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On the vortical structures and behaviour of inclined elliptic jets

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Abstract

A study has been carried out to clarify vortical structures and behaviour resultant from imposing inclined exits along either the major- or minor-plane of an elliptic nozzle. Laser-induced fluorescence (LIF) flow visualizations show production of inclined vortex roll-ups along the incline-planes, with corresponding narrowing of jet columns along the non-inclined planes. Minor-plane inclined nozzles result in significant growths in the jet spread along the incline-plane, while major-plane inclined nozzles produce little variations. Formation of rib structures is observed to be suppressed in minor-plane inclined nozzles and linked to braid vortices inducing the formation of streamwise vortices along the minor-plane. Particle-image velocimetry measurements show that increasing the incline-angle in major-plane inclined nozzles reduce the strengths of the discrete vortex roll-ups, while the opposite occurs in minor-plane inclined nozzles. Although Reynolds shear stress variations correspond well with changes in incline-angle and vortex roll-up strength in major-plane inclined nozzles, they demonstrate a non-monotonic relationship in minor-plane inclined nozzles. LIF visualizations further clarify how strong asymmetric interactions between the inclined vortex roll-ups and braid vortices lead to suppression of axis-switching in major-plane inclined nozzles but not in minor-plane inclined nozzles. The more complex flow behaviour in the latter is responsible for the non-linear relationship in Reynolds shear stress levels observed earlier. Comparisons of the half-jet width profiles confirm the suppression of axis-switching in major-plane inclined nozzles only, while momentum thickness profiles show significant variations in the mixing layer characteristics between major- and minor-plane inclined nozzles.

Keywords: elliptic jet; inclined nozzle; laser-induced fluorescence, particle image velocimetry

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1. Introduction

Imposing axial variations in nozzle length circumferentially have been widely seen as a simple but yet effective technique in which mixing behaviour between a jet exhausting from the nozzle and its surrounding can be manipulated favourably. One of the earliest studies on their basic flow behaviour was made by Wlezien and Kibens (1986, 1988), where they also named such nozzles as indeterminate-origin nozzles, since their geometrical origin cannot be ascertained exactly due to the varying axial extents. Their investigations on inclined and stepped nozzles under subsonic and supersonic conditions revealed systematic changes to the azimuthal energy distributions, trajectories and spread-rates of the resulting jet flows. As a result, further studies [Longmire et al. (1992), Longmire and Duong (1996), Webster and Longmire (1997, 1998), Lim (1998), New et al. (2005), Shu et al. (2005), Cai et al. (2009) and New and Tsovolos (2009)] soon followed with a shift towards the use of increasingly more complex designs. While there are many geometrical possibilities for an indeterminate-origin nozzle, inclined nozzles remain to be of particular interest. It is the simplest possible indeterminate-origin nozzle and produces relatively well-understood behaviour when used on circular nozzles.

On the other hand, less fundamental information on the use of inclined exits in noncircular nozzles is available. New (2009) recently investigated the use of inclined exits along the major-plane of aspect-ratio of three elliptic nozzles to obtain shear layer dye-visualizations and quantify the mean alterations to the flow fields. Results showed that the use of moderate and large incline-angles led to inclination and turning of the elliptic vortex roll-ups respectively, which are similar to inclined circular nozzles. Interestingly, flow visualization images also indicated that axis-switching behaviour had been suppressed. While the study clarified the mean resultant flow fields associated with inclined elliptic nozzles, they also raised questions on how axis-switching behaviour is suppressed by major-plane inclined nozzles. According to the studies carried out by Gutmark and Ho (1986), Ho and Gutmark (1987), Quinn (1989), Hussain and Husain (1989), Husain and Hussain (1991, 1993) and Lee and Baek (1994), axis-switching behaviour in elliptic jets is well characterized by the continuously changing elliptic jet cross-sectional shape which sees the interchanging of the jet major- and minor-axes. Furthermore, the roles played by the braid vortices during their interactions with the elliptic vortex roll-ups is well-documented and studied by Husain and Hussain (1993), in particular their involvement in the production of rib structures. Hence, there is a need to shed some light on how these well-established vortex structures and

behaviour are altered by the presence of inclined exits, and how they are in turn associated with changes in the axis-switching behaviour.

In view of these arguments, a study focusing on the discrete vortex structures and their behaviour resultant from major- and minor-plane inclined elliptic nozzles has been carried out. To carry out the task at hand, detailed flow visualizations on the developments of the jet vortex behaviour and associated cross-sections employing laser-induced fluorescence (LIF) techniques will be presented here. It will provide more detailed information on the internal jet vortex structures and behaviour over the shear layer dye-visualization results presented in the previous study, which revealed only the external flow structures along the jet peripheral. Since the focus in the current paper is on the resultant vortex structures and their behaviour, it is important that the flow fields are clarified in greater detail. In addition, phase-averaged particle-image velocimetry measurements which describe the flow states of the discrete elliptic vortex roll-ups will be provided to assess the initial conditions conferred upon them by the inclined nozzles. Such information will provide then some insights on the interpretations of the vortical behaviour as revealed in the LIF visualization results. Altogether, five elliptic nozzles will be discussed in the paper, which comprise of two major-plane inclined nozzles, two minor-plane inclined nozzles and one reference non-inclined nozzle.

