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A New Method for Calculation of Short Time-step G-functions of Vertical Ground Heat Exchangers

Jianping Wei^{1,2}, Lei Wang^{1,*}, Lei Jia¹, Wenjian Cai³

1. School of Control Science and Engineering, Shandong University, Jinan 250061, China;

2. Shandong Jianzhu University, Key Laboratory of Renewable Energy Utilization Technologies in Buildings of the National Education Ministry, Jinan 250101, China;

3. EXQUISITUS, Centre for E-City, School of Electrical and Electronic Engineering, Nanyang Technological University, Nanyang Avenue, Singapore 639798, Singapore

Highlights:

- A method to simplify the CMILS model of vertical GHE was proposed.
- G-function was easily implemented and quickly calculated by proposed method.
- Experimental verification was made through a reported reference data.
- Comparison studies showed the proposed method had a good performance.

Abstract: The composite medium infinite line source model (CMILS) was recently developed to describe the heat transfer of borehole ground heat exchanges. But this model is very complicated and time-consuming to calculate the solutions, which limit its practical applications. This paper presents a method to simplify the CMILS model and develops short time-step response g-functions using the simplified CMILS model. The advantage of the new short time-step response g-functions is that they are simple in calculation with comparable accuracy compared with CMILS model. Simulation studies show that g-functions calculated by simplified CMILS model match nicely with the experimental data. Detailed comparisons are also carried out between the simplified CMILS model and other numerical or analytical models through simulation.

Keywords: ground heat exchangers; g-function; simplified CMILS model; short time-step response; simulation

Nomenclature	
a_i	Thermal diffusivity, $i = 1, 2, s, g$, (m^2/s)
$a = \sqrt{a_1/a_2}$	Dimensionless thermal diffusivity
B	Distance between the adjacent boreholes, (m)
D	Buried distance of the head of borehole U-tubes, (m)

f, g, φ, ψ	Combined function used in CMILS model
Fo	Fourier number
g	g-function
H	Height of borehole, (m)
J_n	Bessel function of first kind of order n;
k_i	Thermal conductivity, $i = 1, 2, s, g$, ($W / (m \cdot K)$)
$k = k_2 / k_1$	Dimensionless thermal conductivity
* Corresponding author. Tel.: 13075351398; E-mail addresses: lei.wang@sdu.edu.cn	
q_1	Heat flux, (W / m)
(r', θ')	Cylindrical coordinate of infinite line source
r_b	Borehole radius, (m)
R, R'	Ratio of radius
t_s	Characteristic time, (s)
T_b	Average borehole wall temperature, (K)
T_g	Far-field temperature, (K)
T_p	Average U-tube wall temperature, (K)
$T_1(t, r, \theta)$	Temperature at point (r, θ) of region $r \leq r_b$, (K)
$T_2(t, r, \theta)$	Temperature at point (r, θ) of region $r > r_b$, (K)
Y_n	Bessel function of second kind of order n;
$\bar{T}_1$	Integral average temperature of $T_1(t, r, \theta)$, (K)
Subscripts	
1	Region $r \leq r_b$
2	Region $r > r_b$
s	Soil or ground

g	Grout
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1. Introduction

Energy consumption in building HVAC sector, accounting for about 37 percent of total building energy use [1], has received increasing attention in the last decades due to the energy shortage and environmental concerns. Ground source heat pump (GSHP) system, which is one of the cleanest, most energy-efficient and operational cost-effective systems, has been considered as one of the effective way to significantly reduce energy consumption for space air-conditioning of buildings [2]. However, the higher initial cost of ground heat exchangers (GHEs) impedes its wide acceptance. In order to efficiently reduce the length of GHEs and consequently the capital cost of the systems, it is crucial to develop a mathematical model which can accurately predict the heat transfer in and around boreholes to avoid costly over sizing of the GHEs. Among the several model variables, the short time response has significant influence on the GHE performance [3-5]. It is critical to 1) the heat flux build-up stages; 2) during the seasons when both heating and cooling are required; and 3) calculating hourly or sub-hourly thermal energy use.

