0.35S	150 × 300 (5.9 × 11.8)	2750 (108.3)	3T10	2T10	1.51 × 10 ⁵ (1.04 × 10 ⁵)	1.1 (0.25)	2.14 × 10 ⁴ (1.58 × 10 ⁴)	0.005	50.5 (11.4)	48.5 (10.9)	0.96	231.3 (52.0)	239.0 (53.7)	1.03
	2T16+T13			2T13	2.05 × 10 ⁵ (1.41 × 10 ⁵)	0.9 (0.20)	1.99 × 10 ⁴ (1.47 × 10 ⁴)	0.01	90.4 (20.3)	94.5 (21.2)	1.05	281.1 (63.2)	288.2 (64.8)	1.03	
	2T16+T13			2T13	1.83 × 10 ⁵ (1.25 × 10 ⁵)	0.7 (0.16)	1.49 × 10 ⁴ (1.10 × 10 ⁴)	0.006	91.1 (20.5)	99.0 (22.3)	1.09	274.7 (61.8)	271.6 (61.1)	0.99	
	3T13			2T13	1.49 × 10 ⁵ (1.02 × 10 ⁵)	0.8 (0.18)	2.30 × 10 ⁴ (1.70 × 10 ⁴)	0.008	83.7 (18.8)	86.4 (19.4)	1.03	262.8 (59.1)	276.6 (62.2)	1.05	
	2T16+T13			2T13	1.77 × 10 ⁵ (1.21 × 10 ⁵)	0.2 (0.04)	1.72 × 10 ⁴ (1.27 × 10 ⁴)	0.01	91.1 (20.5)	93.1 (20.9)	1.02	305.8 (68.7)	271.1 (60.9)	0.89	
	3T13			2T13	1.59 × 10 ⁵ (1.09 × 10 ⁵)	0.4 (0.09)	2.44 × 10 ⁴ (1.80 × 10 ⁴)	0.008	82.5 (18.5)	82.1 (18.5)	1.00	317.7 (71.4)	273.9 (61.6)	0.86	
Mean value												0.99			1.04
Coefficient of variation												0.05			0.09

*2T10 represents two steel bars with diameter of 10 mm (0.4 in.). Similarly, 3N10 denotes three reinforcing bars with diameter of 10 mm (0.4 in.).

end column stub and horizontal restraints were connected to the column stub to prevent horizontal movement. Meanwhile, reaction forces were measured by means of load cells. Corresponding horizontal displacements were monitored by linear variable differential transducers (LVDTs). Thus, stiffness of horizontal restraints and connection gaps can be quantified from the measured horizontal reaction force and associated displacement. Moreover, rotational stiffness of the support is also determined, as included in Table 1. However, end column stubs of subassemblages tested by FarhangVesali et al.¹⁹ were fixed to bollards anchored to the floor and only the rotational stiffness was measured. Referring to Yu and Tan's analytical study,⁸ horizontal stiffness of each subassemblage is determined to be 1.0×10^6 kN/m (6.9×10^4 kip/ft). In addition to reinforced concrete specimens, precast concrete subassemblages with 75 mm (3.0 in.) thick conventional concrete²¹ and ECC²⁰ topping are also simulated through the model. In the subassemblages, precast beam units were assembled by continuous top reinforcement in cast-in-place structural topping. The test setup in Fig. 8 was used for precast concrete subassemblages. Besides stiffnesses of axial and rotational restraints and connection gaps, free rotation of the end column stub caused by connection gaps is also calculated, as included in Table 1.

Prediction of CAA capacity and horizontal reaction force

Table 1 summarizes the CAA capacities and horizontal compression forces of subassemblages. Comparisons between the analytical and experimental results indicate that the analytical model gives very good predictions of the CAA capacity of beam-column subassemblages. The mean ratio of the analytical and experimental CAA capacities is 0.99, with a coefficient of variation of 0.05. Due to constraints of test setup, horizontal compression force could not be quantified in reinforced concrete beam-column subassemblages tested by FarhangVesali et al.¹⁹ The horizontal compression force in the beams of other subassemblages is slightly overestimated in the model, with an average ratio of 1.04 and a coefficient of variation of 0.09, as it is more sensitive to the boundary conditions of subassemblages than the CAA capacity.

