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Some observations in the vortex-turning behaviour of noncircular inclined jets

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Abstract

An experimental comparison was conducted for $Re=2500$, free elliptic and rectangular jets inclined at 30° and 60° along major- or minor-planes. Regardless of the jet base geometry, minor-plane inclined jets produced vortex roll-ups that remain inclined. In contrast, major-plane inclined jets produce significantly stronger vortex-roll-up turning behaviour.

Interestingly, major-plane inclined rectangular jets exhibit strong vortex overturning behaviour, where the vortex roll-up inclination exceeds the 0° incline-angle considerably.

Vortex-turning extents and rates are compared between major-plane inclined elliptic and rectangular jets here and support present qualitative observations. Closer inspections reveal that the lack of axis-switching phenomenon in major-plane inclined rectangular jets allows vortex overturning behaviour. In addition, jet centreline deflection is most sensitive in minor-plane inclined jets, where increasing the incline-angle leads to a decrease and an increase in the elliptic and rectangular jet deflection respectively.

Keywords: noncircular jets; inclined jets; time-resolved particle-image velocimetry; flow instabilities

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List of symbols/nomenclature

A	Jet cross-section area
D	Seeding particle diameter
D_h	Hydraulic diameter, $4A/P$
D_{major}	Major-plane dimension
D_{minor}	Minor-plane dimension
P	Jet wetted perimeter
t	Time from which inclined vortex roll-up was first observed to form
U_{cl}	Local centerline velocity in the streamwise direction
U_e	Mean jet exit velocity
x	Streamwise distance
y	Cross-stream distance
ρ	Seeding particle density
μ	Water dynamic viscosity
δ	Jet exit shear layer thickness
θ	Nozzle incline-angle
θ_v	Vortex-turning angle
Re	Reynolds number, $U_e D_h / \nu$
Stk	Stokes number
AR	Aspect-ratio of nozzle, $D_{\text{major}}/D_{\text{minor}}$
POD	Proper Orthogonal Decomposition
px	Camera pixel
$TR\text{-}PIV$	Time-resolved particle-image velocimetry

1. Introduction

As a form of passive jet control technique, the use of noncircular jets such as elliptic and rectangular jets has received considerable attention in recent decades. Despite their relatively simple geometries, these noncircular jets would produce vortex filaments that undergo axis-switching behaviour and thereby greatly increase jet-mixing and entrainment levels. Axis-switching behaviour is an unique phenomenon associated with jets and vortex rings issuing from noncircular jet exits/nozzles, whereby their major- and minor-axes interchange regularly until transition to turbulence. Numerous experimental and simulation studies have been conducted to look into the influence of initial conditions on noncircular jets, such as their aspect ratio, initial momentum thickness distributions and the presence of excitation conditions, just to name a few. As a result, flow mechanisms associated with the initiation, evolution, interaction and breakdown of noncircular vortex-ring and jet ring-vortices have been well-established by investigations performed by Gutmark and Ho (1986), Ho and Gutmark (1987), Husain and Hussain (1983, 1991, 1993), Hussain and Husain (1989), Quinn (1989), Lee and Baek (1994) and Gutmark and Grinstein (1999), amongst others.

On the other hand, effects due to the use of inclined exits upon noncircular jets received little attention until studies were conducted on inclined elliptic jets by New (2009), and New and Tsovolos (2011, 2012) recently. These studies were fundamentally motivated by earlier investigations by Wlezien and Kibens (1986), Webster and Longmire (1997, 1998) and Lim (1998), where, depending on the exact incline-angle and initial flow conditions, inclined circular jets were observed to undergo rapid realignment and mutual pairings as a form of self-excitation. For inclined elliptic jets however, such self-excitation behaviour were absent. Instead, major-plane inclined elliptic jets were observed to suppress axis-switching behaviour as the incline-angle becomes sufficiently large, whereas minor-plane inclined elliptic jets

merely **delay** the axis-switching behaviour. Experimental evidences gathered thus far suggested that the inclined jets are highly sensitive towards their basic azimuthal geometry, as well as the exact plane along which the inclination is imposed upon. **In particular, volumetric particle-image velocimetry measurements by Troolin and Longmire (2009) and three-dimensional flow simulations by Le et al. (2011) on inclined circular vortex-rings have revealed surprisingly rich flow details on their formation and breakdown behaviour. These studies lend credence to the notion that jets and vortex-rings emanating from noncircular inclined nozzles should possess equally, if not more, complex near-field flow behaviour.**

