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ABSTRACT

In this paper, we theoretically and experimentally present a precision method to simultaneously estimate the phase and amplitude variations of light based on the weak-value amplification. We demonstrate that the imaginary and real components of the weak-value correspond to the amplification of the phase and amplitude variations, respectively. If both the phase and the amplitude are varied, the phase measurement using the imaginary weak-value is not influenced by the amplitude shift and the amplitude measurement using the real weak value is not influenced by the phase shift. Thus, the tiny changes of the phase and amplitude of light can be obtained simultaneously with different post-selections, and the measurement precision reaches $\sim 10^{-6}$ rad. These findings may be favorable for highly sensitive optical polarization-dependent effects, including optical activity and magneto-optic effects.

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Polarization, including the phase and amplitude, is a very important characteristic of light. Here, the relative variations of the phase and amplitude of two basis vectors correspond to different physical mechanisms, the former is derived from the propagation velocity differences of two eigenvectors and the latter is the absorption coefficient deviations. The estimation of the polarization variation of light has potential applications in many fields, such as birefringence, optical activity, complex optical susceptibility of materials, absorption and dispersion, and magneto optic Faraday and Kerr effects.^{1–6} Generally, the variations of the phase and amplitude always take place at the same time, leading to the variations of the contrast of intensity or polarization rotation. Because both the phase and amplitude of light carry information about the medium in which the light propagates, simultaneous estimations of the phase and amplitude variations are of great significance for understanding the physical process; on the one hand,

it can get full information of the polarization variations (phase and amplitude) efficiently without additional and independent measurement of polarization rotation or ellipticity of light; on the other hand, simultaneous estimation is helpful to avoid introducing additional noises, such as small optical path fluctuations and the change of the biological or chemical medium to be measured.

The standard tool measuring the phase and amplitude variations simultaneously is ellipsometry, which is usually used to measure the thickness and refractive index of thin films.⁷ For the optical activity measurement, Fourier transform spectral interferometry uses the isolated ultrashort pulse to simultaneously retrieve the optical rotatory dispersion and circular dichroism spectra corresponding to the phase and amplitude variations of the left- and right-circularly polarized light.^{1,8} In digital modulation and communication systems, the in-phase and quadrature (IQ) demodulation can simultaneously obtain

the phase and amplitude of the complex signal.^{9–11} How to further improve the measurement precision of the phase and amplitude variations is always of concern in modern physics.

The appearance of the weak-value amplification (WVA) technique provides a chance to solve these issues. The WVA first proposed by Aharonov, Albert, and Vaidman in 1988 has proved to be an effective way for high precision metrology.¹² Based on the WVA scheme, the variation of the signal is enhanced while the technical noise is suppressed.^{13–16} Recently, the WVA has become a powerful tool for the precision measurement of small parameters due to its anomalous amplification, such as beam shifts,^{17–20} optical phases,^{21–24} polarization rotations,²⁵ frequency shifts,¹³ refractive index variations,^{26–29} single photon Kerr non-linearity,^{30–32} and nanometal film thickness.³³ However, previously reported methods based on WVA usually focused on an individual phase or amplitude variation estimation.

In this work, the theoretical model of the WVA formalism and the experimental setup are established to simultaneously estimate the phase and amplitude variations of two eigenvectors. We reveal that the imaginary and real components of the weak-value can be used to amplify the phase and amplitude variations of light. Experimental results demonstrate the high precision and feasibility of the system.

To establish the theoretical model for determining the phase and amplitude based on WVA, we begin with considering the initial state of the system pre-selected in a superposition state $|\psi_i\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$. Here, $|0\rangle$ and $|1\rangle$ are the eigenvectors of $\hat{A}$, where $\hat{A} = |0\rangle\langle 0| - |1\rangle\langle 1|$ is the observable of a two-level system. After introducing tiny changes of the phase and amplitude between the two eigenvectors $|0\rangle$ and $|1\rangle$, which can be represented by an operation $U = \exp[(i\alpha + \beta)\hat{A}/2]$, the polarization state of the whole system evolves into

$$|\psi_f\rangle = U|\psi_i\rangle = \frac{1}{\sqrt{2}} \left[\exp\left(\frac{i\alpha + \beta}{2}\right)|0\rangle + \exp\left(-\frac{i\alpha + \beta}{2}\right)|1\rangle \right], \quad (1)$$