Besides the CAA capacity and horizontal compression force of beam-column subassemblages, the model is able to provide the variations of vertical load with middle joint displacement at the CAA stage, as shown in Fig. 9 and 10. With increasing middle joint displacement, CAA develops in the beam and enhances the flexural resistance of the beam. After attaining its capacity, vertical load decreased slowly due to crushing of concrete in the compression zones of the beam. The analytical model is capable of predicting the vertical load-middle joint displacement curve with reasonably good accuracy. However, rupture of beam bottom reinforce-

Fig. 9—Comparisons of analytical and experimental vertical load-middle joint displacement curves of reinforced concrete subassemblages.^{8,19}

Fig. 10—Comparisons of analytical and experimental vertical load-middle joint displacement curves of precast concrete subassemblages.^{20,21}

ment at the middle joint interface cannot be predicted, as it involves localized strain concentration of reinforcement at the face of middle joint. Thus, at the descending phase of vertical load, sudden drops of vertical load cannot be observed in the predicted vertical load-middle joint displacement curve, as shown in Fig. 9 and 10.

With respect to the horizontal reaction force-middle joint displacement curve, good agreement is reached between the analytical and experimental results, as shown in Fig. 11. At the initial stage, horizontal force is zero due to free rotation of the end column stub. Once connection gap is closed up, horizontal compression force in the bridging beam starts increasing until it attains the maximum value. With increasing middle joint displacement, crushing and spalling

of concrete in the compression zones of the beam results in gradual decrease in the horizontal compression force. The analytical model cannot predict this part of the curve well, as spalling of concrete is not considered in Mander's concrete model. In future work, different concrete models may be used for cover concrete and core region. For the cover concrete, the stress-strain model proposed by Paulay and Priestley²² can be employed so as to take account of spalling of concrete at large strains.

Variation of bending moments

Sagging moment at the middle joint and hogging moment at the end support are calculated using the model. Figure 12 shows the variations of bending moments with middle joint

Fig. 11—Comparisons of analytical and experimental horizontal reaction force-middle joint displacement curves of precast concrete subassemblages.^{20,21}

displacement of ECC subassemblage EMJ-B-1.19/0.59.²⁰ The predicted sagging moment at the middle joint agrees well with the experimental results. In the descending branch of sagging moment, rupture of bottom reinforcement at the middle joint interface led to a drop of sagging moment, as shown in Fig. 12(a). However, this phenomenon cannot be captured by the model. At the end support, the predicted maximum hogging moment is attained earlier than the value obtained from experimental results (refer to Fig. 12(b)).

Estimate of neutral axis depth and reinforcement strain

To investigate the behavior of beam-column subassemblages at the fiber level, Fig. 13(a) through (c) shows the strains of reinforcing bars and concrete fiber at each end of the beam in subassemblage EMJ-B-1.19/0.59.²⁰ Compared to the measured strain of steel reinforcement, the absolute value of the steel strain is substantially underestimated in the model due to the assumption of linear strain profile between the inflection point and beam end sections. However, the overall trend agrees well with the experimental results. Strains of tensile reinforcement keep increasing at the middle joint and end support, as shown in Fig. 13(a). Tensile strain of the bottom reinforcement at the middle joint is several times larger than that of the top reinforcement at the end support. Thus, rupture of bottom reinforcement at the middle joint interface occurs earlier than top reinforcement at the end support, as observed in the experimental test.²⁰ Similar variations of the strains of extreme compression concrete

fibers are obtained, as shown in Fig. 13(b). However, strains of compressive reinforcement increase at the CAA stage and then decrease with increasing middle joint displacement (refer to Fig. 13(c))—in particular, at the middle joint. Moreover, compressive strain of the top reinforcement at the middle joint is considerably smaller than that of the bottom reinforcement at the end support due to a greater compressive reinforcement ratio and a smaller neutral axis depth at the middle joint.

Neutral axis depths at the beam end sections can also be determined in the analytical model. Figure 13(d) shows the variations of neutral axis depths with middle joint displacement. Compared to the end support, compressive reinforcement ratio is greater but tensile reinforcement ratio is smaller at the middle joint. Correspondingly, the neutral axis depth at the middle joint is significantly smaller than that at the end support. It continues to decrease with increasing middle joint displacement at the CAA stage. However, different variation of the neutral axis depth is observed at the end support. At the initial stage, neutral axis depth at the end support is zero due to free rotation of the end column stub, and it attains a peak value when the beam top reinforcement yields. Following the yielding of top tensile reinforcement at the end section, the neutral axis depth decreases slightly with increasing middle joint displacement, and then crushing of compressive concrete increases the neutral axis depth at the end support. Finally, the neutral axis depth decreases with further increase in the middle joint displacement.