With that in mind, a study was conducted here to reveal first-hand, the most important flow differences between jets exhausting from major-plane and minor-plane inclined elliptic and rectangular jets. While rectangular nozzles are fundamentally noncircular by nature, they nonetheless possess sharp corners that produce rather different initial jet conditions (i.e. momentum thickness, turbulence intensities, for example), as compared to elliptic nozzles without any sharp corners. The present study was motivated by three outstanding questions: Firstly, how will the near-field vortex dynamics of inclined rectangular jets differ from equivalent inclined elliptic jets, in terms of the vortex-turning behaviour at sufficiently large incline-angles? Secondly, will the presence of sharp corners results in any discrepancies from observations made for inclined elliptic jets previously? **Thirdly, if discrepancies do exist, how will they impact upon on the jet flow and mixing characteristics within the near-field?** Results from the current investigations will be presented later to shed light upon these questions.

2. Experimental setup and procedures

(a) Jet apparatus and test nozzles

The experimental setup used in the current study was similar to the one used in New (2009), whereby a recirculating Plexiglas-based water tank was used. The transparent water tank walls were 15mm thick and the internal dimensions measured 400mm(W) $\times$ 400mm(H) $\times$ 800mm(L). A centrifugal pump channelled water from a small reservoir into a jet flow-conditioning apparatus, where it comprised of a diffuser, honeycomb section, three layers of fine-screen and a circular-to-rectangular contraction chamber, before the water was exhausted into a quiescent water tank through the test nozzles. The use of these flow-conditioning devices helped to limit the jet exit centre turbulence intensity levels to approximately $u_{rms}/U_e=0.05$ [New and Tsovolos (2013)]. To maintain a constant water level, excess water was redirected back to the reservoir via two pipes installed at the end wall of the water tank. A needle valve and a Flotech electromagnetic flow meter were used to regulate the water rates according to the required jet velocity.

In the present study, aspect-ratio of three (i.e. AR=3) elliptic and rectangular nozzles inclined along either their major- or minor-planes were investigated. Figure 1 shows the design schematics for the reference and inclined rectangular nozzles, while the corresponding details for the reference and inclined elliptic ones can be found in New and Tsovolos (2011). The major and minor internal dimensions of the rectangular nozzles measured $D_{major}=32.6\text{mm}$ and $D_{minor}=10.9\text{mm}$ respectively with a resultant hydraulic diameter of $D_h=16.3\text{mm}$. These internal dimensions were selected such that all the rectangular and elliptic nozzles have similar cross-sectional area for greater consistency (i.e. cross-sectional areas of elliptic and rectangular nozzles are 354.54mm^2 and 355.34mm^2 respectively, with only 0.22% area discrepancy). Additionally, they have similar mean heights and wall thickness (1mm) as the

Fig. 1 Geometrical details of the (a) reference, (b) 30° minor-plane inclined, (c) 60° minor-plane inclined, (d) 30° major-plane inclined and (e) 60° major-plane inclined rectangular nozzles

elliptic nozzles. Two different incline-angles of 30° and 60° were explored to look into their effects. For comparisons, reference non-inclined elliptic and rectangular nozzles were included as part of the investigation as well.

Lastly, low Reynolds number $Re=2500$ free jets were investigated here, unlike some earlier studies which made use of forced jets [Longmire and Truong (1996), Webster and Longmire (1997), New (2009), New and Tsovolos (2009, 2010)]. Flow forcing was not utilised here as it might overwhelm any subtle effects due to the nozzle base geometry, which was the primary motivation of the present study. **Lastly, note that due to the short entrance length**

associated with the jet apparatus and test nozzles, the flows within the latter were not fully-developed prior to their exits.