Heat transfer process of GHEs depends on the soil temperature field. Most existing models were based on either analytical, numerical or combination of the two. There are three fundamental analytical models for outside borehole, i.e. infinite line source model (ILS) [6, 7], cylindrical heat source model (CHS) [8] and finite line source model (FLS) [9, 10]. These models are long time response models as the geometry of the borehole is assumed to be a line or a cylinder without considering the heat transfer features inside the borehole.

The concept of g-function which introduced by Eskilson [11] is adopted in this study. The g-function model, which defines a relationship (Eq.(1)) between the heat flux q extracted from or rejected to the ground at the borehole wall and the borehole wall temperature T_b , can be applied in all cases as it is the thermal response solution of a borehole at its wall. Because the g-function model based on numerical solution treats the borehole as a finite line source, it is only suited to the middle and long time

1 response[3, 12].

$$2 \quad g\left(\frac{t}{t_s}, \frac{r_b}{H}, \frac{B}{H}, \frac{D}{H}\right) = -\frac{2\pi k_s}{q}(T_b - T_g) \quad (1)$$

3 The typical bore field geometry and the parameters using in g-function definition
 4 [12] are described as Fig. 1. The g-function has four parameters: t/t_s , a
 5 non-dimensional time where t_s is a characteristic time ($t_s = H^2 / 9a_s$); r_b/H , the
 6 non-dimensional borehole radius where r_b is borehole radius and H is height of
 7 the borehole; B/H , the bore field aspect ratio where B is the distance between
 8 adjacent boreholes; and D/H , the fourth non-dimensional parameter where D is
 9 buried distance of the head of borehole U-tubes.

10 The g-function approach is considered as the state-of-the-art and has been
 11 implemented in many building energy simulation software including EED, Trnsys,
 12 Energy Plus, and so on.

13 Eskilson's g-functions are applicable for times greater than ($5r_b^2/a$) estimated by
 14 Eskilson which implies times of 3-6 hours for typical boreholes[13]. However,
 15 typically applicable times of short time g-functions are in the time scale of minutes.
 16 Yavuzturk [13] extended Eskilson's concept of non-dimensional temperature response
 17 function and developed a numerical model for short time-step response which
 18 approximated the geometry of U-pipes as pie sector shape as Fig.2. A total steady
 19 state borehole resistance was subtracted from the obtained pipe wall temperature to
 20 adhere to Eskilson's g-function definition (Eq.(1)). For a constant heat transfer rate,
 21 the temperature difference between U-pipe wall and far-field ground was calculated
 22 by the numerical model. This total resistance includes a convective resistance between
 23 fluid and U-pipes, a pipe resistance of U-tubes and the grout resistance. The method
 24 which subtracted out the resistance from the g-function resulted in a negative borehole
 25 wall temperature for times approaching zero [12, 14]. The short time-step model was
 26 validated using actual operating field data from an elementary school building.

27 Other numerical methods using commercial software packages are also widely

used to determine the value of g-functions [14-20].

However, most numerical approaches have the disadvantages of time-consuming and lack of flexibility for different geometry configuration in practical applications. G-functions of GHEs with different geometry configurations should be pre-calculated and stored in the programs as a database. Users are limited to these configurations and need an interpolation in using the database which may lead to computing errors.

Due to above reasons, analytical solutions [12, 21-26] to calculate the short and long time-step response of boreholes have generated a lot of interest from researchers. Sheriff [22] used the FLS model to redraw the g-functions where a different boundary condition was used. Cimmino et al. [12] compared the differences between the g-functions provided by Eskilson and those obtained using analytical solutions, ILS, CHS and FLS as showed in Fig. 3. Young [24] employed the borehole fluid thermal mass (BFTM) model by modifying the classical buried electrical cable (BEC) method and drawing an analogy between buried electric cable and vertical borehole. Beier et al. [27] developed a semi-analytical model, which first solved the borehole heat transfer problem in the Laplace domain and then used a numerical inversion to obtain the time domain solution. Javed and Claesson [4, 28, 29] also developed an analytical solution using the equivalent diameter method to determine the short-term response of borehole heat exchangers.

