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Generalized GNSS Signal Carrier Tracking: Part II - Optimization and Implementation

Rong Yang, Yu Morton, Keck-Voon Ling, and Eng-Kee Poh

Abstract—This paper presents the performance analysis and the optimizations of the generalized phase locked loop (PLL) and the generalized frequency locked loop (FLL). Using the minimum mean square error (MMSE) criterion as a performance measure, the optimal solutions for the generalized PLL and FLL designs under various scenarios: strong and weak signal strength, low and high dynamic, and good and poor oscillator quality are investigated. An adaptive PLL based on automatic adjustment of the integration time and loop parameters in an optimized manner is proposed. An adaptive FLL that operates with the optimal loop parameters is also presented. Simulation results demonstrate the effectiveness of the proposed generalized closed-loop tracking architecture and validate the proposed adaptive tracking schemes.

Index Terms—GNSS receiver, state feedback control, optimization, adaptive phase locked loop, adaptive frequency locked loop.

I. INTRODUCTION

The carrier tracking loop has been widely used in [1] [2] and analyzed [3]–[5] for GNSS signal processing over the past several decades. In recent years, there is a growing demand for GNSS receivers to operate in challenging environments, such as the crowded urban canyons [6], indoors [7], in equatorial and high latitude areas during ionospheric scintillation [8], and on highly dynamic airborne or space-borne platforms [9]. For these applications, the received GNSS signals may experience deep fading and/or high signal dynamics.

Significant effort has been devoted to the design of carrier tracking loops with proportional integral filter (PIF) and Kalman filter (KF) under weak signal conditions [5] [6] [7] [10], for receivers on highly dynamic platforms [9] [11] [12], and in environments where both weak signals and high dynamics conditions co-exist [13]. For a PIF-based tracking loop, the filter coefficients are determined by the product of the noise equivalent bandwidth and integration time. For a KF-based implementation, the signal noise characteristics drives the filter design. These filter designs can be optimized to improve tracking performance. For the PIF-based tracking loop, an optimal bandwidth can be found by adjusting the filter coefficients based on signal carrier-to-noise ratio (C/N_0) [14] [15]. For example, reference [14] presented an adaptive phase locked loop (PLL)

that computes an optimal loop bandwidth based on the discriminator outputs and pre-defined discriminator threshold values. To reduce the computational burden in the real time bandwidth calculation, a look-up table of the input C/N_0 and some pre-defined platform dynamics were used in [15]. Adaptive KF tracking methods with adjustable filter gains controlled by the signal C/N_0 or the equivalent noise bandwidth are discussed in [16] and [17].

The typical strategy for weak signal processing is to increase the integration time. This strategy, however, runs counter to the requirements for tracking signals with high platform dynamics. Moreover, as integration time increases, the oscillator noise effect will accumulate, which also degrades PLL performance. Therefore, a trade-off between oscillator and thermal noise rejection and agile carrier phase tracking must be made. In [18], the optimization of integration time for a KF-based PLL was investigated to improve the tracking sensitivity of low dynamic signals. The receiver oscillator noise and the thermal noise effects were both taken into consideration. However, the optimization analysis of highly dynamic signals for receivers equipped with low quality oscillators was not performed in [18]. Similarly, a PIF-based PLL also needs to balance these effects to achieve better tracking performance.

Although the above studies have improved the performance of PLL to some extent, cycle-slips and the potential loss of phase lock still frequently occur in PLLs. A frequency locked loop (FLL) has the innate ability of tolerating large, sustained frequency errors, and is often employed for the tracking of carrier signals subject to high dynamics [19]. In a FLL, the output from the frequency discriminator (e.g. cross-product discriminator) is equivalent to the difference between two adjacent carrier phase samples within the same data bit interval, and the FLL measurement noise is non-white [19]. Although both PIF [19] and KF [20] have been applied to FLL design, the non-white noise characteristics in FLL measurements cannot guarantee the minimum mean square error (MMSE) performance of the KF implementation.

This paper addresses the issues of performance analysis in the generalized PLL and FLL design under various scenarios: strong and weak signal strength, low and high dynamics, and good and poor oscillator quality. In the companion paper [21], three filters, PIF, WF and KF, were applied in a generalized PLL design and the PIF was used in a generalized FLL design. This paper focuses on the performance analysis and the optimizations of these designs using the MMSE criterion as a performance measure within a generalized theoretical framework. The theoretical tracking sensitivities with respect to

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the 3-sigma rule are obtained and used to guide the tracking loop implementation. The analysis results also lead to an adaptive PLL that automatically adjusts the integration time and loop parameters in an optimized manner. An adaptive FLL that operates with the optimal loop parameters is also presented.

The nomenclatures that will be used in the subsequent sections are listed in the Table I, their detailed descriptions and expressions can be found in the companion paper [21]. The organization of this paper is summarized as follows. Section II and Section III present the optimizations of the generalized PLL and FLL under MMSE criterion, respectively. Section IV discusses an adaptive PLL and FLL implementation. Simulation validation of the optimization performances is presented in Section V. Finally, the conclusions are summarized in Section VI.

TABLE I: Nomenclature

Symbol	Description
$\mathbf{L}$	State estimator gain matrix with 1, 2, and 3-state forms as $\begin{bmatrix} \alpha \end{bmatrix}$, $\begin{bmatrix} \alpha & \beta \end{bmatrix}^T$, and $\begin{bmatrix} \alpha & \beta & \gamma \end{bmatrix}^T$.
$\mathbf{A}_P$	PLL system transition matrix.
$\mathbf{H}_P$	PLL measurement matrix.
$\mathbf{Q}_P$	PLL system noise covariance matrix.
$\mathbf{R}$	PLL measurement noise covariance matrix.
σ_v	PLL measurement noise variance.
$\mathbf{L}_{PIF}$	PLL proportional integral filter gain matrix.
$\mathbf{L}_{WF}$	PLL Wiener filter gain matrix.
$\mathbf{L}_{KF}$	PLL Kalman filter gain matrix.
$\mathbf{A}_F$	FLL system transition matrix.
$\mathbf{H}_F$	FLL measurement matrix.
$\mathbf{Q}_F$	FLL system noise covariance matrix.
$\mathbf{L}_F$	FLL filter gain matrix.
$\mathbf{p}_{\bar{\theta}}$	PLL average phase error variance.
$\mathbf{e}_{\bar{\theta}}$	PLL steady-state dynamic stress phase error.
$\mathbf{p}_{\bar{\omega}}$	FLL average frequency error variance.
$\mathbf{e}_{\bar{\omega}}$	FLL steady-state dynamic stress frequency error.
σ_{PLL}	1-sigma phase jitter.
σ_{FLL}	1-sigma frequency jitter.
C/N_0	Carrier to noise ratio.
T	Coherent integration time.
BN	PLL noise equivalent bandwidth.
BW	FLL noise equivalent bandwidth.
q_a	The power spectral density of the random walk process due to the line-of-sight (LOS) platform acceleration with the unit of $(m^2/s^6)/\text{Hz}$.
q_ϕ	The power spectral density of carrier phase noise due to the oscillator instability.
q_ω	The power spectral density of carrier frequency noise due to the oscillator instability.
j_M	Maximum LOS jerk.
f_L	Carrier frequency.

II. OPTIMIZATION FOR GENERALIZED PLL

Analysis presented in the companion paper [21] shows that the error variance of a PLL, $\mathbf{p}_{\bar{\theta}}$, is determined by signal strength,

platform dynamics, receiver oscillator quality, and tracking loop gain matrix $\mathbf{L}$. The signal characteristics, which are determined by the external factors, can not be controlled. However, the gain matrix $\mathbf{L}$, is determined by the design specifications, such as the values of the integration time T and equivalent noise bandwidth BN (if PIF is adopted), and can be optimized to improve the tracking accuracy. According to the analysis in Section V in the companion paper [21], the PLL can achieve minimum tracking error variance if the appropriate value of T is selected based on signal strength and user dynamics using PIF, WF, and KF designs. Similarly, the optimal value of BN could also be obtained, which enables higher tracking accuracy in the PIF-based PLL. The objective here is to investigate the optimal parameters that minimize the value of $\mathbf{p}_{\bar{\theta}}$ in the three PLL designs employing PIF, WF, and KF for GPS L1 signals.