(b) Shear layer visualisations

Flow visualisations were firstly carried out to obtain first-hand appreciation of the flow structures associated with the reference and inclined **noncircular** jets. Using a gravity-feed technique, coloured-dye was evenly released into the jet shear layer via a 1mm wide circumferential slit setup, which is similar to that used by New (2009). With the coloured-dye highlighting the jet shear layers, a 500W halogen floodlight was used to illuminate the overall jet flow fields and a 25 frames-per-second **JVC 3CCD colour video camera with a resolution of $720 \times 576 \text{ pixel}^2$ was used** to record the experiments. Still flow visualisation images were subsequently extracted from the digital videos to a workstation for further analyses. **Though the captured flow visualization region extended to approximately $8D_h$ downstream of the nozzle mean height location, the presented flow images were cropped to highlight the most important near-field flow features. While it is known that such dyes are scalar by nature and do not track the flow vorticity exactly, it is also generally recognized that dye visualizations remain an effective technique to detect vortex structures/cores, if the visualization analysis is strictly limited to the immediate near-field region of the jet exit and that the underlying vortex formations/motions are significant. In particular, note that dye visualizations have been used successfully by Lim (1998) and Webster and Longmire (1998) to identify vortical behaviour associated with inclined nozzles.**

(c) Time-resolved particle-image velocimetry

To investigate the initiation and evolutions of the jet vortex structures quantitatively, time-resolved particle image velocimetry (i.e. TR-PIV) measurements were performed with a

Coherent 3W, 532nm wavelength, continuous-wave diode-pumped solid-state laser and a Fastec Imaging HiSpec-1 camera. The laser beam was expanded to an approximately 1mm thick light sheet by a plano-concave cylindrical lens to illuminate the jet flows seeded with 50µm diameter polyamide seeding particles with a nominal density of 1.03g/cm³. The ability of the seeding particles to track the jet flows adequately was evaluated through the Stokes number, defined as $Stk = (\rho D^2 / 18\mu) / (\delta / U_{cl})$. In this case, the Stokes number was estimated to be approximately 0.0056 for the current experimental conditions, which is well within the acceptable limit as pointed by Crowe et al. (1988). For the purposes of the present study, the laser sheet was aligned along either the major- or minor-plane of the test nozzles in the streamwise direction. Scattered light from these particles were captured by the 10-bit 1280 × 1024px² camera at 200 frames-per-second. In addition, the camera exposure time was set to 1.5ms throughout the study to avoid any “streaky” particle images. Using a Nikon Micro Nikkor 60mm f2.8 lens, the image spatial resolution was kept at 0.159mm/px for all experiments. A total of 11,438 sequential image frames were captured for each test case and subjected to PIV interrogation techniques.

Note that for the purpose of TR-PIV here, image-pair analysis was performed differently as compared to conventional PIV utilising double-pulsed laser imaging. In this case, images 1 and 2 would be analysed first before images 2 and 3 were processed, and so on and so forth. The maximum particle shift between two consecutive image-frames was less than 8px and the image-sequences were analysed using multi-grid cross-correlation algorithm with an initial and final interrogation size of 64 × 64px² and 32 × 32px² respectively. The interrogation window overlapping was 75% in both horizontal and vertical directions, with a resultant final spatial resolution of 0.882 vector/mm. Raw velocity vectors were firstly validated by the local mean filter scheme and any rejected vectors were subsequently

Nozzle configuration	Elliptic nozzles δ/D_h		Rectangular nozzles δ/D_h	
	Shorter length	Longer length	Shorter length	Longer length
Reference, major plane	0.28		0.28	
Reference, minor plane	0.13		0.27	
30° major-plane inclined	0.20	0.22	0.24	0.14
60° major-plane inclined	0.28	0.25	0.25	0.20
30° minor-plane inclined	0.18	0.16	0.27	0.28
60° minor-plane inclined	0.16	0.17	0.24	0.18

Table 1. Normalized reference and inclined jet boundary layer thicknesses

replaced by a 3-point by 3-point linear interpolation. The velocity measurement uncertainty was estimated to be approximately $\pm 1.7\%$.

(d) Initial jet conditions

Jet exit boundary layer thicknesses along the shorter and longer nozzle length regions for all inclined nozzles here were approximated from mean TR-PIV velocity profiles close to their exits (i.e. about $0.2D_h$ away from the shorter and longer nozzle lengths) and presented in Table 1. Corresponding boundary layer thicknesses for the reference elliptic and rectangular nozzles are provided for comparisons as well. For the reference elliptic nozzle, the boundary layer along its major-plane is thicker than that along its minor-plane, which agrees well with earlier studies on free elliptic jets [Ho and Gutmark (1987), Hussain and Husain (1989)]. On the other hand, those for the reference rectangular nozzle are relatively similar along both major- and minor-planes. Since the most fundamental difference between the elliptic and rectangular nozzles lies in their base geometries, it is clear that the presence of sharp corners plays a significant role in affecting the boundary layer characteristics.