2. Experimental setup and nozzle designs

The study was carried out using a recirculating water tank facility with associated jet apparatus as described in New (2009) and New and Tsovolos (2009) earlier. Hence, only a brief description will be provided here for the sake of brevity and readers are advised to refer to the preceding investigations for fuller details on the setup. The 800mm (L) x 400mm (H) x 400mm (W) horizontal recirculating water tank was constructed entirely from 15mm thick Plexiglas to ensure good optical access for the flow visualization and particle-image velocimetry (PIV) experiments to be carried out. A centrifugal pump was used to channel water from a small reservoir into the jet apparatus, along which a Blue-White Industries F-400 flow meter calibrated against a Fischer and Porter 10DS4111 electromagnetic flow meter prior to the study was employed to determine the flow rate. A piston-driven forcing device based on an LDS Ltd V201/3 electromagnetic actuator was also present along the flow delivery line before the water entered jet apparatus to impart regular physical forcing to the jet flows. As the

water entered the jet apparatus, it would be conditioned by a diffuser, honeycomb structure, three layers of fine screen and a circular-to-elliptic contraction chamber, before exhausting into the stagnant water reservoir. Water level was maintained constant by diverting water overflow out from two PVC tubes located at the end wall of the water tank and in turn, back into the small reservoir.

As the present study focused on the behaviour of the vortex structures and their behaviour, the Reynolds number of the all jet flows was limited to $Re = UD_h/\nu \approx 2500$, where the mean jet velocity, U , was maintained at $U = 0.15\text{m/s}$, D_h is the nozzle hydraulic diameter and ν is the kinematic viscosity of the water at working conditions. All jet flows produced were subjected to regular but mild forcings by the electromagnetic actuator in-conjunction with a TTI TG315 signal generator and LDS Ltd PA100 signal amplifier. The amplitude of forcing was estimated to be 2.2% of the mean jet velocity and just sufficiently high to produce regular rolling-up of the jet shear layers. Based on the actual forcing frequency of $f = 4.6\text{Hz}$, the Strouhal number was determined to be $St = fD_h/U \approx 0.5$. These operating conditions have been found to produce jets with well-defined vortex structures with no observable distortions to the dominant underlying flow mechanisms or productions of extraneous flow structures.

The designs of the reference and four inclined aspect ratio of three (i.e. $AR=3$) elliptic nozzles used here are shown in Fig. 1. Two of the inclined nozzles were inclined along the major-plane, while the other two were inclined along the minor-plane. Two incline-angles of 30° and 60° were used and it is clear from the figure that the discrepancy between the shorter and longer nozzle lengths became significant when 60° incline-angle was used, particularly when the inclined exit was imposed along the major-plane. The major- and minor-diameters of each nozzle measured 36.7mm and 12.3mm respectively, thus giving a hydraulic diameter of approximately $D_h = 16.4\text{mm}$. The origin of the nozzles was located at 40mm from the nozzle base, which resulted in a mean height of $H = 2.4D_h$. As a matter of consistency, all nozzle wall thicknesses were maintained at 1mm throughout as well.

Flow visualizations using laser-induced fluorescence (LIF) were used to understand the vortex structures and behaviour qualitatively. They were realised by premixing the jet fluid with fluorescein disodium dye uniformly

and illuminating it with appropriately located thin laser sheets. The laser sheets were formed using a 532nm wavelength Laser Quantum 5W Elite diode-pumped solid state laser with beam-steering and expanding optics. The thicknesses of the laser sheets were limited to approximately 1.5mm. LIF visualizations were carried out in the streamwise direction along the nozzle centreline, as well as in the cross-stream directions of the jet flows between $x/D_n=1$ to 3 locations. A Panasonic 3CCD video camera with a 17X TV zoom lens was used to record the flow visualizations to a JVC digital video recorder, after which the digital videos would be analysed on a workstation.

For the PIV measurements, a Dantec Dynamics DC-Imaging 2D-PIV system was used. A New-Wave Research Minilase double-pulsed Nd:YAG laser system rated at 50mJ/pulse and **operating at 15Hz double-pulsed mode** was used as the illumination source. The laser beams were expanded into thin laser sheets through beam-expanding optics and limited to a working thickness of approximately 1mm using appropriately-sized slits on the Plexiglas water tank walls. 50micron polyamide seeding particles were premixed with the jet and tank water uniformly before the measurements, by running the jet flows for some time before carrying out the actual experiments. Image-maps of the illuminated particles were captured using an 8-bit grayscale FlowSense CCD camera operating at double-frame mode with a Nikon f2.8 60mm fixed-focal lens at a resolution of 1600px by 1200px. **The time-interval between the two laser pulses was kept at 2millisecond throughout the study.** As the study focused on the behaviour of the resultant vortex structures, the PIV system was operating under **both non-triggered and triggered mode.** **Non-triggered PIV experiments were carried out to determine the temporal flow developments and time-averaged results.** **On the other hand, triggered PIV experiments were used for phase-averaged measurements,** where the signal used for forcing was also used to trigger the PIV system. As such, phase-averaged measurements could then be taken for when the large-scale vortex structures were forming off the nozzle exits. A time-delay of 1millisecond in the operation of the PIV system after every trigger signal was used to capture the jet flows at a satisfactory flow phase.