Fig. 1: T_{opt} and $\sqrt{\mathbf{p}_{\bar{\theta}_{min}}}$ versus C/N_0 for both high and low receiver oscillator qualities for 2-state PLL with PIF, WF, and KF designs under the static condition.

Fig. 2: T_{opt} and $\sqrt{\mathbf{p}_{\bar{\theta}_{min}}}$ versus C/N_0 for both high and low receiver oscillator qualities for 3-state PLL with PIF, WF, and KF designs under the low dynamic condition ($q_a = 0.1(m^2/s^6)/\text{Hz}$).

In the optimization procedure, several typical situations are considered: weak and strong signal strength

Fig. 3: T_{opt} and $\sqrt{\mathbf{p}_{\theta min}}$ versus C/N_0 for both high and low receiver oscillator qualities for 3-state PLL with PIF, WF, and KF designs under the high dynamic condition ($q_a = 10(m^2/s^6)/Hz$).

Fig. 4: BN_{opt} dependency on C/N_0 for static, low, and high signal dynamics, both high and low receiver oscillator qualities in the PIF-based PLL.

($C/N_0 = 10 \sim 44\text{dB-Hz}$), low and high dynamics ($q_a = 0, 0.1, 1, 10, 100, 1000(m^2/s^6)/Hz$), low quality oscillator (LQO) and high quality oscillator (HQO) effects. The value of T varies from 1ms to 1000ms in PIF-, WF-, and KF-based PLLs and the maximum value of $BN \cdot T$ is set at 0.5 in PIF-based PLL. Under these given conditions, the optimal values of T , T_{opt} , can be obtained by minimizing $\mathbf{p}_{\theta}$ in the PIF-, WF-, and KF-based PLLs accordingly. The optimal values of BN , BN_{opt} , for a PIF-based PLL can be obtained by dividing the $BN \cdot T$ that minimizes $\mathbf{p}_{\theta}$ by T_{opt} . With these optimal loop parameters, the minimum phase tracking error variance, $\mathbf{p}_{\theta min}$, in these three PLLs could be calculated. When comparing the values of $\mathbf{p}_{\theta min}$ to the threshold value, such as 15° for data channel and 30° for pilot channel, the lowest signal C/N_0 that can be tracked by a PLL could be obtained. It represents the tracking sensitivity that the tracking loop can maintain lock.

The values of T_{opt} , $\sqrt{\mathbf{p}_{\theta min}}$, and BN_{opt} for several represen-

tative scenarios are plotted in Fig. 1-4.

For a static receiver as shown in Fig. 1, the phase tracking error is dominated by thermal noise and oscillator noise, and T_{opt} should be chosen to balance them. The following items can be observed on Fig. 1:

- i). The value of T_{opt} increases as the signal C/N_0 decreases, especially when the C/N_0 is low.
- ii). T_{opt} for the receiver with a HQO is larger than that of the receiver with a LQO. The higher oscillator quality results in a $\sim 9\text{dB}$ improvement in tracking sensitivity if a 15° threshold is used.
- iii). For T_{opt} and $\sqrt{\mathbf{p}_{\theta min}}$ the difference between PIF- and WF/KF- based PLLs are barely noticeable, which indicates that based on the same cost function, i.e., minimizing $\mathbf{p}_{\theta}$, there is only one optimal design regardless of the specific filter design.

TABLE II: Theoretical PLL tracking sensitivities with thresholds of 15° (data channel) and 30° (pilot channel) values for static, low, and high signal dynamics, both high and low receiver oscillator qualities, and the optimal PIF-, WF-, and KF-based PLL designs (unit: dB-Hz)

$q_a(m^2/s^6)/Hz$	LQO		HQO	
	PIF	WF/KF	PIF	WF/KF
0	22/15	22/15	13/6	13/6
0.1	23/17	23/16	21/15	21/14
1	24/18	23/17	23/17	23/16
10	26/19	25/19	25/19	25/18
100	27/21	27/21	27/21	27/21
1000	29/23	29/22	29/23	29/22

Note: */* represent the corresponding PLL sensitivity with respect to $15^\circ/30^\circ$ threshold;

For a receiver on a dynamic platform, the following items can be observed on Fig. 2 and Fig. 3:

- i). PLL is less sensitive to the oscillator noise effect with receiver dynamic motion as compared to the static scenario. T_{opt} and $\sqrt{\mathbf{p}_{\theta min}}$ for receivers with LQO and HQO gradually overlap for both the low and high dynamic cases as the C/N_0 decreases.
- ii). As the dynamics increase, T_{opt} decreases while $\sqrt{\mathbf{p}_{\theta min}}$ increases. In a receiver with HQO at 10dB-Hz C/N_0 , a WF/KF-based PLL operating with T_{opt} at a $\sim 80\text{ms}$ can maintain a $\sim 50^\circ$ MMSE performance under low dynamic conditions, while under high dynamic conditions T_{opt} is $\sim 50\text{ms}$, and the corresponding minimum phase tracking error is $\sim 80^\circ$, indicating that higher dynamic limits integration times, degrades tracking accuracy, as well as tracking sensitivity.

- iii). The minimum tracking errors in PIF-based PLLs are slightly larger than that in WF- and KF-based PLLs under both low and high dynamic conditions. As was discussed in Section III D in the companion paper [21], for $\mathbf{L}_{PIF}$, its α , β , and γ components are determined by the values of BN and T , while for $\mathbf{L}_{WF}$ and $\mathbf{L}_{KF}$, their corresponding α and β components are mainly determined by the oscillator h -parameters and T , and γ is determined by q_a and T . The value of γ is coupled with α and β in $\mathbf{L}_{PIF}$, but is independent of α and β in $\mathbf{L}_{WF}$ and

TABLE III: T_{opt} and BN_{opt} parameter values for static, low, and high signal dynamics, both high and low receiver oscillator qualities, and PIF-, WF-, and KF-based PLL designs

q_a (m^2/s^6)/Hz	Osci. type	PLL type	b_1	μ_1	b_2	μ_2
0	Low	PIF	0.257	0.559	1.191	0.288
		WF/KF	0.272	0.567	N/A	
	High	PIF	0.700	0.583	0.313	0.258
		WF/KF	0.719	0.588	N/A	
0.1	Low	PIF	0.166	0.527	1.677	0.278
		WF/KF	0.247	0.555	N/A	
	High	PIF	0.190	0.487	1.981	0.179
		WF/KF	0.279	0.526	N/A	
1	Low	PIF	0.140	0.501	2.212	0.251
		WF/KF	0.211	0.537	N/A	
	High	PIF	0.140	0.473	2.858	0.181
		WF/KF	0.213	0.514	N/A	
10	Low	PIF	0.103	0.467	3.597	0.213
		WF/KF	0.163	0.511	N/A	
	High	PIF	0.104	0.459	4.110	0.183
		WF/KF	0.163	0.502	N/A	
100	Low	PIF	0.076	0.448	5.662	0.193
		WF/KF	0.121	0.492	N/A	
	High	PIF	0.074	0.438	5.910	0.184
		WF/KF	0.124	0.493	N/A	
1000	Low	PIF	0.052	0.416	8.271	0.191
		WF/KF	0.092	0.482	N/A	
	High	PIF	0.052	0.415	8.433	0.187
		WF/KF	0.090	0.473	N/A	

L_{KF} . This allows a higher degree of freedom in the WF- and KF-based PLL designs. Hence, PIF-based PLL only achieves a sub-optimal performance, while the optimizations of WF- and KF-based PLLs can realize the MMSE performance.