where $\alpha \ll 1$ and $\beta \ll 1$ denote the tiny variations of the phase and amplitude, respectively. Next, the light beam is post-selected in

$$|\psi_f\rangle = \cos\left(\frac{\pi}{4} - \gamma\right) \exp(-i\chi)|0\rangle - \sin\left(\frac{\pi}{4} - \gamma\right) \exp(i\chi)|1\rangle, \quad (2)$$

where $\gamma \ll 1$ and $\chi \ll 1$ are the post-selected parameters, describing the deviation from the orthogonal state of $|\psi_i\rangle$. Then, the weak-value of the observable $\hat{A}$ can be expressed as

$$A_w = \frac{\langle\psi_f|\hat{A}|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle}, \quad (3)$$

where the imaginary and real parts of the weak-value, respectively, take the forms

$$\text{Im}(A_w) \simeq -\frac{\tan\chi}{\tan^2\gamma + \tan^2\chi}, \quad (4)$$

$$\text{Re}(A_w) \simeq \frac{\tan\gamma}{\tan^2\gamma + \tan^2\chi}. \quad (5)$$

Note that χ and γ generate the imaginary and real components of the weak-value. The intensity of the post-selected light is given by

$$I = I_0 \left| \langle\psi_f|\exp\left(\frac{i\alpha + \beta}{2}\hat{A}\right)|\psi_i\rangle \right|^2 \simeq I_0 |\langle\psi_f|\psi_i\rangle|^2 [1 - \alpha \text{Im}(A_w) + \beta \text{Re}(A_w)], \quad (6)$$

where I_0 denotes the initial intensity of light without post-selection. The approximation in Eq. (6) is feasible with $|A_w|^2(\alpha^2 + \beta^2)/4 \ll 1$. The intensity of the post-selected light is proportional to the post-selected probability $|\langle\psi_f|\psi_i\rangle|^2$, and the variation of the intensity is closely related to the weak-value. As we can see, the variations of the phase and amplitude can be amplified by the imaginary and real parts of the weak-value.

Therefore, by choosing different post-selection states, we can construct imaginary and real weak-values to estimate the phase and amplitude variations. As shown in Fig. 1, the light is simultaneously post-selected by different post-selection states. For the phase variation estimation, by making $\gamma = 0$ in Eq. (2), the post-selection state evolves into

$$|\psi_{f(\alpha)}^\pm\rangle = \frac{1}{\sqrt{2}} [\exp(\mp i\chi)|0\rangle - \exp(\pm i\chi)|1\rangle], \quad (7)$$

and a pure imaginary weak-value is obtained with

$$A_{w(\alpha)}^\pm = \mp i \cot\chi. \quad (8)$$

For the amplitude variation estimation, by making $\chi = 0$ in Eq. (2), the post-selection state is

$$|\psi_{f(\beta)}^\pm\rangle = \cos\left(\frac{\pi}{4} \mp \gamma\right)|0\rangle - \sin\left(\frac{\pi}{4} \mp \gamma\right)|1\rangle, \quad (9)$$

and a pure real weak-value is obtained with

$$A_{w(\beta)}^\pm = \pm \cot\gamma. \quad (10)$$

In the above analysis, $\pm$ denotes two symmetry post-selection states with respect to the polarization direction being orthogonal to the pre-selection. Finally, the corresponding measured intensities of the post-selected light can be expressed as

FIG. 1. The detection diagram for estimating the phase and amplitude shifts simultaneously. PBS, polarizing beam splitter; BS, beam splitter. Tiny phase and amplitude variations (α and β) are introduced between the states $|0\rangle$ and $|1\rangle$. The estimations of α and β are obtained by two different post-selection states which are used to construct pure imaginary and real weak-values.