For elliptic and rectangular major-plane inclined nozzles, both the boundary layers along both the shorter and longer nozzle lengths become thicker as the incline-angle is increased from 30° to 60° . On the other hand, for elliptic and rectangular minor-plane inclined nozzles, boundary layer thickness along both shorter and longer nozzle lengths tend to reduce as the incline-angle increases 30° to 60° . Since the shorter and longer nozzle lengths become even shorter and longer respectively after the increase in incline-angle, the results deviate from what would have been expected for a non-fully-developed boundary layer where its thickness solely depends upon its flow length (and hence nozzle length). Instead, it should be clear that there should be other factors that need to be taken into account, other than the nozzle length. For instance, circumferential boundary layer thickness distribution of elliptic jets are sensitive towards their aspect-ratios. In particular, nozzle aspect-ratio dictates how close the sharp corners are to one another for rectangular nozzles and with an aspect-ratio of three nozzles used here, its influence is not inconsiderable.

3. Results and discussions

(a) General near-field vortical flow behaviour

Figures 2 and 3 show instantaneous visualisation results for reference and inclined elliptic and rectangular jets respectively, taken along both major- and minor-planes of their respective nozzles. Earlier studies (New and Tsovolos 2011, 2012) had already shown that inclined nozzles tend to confer more significant and interesting flow changes along the incline-planes (i.e. major-planes for major-plane inclined jets, and minor-planes for minor-plane inclined jets). Therefore, for the sake of brevity, visualisation results of inclined jets along planes orthogonal to their incline-planes will not be presented here.

Fig. 2 Instantaneous jet shear layer visualisations taken for the reference, 30° and 60° major- and minor-plane inclined elliptic jets

Starting with the visualisation results of reference elliptic and rectangular jets taken along their major- and minor-planes (i.e. Figs. 2(a), 2(d), 3(a) and 3(d) respectively), it can be readily seen that their general flow behaviour is relatively similar. For instance, natural wave instabilities exist along the jet shear layers, **which eventually lead to the shear layers “rolling-up” into distinct large-scale azimuthal vortex roll-ups**. Along the major-plane, the vortex roll-ups consistently move towards the jet centreline and as a result, rib-structures form regularly at both ends of the major-plane. Along the minor-plane however, vortex roll-ups move away from the jet centreline with no evidence of regular rib-structure formations. These shared flow features attest to some of the basic flow similarities shared by noncircular jets in general (Gutmark and Grinstein 1999).

For the major-plane inclined elliptic jets shown in Figs. 2(b) and 2(c), rapid turning of the initially inclined vortex roll-ups occurs very shortly after they emanate from the nozzle exits.

Fig. 3 Instantaneous jet shear layer visualisations taken for the reference, 30° and 60° major- and minor-plane inclined rectangular jets

The vortex roll-up section along the shortest nozzle length location convects significantly faster than that at the longest nozzle length location, causing the entire vortex-roll-up to reduce its inclination and realign rapidly to become relatively perpendicular to the jet flow direction. Note that Wlezien and Kibens (1986), Webster and Longmire (1998) and New (2009) had reported upon such vortex-turning behaviour in inclined circular and elliptic jets previously and hence, their detailed behaviour will not be elaborated here.

As for major-plane inclined rectangular jets shown in Figs. 3(b) and 3(c), similar rapid turning of the vortex roll-ups also occurs. What is particularly intriguing however, is that these vortex roll-ups are turning significantly faster and through larger angles as compared to those for major-plane elliptic jets at similar inclination angles. In fact, closer inspections will reveal that the inclined vortex roll-ups will actually “overturn” significantly beyond the cross-stream axis as they convect away from the inclined nozzle exit. “Over-turning” refers to the

situation whereby the turning of the inclined vortex roll-up not only leads to a reduction of its inclination to 0° (i.e. parallel to cross-stream axis), but increases its inclination further beyond the cross-stream axis as well. Furthermore, as the incline-angle increases from 30° to 60° , the overturning behaviour of the vortex-roll-ups accentuates further. In fact, the vortex roll-ups incur significant overturning even before they completely detach from the 60° inclined nozzle. Compared to the major-plane inclined elliptic jets, the vortex-turning behaviour in major-plane inclined rectangular jets is considerably more aggressive.