Lamarche et al. [25, 26] used an equivalent diameter approach (Fig. 4) and developed an analytical solution for the short term response of vertical boreholes that took into account the different thermal properties of grout and soil. A closed analytical form (Eq.(12)) of thermal response in the form of g-function can be obtained. This model gave a good agreement with COMSOL software[30] in which an actual U-tube configuration model of COMSOL was made.

Li et al. [31-34] developed a new composite media line source (CMILS) model based on Jaeger's infinite instantaneous line source solution in composite cylindrical media. This model was appropriate for the short time response as it took the grout into consideration. However, the CMILS model is fairly sophisticated, and its computation is time-consuming even with advanced numerical integration methods.

In this paper, a valuable simplified method is proposed to improve the calculation speed of short time-step g-functions of vertical GHE for the CMILS model. Although the simplified CMILS model looks also a bit complicated, it can be easily implemented by a computer program, and its calculating speed is faster than CMILS model. The effectiveness of the method is evaluated by comparison with experimental data and other g-functions obtained by other noteworthy analytical and numerical solutions through simulation. It shows that the g-functions calculated with the simplified CMILS model display a satisfying consistency with experimental data than those calculated by numerical and the other noteworthy analytical models for the short time-step response.

2. Composite media line source model and its simplification

2.1. Composite media line source model

In cylindrical coordinate (Fig. 5), there is an infinite line source in the axial direction. The infinite line source is located through a point (r', θ') with a constant heat flux $q_l (W/m)$. The region $r \leq r_b$ is of one medium having conductivity k_1 and diffusivity a_1 , and region $r > r_b$ is another, the corresponding quantities are k_2 and a_2 . The initial temperature was assumed constant and uniform for all mediums, i.e. fluid, grout, and soil. The infinite line source in composite media will continuously release heat flux into the composite solid from the initial time, and the temperature response at point (r, θ) can be obtained from these expressions:

$$\begin{cases} T_1(t, r, \theta) = \frac{q_l}{2\pi k_1} \sum_{n=-\infty}^{+\infty} \cos n(\theta - \theta') \int_0^{+\infty} (1 - e^{-v^2 Fo}) \frac{J_n(vR) J_n(vR') (\varphi g - \psi f)}{v(\varphi^2 + \psi^2)} dv & , r \leq r_b \\ T_2(t, r, \theta) = \frac{q_l}{\pi^2} \sum_{n=-\infty}^{+\infty} \cos n(\theta - \theta') \int_0^{+\infty} (1 - e^{-v^2 Fo}) \frac{J_n(vR') [\psi J_n(avR) - \varphi Y_n(avR)]}{v^2(\varphi^2 + \psi^2)} dv & , r > r_b \end{cases} \quad (2)$$

Where

$$Fo = \frac{a_1 t}{r_b^2}, R = \frac{r}{r_b}, R' = \frac{r'}{r_b}$$

And

$$\begin{aligned}
 \varphi &= akJ_n(v)J'_n(av) - J'_n(v)J_n(av) \\
 \psi &= akJ_n(v)Y'_n(av) - J'_n(v)Y_n(av) \\
 f &= akY_n(v)J'_n(av) - Y'_n(v)J_n(av) \\
 g &= akY_n(v)Y'_n(av) - Y'_n(v)Y_n(av)
 \end{aligned}$$

Subscripts 1 and 2 denote regions of $r \leq r_b$ and $r > r_b$; J_n and Y_n denote the Bessel functions of first kind and second kind of order n ; k_i and a_i ($i = 1, 2$) are thermal conductivity and thermal diffusivity respectively; a and k are dimensionless variables $k = k_2/k_1$, $a = \sqrt{a_1/a_2}$.

The formulation of g-functions can be expressed according to the definition of Eskilson (Eq.(1)):

$$g = \frac{2\pi k_2}{q_l} T_1(t, r_b, \theta) = \frac{k_2}{k_1} \sum_{n=-\infty}^{+\infty} \cos n(\theta - \theta') \int_0^{+\infty} (1 - e^{-v^2 F \theta}) \frac{J_n(vR) J_n(vR') (\varphi g - \psi f)}{v(\varphi^2 + \psi^2)} dv \quad (3)$$

2.2. Simplifying the composite media line source model

The expression of g-functions (Eq.(3)) based on CMILS model is so complicated that it is difficult to calculate the values because it involves the infinite integration and the first and second kind of Bessel functions with the infinite order.