The captured pairs of image-maps were transferred to and analyzed by a workstation using Dantec FlowManager software. To derive the velocity field vectors from these image-pairs, a two-pass multi-grid cross-correlation scheme was used, where initial and final interrogation window sizes of 128px by 128px (i.e. 14.1mm x 14.1mm) and 32px by 32px (i.e. 3.5mm x 3.5mm) respectively were used. Typically, about 8-12

particles could be detected within the final 32px by 32px interrogation window, and occasionally more nearer to the jet exit. 50% overlaps were used in both horizontal and vertical directions during the cross-correlations. To eliminate spurious vectors, the resultant raw velocity field vectors were then validated using global and local validation schemes, such as peak and moving-average validations. After that, these validated velocity field vectors were further subjected to a three-point by three-point moving average smoothing filter to arrive at the final velocity vector maps. For time- and phase-averaged results, 500 instantaneous final velocity vector maps were used in calculations for each case. For corresponding phase-averaged Reynolds shear stress distributions, non-dimensionalized Reynolds shear stress (i.e. defined here as $\overline{u'v'}/U^2$) distribution was calculated from each of the 500 instantaneous velocity vector map, before they were averaged for the results. Using methodologies in Keane and Adrian (1992), the measured velocity components were expected to be within $\pm 1\%$ of the actual velocity components. Experimental uncertainties for the derived vorticity and Reynolds shear stress results, based on Moffat (1988) and Lund (1999) were estimated to be approximately $\pm 3.5\%$ and $\pm 4\%$ respectively.

3. Results and discussions

(a) Streamwise vortex structures and behaviour

Figures 2 and 3 show the LIF cross-sections obtained along the nozzle centreline in the streamwise direction for the five nozzles studied here. LIF images for the reference nozzle are present in both figures to ease comparisons. In Fig. 2 where major-plane inclined nozzles are shown, it can be seen that the formation of the vortex roll-ups for the reference nozzles is in-line with experimental flow visualizations made previously by Gutmark and Ho (1986) and Hussain and Husain (1989). It was observed during the experiments that for the reference nozzle, the rolling-up of the elliptic jet shear layer consistently occurred earlier along the minor-plane than the major-plane, before travelling circumferentially to the ends of the major-plane to complete the rolling-up process. Despite the differences in the sensitivities of the jet shear layer along the major- and minor-planes towards rolling-up, the vortex roll-up remains parallel to the flat nozzle exit once it is fully formed, as can be seen in Figs. 2(a) and 2(d). Subsequently, the vortex roll-up will undergo axis-switching behaviour and that streamwise-aligned rib structures will be produced along the major-plane.

When the 30° major-plane inclined nozzle is used as seen in Fig. 2(b), the resultant vortex roll-up becomes inclined and parallel to the inclined exit, at least in the near-field. Further increase in the incline-angle to 60° as shown in Fig. 2(c) will see the significantly inclined vortex roll-up undergoing rapid turning along the incline-plane, such that its inclination reduces drastically as it convects downstream. It is worthwhile to note that the production of rib structures along the major-plane does not appear to be significantly influenced by the use of the major-plane inclined nozzles. Furthermore, the flow behaviour along the minor-plane demonstrates systematic flow changes with the use of increasingly more inclined nozzles. Take for example flow visualization images taken along the minor-plane as shown in Figs. 2(e) and 2(f), they show not only gradual reductions in the lateral extent of the jet column but the formation of the vortex roll-ups appears later and less coherent. Hence, it can be inferred that the flow influences exerted by inclined exits are not limited to flow characteristics along the incline-plane, but extends to the entire jet flow field.

On the other hand, Fig. 3 shows corresponding LIF results obtained for minor-plane inclined nozzles. Other than the expected formation of inclined vortex roll-ups, the jet spreads for these inclined nozzles are also evidently wider than the reference nozzle, as shown in Figs. 3(b) and 3(c). While there remain detectable turning of the inclined vortex roll-ups for the 60° inclined nozzle, its extent is less than that observed for its major-plane inclined counterpart and hence more gradual. Along the major-plane of these nozzles, one of the distinctive features, or the lack of, is the distinct production of rib structures along the major-plane ends. The use of inclined exits along the minor-plane has suppressed their formation to such an extent that the overall jet spreads along that plane reduced drastically, as can be seen in Figs. 3(e) and 3(f). The lack of rib structures also meant that the formation of the vortex roll-ups can now be seen more clearly. Intriguingly, the flow images show that there exists persistent narrowing of the jet columns at about three to four hydraulic diameters downstream from the nozzle exits for both 30° and 60° inclined nozzles. Furthermore, the location where the jet column narrowing occurs moves upstream when the incline-angle increases from 30° to 60°.