On the other hand, vortex roll-ups of minor-plane inclined elliptic and rectangular jets do not undergo significant turning (see Figs. 2(e), 2(f), 3(e) and 3(f)). For 30° minor-plane inclined elliptic and rectangular jets, the inclined vortex roll-ups retain their original inclination till they transit to incoherence. When the incline-angle increases to 60° , some mild vortex-turning behaviour occurs. However, it is important to highlight here that the vortex-turning appears to reach an asymptotic state within two to three hydraulic diameters downstream of the nozzle exits, such that the vortex roll-up inclinations become relatively constant before becoming incoherent. This is true regardless of whether the nozzle base geometry is elliptic or rectangular. Curiously, unlike the significant differences in the vortex-turning behaviour observed along the major-planes of major-plane inclined elliptic and rectangular jets, the overall vortex roll-up inclinations observed for both minor-plane inclined elliptic and rectangular jets are relatively similar.

(b) Vortex overturning behaviour

It is intriguing that major-plane inclined elliptic and rectangular jets exhibit significantly different vortex-turning behaviour. In particular, the vortex-turning impacts upon the flow strain of the inclined elliptic/rectangular ring-vortices and intuitively, this will in turn affect

Fig. 4 Detailed time-sequenced views of the 60° major-plane inclined elliptic vortex roll-up

the extent to which dissimilar jet mixing characteristics exist along the shortest and longest nozzle length locations. Therefore, it is worthwhile to take a closer look at the near-field vortex dynamics associated with their respective vortex-turning behaviour. To that end, time-sequenced close-up views of the vortex filaments emanating from both 60° inclined elliptic and rectangular jets are shown in Figs. 4 and 5 respectively.

For the inclined elliptic jet shown in Fig. 4, the vortex filament begins to roll up only after it has realigned more or less perpendicularly to the streamwise direction. Note that the vortex roll-up forms close to the jet centreline and thus, similar to non-inclined elliptic jets. During its formation, it incurs a bend along its centre, which appears to move relatively slower than the rest of the vortex roll-up and indicates that axis-switching phenomenon continues to exist. To illustrate the flow behaviour better, Fig. 6(a) shows the time-sequenced outlines of the key vortex structures traced out from Fig. 4 for clarity. Note that in this figure, solid black lines

Fig. 5 Detailed time-sequenced views of the 60° major-plane inclined rectangular vortex roll-up

represents the vortex roll-ups, while dashed grey lines represents parts of the shear layer instabilities which have not rolled-up fully yet. It has been observed previously [New (2009)] that rolling-up of the inclined elliptic jet shear layer propagates at a slower speed towards the longer nozzle length region, as compared to the propagation speed towards the shorter nozzle region. Hence, Fig. 6(a) depicts a situation whereby instances of shear layer instabilities are still present along the longer nozzle length region. On the other hand, dashed black lines represent braid vortices along the shorter nozzle length regions that are stretched by vortex roll-ups turning rapidly towards the downstream direction. It can be discerned that, despite the rather convoluted near-field vortical behaviour, the entire vortex roll-up generally incurs only slight vortex-overturning.

On the other hand, the near-field vortex dynamics for the major-plane inclined rectangular nozzle is less convoluted than its elliptic counterpart. Once the vortex roll-up is produced, it

Fig. 6 Comparison of the vortex roll-up outlines between (a) 60° major-plane inclined elliptic and (b) 60° major-plane inclined rectangular jets

remains relatively free of any significant flow distortions and its inclination is clearer compared to the inclined elliptic jet. In particular, **little bending of the vortex roll-up can be observed, compared to its inclined elliptic counterpart.** Interestingly though, while the formation of the vortex roll-ups appears to proceed at relatively similar time-scales along the shortest and longest nozzle length regions for the inclined elliptic jet, it does not seem to be the case for the inclined rectangular jet. For instance, it can be discerned from Fig. 5(f) onwards that vortex roll-up formation proceeds at a significantly faster rate along the shortest nozzle length region, which would explain the ability of inclined rectangular jets to undergo vortex-overturning. Furthermore, the absence of **any significant bending** along its centreline also means that the vortex-overturning is comparatively more stable. For comparison with the inclined elliptic jet, outlines of the vortex filament/roll-up for the inclined rectangular jet are presented in Fig. 6(b).