The up and down legs of a single U-shape tube can be treated as two infinite line sources separately located at $(r', 0)$ and (r', π) (Fig. 6) with constant fluxes of $q_l/2$. Note that

$$\int_0^{2\pi} \cos n(\theta - \theta') d\theta = \begin{cases} 2\pi & , n = 0 \\ 0 & , n \neq 0 \end{cases} \quad (4)$$

the average temperature $\bar{T}_1$ of source (r, θ) can be obtained by integrating the temperature $T_1(t, r, \theta)$ along the circumference of the radius r .

For the source at $(r', 0)$, the temperature expression is

$$\begin{aligned}
 \bar{T}_1 \Big|_{\theta'=0} &= \frac{1}{2\pi} \int_0^{2\pi} T_1(t, r, \theta) \Big|_{\theta'=0} d\theta \\
 &= \frac{1}{2\pi} \int_0^{2\pi} \frac{q_l}{2\pi k_1} \sum_{n=-\infty}^{+\infty} \cos n(\theta - 0) \int_0^{+\infty} \left(1 - e^{-v^2 Fo}\right) \frac{J_n(vR) J_n(vR')(\varphi g - \psi f)}{v(\varphi^2 + \psi^2)} dv d\theta \\
 &= \frac{1}{2\pi} \frac{q_l}{2\pi k_1} \sum_{n=-\infty}^{+\infty} \left(\int_0^{2\pi} \cos n(\theta - 0) d\theta \right) \int_0^{+\infty} \left(1 - e^{-v^2 Fo}\right) \frac{J_n(vR) J_n(vR')(\varphi g - \psi f)}{v(\varphi^2 + \psi^2)} dv \quad (5) \\
 &= \frac{1}{2\pi} \frac{q_l}{2\pi k_1} \sum_{n=-\infty}^{+\infty} \left(\begin{cases} 2\pi, n=0 \\ 0, n \geq 1 \end{cases} \right) \int_0^{+\infty} \left(1 - e^{-v^2 Fo}\right) \frac{J_n(vR) J_n(vR')(\varphi g - \psi f)}{v(\varphi^2 + \psi^2)} dv \\
 &= \frac{q_l}{4\pi k_1} \int_0^{+\infty} \left(1 - e^{-v^2 Fo}\right) \frac{J_0(vR) J_0(vR')(\varphi g - \psi f)}{v(\varphi^2 + \psi^2)} \Big|_{n=0} dv
 \end{aligned}$$

And for (r', π) , it is

$$\bar{T}_1 \Big|_{\theta'=\pi} = \frac{q_l}{4\pi k_1} \int_0^{+\infty} \left(1 - e^{-v^2 Fo}\right) \frac{J_0(vR) J_0(vR')(\varphi g - \psi f)}{v(\varphi^2 + \psi^2)} \Big|_{n=0} dv \quad (6)$$

Adding these two temperatures, the average temperature can be obtained for the region $r \leq r_b$,

$$\bar{T}_1(t, r, \theta) = (\bar{T}_1 \Big|_{\theta'=0} + \bar{T}_1 \Big|_{\theta'=\pi}) = \frac{q_l}{2\pi k_1} \int_0^{+\infty} \left(1 - e^{-v^2 Fo}\right) \frac{J_0(vR) J_0(vR')(\varphi g - \psi f)}{v(\varphi^2 + \psi^2)} \Big|_{n=0} dv \quad (7)$$

So the g-functions of single U-shape tubes can be expressed as

$$\begin{aligned}
 g_{s-CMILS-s} &= \frac{2\pi k_2}{q_l} \bar{T}_1(t, r_b, \theta) \\
 &= k \int_0^{+\infty} \left(1 - e^{-v^2 \left(\frac{1}{9} \left(\frac{r_b}{H} \right)^{-2} \frac{t}{t_s} \right)} \right) \frac{J_0(v) J_0(vR')(\varphi g - \psi f)}{v(\varphi^2 + \psi^2)} \Big|_{n=0} dv \quad (8)
 \end{aligned}$$