The following items can be observed on Fig.4:

- i). BN_{opt} increases as the signal C/N_0 increases.
- ii). BN_{opt} in a receiver with LQO is larger than that in a receiver with HQO.
- iii). BN_{opt} increases as receiver dynamics increase. These trends are opposite to that of T_{opt} in Fig.1-3. This is because for a stronger signal, a shorter integration time is needed, so a larger loop bandwidth can be accommodated to handle the higher dynamics and the larger oscillator noise.

The theoretical tracking sensitivities of the PLLs with optimal loop parameters T_{opt} and BN_{opt} according to the 3-sigma rule are summarized in Table II. Table II shows that:

- i). Under the static scenario ($q_a = 0(m^2/s^6)/Hz$), a PLL tracking sensitivity mainly depends on the receiver oscillator quality. The theoretical tracking sensitivity for a receiver with a HQO can reach 13dB-Hz and 6dB-Hz for data channels and pilot channels, respectively, which is ~ 9 dB higher than that for a receiver with LQO. These are the theoretical performance bounds of PLL tracking sensitivity.
- ii). Under the dynamic scenario ($q_a \neq 0(m^2/s^6)/Hz$), PLL is more sensitive to the platform dynamics than the influence

of oscillator quality as the platform dynamics increases. The tracking sensitivity for a receiver with a LQO is the same as one with a HQO when $q_a > 1(m^2/s^6)/Hz$. The higher the platform dynamics, the worse the tracking sensitivity becomes.

iii). Although a PLL with L_{PIF} is suboptimal, it can still achieve nearly the same tracking sensitivity as the ones with L_{WF} and L_{KF} . This indicates that under the uniform optimization criteria, the PLL performance is mainly limited by the signal characteristics and receiver hardware quality, such as oscillator quality and platform dynamics, regardless of what filter design is used.

iv). There is a general 6dB to 7dB improvement in pilot channels over data channels. The potential advantages of pilot channels over data channels are not greatly affected by the receiver oscillator quality and platform dynamics, as well as filter designs. These results are consistent with the conclusions in [1].

Fig. 5: Trends of b_1 and μ_1 versus signal dynamics for high and low receiver oscillator quality, and PLL with PIF, WF, and KF designs.

Fig. 6: Trends of b_2 and μ_2 versus signal dynamics for high and low receiver oscillator qualities in the PIF-based PLL.

To calculate T_{opt} and BN_{opt} for various scenarios, we obtain $\mathbf{p}_{\bar{\theta}_{min}}$ (details can be seen in the companion paper [21]) for T

from 1ms to 1000ms, $BN \cdot T$ from 0.0001 to 0.5, and C/N_0 from 10dB-Hz to 44dB-Hz for each specified dynamics. These optimal values are uniquely associated with a given set of and receiver parameters which can be pre-determined. Therefore, for practical applications, the relationships between T_{opt} and C/N_0 , BN_{opt} and C/N_0 can be established beforehand. Such relationships also provide insights into the impact of the signal and receiver parameters on the design choices. The relationships can be summarized below:

$$T_{opt} = b_1 (C/N_0)^{-\mu_1} (s) \quad (1)$$

$$BN_{opt} = b_2 (C/N_0)^{\mu_2} (Hz) \quad (2)$$

where the parameters b_1 , μ_1 , b_2 , and μ_2 for various signal dynamics, receiver oscillator qualities, and PLL designs are obtained through curve fitting of numerical calculations and listed in Table III. Equations (1) and (2) provide the optimal parameter designs in the generalized PLL for diverse dynamic and weak signal scenarios. To clearly show the trends of these parameters versus different signal dynamics for typical low and high receiver oscillator qualities, and PLL designs, their values are plotted in Fig.5 and Fig.6.

Fig.5 shows that as the dynamic parameter q_a increases, b_1 and μ_1 will decrease for both LQO and HQO. Fig.6 shows that increasing q_a is associated with a slight decline in μ_2 and a sharp rise in b_2 . This indicates that b_2 is more sensitive to the changing signal dynamics. Similarly, the difference between b_2 and μ_2 for LQO and HQO is reduced with increasing signal dynamics.

III. OPTIMIZATION FOR GENERALIZED FLL

Similar to the case with PLL, the objective of optimization for generalized FLL is to determine system parameters that minimize $\mathbf{p}_{\bar{\omega}}$. The minimum frequency tracking error variance, $\mathbf{p}_{\bar{\omega}_{min}}$, for a FLL can be achieved if the appropriate values of loop parameters, such as integration time T and equivalent noise bandwidth BW , are selected for specified signal strength, receiver dynamic scenario, and oscillator quality. For a FLL operating on a data channel, at least two integrate-and-dump samples must be taken between data bit transitions. Therefore, navigation data rate limits the length of the integration time T in FLL, and the maximum value of T for GPS L1 signal frequency tracking is 10ms. Even for the pilot channel, a shorter integration time is preferred because the frequency tracking error threshold decreases as T increases. Two cases with $T = 1ms$ and $T = 10ms$ are considered for FLL optimization, and their corresponding optimal parameter, BW_{opt} , will be analyzed.

A. 1-state FLL

Substituting $\mathbf{A}_F$, $\mathbf{H}_F$, $\mathbf{Q}_F$, $\mathbf{R}$ and $\mathbf{L}_F$ into $\mathbf{p}_{\bar{\omega}}$ (details can be seen in the companion paper [21]), we obtain:

$$\mathbf{p}_{\bar{\omega}} = \frac{(2\pi f_L)^2 T q_{\omega} + 2\sigma_v^2 / T^2 \alpha^3}{2\alpha - \alpha^2}. \quad (3)$$

The extreme values of a function occur where its partial derivatives are zero. Taking a derivative of (3) with respect to α , we

obtain:

$$\frac{d\mathbf{p}_{\bar{\omega}}}{d\alpha} = \frac{-2\sigma_v^2 / T^2 \alpha^4 + 8\sigma_v^2 / T^2 \alpha^3 + 2(2\pi f_L)^2 T q_{\omega} (\alpha - 1)}{\alpha^2 (\alpha - 2)^2} \quad (4)$$

The derivative becomes zero when:

$$-2\sigma_v^2 / T^2 \alpha^4 + 8\sigma_v^2 / T^2 \alpha^3 + 2(2\pi f_L)^2 T q_{\omega} (\alpha - 1) = 0. \quad (5)$$

Let $b = (2\pi f_L)^2 T^3 q_{\omega} / \sigma_v^2$, then equation(5) becomes $\alpha^4 - 4\alpha^3 - b\alpha + b = 0$. The solution of α that minimizes $\mathbf{p}_{\bar{\omega}}$ is (see derivation in Appendix A):

$$\alpha_{opt} = 1 + \frac{1}{2} \sqrt{4 + \sqrt[3]{b^2 + 16b}} - \frac{1}{2} \sqrt{8 - \sqrt[3]{b^2 + 16b} + \frac{2b + 16}{\sqrt{4 + \sqrt[3]{b^2 + 16b}}}}. \quad (6)$$

According to the relationship between α and BW (details can be seen in the companion paper [21]), we can obtain:

$$1 - \text{state} : BW_{opt} = \frac{\alpha_{opt}}{4T} (Hz) \quad (7)$$

B. 2-state FLL

Due to the complexity involved in obtaining the analytical solutions of BW_{opt} for the 2-state case, the numerical solutions are provided here instead.

The following scenarios are considered: $C/N_0 = 10 \sim 44$ dB-Hz, $q_a = 0.1, 1, 10, 100, 1000(m^2/s^6)/Hz$, and LQO and HQO with parameters provided in the companion paper [21]. T is set at 1ms and 10ms, and the maximum value of $BW \cdot T$ is 0.5. To calculate the optimal values of BW_{opt} for each scenario, we obtain $\mathbf{p}_{\bar{\omega}_{min}}$ (Expression of $\mathbf{p}_{\bar{\omega}}$ can be found in the companion paper [21]) over the search space of $BW \cdot T$ from 0.0001 to 0.5, and C/N_0 from 10dB-Hz to 44dB-Hz for each specified platform dynamics. Then, the corresponding values of BW_{opt} can be obtained by dividing the product $BW \cdot T$ that minimizes $\mathbf{p}_{\bar{\omega}}$ by the corresponding T values. Figs. 7 and 8 plot BW_{opt} and $\sqrt{\mathbf{p}_{\bar{\omega}_{min}}}$ for the 2-state FLLs with $T = 1ms$ and $10ms$ under different dynamics, respectively. The solutions for 1-state FLLs with $T = 1ms$ and $10ms$ under static ($q_a = 0(m^2/s^6)/Hz$) are also plotted for comparison purposes.