Fig. 7 Time-sequences of POD-reconstructed vorticity maps for (a) 60° major-plane inclined elliptic and (b) 60° major-plane inclined rectangular jets

To validate the preceding qualitative observations, corresponding time-sequenced vorticity fields of the 60° major-plane inclined elliptic and rectangular jets are presented in Fig. 7. Vorticity fields directly derived from TR-PIV measurements were subjected to Proper Orthogonal Decomposition (POD) analysis [Lumley (1967) and Kim and Rockwell (2005)], whereby the first 30 POD modes were used to generate the reconstructed vorticity fields. Since approximately 80% of the flow energy resides within the first 30 POD modes here, the reconstructed vorticity fields will allow better identification of the dominant vortex structures and behaviour. From the figure, it can be observed that not only both noncircular inclined jets undergo vortex-turning, but that the vortex-turning rate for the inclined rectangular jet is indeed faster than that of its elliptic counterpart. Therefore, these quantitative results agree well with the earlier flow visualization observations and reinforce the notion that the near-field vortex dynamics of inclined jets is highly sensitive towards nozzle base geometry.

Based on the flow behaviour seen so far, it would appear that the critical lack of axis-switching phenomenon in the 60° inclined rectangular jet allows its vortex roll-ups to overturn. The lack of significant distortions and associated motions to the vortex roll-ups implies that effects due to circumferential self-induced velocities are minimal. Without the latter extraneous flow effects disrupting the vortex-turning process resulting from the initially highly-inclined vortex roll-ups, the vortex roll-ups will logically be able to sustain larger turning-angles than those seen in 60° inclined elliptic jet vortex roll-ups. In contrast, experimental evidence presented in Fig. 4 indicates that 60° inclined elliptic jet vortex roll-ups undergo strong vortex distortions closely associated with axis-switching phenomenon. As a result of highly three-dimensional and non-uniform self-induced velocities typically linked to these vortex distortions, vortex-turning behaviour will be less sustainable and more limited.

(c) Comparison of vortex-turning rate

To explore and quantify the vortex-turning behaviour further, time-sequenced dye-visualisation images were extracted from the flow visualisation videos for additional analysis. To accomplish this, firstly, a clear video segment of a selected vortex roll-up forming at the inclined nozzle exit and travelling downstream until it transits to turbulence was identified and isolated from the video files. Subsequently, instantaneous flow image sequences were extracted from the video segments and changes to the vortex roll-up inclination were estimated manually from all the extracted flow images, as shown in Fig. 6(b). Note that only the vortex roll-up inclinations were estimated, without taking into account the inclinations of shear layer vortices that have not developed into complete vortex roll-ups and braid vortices, Incorporating them into the estimations would have grossly distorted the actual vortex roll-up inclinations.

Additionally, estimations of the inclination angles were not entirely based on the upper and lower locations of the inclined vortex roll-ups, though they did serve as important guides. Instead, the overall inclination of the vortex roll-up as it moved away from the nozzle exit was estimated instead. Depending on the exact vortex roll-up outline, this inclination angle could be quite different from that based on the inclination angle of a hypothetical straight line joining the upper and lower ends of the vortex roll-up. While there was some level of uncertainty associated with such a procedure, it remained satisfactory for a first-hand appreciation of the different vortex-turning rates between inclined elliptic and rectangular jets [here](#). Due to the rather laborious technique of extracting the vortex roll-up inclinations manually, these procedures were repeated for five complete vortex roll-up flow cycles and the five vortex roll-up inclinations at each time-stamp were averaged to arrive at the final inclination. The first vortex filament inclination to be measured is located approximately at the mean height position (i.e. $x/D_h=0$).

Figure 8 shows the variations in the vortex roll-up inclination angle with time for all major-plane inclined elliptic and rectangular jets. Starting with the inclined elliptic jets, it can be seen that the vortex-turning trends are consistent with Fig. 2 presented earlier. While their vortex roll-ups undergo turning, no overturning behaviour occurred with the exception of the 60° inclined elliptic jet. Even then, the maximum overturned vortex inclination reaches a maximum of only approximately $\theta_v/\theta=-0.18$. It should be noted that the 60° inclined elliptic jet vortex roll-ups undergo rapid vortex-turning behaviour within the immediate vicinity of the nozzle, as the first discernible vortex roll-up inclination is already approximately $\theta_v/\theta=0.2$. Clearly, this inclination is significantly smaller than the physical 60° inclined elliptic nozzle. In contrast, first discernible vortex roll-up inclination for the 30° inclined elliptic jet is approximately $\theta_v/\theta=0.8$. Lastly, the rate of the vortex-turning behaviour is slower for the 60°

Fig. 8 Temporal variations of vortex roll-up inclination angle for 30° and 60° major-plane inclined elliptic and rectangular jets

inclined elliptic jet, presumably due to the initially larger vortex-turning rate which reduces the flow strain more than its 30° inclined counterpart.