(a) Sagging moment at the middle joint

(b) Hogging moment at the end support

Fig. 12—Variations of bending moments at middle joint and end support.²⁰

(a) Tensile reinforcement

(b) Extreme compression concrete fibre

(c) Compressive reinforcement

(d) Neutral axis depths

Fig. 13—Variations of calculated strains and neutral axis depths with middle joint displacement.

EFFECT OF STIFFNESS OF HORIZONTAL RESTRAINT

Horizontal restraints dominate the development of CAA in reinforced concrete slabs.² Likewise, the capacity of CAA in beam-column subassemblages highly depends on the stiffness of horizontal restraints under column removal scenarios. To investigate the effect of horizontal restraints on the behavior of beam-column subassemblage at the CAA stage, beam-column subassemblage EMJ-B-1.19/0.59²⁰ with ECC in the structural topping and beam-column joints is selected

as a benchmark. Moreover, restraint stiffness is normalized by the axial stiffness of the bridging beam,⁵ as expressed in Eq. (28). As for EMJ-B-1.19/0.59, γ_a is equal to 0.4.

$$\gamma_a = \frac{K_a I}{bhE_c} \quad (28)$$

Figures 14(a) and (b) show the variations of vertical loads and horizontal reaction forces with middle joint displacement when different stiffnesses of horizontal restraints are

Fig. 14—Comparisons of force-displacement curves with different horizontal restraints.

Fig. 15—Comparisons of neutral axis depths with different horizontal restraints.

provided for the subassembly. When γ_a is reduced from 0.4 to 0.02, the CAA capacity is reduced by 15% from 99.0 to 84.5 kN (22.3 to 19.0 kip), but associated vertical displacement is substantially increased from 99 to 143 mm (3.9 to 5.6 in.) (refer to Fig. 14(a)). Based on plastic hinge mechanism without axial compression force in the beam, the flexural capacity of the subassembly is determined as 79.4 kN (17.8 kip). Thus, compared to the flexural capacity, the CAA capacity of the subassembly is only 6% greater when γ_a is 0.02. By increasing γ_a from 0.4 to 10, the CAA capacity is increased by 8% from 99.0 to 106.6 kN (22.3 to 24.0 kip) and it is 34% greater than the calculated flexural capacity. However, the middle joint displacement corresponding to the CAA capacity is reduced from 99 to 93 mm (3.9 to 3.7 in.). A more significant effect of horizontal restraints is observed on horizontal reaction forces, as shown in Fig. 14(b). By reducing γ_a from 0.4 to 0.02, the maximum compression force in the beam is reduced by 82%. Similarly, the compression force is increased by 35% when γ_a is increased from 0.4 to 10. Therefore, stiffer horizontal restraints are necessary to mobilize significant CAA in the beam-column subassembly.

Besides the CAA capacity, the effect of horizontal restraints on the pseudo-static resistance of the subassembly has to be investigated in accordance with the energy balance method proposed by Izzudin et al.²³ In the method, the work done by external loads, equivalent to the area under the quasi-static load-displacement curve, is assumed to be equal to the strain energy stored in the subassembly when

the same vertical displacement is attained under quasi-static and dynamic loading conditions, as expressed in Eq. (29). Thus, pseudo-static resistance of the subassembly can be determined accordingly.

$$P_d u_d = \int_0^{u_d} P(u) du \quad (29)$$

where $P(u)$ and u_d are the quasi-static resistance and vertical displacement of the subassembly and P_d is the pseudo-static resistance at a displacement of u_d .

Figure 14(c) shows the pseudo-static resistances of the subassembly. At the initial stage, the subassembly with relatively weak horizontal restraints develops much lower pseudo-static resistance than that with strong restraints. However, with increasing middle joint displacement, the pseudo-static resistances of the subassembly with different horizontal restraints, come close to each other. Thus, the stiffness of horizontal restraints does not have significant influence on the pseudo-static load capacity of the subassembly. For instance, when γ_a is increased from 0.4 to 10, the pseudo-static load capacity is only increased from 74.3 to 75.3 kN (16.7 to 16.9 kip) by 1%, as shown in Fig. 14(c). Likewise, the value is reduced from 74.3 to 72.1 kN (16.7 to 16.2 kip) by 3% if γ_a is reduced from 0.4 to 0.02. However, by increasing the stiffness of horizontal restraints, the vertical displacement corresponding to the pseudo-static load capacity is substantially reduced.