For $T = 1ms$, the following items can be observed on Fig. 7(a):

- i). BW_{opt} increases as C/N_0 increases.
- ii). For a static receiver, BW_{opt} for the receiver with a LQO is larger than that of the receiver with a HQO. The reason is that the larger oscillator noise in LQO will lead to larger frequency tracking error than in HQO. A larger bandwidth is therefore needed for the receiver with LQO. However, for a receiver on a higher dynamic platform, FLL is more sensitive to the platform dynamics than the oscillator noise effect. BW_{opt} for the LQO and HQO for the high dynamic case overlap with each other.
- iii). As the platform dynamics increase, BW_{opt} also increases, indicating that the FLL needs a wider bandwidth to handle the higher dynamics.

The following items can be observed on Fig. 7(b):

- i). $\sqrt{\mathbf{p}_{\bar{\omega}_{min}}}$ decreases as C/N_0 increases.

Fig. 7: BW_{opt} and $\sqrt{p_{\omega_{min}}}$ versus C/N_0 for static, low, and high signal dynamics, and both high and low receiver oscillator qualities in the PIF-based FLL with $T = 1ms$

Fig. 8: BW_{opt} and $\sqrt{p_{\omega_{min}}}$ versus C/N_0 for static, low, and high signal dynamics, and both high and low receiver oscillator qualities in the PIF-based FLL with $T = 10ms$

ii). $\sqrt{p_{\omega_{min}}}$ is slightly larger for LQO than for HQO for the static scenario. As with PLL, FLL is also more sensitive to the platform dynamics than the oscillator noise effect. The frequency tracking error differences between the receivers with LQO and HQO are smaller than 2Hz when $C/N_0 = 10\text{dB-Hz}$ and under a static condition for $T = 1ms$ in Fig. 7(b). While under the same signal strength and static condition, the phase tracking error difference between the receivers with LQO and HQO is $\sim 30^\circ$, as shown in Fig.1. Hence, the difference between LQO

and HQO is negligible for all scenarios in a FLL, while for a PLL the difference is only negligible when platform dynamics are relatively high. It indicates that the frequency tracking is less affected by the oscillator noise and more robust than carrier phase tracking.

iii). $\sqrt{p_{\omega_{min}}}$ increases as receiver dynamic increases. This is because for a more dynamic signal, a larger loop bandwidth is used to handle the higher dynamics, which results in a reduced tracking accuracy.

A similar trend is presented in Fig. 8 for $T = 10ms$. Comparing Fig. 7 and Fig. 8, the frequency tracking errors for $T = 1ms$ and $10ms$ are $\sim 38\text{Hz}$ and $\sim 10\text{Hz}$, respectively when $C/N_0 = 10\text{dB-Hz}$ and $q_a = 1000(m^2/s^6)/\text{Hz}$, indicating that with a longer integration time, a FLL achieves better tracking accuracy. This observation is consistent with that for a PLL in Section II. However, even for a pilot channel without data modulation, T should be relative small to maintain robust tracking performance in a FLL because the frequency tracking threshold decreases when T increases, as shown by the dash red lines in Fig. 7 and Fig. 8.

TABLE IV: Theoretical FLL tracking sensitivities with thresholds of $\frac{1}{24T}$ (data channel) and $\frac{1}{12T}$ (pilot channel) for static, low, and high signal dynamics, both high and low receiver oscillator qualities, and optimal FLL designs (unit: dB-Hz)

$q_a(m^2/s^6)/\text{Hz}$	LQO		HQO	
	$T = 1ms$	$T = 10ms$	$T = 1ms$	$T = 10ms$
0	$< 0 / < 0$	$< 0 / < 0$	$< 0 / < 0$	$< 0 / < 0$
0.1	$< 0 / < 0$	$3 / < 0$	$< 0 / < 0$	$3 / < 0$
1	$< 0 / < 0$	$7 / 1$	$< 0 / < 0$	$7 / 2$
10	$3 / < 0$	$10 / 5$	$3 / < 0$	$10 / 5$
100	$6 / 1$	$14 / 8$	$7 / 1$	$14 / 9$
1000	$10 / 5$	$18 / 12$	$10 / 5$	$18 / 12$

Note: < 0 represents the theoretical sensitivity is close to 0dB-Hz ; $*/$ represent the corresponding FLL sensitivity with respect to $\frac{1}{24T}(\text{Hz}) / \frac{1}{12T}(\text{Hz})$ threshold.

The theoretical tracking sensitivities of the generalized FLLs with $T = 1ms$ and $10ms$ and their corresponding BW_{opt} are summarized in Table IV. It shows that:

- As dynamic increases, the tracking sensitivity deteriorates.
- The tracking sensitivity in FLL with $T = 10ms$ is worse than that with $T = 1ms$ due to the FLL threshold being inversely proportional to the integration time, although the tracking accuracy is better with the greater integration time.
- The FLL tracking sensitivity difference between receivers with LQO and HQO is negligible.
- There is a 5dB to 6dB tracking sensitivity improvement in pilot channels over data channels, which is consistent with the conclusions for PLL, as well as the conclusions from reference [1].

v). Comparing Table IV and Table II, it can be noted that tracking sensitivity of a FLL is much higher than a PLL. The tracking sensitivity limit in a FLL is close to 0dB-Hz , which is 6dB better than that of a PLL, indicating that the frequency

tracking is more robust than carrier phase tracking under weak signal conditions.

Similar to PLLs, the relationships between BW_{opt} and C/N_0 in 2-state FLLs with $T = 1ms$ and $10ms$ are formulated as:

$$BW_{opt} = b_3(C/N_0)^{\mu_3} (Hz). \quad (8)$$

The parameters b_3 and μ_3 are computed through curve fitting of simulation results and are listed in Table V for various signal dynamics and receiver oscillator qualities for practical implementation purposes. Table V shows that there is only a slight difference between LQO and HQO in a low dynamic scenario, while in a high dynamic scenario the difference is negligible. Equations (7) and (8) together with b_3 and μ_3 values in Table V provide the optimal designs in the generalized FLL under diverse dynamic and weak signal scenarios.

TABLE V: BW_{opt} parameter values for low and high signal dynamics, and both high and low receiver oscillator qualities in the 2-state PIF-based FLL

$q_a(m^2/s^6)/Hz$	Oscillator	$T = 1ms$		$T = 10ms$	
		b_3	μ_3	b_3	μ_3
0.1	LQO	0.133	0.314	0.298	0.30
	HQO	0.167	0.264	0.397	0.22
1	LQO	0.246	0.279	0.556	0.25
	HQO	0.263	0.264	0.628	0.22
10	LQO	0.420	0.264	1.023	0.22
	HQO				
100	LQO	0.659	0.265	1.57	0.22
	HQO				
1000	LQO	1.049	0.264	2.46	0.22
	HQO				

IV. ADAPTIVE TRACKING PROCESS

Based on the optimization analysis for PLL and FLL in Sections II and III, we propose an adaptive tracking scheme in which the loop parameters are adjusted according to the variation of signal strength and dynamics to improve the tracking sensitivity and accuracy.

The proposed scheme requires knowledge of the signal C/N_0 and dynamic characteristics. C/N_0 can be obtained by using a real time signal-to-noise ratio (SNR) estimator at regular intervals [22]–[24] which is typically on the order of a navigation data bit, such as 20ms for GPS L1 signal, to achieve the lowest expected measurable C/N_0 estimates at around 17dB-Hz [24].