$$I_{\alpha}^{\pm} = \frac{1}{2} I_0 \sin^2 \chi \left[\cos \alpha \left(1 - |A_{\omega(\alpha)}^{\pm}|^2 \right) + \cosh \beta \left(1 + |A_{\omega(\alpha)}^{\pm}|^2 \right) - 2 \operatorname{Im} \left(A_{\omega(\alpha)}^{\pm} \right) \sin \alpha \right] \\ \simeq I_0 \sin^2 \chi \left[1 - \alpha \operatorname{Im} \left(A_{\omega(\alpha)}^{\pm} \right) \right], \quad (11)$$

$$I_{\beta}^{\pm} = \frac{1}{2} I_0 \sin^2 \gamma \left[\cos \alpha \left(1 - |A_{\omega(\beta)}^{\pm}|^2 \right) + \cosh \beta \left(1 + |A_{\omega(\beta)}^{\pm}|^2 \right) + 2 \operatorname{Re} \left(A_{\omega(\beta)}^{\pm} \right) \sinh \beta \right] \\ \simeq I_0 \sin^2 \gamma \left[1 + \beta \operatorname{Re} \left(A_{\omega(\beta)}^{\pm} \right) \right]. \quad (12)$$

From Eqs. (11) and (12), by choosing the appropriate post-selection states, the pure imaginary weak-value is used to amplify the phase shift while the pure real weak-value is used to enhance the amplitude shift. Subsequently, we obtain the differential intensities of both the phase and amplitude shifts, which take the forms $I_{\alpha}^{\delta} = I_{\alpha}^{+} - I_{\alpha}^{-} = 2 \sin \alpha |A_{\omega(\alpha)}| I_0 \sin^2 \chi$ and $I_{\beta}^{\delta} = I_{\beta}^{+} - I_{\beta}^{-} = 2 \sinh \beta |A_{\omega(\beta)}| I_0 \sin^2 \gamma$. However, compared with the output intensity $I_{\alpha, \beta}$, the signal variation $I_{\alpha, \beta}^{\delta}$ may be pretty small and is possible to be covered by the background noise. In this case, it is not always feasible to estimate the phase and amplitude shifts by the differential intensities in practical applications. Therefore, the intensity contrast ratios (ICRs) are introduced to reveal the role of the WVA in this scheme, which are, respectively, given by

$$\eta_{\alpha} = \frac{I_{\alpha}^{\delta}}{I_{\alpha}^{\text{ave}}} = \frac{4 \sin \alpha |A_{\omega(\alpha)}|}{\cos \alpha \left(1 - |A_{\omega(\alpha)}|^2 \right) + \cosh \beta \left(1 + |A_{\omega(\alpha)}|^2 \right)} \simeq 2 \alpha |A_{\omega(\alpha)}|, \quad (13)$$

$$\eta_{\beta} = \frac{I_{\beta}^{\delta}}{I_{\beta}^{\text{ave}}} = \frac{4 \sinh \beta |A_{\omega(\beta)}|}{\cos \alpha \left(1 - |A_{\omega(\beta)}|^2 \right) + \cosh \beta \left(1 + |A_{\omega(\beta)}|^2 \right)} \simeq 2 \beta |A_{\omega(\beta)}|, \quad (14)$$

where $I_{\alpha, \beta}^{\text{ave}} = (I_{\alpha, \beta}^{+} + I_{\alpha, \beta}^{-})/2$ is the average intensity of the post-selected light, which nearly always keeps constant for a given post-selection parameter. The ICR describes the ratio of the differential intensity $I_{\alpha, \beta}^{\delta}$ (detected signal) to the average intensity of the post-selected light $I_{\alpha, \beta}^{\text{ave}}$. In recent years, much attention has been focused on the effectiveness of WVA,^{34–36} and it has been shown that the intrinsic precision cannot be improved by the WVA technique in the case of without any technical noise. In general, there are a variety of technical noises in the practical measurements. In our scheme, the main technical noise is relative intensity noise (RIN) which is proportional to the post-selected intensity,²³ such as power fluctuation of light and air turbulence. Thus, to a certain degree, the ICR can be regarded as the ratio of the signal variation induced by the phase or amplitude shift to the magnitude of the noise [i.e., the signal-to-noise ratio (SNR)]. From Eqs. (13) and (14), the ICRs are proportional to the corresponding weak-values, so a larger ICR can be obtained by constructing a larger weak-value, which implies a higher measurement precision. However, it should be noted that the ICR is not completely equivalent to the SNR because of the existence of other noise sources, such as electric noise of the detector and shot noise. In addition, owing to the large reduction of the light intensity by the post-selection process, we can use a highly sensitive detector with low saturation intensity and electronic noise to measure the light intensity.