Under column removal scenarios, the pseudo-static resistance of the subassembly is related to its CAA capacity

and ductility.²³ By increasing the stiffness of horizontal restraint, the CAA capacity is significantly increased (refer to Fig. 14(a)). However, its ductility is reduced due to deeper neutral axis depths at the end support and the middle joint, as shown in Fig. 15(a) and (b). Thus, a limited increase is obtained in the pseudo-static load capacity of the subassembly. When weak horizontal restraints are provided for the subassembly, say $\gamma_a = 0.02$, the CAA capacity is reduced, but the ductility is improved due to lower neutral axis depths at the beam ends, as shown in Fig. 15. Correspondingly, the pseudo-static load capacity is slightly reduced, but vertical displacement associated with the pseudo-static resistance is increased. Therefore, when weak horizontal restraints are used for the subassembly, ductility demand on the subassembly is comparatively high to attain the pseudo-static load capacity.

DISCUSSIONS

CAA acts as a primary mechanism to resist vertical load on a horizontally restrained beam subjected to quasi-static loads. However, with axial compression force developed in the beam, ductility of the beam is substantially reduced due to greater neutral axis depths at beam ends. Therefore, compared to a beam under pure flexural action or with weak horizontal restraints, pseudo-static resistance of the beam is not significantly enhanced by CAA when sufficiently strong restraints are provided. Nonetheless, development of CAA in the beam reduces the middle joint displacement required to achieve the same pseudo-static load capacity. Thus, in terms of deformation control, CAA may still be beneficial under pseudo-static loading conditions. In addition to strong horizontal restraints, longitudinal reinforcement has to be properly designed and placed in the beam such that the maximum neutral axis depth, normally at the end support, is reduced and ductility of the beam is improved. In concept, reinforcement detailing of the beam subjected to seismic loads is also applicable to pseudo-static loading conditions because it intends to control the neutral axis depths at the beam ends for the sake of better ductility.²²

CONCLUSIONS

Based on the rigid-plastic assumption, an analytical model is proposed to predict the CAA capacity of beam-column subassemblies with horizontal restraints under column removal scenarios. A linear variation of reinforcement strains along the beam length is assumed in the model. Instead of employing equivalent rectangular concrete stress block, Mander's stress-strain model for concrete is incorporated to take account of the actual stress of concrete in compression. Tensile strength and strain capacity of ECC in tension can also be considered if needed. The model is calibrated by experimental results of reinforced concrete and ECC beam-column subassemblies and is able to estimate the CAA capacity with reasonably good accuracy.

A parametric study is conducted through the analytical model on the effects of horizontal restraints on CAA of subassemblies. Horizontal restraints with higher stiffness increase CAA capacity but reduces ductility of subassemblies. Therefore, when the quasi-static resistance of

subassemblies is converted to the pseudo-static resistance through the energy balance method, the effect of stiffer horizontal restraints on the resistance of subassemblies becomes limited. Nonetheless, the vertical displacement corresponding to the pseudo-static resistance is substantially reduced by stiffer horizontal restraints.

Limitations exist in the analytical model. The assumption of linear strain profile between the inflection point and beam end sections substantially underestimates the strain of top and bottom steel reinforcement at locations where fracturing of reinforcement occurs. Thus, rupture of tension reinforcement cannot be accurately predicted in the model.

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APPENDIX: SAMPLE CALCULATION

Beam-column sub-assemblages with ECC in cast-in-situ structural topping is analysed through the proposed model.