The dynamics parameter q_a can be estimated directly using receiver measurements or other on-board sensors. When the receiver is moving with a nearly constant acceleration, q_a should be set to a small value. For receivers on more dynamic platforms where acceleration may change in the receiver-satellite line-of-sight (LOS) direction over a short range, the maximum changes over a sampling period T should be on the order of $\sqrt{q_a T}$ [25]:

$$\sqrt{q_a T} \propto j_M T \quad (9)$$

where j_M is maximum LOS jerk. A practical range for $\sqrt{q_a T}$ is [25]:

$$0.5j_M T \leq \sqrt{q_a T} \leq j_M T \quad (10)$$

Note that $j_M T$ should be relatively small compared to the actual acceleration levels. Thus, by using this relationship we can set the corresponding value of q_a according to the empirical knowledge of the platform's dynamics. If an inertial measurement unit (IMU) is used, the value of j_M could be estimated in real time to assist the adaptive tracking loop implementations.

Algorithm 1 Adaptive PLL algorithm

- 1: **Initialization:**
- 2: Set $T = 1ms$ and calculate the phase error $\Delta\theta_k$ from the phase discriminator once per 1ms;
- 3: Set $BN = 50Hz$ to make the tracking loop fast convergent to steady-state;
- 4: Obtain the value of C/N_0 estimation, c/n_0 , through the SNR estimator;
- 5: **Optimal tracking:**
- 6: Set $T = T_{opt}$, where $T_{opt} = b_1(c/n_0)^{-\mu_1}$;
- 7: Set $BN = BN_{opt}$, where $BN_{opt} = b_2(c/n_0)^{-\mu_2}$ (if PIF is adopted);
- 8: Calculate the loop gain matrix $\mathbf{L}$, such as $\mathbf{L}_{PIF}$, $\mathbf{L}_{WF}$, and $\mathbf{L}_{KF}$;
- 9: Calculate the phase error $\Delta\theta_k$ from the phase discriminator once per $T_{opt}ms$;
- 10: Estimate the state $\hat{\mathbf{x}}_{k+1} = \mathbf{A}_P \hat{\mathbf{x}}_k + \mathbf{A}_P \mathbf{L} \Delta\theta_k$ once per $T_{opt}ms$;
- 11: Generate $T_{opt}ms$ carrier signals for correlation and estimate the signal C/N_0 ;
- 12: **Update:**
- 13: **if** c/n_0 is not changed **then**
- 14: go to step 9;
- 15: **else**
- 16: go to step 6;
- 17: **endif**

A. Adaptive PLL

A commonly used PLL can be classified as a fixed $\mathbf{L}$ and fixed T (FL-FT) algorithm or an adaptive $\mathbf{L}$ and fixed T (AL-FT) algorithm. The FL-FT approach corresponds to the traditional PIF-based PLL, where $\mathbf{L}$ is determined by the predefined value of BN and T . The AL-FT approach corresponds to the KF-based PLL, where $\mathbf{L}$ is adjustable, based on the signal strength and platform dynamics estimations, while T is a predefined constant. In our proposed adaptive PLL algorithm (see Algorithm 1), T and other loop parameters are adaptively tuned according to the estimations of C/N_0 and q_a to achieve the optimal tracking loop sensitivity and accuracy. Below is a summary of the processes in Algorithm 1:

1) Initialization. The tracking loop is initialized with Doppler frequency, carrier phase, and C/N_0 estimations from the acquisition process. In this initial stage, PLL is operating with a

1ms integration time and a large loop bandwidth (for example $BN \geq 50\text{Hz}$) to ensure fast convergence to the steady state.

2) Optimal tracking. When the PLL operates in a steady state, the T_{opt} and BN_{opt} are obtained according to C/N_0 and q_a estimations obtained through equations (1) and (2). The corresponding loop gain $\mathbf{L}_{PIF}/\mathbf{L}_{WF}/\mathbf{L}_{KF}$ is calculated and used to update the state estimations, which are subsequently used to generate T_{opt} ms carrier signals (we assume that the data is wiped off when $T > 1\text{ms}$ on data channel) for correlation with new incoming signals, for phase discriminator calculation, and for estimating the signal C/N_0 in preparation for the next tracking iteration.

Algorithm 2 Adaptive FLL algorithm

```

1: Initialization:
2:   Set  $T = 1\text{ms}$  and calculate the frequency error  $\Delta\varpi_k$ 
   from the frequency discriminator once per 1ms;
3:   Set  $BW = 50\text{Hz}$  to make the tracking loop fast conver-
   gent to steady-state;
4:   Obtain the value of  $C/N_0$  estimation,  $c/n_0$ , through the
   SNR estimator;
5: Optimal tracking:
6:   Set  $T = 1\text{ms}$  or  $T = 10\text{ms}$ ;
7:   Set  $BW = BW_{opt}$ , where  $BW_{opt} = b_3(c/n_0)^{-\mu_3}$  if  $q_a \neq$ 
    $0(m^2/s^6)/\text{Hz}$ , or  $BW_{opt} = \alpha_{opt}/T$  if  $q_a = 0(m^2/s^6)/\text{Hz}$ ;
8:   Calculate the loop gain matrix  $\mathbf{L}_F$ ;
9:   Calculate the frequency error  $\Delta\varpi_k$  from the frequency
   discriminator once per  $T$ ms;
10:  Estimate the state  $\hat{\mathbf{x}}_{k+1} = \mathbf{A}_F\hat{\mathbf{x}}_k + \mathbf{A}_F\mathbf{L}_F\Delta\varpi_k$  once per
    $T$ ms;
11:  Generate  $T$ ms carrier signals for correlation and esti-
   mate the signal  $C/N_0$ ;
12: Update:
13:   if  $c/n_0$  is not changed then
14:     go to step 9;
15:   else
16:     go to step 7;
17:   endif

```

3) Update. The integration time and loop parameters computed from 2) are updated when a new C/N_0 estimation is obtained in each new epoch. Otherwise, the loop parameters are left unchanged.

Consequently, the adaptive PLL adjusts the values of T and $\mathbf{L}$ in a time-varying manner aimed to achieve the theoretical MMSE performance.

B. Adaptive FLL

Similar to PLL, the estimations of C/N_0 and q_a are used to update the adaptive tuning scheme for a FLL as shown in algorithm 2.

V. SIMULATION VERIFICATION

Simulations under various dynamics, signal attenuations, and oscillator noise effects are presented in this section. A Spirent

GSS8000 GNSS signal simulator was used to generate GPS L1 signals experiencing controlled platform dynamics and attenuations. Two RF front-ends each with a LQO and with a HQO are used to collect the data. The h -parameters are set as $h_0 = 1 \times 10^{-21}(s^2/\text{Hz})$, $h_{-2} = 2 \times 10^{-20}(1/\text{Hz})$ for the front-end with LQO, and $h_0 = 6.4 \times 10^{-26}(s^2/\text{Hz})$, $h_{-2} = 4.3 \times 10^{-23}(1/\text{Hz})$ for the front-end with HQO, respectively. These values are similar to typical values that can be found in [5] [18]. Finally, these data were processed to verify our theoretical optimization analysis and to evaluate the performance of the proposed adaptive PLL and FLL.

A. Static weak signal scenario

In this scenario, the receiver is statically located at (N39°, W82°) and 8 channels are simulated. The C/N_0 in all 8 channels are set to 46dB-Hz initially. After the first two minutes the signal power is decreased 1dB per minute for 6 minutes. Then, the attenuation rate changes to 2dB per minute until the maximum attenuation reaches 29dB. The RF front-end with HQO is used to collect the data. It down-converts the input signal to baseband with an IF of 0MHz, sampled at 100MHz, and stored with 16-bit resolution for post-processing. Additionally, a NovAtel receiver was also connected to the simulator to record data simultaneously with the RF front-end for comparison purpose.

1) *Adaptive PLL*: Three different types of 2-state PLLs are evaluated for static weak signals. They are:

i). A PIF-based PLL with 1ms and 20ms integration time and 50Hz and 15Hz noise equivalent bandwidth in transit state and steady state, respectively.

ii). A WF/KF-based PLL with 1ms and 20ms integration time.

iii). The proposed adaptive PIF and WF/KF-based PLL with T_{opt} or BN_{opt} computed according to equations (1) and (2) and Table III for HQO and $q_a = 0(m^2/s^6)/\text{Hz}$.