As we can see, Eq. (13) shows that the ICR η_{α} is associated with the phase shift while it is independent of the amplitude shift. Remarkably, Eq. (14) shows that the ICR η_{β} is associated with the amplitude shift while it is independent of the phase shift. Therefore, both the phase and amplitude variations of light can be estimated simultaneously with the ICR detection by different post-selections. This is the main result of this work.

In the case without any approximation in Eqs. (13) and (14), Fig. 2(a) shows η_{α} changing with the phase shift under the condition of different amplitude shifts, while Fig. 2(b) shows η_{β} varying with the amplitude shift in the case of different phase shifts. From Fig. 2, we can see that these curves are almost coincident, which shows that the estimations of the phase and amplitude shifts are not influenced by each other. These results provide a chance to estimate the phase and amplitude shifts simultaneously, see Fig. 1. However, with the phase or amplitude variation further expanded, the approximation $|A_{\omega(\alpha, \beta)}|^2 (\alpha^2 + \beta^2)/4 \ll 1$ is no longer satisfied, ICR of the phase (or amplitude) becomes dependent on amplitude (or phase).

In order to verify the principle of this WVA method, we design an experiment to estimate the variations of the phase and amplitude. The experimental setup is shown in Fig. 3(a), where we consider the horizontal and vertical polarization states ($|H\rangle$ and $|V\rangle$) as an example of the eigenvectors ($|0\rangle$ and $|1\rangle$). The light source is a He-Ne laser (Thorlabs HNL210) at a wavelength of 632.8 nm. Then, the light beam passes through the binary compound zero-order half wave plate (HWP) for adjusting the light intensity. In our case, the first Glan polarizer (P1) and the first quarter wave plate (QWP1) play the role of the pre-selection with their optical axes placed at the angle of $\pi/4$ with respect to the horizontal direction. In the presence of a slight rotation of P1 or QWP1, the tiny variations of the phase and amplitude between the horizontal and vertical polarization components will appear, see Fig. 3(b). In the case of the existence of QWP1, $\alpha \neq 0$ and $\beta = 0$ are obtained by rotating P1, and $\alpha = \beta = \kappa$ can be produced by rotating QWP1. For the case without QWP1, by rotating P1, $\alpha = 0$ and $\beta \neq 0$ are selected.

Next, the combination of the second quarter wave plate (QWP2) and the second Glan polarizer (P2) is used to prepare the different post-selections, as shown in Fig. 3(c). Under the condition of the existence of QWP2, where the optical axes of the QWP2 and P2 are both rotated by $-\pi/4$ with respect to the horizontal direction, by rotating P2, the post-selection in Eq. (7) is constructed, and thus, a purely imaginary weak-value described by Eq. (8) is obtained. For the case without QWP2, by rotating P2, the post-selection in Eq. (9) is

FIG. 2. (a) The theoretical predictions of the ICR η_{α} versus the phase shift α for different amplitude shifts β . (b) The theoretical predictions of the ICR η_{β} versus the amplitude shift β for different phase shifts α . The post-selected parameters are chosen as $\chi, \gamma = 0.01$ rad.

FIG. 3. (a) Experimental setup. Light source, He-Ne laser; HWP, half-wave plate; P1 and P2, Glan laser polarizers; QWP1 and QWP2, binary compound zero-order quarter wave plates; CCD, Charge-coupled device. (b) Three different incident polarizations to be estimated. (c) Two different post-selection states for the phase and amplitude shift estimations.

constructed, and thus, a purely real weak-value deduced from Eq. (10) is obtained. Finally, the intensity of the post-selected light is recorded by the CCD (Thorlabs BC106NVIS).