Step 1: free rotation stage of end support

Due to the presence of connection gap between the beam and the end support, compatibility condition of the beam is not satisfied when rotation φ of the beam is smaller than free rotation angle Θ_0 of the end support. Thus, only force equilibrium is considered, in which axial compression force in the beam is zero. At a given rotation φ , neutral axis depth c_2 at section 2 is assumed and l_2 is equal to l , as shown in Fig. 2(a). Substituting φ , c_2 and l_2 into Eq. (8) and rearranging give

$$\varepsilon_{s2}' = \frac{2(c_2 - a_{s2}') \tan \varphi}{l_2} \quad (30)$$

Accordingly, tensile strain ε_{s2} of bottom reinforcement and strain ε_{c2} of extreme compression concrete fibre are determined via Eqs. (12) and (14). Thereafter, stresses of concrete fibres and steel reinforcement can be obtained from the respective stress-strain models for concrete and steel bars. It is noteworthy that compression force C_{c2} sustained by concrete is calculated by integrating the compressive stresses across the compression zone at section 2 via Eq. (8). Substituting forces C_{c2} , C_{s2} and T_{s2} sustained by concrete, compressive and tensile reinforcement, respectively, into Eq. (20) yields

$$C_{c2} + C_{s2} = T_{s2} \quad (31)$$

Once equilibrium expressed in Eq. (31) is satisfied, sagging moment M_2 at section 2 can be computed from Eq. (22). Thus, vertical load applied on the middle joint is calculated from Eq. (16). Associated displacement δ of the joint can be determined as

$$\delta = l \tan \varphi \quad (32)$$

Step 2: CAA stage of beam

When rotation φ of the beam surpasses free rotation angle Θ_0 of the end support, net compression force develops in the beam, and thus compatibility and equilibrium have to be satisfied.

(a) *Equilibrium at section 1*

For a given rotation φ of the beam, neutral axis depth c_1 and hogging moment M_{10} are assumed at section 1. Sagging moment M_2 at section 2 calculated from the previous step is used to determine l_1 and l_2 from Eqs. (9) and (10). Support rotation Θ can be determined from Eq. (4). Substituting c_1 , l_1 and Θ into Eq. (7) yields

$$\varepsilon_{s1} = \frac{2(h - c_1 - a_{s1}) \tan(\varphi - \Theta)}{l_1} \quad (33)$$

Based on the plane-section assumption, strain profile at section 1 can be determined from Eqs. (11), (13) and (15). Corresponding stresses of concrete, steel reinforcement and ECC are obtained from material constitutive models, as expressed in Eqs. (24), (25) and (26). Thereafter, internal forces C_{c1} in concrete, C_{s1} in bottom steel reinforcement, T_{s1} in top reinforcement and T_{t1} in ECC topping can be calculated. Substituting C_{c1} , C_{s1} , T_{s1} and T_{t1} into Eq. (20) gives

$$N_1 = C_{c1} + C_{s1} - T_{s1} - T_{t1} \quad (34)$$

Besides axial compression force N_1 , bending moment M_1 at section 1 can be computed from Eq. (21). Iteration procedure continues until equilibrium is satisfied at section 1, namely, calculated hogging moment M_1 is approximately equal to assumed value M_{10} within a limited tolerance ΔM_{tol} .

(b) Equilibrium at section 2

At section 2, neutral axis depth c_2 is assumed and strain ε'_{s2} of beam top reinforcement is determined from Eq. (30). Thus, strains ε_{s2} of bottom reinforcement and ε_{c2} of extreme concrete fibre can be quantified from the plane-section assumption, as expressed in Eqs. (12) and (14). Stress-strain models for ECC and steel reinforcement are employed to calculate internal forces C_{c2} , C_{s2} and T_{s2} acting on section 2. Corresponding axial force N_2 at section 2 can be calculated from Eq. (35).

$$N_2 = C_{c2} + C_{s2} - T_{s2} \quad (35)$$

Once equilibrium at section 2 is satisfied, namely, N_2 is close to N_1 at section 1 within a small tolerance ΔN_{tol} , sagging moment M_2 at section 2 can be computed.

(c) Compatibility of beam

Besides equilibrium, compatibility of the beam has also to be satisfied. Axial compressive strain ε_b of the beam is determined from Eq. (23). Rearranging Eq. (23) yields

$$\varepsilon_b = \frac{N}{bhE_c} \quad (36)$$

Support movement t can be calculated from Eq. (3).

Substituting ε_b , t , c_1 and c_2 into Eqs. (2) and (5) and rearranging give

$$l + 0.5\varepsilon_b(\beta - 2)l + t + t_0 - 0.5h \tan \Theta = (l(1 - \varepsilon_b) + (h - c_1) \tan(\varphi - \Theta) - c_2 \tan \varphi) \cos \varphi \quad (37)$$

Thus, neutral axis depths c_1 and c_2 are determined. Vertical load applied to the middle joint and associated displacement can be determined from Eqs. (16) and (6), respectively.