Using the PRN 19 for illustration purposes, the tracking results using the 2-state PLLs with $\mathbf{L}_{PIF}$, $\mathbf{L}_{WF}$ and $\mathbf{L}_{KF}$ implementations are plotted in Fig. 9(a)-(d). C/N_0 is estimated based on the variance summing method [22] and was initially computed at 1Hz rate. Starting at 900s the signal strength is too weak to be measured accurately; 20ms averaging times are used and C/N_0 is estimated for every 5s. From Fig.9(a) we can observe that the estimations of C/N_0 in the adaptive PLLs generally follow the actual signal strength variation. The mismatches between estimated and actual C/N_0 in the other PLL implementations are most likely due to their large frequency tracking errors. Fig.9(b) shows that the values of T_{opt} used in the adaptive PIF- and KF-based PLLs increase as the estimated C/N_0 decreases. The maximum value of T for adaptive PIF- and KF-based PLLs is $\sim 60\text{ms}$ when the estimated C/N_0 is at 17dB-Hz level. This is 10ms shorter than the theoretical value of T_{opt} obtained from the actual C/N_0 . This 10ms discrepancy is most likely due to the implementation manner (i.e., the integration time is set on the order of a navigation data bit when it is larger than a period of a navigation data bit). Fig.9(c) shows that the values of BN_{opt} used in the adaptive PIF-based PLL decrease as the estimated C/N_0 decreases. The minimum value of BN for adaptive PLL is between 0.6Hz and 1Hz when the

Fig. 9: Tracking results in 2-state PLLs for PRN 19 satellite signal after 600s ($C/N_0 < 35\text{dB-Hz}$) under static weak signal condition. (a) C/N_0 estimations. The estimated C/N_0 is used to tune T_{opt} and BN_{opt} in adaptive PLLs as well as measurement noise covariance matrix $\mathbf{R}$ in WF/KF-based PLLs. (b) T_{opt} for adaptive PIF- and WF/KF-based PLLs. (c) BN_{opt} for adaptive PIF-based PLL. (d) Doppler frequency estimations in the 2-state PLLs. The proposed adaptive PLLs are able to maintain tracking throughout this very challenging time period while other PLLs lose lock when the signal strength drops below certain values as shown.

estimated C/N_0 is at the 17dB-Hz level, which is quite close to the theoretical value of BN_{opt} ($\sim 0.8\text{Hz}$) obtained from the actual C/N_0 .

From Fig.9(d) we can see that for FL-FT algorithms, the PIF-based PLL with $T = 1\text{ms}$ can maintain lock until 900s when the signal C/N_0 drops to about 25dB-Hz. However, a 1ms integration time is too short for the weak signal tracking after about 900s. By increasing the integration time to 20ms, the PIF-based PLL has a slight advantage over its 1ms implementation, but still loses lock at about 920s. This indicates that without properly designed filter parameters, the noise rejection caused by increasing the integration time may not be effective.

For AL-FT algorithms, the PLL with WF/KF is slightly better because the loop gain $\mathbf{L}_{WF}/\mathbf{L}_{KF}$ is automatically adjusted according to the signal strength. The WF/KF-based PLLs with

$T = 1\text{ms}$ and $T = 20\text{ms}$ maintain tracking until 920s and 1000s, which are respectively 20s and 80s longer (or 2dB better) than that in PIF-based PLL implementations. The adaptive PIF/WF/KF-based PLLs improve the tracking sensitivity by at least 6dB due to their adaptively self-adjusting integration time and gain matrix. An even longer integration time ($>40\text{ms}$) was invoked in the adaptive PLLs to maintain tracking of the weak signal until the end of the data sequence when C/N_0 reached the 17dB-Hz level. This result also shows that even through accurate estimation of C/N_0 is difficult to achieve for weak signals, the error in C/N_0 does not appear to critically impact the adaptive tracking algorithm performance.

The NovAtel receiver lost lock at an attenuation level of -19dB ($C/N_0 = 27\text{dB-Hz}$) at about 840 seconds. The results indicate that the adaptive PLLs outperforms FL-FT and AL-FT algorithms, validating the state space design and optimization analysis for generalized PLL under weak signal conditions for a receiver with HQO.

Fig. 10: Tracking results in 1-state FLLs for PRN 19 satellite signal after 600s ($C/N_0 < 35\text{dB-Hz}$) under static weak signal condition. (a) C/N_0 estimations. The estimated C/N_0 is used to tune BW_{opt} in adaptive FLLs as well as measurement noise covariance matrix $\mathbf{R}$ in KF-based FLLs. (b) BW_{opt} for adaptive PIF-based FLL. (c) Doppler frequency estimations for the 1-state FLLs. The optimized FLLs are better than the KF-based FLLs for both $T = 1\text{ms}$ and 10ms .

2) *Adaptive FLL*: The 1-state adaptive FLLs with $T = 1\text{ms}$ and 10ms and their corresponding BW_{opt} values are adopted

to process this static weak signal. The KF-based FLLs with $T = 1\text{ms}$ and 10ms in [20] are also shown in the figure. Note that this KF-based FLL does not take into consideration the non-white characteristics of frequency estimation noise. It is shown here for comparison purposes.

Using the PRN 19 for illustration, Fig.10(a) shows that the estimations of C/N_0 with $T = 10\text{ms}$ follow the real signal strength trend more accurately than other approaches, especially when the signal strength is low. Fig.10(b) shows that BW_{opt} for both $T = 1\text{ms}$ and $T = 10\text{ms}$ adaptively decreases as C/N_0 decreases, with BW_{opt} being slightly lower for $T = 1\text{ms}$ than for $T = 10\text{ms}$ especially when the signal is weak. Fig.10(c) shows that as signal strength decreases, the frequency error increases. This trend is particularly obvious for $T = 1\text{ms}$ after about 700 seconds ($C/N_0=33\text{dB-Hz}$) in the KF-based FLL and 1000 seconds ($C/N_0=23\text{dB-Hz}$) in the adaptive PIF-based FLL. This is in disagreement with the theoretical analysis in Table IV which shows that the adaptive FLL could track a weak signal as low as 0dB-Hz even when $T = 1\text{ms}$. It is known that the maximum frequency error in the 1ms FLL can be as much as 42Hz when compared to the threshold. However, this large frequency error may degrade the C/N_0 estimations especially when the signal is extremely weak in real implementations. That leads to the disagreement between real tracking performance and theoretical tracking performance. To improve the tracking accuracy and to reduce C/N_0 estimation errors when the signal is weak, longer integration times such as $T = 10\text{ms}$ should be used. Using $T = 10\text{ms}$, the KF-based FLL lost lock at 1000 seconds ($C/N_0=23\text{dB-Hz}$), while the adaptive PIF-based FLL maintained tracking throughout the time period.

The above result demonstrates that the adaptive PIF-based FLL is superior to a KF-based FLL and validates the state space design and optimization analysis for a generalized FLL under the static and weak signal conditions for a receiver front-end with HQO. Comparing Fig.10(c) to Fig.9(d), the adaptive PIF-based FLL with $T = 10\text{ms}$ is equivalent to the adaptive PIF-based PLL with a maximum 60ms integration time and is better than the KF-based PLL with $T = 20\text{ms}$. It shows that even with a shorter integration time, a well-designed FLL is superior to or at least equivalent to a well-designed PLL in weak signal processing.