On the other hand, the same figure shows that inclined rectangular jet vortex roll-ups undergo significant vortex overturning, regardless of the exact nozzle inclination. For instance, the 30° and 60° inclined rectangular vortex roll-ups attain maximum overturning inclinations of approximately $\theta_v/\theta = -0.65$ and -0.5 respectively, which far exceeds those observed for their corresponding inclined elliptic counterparts. Interestingly, while the 30° inclined rectangular jet starts off with relatively similar initial vortex roll-up inclination as the 30° inclined elliptic jet (i.e. $\theta_v/\theta = 0.8$), its 60° counterpart has a larger initial vortex roll-up inclination than the 60° inclined elliptic jet. This suggests that, for an inclined nozzle with a large inclination, vortex-turning behaviour within close proximity of an inclined nozzle will be significantly lower for a rectangular nozzle geometry than an elliptic one. Additionally, the rates of vortex-turning

Fig. 9 Comparisons of mean jet centreline trajectories for 30° and 60° major- and minor-plane inclined (a) elliptic and (b) rectangular jets

for both 30° and 60° inclined rectangular vortex roll-ups are relatively similar before the latter begins to overturn. After they proceed to overturn, the 60° inclined jet will incur a slower rate of vortex turning than the 30° inclined jet.

(d) Mean jet centreline deflection

Earlier studies on inclined circular vortex-rings by Webster and Longmire (1997), inclined elliptic jets under forced flow conditions by New and Tsovolos (2011) and inclined circular coaxial jets by New and Tsioli (2011) had demonstrated the tendency for the vortex-rings and jets to gradually deviate from the nozzle centreline as they propagate from the inclined nozzle exits. The availability of TR-PIV measurements for both inclined elliptic and rectangular jets here provides an opportunity to accurately compare the effects of nozzle base geometry upon the mean centreline deflections of the inclined jets. To do that, time-averaged TR-PIV velocity vector results were first plotted and mean streamlines originating from the inclined nozzle centerlines were subsequently determined and plotted in Fig. 9. These streamlines

would represent the mean trajectories incurred by fluid elements emanating from the jet centreline and provides some indications upon the mean jet deflections. Note that this method has been used successfully by New and Tsioli (2011) to assess the deflections incurred by inclined coaxial jets under a variety of initial flow and physical conditions.

For major-plane inclined elliptic jets, increasing the nozzle incline-angle from 30° to 60° leads to a significant shift of the jet trajectory towards the shortest nozzle length region. In contrast, the same increment in the nozzle incline-angle for minor-plane inclined elliptic jets yields an even larger shift of the jet trajectory towards the longer nozzle length region. In comparison, increasing the incline-angle produces consistent shifts in the jet trajectories towards the shortest nozzle length region for all inclined rectangular jets, regardless of whether they are major- or minor-plane inclined. In particular, minor-plane inclined rectangular jet achieves the largest shift when the incline-angle was increased from 30° to 60° . As for 60° major-plane inclined rectangular jets, the corresponding change in jet trajectory is significantly smaller, even when compared to 60° major-plane inclined elliptic jets. This agrees well with the persistent vortex overturning behaviour in major-plane inclined rectangular jets, where its motions towards the jet centreline serve to limit the jet-spread severely.

It is also interesting to note that when the incline-angle is moderate at 30° and for the same incline-plane used, the exact nozzle base geometry does not impact significantly upon the jet trajectories. However, the use of inclined exits along minor-planes of noncircular nozzles confers much more significant flow effects than varying their base geometry. This is an intriguing observation, since earlier flow visualisation results do not show significant differences in the vortex structures and behaviour between minor-plane inclined elliptic and

rectangular jets. However, note that it has been shown in earlier studies on noncircular jets that compared to momentum thickness along the major-planes, those along the minor-planes are typically smaller and therefore more sensitive towards the imposition of any inclined exits.