Streamwise LIF flow images for the major-plane inclined nozzles agree well with earlier flow observations by New (2009) using shear layer dye-visualizations, particularly with respect to the formation of inclined vortex roll-ups and their turning at large incline-angles. More importantly, corresponding flow images for the minor-

plane inclined nozzles are available here for direct comparisons. The present LIF flow images also complement the earlier study by providing information on the more subtle flow features not revealed by the earlier studies by virtue of the cross-sectional nature of the results. So far, the results suggest that the presence of the inclined exits is likely to confer different vortex-stretching effects upon the vortex roll-ups, especially when turning at large incline-angles occurs. In addition, the vortex-stretching behaviour is also expected to be sensitive towards the exact incline-plane used. Hence, the influences upon the highly-strained braid regions between consecutive vortex roll-ups and in turn, jet entrainment behaviour, are expected to be significant and non-trivial. Before discussing how the differently inclined nozzles affect the vortex roll-up formations and their subsequent interactions with the braid vortices, it will be useful to have a good appreciation of the impact on the quantifiable characteristics of the vortex roll-ups due to the inclined exits.

(b) Influence of inclined nozzles on vortex roll-up characteristics

It is clear from the preceding results that inclined exits in elliptic nozzles will exert considerable influences upon their mixing and entrainment behaviour, especially in the immediate downstream vicinity of the nozzles, through distortions to the formation mechanisms of the vortex roll-ups and rib structures. To assess the flow alterations arising from the use of these inclined nozzles quantitatively, phase-averaged vorticity and Reynolds shear stress distributions were determined from PIV measurements. Since the use of inclined exits will lead to additional dissimilar circumferential redistributions of the original elliptic jet momentum thicknesses and vorticity levels, they will allow us to quantify the immediate influence upon the discrete vortex structures caused by the inclined exits.

It can be appreciated from Fig. 4 that for major-plane inclined nozzles, the behaviour of the discrete vortex roll-ups along both the major- and minor-planes is in good agreement with Fig. 2. For instance, the production of inclined vortex roll-ups and their subsequent turning at 60° incline-angle along the major-plane, as well as the delay in vortex roll-up formation along the minor-plane, are well-captured. For the reference nozzle, other than exhibiting a highly symmetric flow field, the strengths of the discrete vortex roll-ups are notably higher along the minor-plane than along the major-plane as well. This is in-line with observations made by Ho and Gutmark (1987), where they noted earlier rolling-up of the jet shear layer occurs along the plane with a

thinner momentum thickness, which in turn produces higher strengths for the resultant vortex roll-up. On the other hand, when the inclined nozzles are used, strengths of the vortex roll-ups along both shorter and longer nozzle lengths decrease, the reduction being larger along the shorter nozzle lengths. This decrease in vortex strengths can also be seen along the minor-plane. However, it should be noted that the vortex roll-up strengths remain higher along the minor-plane, which explains why corresponding dye-visualization results in New (2009) showed earlier rolling-up of the jet shear layer along the minor-plane. Hence, these observations imply that imposing inclined exits along the major-plane here essentially reduce the strengths of the resultant vortex roll-ups along their entire circumferences.

Corresponding results for the minor-plane inclined nozzles are shown in Fig. 5 and the depicted vortex behaviour along the incline-plane is quite similar to that shown in Fig. 3. In contrast to the major-plane inclined nozzles however, inclining the nozzles along the minor-plane serves to increase the strengths of the vortex roll-ups produced along both the shorter and longer lengths. Furthermore, the increase is noted to be larger along the longer nozzle length. Along the major-plane, the vortex roll-ups can also be observed to increase in strength as the incline-angle increases as well. These observations are clearly in direct contrast with the flow effects caused by the major-plane inclined nozzles as discussed above. As pointed out previously, the vortex roll-up strengths decrease with increases in the incline-angle along both major- and minor-planes for major-plane inclined nozzles. Hence, it can be seen that the presence of the inclined exits increases and decreases the vortex-stretching level when they are imposed along the minor- and major planes respectively. An isolated elliptic vortex roll-up produces dissimilar self-inductions along its circumference, which lead to non-uniform vortex-stretching levels. Imposing inclinations upon it along either major- and minor-plane will then further accentuate these non-uniform vortex-stretching levels. And clearly, these associated flow behaviour will lead to corresponding changes in their mixing characteristics. To investigate further, Figs. 6 and 7 show the Reynolds shear stress distributions associated with the discrete vortex roll-ups.