B. Dynamic weak signal scenario

In this simulation, signal attenuation and dynamics are applied simultaneously. For signal attenuation, the signal C/N_0 started at the nominal 46dB-Hz level and was decreased by 1dB per 5s starting at 20s . The maximum attenuation reached 20dB at $t = 120\text{s}$ and was maintained at this level for 60s , followed by a recovery period with 1dB per 5s until it returned back to the nominal level. For a signal dynamics simulation, the receiver was static in the first 20s , then it started moving to the east with an acceleration of 50m/s^2 for 100s . When $t = 120\text{s}$, the acceleration was at zero with a negative jerk of 50m/s^3 . After this, the receiver remained at a constant velocity until $t = 180\text{s}$; then it slowed down with negative acceleration for 100s and stopped at $t = 280\text{s}$. The RF front-end with LQO was used to

collect the data with an IF at 4.309MHz and sampling frequency at 12MHz , and stored in 1-bit resolution for post-processing.

1) *Adaptive PLL*: Three types of 3-state PLLs are evaluated for dynamic weak signal processing. They are:

i). A PIF-based PLL with 1ms and 20ms integration times. A 50Hz noise equivalent bandwidth is used in both the transient- and steady-state.

ii). A WF/KF-based PLL with 1ms and 20ms integration times and corresponding q_a values are estimated using equation (10) as $1(\text{m}^2/\text{s}^6)/\text{Hz}$ and $20(\text{m}^2/\text{s}^6)/\text{Hz}$ for the dynamic scenarios described above.

iii). The proposed adaptive PIF and WF/KF-based PLL with T_{opt} or BN_{opt} computed according to equations (1) and (2) and Table III for the case of $q_a = 10(\text{m}^2/\text{s}^6)/\text{Hz}$ and a receiver with LQO.

Fig.11(a) shows that the estimated C/N_0 in all three adaptive PLLs in general follows the actual signal strength variations. The mismatch between estimated and actual C/N_0 in all non-adaptive PLLs is most likely due to the large frequency tracking errors. Fig.11(b) and (c) show that the values of T_{opt} and BN_{opt} in the adaptive PLLs are automatically tuned according to C/N_0 estimations, although there are some discrepancies between the theoretically computed T_{opt} and the actual T_{opt} adopted by the tuning process. For example, the theoretical T_{opt} is 7ms and 8ms for the PIF- and WF-based adaptive PLLs respectively when C/N_0 is 26dB-Hz . In the real implementation, T_{opt} is about 10ms due to inaccurate C/N_0 estimations when the signal is weak. Fig.11(c) shows that the minimum value of BN for adaptive PIF-based PLL is between 11Hz and 15Hz in real implementations, which is quite close to the theoretical value of BN_{opt} ($\sim 12\text{Hz}$) obtained based on the actual C/N_0 . Hence, the error in the C/N_0 estimation does not appear to critically impact the optimization results for T_{opt} and BN_{opt} .

Fig.11(d) shows the Doppler frequency estimations of the 3-state PLLs, where the maximum changes of Doppler frequency are about 15000Hz during acceleration from 20s to 120s and 14000Hz during deceleration from 180s to 280s . A loss-of-lock occurs in the PIF-based PLL with $T = 1\text{ms}$ at about 110s . Using a longer time integration, such as $T = 20\text{ms}$ enables the PIF-based PLL to maintain lock for 10s longer (2dB better in tracking sensitivity). The WF/KF-based PLL with a variable gain demonstrates improved performance. For example, for $T = 1\text{ms}$, the WF/KF-based PLL is about 2dB better than the PIF-based PLL in tracking sensitivity. This observation shows that WF/KF has the potential to achieve better noise performance due to its narrow equivalent noise bandwidth than the model-free approach PIF. However, the same conclusion doesn't hold any more when jerk starts at 120s . The WF/KF-based PLL with $T = 20\text{ms}$ loses lock almost at the same time with 20ms PIF-based PLL. As C/N_0 decreases, the noise equivalent bandwidth in WF/KF-based PLL decreases. Hence, the narrower noise equivalent bandwidth in WF/KF degrades the dynamic adaptability when dynamic stress occurs. Moreover, a longer integration time, such as $T = 20\text{ms}$, makes things worse in dynamic signal tracking. Our proposed adaptive PLLs are able to maintain tracking, indicating that the appropriate values of T and $\mathbf{L}$ are necessary when the signal is both weak and highly

Fig. 11: Tracking results in 3-state PLLs for PRN 14 satellite signal under the dynamic weak signal condition. (a) C/N_0 estimations. The estimated C/N_0 is used to tune T_{opt} and BN_{opt} in adaptive PLLs as well as measurement noise covariance matrix $\mathbf{R}$ in WF/KF-based PLLs. (b) T_{opt} for adaptive PIF- and WF/KF-based PLLs. (c) BN_{opt} for adaptive PIF-based PLL. (d) Doppler frequency estimations. Only the proposed adaptive PLLs are able to maintain tracking while others have lost lock. (e) Doppler frequency rate estimations. Only the Doppler frequency rate estimations in the proposed adaptive PLLs generally follow the signal dynamic, while others have diverged after 120s.

Fig. 12: Tracking results of 2-state FLLs for PRN 14 satellite signal under dynamic weak signal condition. (a) C/N_0 estimations. The estimated C/N_0 is used to tune BW_{opt} in adaptive PIF-based FLLs as well as measurement noise covariance matrix $\mathbf{R}$ in KF-based FLLs. (b) BW_{opt} for adaptive PIF-based FLLs. (c) Doppler frequency estimations. Only the proposed adaptive FLLs are able to maintain tracking while KF-based FLLs with $T = 1\text{ms}$ and 10ms respectively lost of lock after 120s and 180s. (d). Doppler frequency rate estimations. Only the Doppler frequency rate estimations in the proposed adaptive PIF-based FLLs the generally follow the signal dynamic while KF-based FLLs with $T = 1\text{ms}$ and 10ms respectively diverge after 120s and 180s.

dynamic. Fig.11(e) shows that the estimated Doppler rates are consistent with the actual rates for the adaptive schemes, while the estimations from non-adaptive approaches deviate from the actual values at different times. Note that the receiver's jerks at 20s, 120s, 180s and 280s can be properly estimated in the adaptive schemes.

The superior performance of the adaptive PLLs over FL-FT algorithm and AL-FT algorithm validates the state space design and optimization analysis for generalized PLL for dynamic weak signals using a receiver with LQO.

2) *Adaptive FLL*: The 2-state adaptive FLLs with $T = 1ms$ and $10ms$ are adopted to process this dynamic weak signal. KF-based FLLs with $T = 1ms$ and $10ms$ are also tested for performance comparison purposes. The values of q_a are set as $1(m^2/s^6)/Hz$ and $10(m^2/s^6)/Hz$ to represent signal dynamics described by equation (10) for $T = 1ms$ and $10ms$, respectively.

Fig.12(a) shows that the estimations of C/N_0 in the adaptive FLLs generally follow the real signal strength variation. The mismatch between the estimated and actual C/N_0 in the KF-based FLLs is most likely due to the large frequency tracking errors. Fig.12(b) shows that the values of BW_{opt} used in the adaptive FLLs are automatically tuned according to the real time C/N_0 estimations, and these values generally follow the theoretical values that obtained for the actual C/N_0 . Again, BW_{opt} with $T = 1ms$ is smaller than that with $T = 10ms$. Fig.12(c) shows the KF-based FLLs with $T = 1ms$ and $10ms$ lost lock at about 120s and 180s, respectively, while the adaptive FLLs maintain tracking throughout the time period. However, there are some frequency errors when $T = 1ms$ due to the inaccurate measurements when the signal is weak from 120s to 180s. Fig. 12(d) shows that the Doppler frequency rate estimations in the KF-based FLLs with $T = 1ms$ and $10ms$ deviate from the truth after 120s and 200s, respectively. The estimations in the adaptive PIF-based FLL with $T = 1ms$ follow the signal dynamics in general, but with some disturbances from 80s to 220s when C/N_0 drops to 26dB-Hz. A FLL with short integration time may be adequate to satisfy the weak and dynamic signal tracking requirement, but may be at the risk of having a lower tracking accuracy. With a longer integration time, such as $T = 10ms$, the estimations in the adaptive FLL are more accurate throughout the time period.