For double U-shape tubes, its four legs can be treated as four infinite line sources separately located at $(r', 0)$, $(r', \pi/2)$, (r', π) and $(r', 3\pi/2)$ (Fig. 7) with constant flux of $q_l/2$ for parallel connection. Its average temperature is

$$\begin{aligned} \bar{T}_1(t, r, \theta) &= \left(\bar{T}_1 \Big|_{\theta'=0} + \bar{T}_1 \Big|_{\theta'=\frac{\pi}{2}} + \bar{T}_1 \Big|_{\theta'=\pi} + \bar{T}_1 \Big|_{\theta'=\frac{3\pi}{2}} \right) \\ &= \frac{q_l}{\pi k_1} \int_0^{+\infty} \left(1 - e^{-v^2 F_0} \right) \frac{J_0(vR) J_0(vR') (\varphi g - \psi f)}{v (\varphi^2 + \psi^2)} \Big|_{n=0} dv \end{aligned} \quad (9)$$

So the g-functions of double U-shape tubes can be expressed as

$$\begin{aligned} g_{s-CMILS-d} &= \frac{2\pi k_2}{q_l} \bar{T}_1(t, r_b, \theta) \\ &= 2k \int_0^{+\infty} \left(1 - e^{-v^2 \left(\frac{1}{9} \left(\frac{r_b}{H} \right)^{-2} \frac{t}{t_s} \right)} \right) \frac{J_0(vR) J_0(vR') (\varphi g - \psi f)}{v (\varphi^2 + \psi^2)} \Big|_{n=0} dv \end{aligned} \quad (10)$$

By simplification the expressions of g-functions are reduced to zero and first order Bessel functions. The simplification will accelerate the computation reduction great deal. The formulations can be used in programming software for design and simulation.

3. Simulation and verification

3.1. Experimental verification

Beier et al. [27] reported a set of reference data which were obtained from a medium-scale sandbox experiment. The geometric and input parameters for the simulation are listed in Table 1.

From the experimental data we can obtain the value of the ground thermal conductivity k_s , the grout thermal conductivity k_g , the far-field temperature T_g , the borehole wall temperature T_b and the heating rate per-meter q_l/H . Using the definition of Eq. (1), Fig. 8 shows the plot of the g-functions: 1) from the experimental data; 2) by two-dimensional finite volume model of Yang [35], which is used to examine the short-time performance of CMILS model using the same reference data set; 3) by simplified CMILS model. It can be observed that the simplified CMILS model and Yang's numerical model fit the experimental data in short time-step response relatively well. The error between the simplified CMILS model and Yang's model decreases along time increasingly. But errors between experimental data increase over time. They agree with each other within 1.5% for the

1 short time-step response.

2 3.2. Model comparison and analysis

3 The g-functions generated by ILS, CHS and FLS model are plotted in Fig. 9 for
 4 comparing with the simplified CMILS model. From Fig. 9 the fact can be observed
 5 that the temperature rise resulted from the simplified CMILS model has a relatively
 6 smaller value in short time response than other models because the heat capacity of
 7 grout can be fully considered, and has a good agreement in middle time. The
 8 influence of heat capacity of the grout can be seen from Fig. 11 where the thermal
 9 conductivities of grout and ground are the same. So it can be concluded that the
 10 capacity of grout has a strong influence on the g-functions especially in a short time
 11 and the simplified CMILS model is good at short time response because of its
 12 consideration of the thermal properties of the grout. Error analysis from Fig. 10 shows
 13 that the errors between ILS, FLS and simplified CMILS are smaller because they all
 14 approximate the U-tube as a line source. But because the influences of the ground
 15 surface are not considered, the response g-function of simplified CMILS model is
 16 similar as ILS and CHS that all continue to increase indefinitely while the FLS
 17 converges to a steady state value in a great time. So the error between FLS and
 18 simplified CMILS will increase over time.