The strong influences of the underlying vortex structures on the Reynolds shear stress distributions are evident, even for the reference nozzle. In Figs. 6(a) and 6(d), symmetric distributions can be seen for the reference nozzle along both major- and minor-planes due to the corresponding symmetric formation of the vortex roll-ups shown in Figs. 2(a) and 2(d) earlier. Furthermore, the stress distributions reflect the vortex structures and

their motions incurred under the resultant axis-switching behaviour, particularly where they tend to redistribute towards the nozzle centreline along the major-plane. Before the Reynolds shear stress levels are compared, note that care should be taken during the comparisons due to the less distinctive magnitude differences here, as well as the uncertainties in the determination of the Reynolds shear stress levels. In terms of the maximum stress levels, the attained level along the major-plane appears to be comparable to that associated with the minor-plane. On the other hand, increasingly asymmetric stress distributions will result along the major-plane when the inclined exit is imposed along that plane and its incline-angle increases, as shown in Figs. 6(b) and 6(c). This attests to the fact that the mixing characteristics are strongly driven by the formation of inclined vortex roll-ups and their turning at large incline-angles. In contrast, along the minor-plane where flow symmetry is expected, no such behaviour is observed, as can be seen in Figs. 6(e) and 6(f). It should also be noticeable that the maximum stress levels tend to reduce along the shorter and longer nozzle lengths as the incline-angle increases, similar to what is observed along the minor-plane. These findings agree well with the overall decreasing trends of the vortex strengths associated with the use of inclined exits along the major-plane as discussed earlier.

The Reynolds shear stress distributions for minor-plane inclined nozzles are shown in Fig. 7 and the close relationship between the behaviour of the vortex roll-ups and their resultant mixing characteristics can again be discerned. For example, the correlation between the production of the inclined vortex roll-ups and the stress distributions is strong along the minor-plane, even at an incline-angle of 60° . This is due to the increasingly higher strengths of the inclined vortex roll-ups with incline-angle increments, as well as the more gradual rate of vortex roll-up turning at 60° as compared to major-plane inclined nozzles. For instance, comparing with Fig. 6(c), where vortex roll-up turning is much more rapid, it can be seen that the stress distribution is more disorganized in that case. Along the major-plane of minor-plane inclined nozzles, the stress distributions of the inclined nozzles grossly resemble that of the reference nozzle, though the regions of high stress levels are not as concentrated at some certain locations as the reference nozzle. Intriguingly, the maximum stress levels attained show less straight forward developments than the case for major-plane inclined nozzles.

Instead of demonstrating monotonic changes as the incline-angle increases, minor-plane inclined nozzles appear to produce variations which depend upon the exact incline-angle used. For instance, when the 30° incline-angle is used, the stress level along the shorter and longer nozzle length slightly reduces and remains comparable respectively. However, increasing the incline-angle to 60° will see a slight increase in the stress level along the shorter nozzle length back to a level similar to the reference nozzle, while the stress level along the longer nozzle length decreases substantially below that of the reference nozzle. From the magnitudes of the Reynolds shear stress changes, it is reasonable to deduce that increasing the incline-angle does not have appreciable impact upon the maximum stress levels along the shorter nozzle length. Furthermore, corresponding changes in the incline-angle do not influence the maximum stress levels until it is significant (i.e. 60°). Similarly, non-linear stress level behaviour can also be seen along the major-plane. In this instance, the maximum stress level at 30° incline is comparable to that of the reference nozzle, before decreasing below that when the incline-angle increases to 60°. As will be seen later, significantly more complex flow fields produced by minor-plane inclined nozzles are responsible for this observation.

(c) Interactions between inclined vortex roll-ups and braid vortices

To investigate the vortex structures as they convect downstream, LIF cross-sections are taken along the cross-stream direction at various downstream locations for all nozzles used here and presented in Figs. 8 to 12. To ease reader understanding, the locations where the cross-sections are taken are indicated in Figs. 2(a) to 2(c), 3(b) and 3(c) for each corresponding nozzle. To prevent any misinterpretations due to the differences in vorticity and scalar transport, the discussions will be limited to results obtained within the near-field at no more than three hydraulic diameters away from the nozzle origins in the streamwise direction. In all the figures, the major- and minor-planes are aligned along the horizontal and vertical directions respectively.

To start things off, Fig. 8 shows the instantaneous cross-sections for the reference nozzle taken at $x/D_h=0.5$ to 3.0 downstream locations. In the near-field region, the formation of the elliptic vortex roll-up and the rib structures can be observed from $x/D_h=1.0$ to 1.5 as shown in Figs. 8(b) and 8(c). Figure 8(d) shows that as the visualization plane moves to $x/D_h=2.0$, the braid vortices can be seen to be located just within the inner boundary of the vortex roll-up, by means of visualizing the entrainment of non-fluorescent surrounding fluid

into the elliptic jet body. Their rotational senses and the fact that pairs of them are observed to move outwards and beyond the jet cross-section along the major-plane to form the rib structures regularly, agree exactly with the analysis carried out by Husain and Hussain (1993) where the formation of rib structures was discussed in detail. At this point, the overall shape of the vortex roll-up remains elliptic. However, that changes when the visualization moves further downstream to $x/D_h=2.5$ as shown in Fig. 8(e). At this location, it can not only be seen that the rib structures and braid vortices have intensified, but the overall cross-sectional shape of the vortex roll-up has also transited to a circular one. This indicates that the jet is undergoing an intermediate stage of axis-switching, where the shape is a result of the elliptic vortex roll-up bending towards the downstream direction under self-induction. It is also worthwhile to note that weak rib structures are beginning to manifest along the minor-plane.