At first, we only measured the phase (or amplitude) shift in the absence of the amplitude (or phase) shift (i.e., $\beta = 0$ or $\alpha = 0$). The post-selected parameters are chosen as $\chi, \gamma = 0.007, 0.01$, and 0.02 rad. Figures 4(a) and 4(b) show the differential intensities I_{α}^{δ} and I_{β}^{δ} as functions of the phase shift α and amplitude shift β , respectively. The corresponding ICRs (η_{α} and η_{β}) have been calculated by using the measured intensities, see Figs. 4(c) and 4(d). Both the differential intensities (I_{α}^{δ} and I_{β}^{δ}) and ICRs (η_{α} and η_{β}) show high sensitivity to the variations of the phase and amplitude, respectively. In addition, it is worth noting that a larger ICR can be achieved by choosing a smaller post-selected parameter.

Then, we also measured the phase (or amplitude) shift in the presence of the amplitude (or phase) shift (i.e., $\alpha = \beta = \kappa$). Interestingly, the experimental results from Fig. 5 are similar to those from Fig. 4, which shows that the existence of the amplitude (or phase) shift has no influence on the measurement of the phase (or amplitude) shift by using a pure imaginary (or real) weak-value. This is an obvious advantage of the presented method based on WVA. In addition, the experimental results are also satisfied with the theoretical analysis from Fig. 2.

FIG. 4. Experimental and theoretical results for the individual measurement of the phase or amplitude shift. (a) and (b) show the differential intensity I_{α}^{δ} and I_{β}^{δ} changing with the phase and amplitude shifts in the case of $\beta = 0$ and $\alpha = 0$, respectively. The post-selected angles χ and γ are both chosen as $0.007, 0.01$, and 0.02 rad. (c) and (d) describe the corresponding ICRs η_{α} and η_{β} for different phase and amplitude shifts, respectively. The lines and dots denote the theoretical predictions and experimental results, respectively.

FIG. 5. Experimental and theoretical results for simultaneous determination of the phase and amplitude shifts. (a) and (b) stand for the differential intensity I_{α}^{δ} and I_{β}^{δ} changing with the phase and amplitude shifts in the case of $\alpha = \beta = \chi$, respectively. The post-selected angles χ and γ are both chosen as 0.007, 0.01, and 0.02 rad. (c) and (d) show the corresponding ICRs η_{α} and η_{β} for different phase and amplitude shifts. The lines and dots denote the theoretical predictions and the experimentally measured values.

Finally, we discuss the measurement precision of the phase and amplitude changes, which could be estimated by using $\Delta\alpha = \Delta I_{\alpha}^{\delta} / (\partial I_{\alpha}^{\delta} / \partial \alpha)$ and $\Delta\beta = \Delta I_{\beta}^{\delta} / (\partial I_{\beta}^{\delta} / \partial \beta)$, respectively. In our experiment, the intensity of the post-selected light was recorded 50 times by the CCD, and the standard deviation of the experimentally measured values was $\Delta I_{\alpha}^{\delta}$ and $\Delta I_{\beta}^{\delta}$, which are closely correlated with the technical noise. For three different post-selected parameters (0.007, 0.01, and 0.02 rad), the corresponding average intensities of the post-selected light ($I_{\alpha, \beta}^{\text{ave}}$) are always about 0.20, 0.46, and 1.84 uW, as well as the standard deviations ($\Delta I_{\alpha}^{\delta}$ and $\Delta I_{\beta}^{\delta}$) are always, respectively, less than 0.15, 0.3, and 1.17 nW. Therefore, the corresponding measurement precision of the phase and amplitude is calculated to be 2.7×10^{-6} , 3.6×10^{-6} , and 7×10^{-6} rad, which is nearly two orders of magnitude higher than that of the wide spectrum light phase weak measurements.^{21,22} Moreover, the measurement precision is positively correlated with the ICR.

To conclude, we have provided a high precision measurement method for simultaneous estimations of the phase and amplitude variations of light based on WVA. The WVA technique significantly enhances the detected signal with respect to the relative intensity noise. By using different post-selection techniques, a pure imaginary weak-value is constructed to estimate the phase shift, and the corresponding ICR η_{α} is independent of the amplitude shift, while a pure real weak-value is constructed to estimate the amplitude shift, and the corresponding ICR η_{β} is independent of the phase shift. Therefore, the complete information of the polarization state variation of light can be deduced from our WVA scheme. The phase and amplitude variations can be estimated with precision of the order of $\sim 10^{-6}$ rad. This method may open the possibility of accurately determining tiny signals from the circular dichroism and optical rotatory dispersion.

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