The above results show that the adaptive FLL perform better than the KF-based FLL for dynamic weak signals using a receiver front-end with a LQO. Comparing the Doppler frequency and frequency rate estimations in adaptive PIF-based PLL in Fig. 11(d)-(e) with that in adaptive PIF-based FLL in Fig. 12(c)-(d) for $C/N_0 = 26dB-Hz$ shows that the 2-state PIF-based FLL with $T = 10ms$ and $BW_{opt} \sim 4Hz$ achieves equivalent tracking performance with that of the 3-state PIF-based PLL with $T = 10ms$ and $BN_{opt} \sim 12Hz$ and T_{opt} at 10ms. Both tracking loops perform better than the 2-state PIF-based FLL with $T = 1ms$ and BW_{opt} at about $\sim 1.8Hz$.

VI. CONCLUSIONS

In this paper, the generalized PLL and FLL architectures presented in the companion paper [21] are analyzed for performance optimization and practical implementation.

Based on the MMSE criteria, the PLL with PIF, WF, and KF implementations were optimized to improve the tracking sensitivity and dynamic responses. The carrier phase tracking sensitivity limitations were obtained in terms of the 3-sigma rule. The analysis demonstrated that the minimum C/N_0 level that can be tracked in PLLs with PIF, WF, and KF implementations is 6dB-Hz in the pilot channel and 13dB-Hz in the data channel for static receivers with HQO, respectively. These levels are 9dB better than for receivers with LQO. Additionally, it confirmed that receiver platform dynamics clearly impact the sensitivity performance. For the convenience of real implementations, the relationships between T_{opt} and C/N_0 , BN_{opt} and C/N_0 for different levels of platform dynamics and two types of receiver front-end oscillator qualities were established. Optimal PLL parameters were derived to facilitate their implementations.

Similarly, the FLL with PIF was optimized based on the MMSE criteria. The theoretical analysis shows that a FLL is less affected by the oscillator noise effect, but is more sensitive to the integration time than a PLL. The optimized FLL has at least a 6dB improvement in tracking sensitivity compared to the optimized PLL. For the convenience of real FLL implementations, the relationships between BW_{opt} and C/N_0 for different levels of receiver dynamics with 1ms and 10ms integration times were obtained.

Building on this theoretical analysis, the adaptive PLL and the adaptive FLL to track weak and high dynamic signals were proposed. Two case studies using simulator generated signals with high receiver platform dynamics and low signal power were presented to verify the theoretical analysis, as well as the adaptive tracking schemes. The simulation results confirmed that (1) the adaptive PLL is superior to the traditional PLL; (2) the adaptive FLL is superior to the KF-based FLL; (3) the 1-state adaptive FLL with a 10ms integration time achieves almost equivalent tracking performance with the 2-state adaptive PLL with adaptive integration time (60ms maximum) for the C/N_0 as low as 17dB-Hz under the static scenario; (4) the 2-state adaptive FLL with a 10ms integration time and BW_{opt} at about 4Hz achieves an equivalent tracking performance to that of the 3-state PIF-based PLL with $T_{opt} = 10ms$ and $BN_{opt} \sim 12Hz$. Both tracking loops perform better than the 2-state PIF-based FLL with $T = 1ms$ and $BW_{opt} \sim 1.8Hz$ for C/N_0 as low as 26dB-Hz under a maximum $50m/s^3$ jerk dynamic condition.

There are several areas of future improvement in this project. We list them below.

1). The performance of the proposed adaptive PLL and FLL depends on good signal C/N_0 estimation. However, in the presence of multipath fading or interference, the C/N_0 estimation may be affected by large signal level fluctuations due to constructive interference. The adaptive filter weighs these unreliable estimates more strongly due to the high C/N_0 and the tracking loop quickly can become unstable. Therefore, to improve the carrier tracking robustness and reliability, robust C/N_0 estimation methodology and improved carrier tracking algorithms to mitigate interference and multipath effects need to be further investigated.

2). The selection of $\mathbf{B} = \mathbf{I}$ and $\mathbf{K} = \mathbf{A}$ has been used in

the companion paper [21] to simplify the error state estimation so as to cast the traditional PLL design onto the state space framework. However, from a control system design perspective, other possible $\mathbf{B}$ and $\mathbf{K}$ selections that ensure system controllability and stability can be adopted as well to satisfy the specific system performance, such as robustness and anti-interference, etc. The different $\mathbf{B}$ and $\mathbf{K}$ selections will need to be discussed and studied in the future.

3). The optimization in this paper shows that the limits of carrier tracking loop design is ultimately determined by signal characteristics regardless of what filter design approach is used. Therefore, an accurate signal model is key to improving the tracking performance. This paper only considers the general signal model found in the literature. However, the time-correlated clock errors [26], which represent the correlation between measurement noise and process noise, will affect the system performance, particularly for longer integration times. Additionally, there are various error sources, such as man-made RF interference [27] and ionospheric scintillation [28], that corrupt and distort signal carrier parameters in real world applications. These errors should be considered in future signal models to improve the carrier tracking performance.

4). The generalized state space framework enables not only the single-input, single-output tracking loop designs, such as PIF-, WF-based PLL and PIF-based FLL, but also the multiple-input, multiple-output tracking loop designs. Hence, many other filtering techniques and estimation approaches, such as moving horizon estimation (MHE) [29] based on the multi-epoch phase error or frequency error measurements, could be used to design the state estimator in the generalized carrier tracking loop. In addition, the FLL-assist-PLL that utilizes both the phase error or frequency error measurements can be designed, analyzed, and optimized based on the state space design approach to improve future carrier tracking capability.

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APPENDIX A

OPTIMAL SOLUTION FOR 1-STATE FLL

To solve the quartic equation $\alpha^4 - 4\alpha^3 - b\alpha + b = 0$, we denote $\alpha = x + 1$ to eliminate the term α^3 in this quartic equation and then we have:

$$x^4 + 6x^2 - (b + 8)x - 3 = 0. \quad (\text{A.1})$$

Applying the Ferrari's solution, the four solutions of the above equation can be obtained as:

$$x_1 = \frac{1}{2} \sqrt{4 + \sqrt[3]{b^2 + 16b}} - \frac{1}{2} \sqrt{8 - \sqrt[3]{b^2 + 16b} + \frac{2b + 16}{\sqrt{4 + \sqrt[3]{b^2 + 16b}}}} \quad (\text{A.2})$$

$$x_2 = \frac{1}{2} \sqrt{4 + \sqrt[3]{b^2 + 16b}} + \frac{1}{2} \sqrt{8 - \sqrt[3]{b^2 + 16b} + \frac{2b + 16}{\sqrt{4 + \sqrt[3]{b^2 + 16b}}}} \quad (\text{A.3})$$

$$x_3 = -\frac{1}{2} \sqrt{4 + \sqrt[3]{b^2 + 16b}} - \frac{1}{2} \sqrt{8 - \sqrt[3]{b^2 + 16b} - \frac{2b + 16}{\sqrt{4 + \sqrt[3]{b^2 + 16b}}}} \quad (\text{A.4})$$

$$x_4 = -\frac{1}{2} \sqrt{4 + \sqrt[3]{b^2 + 16b}} + \frac{1}{2} \sqrt{8 - \sqrt[3]{b^2 + 16b} - \frac{2b + 16}{\sqrt{4 + \sqrt[3]{b^2 + 16b}}}} \quad (\text{A.5})$$

The corresponding solutions of α can be obtained through the relation $\alpha = x + 1$. It is noted that the tracking error variance, $\mathbf{p}_{\bar{\omega}}$, should be always positive. The condition that $0 < \alpha < 2$ should be satisfied. Hence, the optimal value of α that minimizes $\mathbf{p}_{\bar{\omega}}$ is

$$\alpha_{opt} = 1 + \frac{1}{2} \sqrt{4 + \sqrt[3]{b^2 + 16b}} - \frac{1}{2} \sqrt{8 - \sqrt[3]{b^2 + 16b} + \frac{2b + 16}{\sqrt{4 + \sqrt[3]{b^2 + 16b}}}}. \quad (\text{A.6})$$

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