Comparisons with the simulated deformations of an isolated, slender elliptic vortex filament by Hussain and Husain (1989), as well as numerical simulations on elliptic jets carried out by Miller et al. (1994), show good agreements in the changes to the vortex roll-up cross-sectional shape. Based on the flow visualizations presented earlier, as well as the numerical results from Hussain and Husain (1989), the formation of the rib structures along the minor-plane at this point can be explained. At this stage of axis-switching where the vortex roll-up is bending increasingly towards the downstream direction along the major-plane, the curvature of the vortex roll-up along the minor-plane increases as well. As a result, it will begin to bend towards the downstream along the minor-plane under self-induction and in doing so, rib structures will be formed in the same manner as what has been observed along the major-plane earlier. Along the final visualization plane at $x/D_h=3.0$ shown in Fig. 8(f), flow field relatively similar to that observed at $x/D_h=2.5$ occurs, except that the overall flow field has intensified further and the rib structures along the minor-plane are expectedly more developed than before. LIF cross-sections taken at further locations downstream show the flow transiting towards turbulence which yields little information on the coherent vortex structures. Hence, they will not be shown here as a matter of brevity.

When the 30° major-plane inclined nozzle is used, cross-stream LIF cross-sections taken at $x/D_h=0.5$ to 3.0 locations and presented in Fig. 9 show an asymmetric flow field due to the formation of the inclined vortex roll-ups. In the figure, the shorter and longer nozzle lengths are located on the left- and right-hand sides of the

elliptic jet body. At $x/D_h=0.5$ location shown in Fig. 9(a), the partially visualized cross-section of an inclined vortex roll-up can be detected along the shorter nozzle length with no evidence of rib structure formations as yet. However, the latter can be observed further downstream in Fig. 9(b) at $x/D_h=1.0$, together with a clearer partial cross-section of the inclined vortex roll-up. As the visualization plane moves downstream to $x/D_h=1.5$ and 2.0 as shown in Figs. 9(c) and 9(d), additional large- and small-scale vortex structure formation become more apparent. In particular, Fig. 9(d) shows that at this location, the formation of the braid vortices can now be clearly discerned. As what was observed earlier on in the reference nozzle, the braid vortices are contained within the inner boundary of the vortex roll-up, despite their inclined nature. Furthermore, even though the braid vortices look symmetric in the cross-section image, their behaviour beyond that stage is asymmetric due to their interactions with the adjacent inclined vortex roll-up.

When a larger incline-angle of 60° is used, the gross flow features remain relatively similar, as can be seen in Fig. 10. For the sake of brevity, readers are advised to refer to the figure for more visual details. Since the highly-inclined vortex roll-ups undergo turning shortly after they are formed, partial cross-sections of the turning vortex roll-ups can be clearly discerned. The vortex roll-up turning causes the vortex section along the shorter nozzle length to surge downstream before its counterpart along the longer nozzle length, therefore small but regular and symmetric fluorescent “spots” can be identified at the expected locations, as shown in Fig. 10(d). Flow developments further downstream shown subsequently in Figs. 10(e) and 10(f) also resemble those seen earlier for the 30° inclined nozzle, although they are considerably more incoherent as the flow undergoes transition to turbulence earlier here.

As for the minor-plane inclined nozzles, Fig. 11 shows the LIF cross-sections for the 30° inclined nozzle taken at $x/D_h=0.5$ to 3.0 locations. The shorter and longer nozzle lengths of the nozzles are located at the top and bottom of the jet cross-sections respectively. Similar to the major-plane inclined nozzles, the rolling-up of the jet shear layer can be seen to initiate earlier along the shorter nozzle length at $x/D_h=0.5$ location as depicted in Fig. 11(a), which results from the formation of inclined vortex roll-ups as observed before. As the visualization plane moves to $x/D_h=1.0$ as shown in Fig. 11(b), evidence of braid vortices appearing within the inner boundary of the vortex roll-up along the shorter nozzle length can be found. Interestingly, a small-scale counter-rotating vortex pair aligned in the streamwise direction (i.e. abbreviated as streamwise vortices) can be observed to

form directly above the small cross-section of the inclined vortex roll-up just before it fully exits the visualization plane. Further downstream at $x/D_h=1.5$ shown in Fig. 11(c), the braid vortices can be seen to be better defined than before. Interesting though, vortical interactions between these braid vortices appear to produce streamwise vortices along the shorter nozzle length regularly by ejecting jet fluid outwards along the minor-plane.

The streamwise vortices are likely to be closely related to the rib structures formed along the minor-plane seen in the reference nozzle earlier on, though present flow visualization results cannot ascertain that. Recall that for minor-plane inclined nozzles, the curvatures of vortex roll-ups along their minor-plane are greater than that of the reference nozzle to begin with. Hence, it is possible that they will induce the formation of rib structures along the minor plane much earlier than the reference nozzle, as their radii of curvature will reach the critical level for doing so faster. This agrees well with the observation that these rib structures appear at $x/D_h=1.5$ and 1.0 locations for the 30° and 60° minor-plane inclined nozzles respectively [see Figs. 11(c) and 12(b)], as opposed to at $x/D_h=2.5$ location for the reference nozzle [see Fig. 8(e)]. Furthermore, such a flow scenario will become increasingly more likely if flow interactions between the vortex roll-ups and braid vortices confer additional effects conducive towards the formation of streamwise vortices.