19 In order to make a comparison for convenience, the expressions of the CHS
 20 model and Lamarche's model at r_b are written below,

$$21 \quad g_{CHS} = \frac{1}{\pi^2} \int_0^{+\infty} \left(1 - e^{-v^2 \left(\frac{1}{9} \left(\frac{r_b}{H} \right)^{-2} \frac{t}{t_s} \right)} \right) \frac{J_1(v)Y_0(v) - J_0(v)Y_1(v)}{v^2 (J_1^2(v) + Y_1^2(v))} dv \quad (11)$$

$$22 \quad g_{Lam} = \frac{8k^2}{\pi^3 \delta^2} \int_0^{+\infty} \left(1 - e^{-v^2 \left(\frac{1}{9} \left(\frac{r_b}{H} \right)^{-2} \frac{t}{t_s} \right)} \right) \frac{[Y_0(v\delta)J_1(v) - J_0(v\delta)Y_1(v)]}{v^4 (\phi^2 + \psi^2)} dv \quad (12)$$

23 Where,

$$24 \quad \begin{aligned} \phi &= Y_1(v) [Y_0(v\delta a)J_1(v\delta) - Y_1(v\delta a)J_0(v\delta)ka] \\ &\quad - J_1(v) [Y_0(v\delta a)Y_1(v\delta) - Y_1(v\delta a)Y_0(v\delta)ka] \end{aligned}$$

$$\psi = J_1(v) \left[J_0(v\delta a) Y_1(v\delta) - J_1(v\delta a) Y_0(v\delta) ka \right] \\ - Y_1(v) \left[J_0(v\delta a) J_1(v\delta) - J_1(v\delta a) J_0(v\delta) ka \right]$$

And

$$k = k_s / k_g, a = \sqrt{a_g / a_s}, \delta = r_b / r_e;$$

J_0, J_1, Y_0 and Y_1 are the Bessel functions of first kind and second kind of order 0 and 1.

Compared with equations (8), (11) and (12), it can be observed that the simplified CMILS model, CHS model and Lamarche's model have the same form with the same integration ranges and almost identical for the first parts except for Eq. (12). The second parts of integrations are the combination of Bessel functions, the integrations of second parts, however, are different which will cause some different behaviors. The properties of grout are considered by Lamarche's model and simplified CMILS model, but CHS model is not. The difference between them in short time response occurs as revealed in Fig. 12. At short time, Lamarche's model and simplified CMILS model have a lesser value than the CHS model. But for large time, there are almost no differences between them. Therefore, the simplified CMILS model is mainly for improving the short time response performance.

Both Lamarche's and simplified CMILS models are suitable for short time response. However, Lamarche's model requires equivalent radius due to the use of equivalent diameter. The influence of equivalent radius has a huge impact on Lamarche's model which can be observed from Fig. 13 where four methods of equivalent radius (Bose (Eq.(13)) [36], Kavanaugh (Eq.(14)) [8], Sutton et al. (Eq.(15)) [37] and Gu and O'Neal (Eq.(16)) [38]) are used to calculate different g-function value [25, 26].

$$r_{eq} = \sqrt{2} r_0 \quad (13)$$

$$r_{eq} = \sqrt{2} r_0 + x \quad (14)$$

$$r_{eq} = r_b \exp \left\{ \frac{2\pi}{\beta_0 \left(\frac{r_b}{r_0} \right)^{\beta_1}} \right\} \quad (15)$$

$$r_{eq} = \sqrt{r_0 (2r_0 + x)} \quad (16)$$

where r_0 is radius of U-pipes, x is the distance (Fig. 4) between two legs of U-tube and β_0 β_1 are coefficients.

However, the biggest disadvantage of the Lamarche's method is that there is no relationship between the equivalent radius and the shank spacing of U-tube legs which often changes in the field due to the installation error. While simplified CMILS model can display the effect of the distance of U-tube legs clearly as shown in Fig. 14. The greater the distance of U-tube legs, the closer the U-tube legs to borehole wall. The heat transfer will strengthen between the U-tube and the borehole wall. So the g-function will become large with the distance increasing as shown in Fig. 14.