4. Conclusions

Near-field flow visualisations reveal **that** major-plane inclined elliptic jet vortex roll-ups undergo turning as observed in earlier studies with little overturning. In contrast, major-plane inclined rectangular jets incur significant overturning of the vortex roll-ups at similar incline-angles. Closer inspection also suggests that the inclined elliptic jet vortex roll-ups become so distorted by the axis-switching phenomenon, such that any potential vortex overturning is mitigated. In comparison, inclined rectangular jet vortex roll-ups do not suffer much flow distortions, even at the largest 60° incline-angle studied here. Minor-plane inclined jets, regardless of whether they are elliptic or rectangular, lead to vortex roll-ups that do not undergo discernible vortex-turning. Vortex-turning behaviour quantified from flow images show that major-plane inclined rectangular jets consistently lead to significantly larger vortex-turning angles and rates than their elliptic counterparts. Lastly, minor-plane inclined elliptic and rectangular jets are more sensitive towards incline-angle variations than their major-plane inclined counterparts. Interestingly, minor-plane inclined elliptic and rectangular jets incur a reduction and increment in their centreline deflections respectively when the incline-angle increases from 30° to 60° .

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References

- Crowe CT, Chung JN, Troutt TR (1988) Particle mixing in free shear flows. *Prog Energy Combust Sci* 14:171–194
- Gutmark E, Ho C-M (1986) Visualization of a forced elliptic jet. *AIAA J* 24:684–685
- Gutmark E, Grinstein F (1999) Flow control with noncircular jets. *J Fluid Mech* 31:[239–272](#)
- Ho C-M, Gutmark E (1987) Vortex induction and mass entrainment in a small-aspect-ratio elliptic jet. *J Fluid Mech* 179:383–405
- Husain HS, Hussain F (1983) Controlled excitation of elliptic jets. *Phys Fluids* 26:2763–2766
- Hussain F, Husain HS (1989) Elliptic jets. Part 1. Characteristics of unexcited and excited jets. *J Fluid Mech* 208:257–320
- Husain HS, Hussain F (1991) Elliptic jets. Part 2. Dynamics of coherent structures: pairing. *J Fluid Mech* 233:439–482
- Husain HS, Hussain F (1993) Elliptic jets. Part 3. Dynamics of preferred mode coherent structures. *J Fluid Mech* 248:315–361
- [Kim Y, Rockwell D \(2005\) Vortex buffeting of aircraft tail: interpretation via proper orthogonal decomposition. *AIAA J* 43:550-559](#)
- [Lumley JL \(1967\) Atmospheric turbulence and radio wave propagation. Edited by A. M. Yaglom and V.I. Tatarski, Navko, Moscow, 160-178](#)
- Lee SJ, Baek SJ (1994) The effect of aspect ratio on the near-field turbulent structure of elliptic jets. *Flow Meas Instrum* 5:170–180
- Longmire EK, Duong LH (1996) Bifurcating jets generated with stepped and sawtooth nozzles. *Phys Fluids* 8:978–992

- Lim TT (1998) On the breakdown of vortex rings from inclined nozzles. *Phys Fluids* 10:1666–1671
- Le TB, Borazjani I, Kang S, Sotiropoulos F (2011) On the structure of vortex rings from inclined nozzles, *J Fluid Mech* 686:451–483
- New TH (2009) An experimental study on jets issuing from elliptic inclined nozzles, *Exp. Fluids* 46:1139–1157
- New TH, Tsovolos D (2011) On the vortical structures and behaviour of inclined elliptic jets, *Eur J Mech B Fluids* 30:437-450
- New TH, Tsioli E (2011) An experimental study on the vortical structures and behaviour of jets issuing from inclined coaxial nozzles. *Exp. Fluids* 51:917-932
- New TH, Tsovolos D (2012) On the flow characteristics of minor-plane inclined elliptic jets. *Exp Therm Fluid Sci* 38:94-106
- Quinn WR (1989) On mixing in an elliptic turbulent free jet. *Phys Fluids A* 1:1716–1722
- Troolin D, Longmire E (2009) Volumetric velocity measurements of vortex rings from inclined exits. *Exp. Fluids* 48:409-420
- Webster DR, Longmire EK (1997) Vortex dynamics in jets from inclined nozzles. *Phys Fluids* 9:655–666
- Webster DR, Longmire EK (1998) Vortex rings from cylinders with inclined exits. *Phys. Fluids* 10:400–416
- Wlezien RW, Kibens V (1986) Passive control of jets with indeterminate origins. *AIAA J.* 24:1263–1270