Going back to Fig. 11(c), the cross-section of the vortex roll-up along the longer nozzle length can now also be seen. But perhaps the most interesting observation in the image is that suppressed formation of the rib structures appears to be related to the differences in the manner braid vortices behave at the jet major-axis ends. Instead of moving outwards rapidly to form rib structures as demonstrated in the case of the reference nozzle in Figs. 8(c) and 8(d), these braid vortices appear to remain within the vortex roll-up inner boundary along the major-plane. In Fig. 11(d) where the cross-section at $x/D_h=2.0$ location is shown, the flow field has intensified significantly and makes differentiation of the various vortex structures and their behaviour difficult. Nonetheless, pairs of braid vortices can be detected near the centre of the jet, while streamwise vortices can now be observed to form significantly at the top and bottom of the jet cross-section. Incidentally, the jet cross-section shape is also beginning to deviate noticeably away from its original elliptic shape. The flow field remains grossly similar at $x/D_h=2.5$ and from the image shown in Fig. 11(e), the flow field has begun to lose its coherence. The only flow features which can be positively identified are the braid vortices at the jet centre

and the vortex roll-ups along the cross-section peripheral. At this location, the jet cross-section shape also takes on an almost circular shape as well. At the last downstream location of $x/D_h=3.0$ shown in Fig. 11(f), the jet cross-section shape can be discerned to have elongated along the minor-plane, due to the continuing production of the streamwise vortices. The gradually changing of the jet cross-section shape is relatively similar to what was previously in the reference nozzle, which indicates that axis-switching behaviour continues to exist for this 30° inclined nozzle.

For the 60° minor-plane inclined nozzle, its LIF cross-sections are shown in Fig. 12. Generally speaking, the observed vortical behaviour and motions of the flow structures are grossly similar to those encountered in its 30° inclined counterpart but more intense. This can be readily discerned in Figs. 12(a) to 12(c), where the formation of the inclined vortex roll-up, braid vortices, streamwise vortices, as well as the suppression of rib structures along the major-plane, can be observed. Intriguingly, Fig. 12(d) shows that while the pair of streamwise vortices along the shorter nozzle length intensifies, three additional pairs are formed along the longer nozzle length and interacting with vortex roll-up. This situation continues to exist at $x/D_h=2.5$ location depicted in Fig. 12(e) and the increase in flow activities (due to the additional streamwise vortices) as compared to the shorter nozzle length is apparent. Further downstream, similar to its 30° inclined counterpart, the jet cross-section is again significantly elongated along the minor-plane as shown in Fig. 12(f). In contrast, Figs. 12(d) to 12(f) show that more pairs of streamwise vortices are formed along the longer nozzle length here - three as opposed to only one for its 30° minor-plane inclined counterpart as shown in Fig. 11(d). More convoluted interactions between the inclined vortex roll-ups and braid vortices due to the large incline-angle are likely to be responsible for this observation.

(d) Influence on axis-switching behaviour and mixing layers

Not only do the preceding cross-stream LIF flow visualizations clarify the differences in the formation of, as well as the interactions between the inclined vortex roll-ups and braid vortices between major-plane and minor-plane inclined nozzles, they also provide further evidence that major-plane inclined nozzles are able to suppress axis-switching behaviour, while minor-plane inclined nozzles do not. To verify that conclusively, mean half-jet widths for all nozzles along their major- and minor-planes are determined from time-averaged

PIV results and presented in Fig. 13. For the reference nozzle half-jet width profiles taken along its major- and minor-planes shown in Fig. 13(a)(i), it can be observed that axis-switching occurs with a cross-over location at approximately $x/D_h=5.7$ location. It should be mentioned here that half-jet width profiles are determined separately along the shorter and longer lengths of the inclined nozzles instead of averaging them out. This is to further discern any differences in the jet spreads due to the reduction and elongation of the nozzle lengths respectively caused by the inclined exits.

For the 30° major-plane inclined nozzle shown in Fig. 13(a)(i), it can be readily observed that the jet-widths are consistently larger along the shorter nozzle length. This is consistent with Fig. 9 shown earlier where the rolling-up of the jet shear layer and formation of the rib structures are first observed along the shorter nozzle length region. However, it is intriguing that the growth rates of the jet-widths along both shorter and longer nozzle lengths are practically similar within the present measurement range. As for the half-jet widths along the minor-plane, they remain smaller than those along the major-plane here. No evidence of axis-switching behaviour can be observed due to the absence of any cross-over points between the minor-plane half-jet width profile and any of those along the major-plane. Increasing the incline-angle to 60° will cause the discrepancies between the shorter and longer nozzle length half-jet widths to widen significantly. This is unsurprising as the use of larger incline-angle would cause the jet to spread out even earlier along the shorter nozzle length and later along the longer nozzle length. However, note that there exists a cross-over location between the minor-plane and longer nozzle length half-jet width profiles. Upon closer inspection of the figure will reveal that despite the occurrence of the cross-over point, the overall cross-stream extent of the elliptic jet remains wider along the major-plane instead of the minor-plane. This is supported by Fig. 10 and hence, no axis-switching behaviour is actually present for this nozzle.