The comparison between short time-step g-function of Yavuzturk [13], long time-step g-function of Eskilson [11] and the simplified CMILS model is shown in Fig. 15. When the dimensionless time $\ln(t/t_s)$ is less than -8.5, the g-function is calculated with the short time-step method of Yavuzturk, while others were generated using long time-step g-function method of Eskilson. It can be seen that the short time-step g-function developed by Yavuzturk becomes negative when the non-dimensional time is less than -13 as the total resistance was subtracted. When it is larger than 0, the curve of long time-step g-function of Eskilson reaches a plateau while the simplified CMILS model is kept rising. The difference is caused by the difference in the models used, i.e. a finite line source and an infinite line source. When non-dimensional time is between -12 and -3, it shows good agreement between them. Therefore, the simplified CMILS model is validated for short and middle time-step response g-functions.

4. Conclusions

This paper develops a new valuable method for the calculation of short time-step g-functions of vertical ground heat exchangers by simplifying the composite-medium infinite line-source model. Detailed simplifying procedures are provided and particular comparisons with other typical numerical or analytical models are performed. The performance of simplified CMILS model is evaluated by use of a reported reference experimental data, and the main conclusions drawn from this research are that

1) By simplifying, the CMILS model is easily implemented by numerical integration methods and its numerical evaluations are expected to be very quick.

2) The simplified CMILS model is a new and applicable method to calculate the short time-step g-functions of vertical GHE with acceptable accuracy compared with the other noteworthy analytical models, such as ILS, CHS and FLS models, because it takes the consideration of grout.

3) Comparison with Lamarche's model using the equivalent diameter method, simplified CMILS model can evaluate the effect of heat transfer explicitly when the shank spacing of U-tube legs is changed.

4) For thermal response test (TRT) simplified CMILS model may reduce the TRT duration time and approximate the parameters of grout and ground at the same time in one test.

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11

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Fig. 1. Schematic layout of typical bore field geometry[12]

Fig. 2. Yavuzturk's numerical model using simplified pie-sector representation for single U-tube pipes [13]

Fig. 3. G-functions for a single borehole calculated by Eskilson and three analytical models[12]

Fig. 4. Equivalent cylinder used by Lamarche

Fig. 5. Schematic layout of the CMILS model

Fig. 6. Two line sources located at $(r', 0)$ and (r', π) for single U-tubes

Fig. 7. Four line source for double U-tubes

Fig. 8. Difference between Yang[35], experimental data[27] and Simplified CMILS model

Fig. 9. G-functions generated by ILS[6, 7], CHS[8], FLS[9, 10] and simplified CMILS model

Fig. 10 Errors between ILS, FLS, CHS and simplified CMILS model

Fig. 11. G-functions of different models with same thermal conductivities of grout and ground

Fig. 12. Comparison between CHS[8], simplified CMILS and Lamarche's Models[26]

Fig. 13. Lamarche's model with different method of equivalent radius

Fig. 14. CMILS and Lamarche's models[26] with changing U-tube legs

Fig. 15. Comparison between models of Yavuzturk[3], Eskilson [11] and simplified CMILS

1 Table 1. Parameters used in the model validation [27, 32].

Parameters	Description	Value
T_g	Initial temperature of sand and grout	22(°C)
r_b	Outer radius of aluminum pipe (modeled bore)	6.5(cm)
H	Length of bore and U-shaped tube	18.3(m)
B	Half distance between centers of U-tube pipes	2.65(cm)
k_p	Thermal conductivity of U-shaped pipe	0.39($W \cdot m^{-1} \cdot K^{-1}$)
k_s	Thermal conductivity of used wet sand	2.82($W \cdot m^{-1} \cdot K^{-1}$)
k_g	Thermal conductivity of used grout	0.73($W \cdot m^{-1} \cdot K^{-1}$)
G_f	Average volumetric flow rate of circulating fluid	0.197($L \cdot s^{-1}$)
q_l	Average heating rate of electric heater	1056(W)
C_s	Estimated volumetric heat capacity of wet sand	3.2E+6($J \cdot m^{-3} \cdot K^{-1}$)
C_g	Estimated volumetric heat capacity of grout	3.8E+6($J \cdot m^{-3} \cdot K^{-1}$)

2