On the other hand, for the minor-plane inclined nozzles, it can be observed that the half-jet profiles resemble closely to those of the reference nozzle [included in Fig. 13(b)(i) for comparison]. For the 30° minor-plane inclined nozzle, due to the much smaller but nonetheless distinct differences in the jet width growth rates along the shorter and longer nozzle lengths, two cross-over points can be found at approximately $x/D_h=4.8$ and 5.3 locations. Increasing the incline-angle to 60° similarly leads to two cross-over points, though they shift

upstream to approximately $x/D_h=3.8$ and 4.7 locations. Comparisons with Figs. 11 and 12 show also good agreement between the behaviour of the visualized elliptic jet cross-sections and the half-jet widths. Therefore, it can be concluded that axis-switching behaviour remains present in minor-plane inclined nozzles. In addition, the narrowing of the jet column along the major-plane as captured earlier in Fig. 3(f) when the incline-angle increases from 30° to 60° also corresponds to the reduction of the half-jet widths along the major-plane in Fig. 13(b).

For the sake of completeness and to evaluate the differences in the mixing layers resulting from the reference and inclined nozzles, momentum thicknesses were determined and shown in Fig. 14. It can be appreciated instantly that the mixing layers are significantly altered by the major-plane inclined nozzles as depicted in Fig. 14(a). For both 30° and 60° major-plane inclined nozzles, while momentum thicknesses along the longer nozzle lengths are initially suppressed along the longer nozzle lengths, they eventually grow much thicker than that of the reference nozzle along the major-plane. However, they are not as sensitive towards an increment in the incline-angle as those taken along shorter nozzle lengths. Nonetheless, the significant increase in the momentum thickness along the shorter nozzle length over that of the longer nozzle length in the near-field accompanying the increase in the incline-angle cannot be ignored. This is the result of a much faster growth of the mixing layer along the shorter nozzle when the incline-angle used is large, which in turn is associated with the much earlier formation of the inclined vortex roll-ups and rib structures, as noted in the flow visualization and half-jet width results presented earlier. In contrast, the momentum thickness along the minor-plane of the major-plane inclined nozzles is only slightly smaller than the reference nozzle and retains similar growth trend.

As for minor-plane inclined nozzles, it is interesting to observe that the overall momentum thickness along their major-plane reduces as the incline-angle increases from 30° to 60° - a trend noted and attributed to the narrowing of the jet column caused by the absence of rib structures earlier. On the other hand, momentum thicknesses along both shorter and longer nozzle lengths significantly exceed those along the reference nozzle minor-plane. In particular, drastic growth of momentum thickness is noticed along the shorter nozzle length when the incline-angle increases. This suggests much greater growth in the overall extent of the mixing layer associated with minor-plane inclined nozzles along their minor-planes. A comparison against Figs. 3(a)-3(c) will

show that this is indeed the case. The significant deviations observed in the momentum thickness distributions of minor-plane inclined nozzles when compared against the reference nozzles appear to be counter-intuitive at first glance. However, it should be reminded that Figs. 11 and 12 have already shown earlier the very different behaviour of the braid vortices [as compared to those observed in the reference nozzle here and earlier work by Husain and Hussain (1993)] associated with use of minor-plane inclined nozzles. On top of this, the earlier and pronounced formations of the streamwise vortices along the minor-plane, especially multiple instances for the 60° minor-plane inclined nozzle, are likely to be responsible for the discrepancy.

4. Conclusions

The present study investigates the effects of inclined elliptic nozzles on the formation and behaviour of the resultant vortex structures in association with their roles in the alterations of the axis-switching behaviour. As compared to the reference nozzle, PIV measurements show that major- and minor-plane inclined nozzles produce inclined vortex roll-ups which are lower and higher in vortex strengths respectively. One plausible explanation is that the vortex stretching levels decrease and increase correspondingly in major- and minor-plane inclined vortex roll-ups. Reynolds shear stress distributions for major-plane inclined nozzles are observed to correlate well with the vortex roll-up behaviour and strengths which show almost similar changes with incline-angle variations. In contrast, minor-plane inclined nozzles produce largely non-linear changes in the Reynolds shear stress distributions with increases in the incline-angle. These observations first suggest that more convoluted flow processes may be governing jets issuing from minor-plane inclined nozzles than their counterparts from major-plane inclined nozzles.

LIF visualizations reveal that for major-plane inclined nozzles, the inclined vortex roll-ups do not undergo flow processes leading to axis-switching behaviour. Instead, they consistently elongate the jet cross-section along the major-plane within the visualization range. Braid vortices are also observed to form closer to the shorter nozzle lengths due to the inclined vortex roll-ups which result in earlier formation of the rib structures along the shorter nozzle length. On the other hand, minor-plane inclined nozzles produce jets which undergo flow changes that lead to axis-switching behaviour, albeit in a more convoluted manner. Instead of moving away

from the minor-plane to form the rib structures in the same manner as the reference nozzle, the braid vortices remain within the jet cross-section and converge towards each other along the minor-plane to produce streamwise vortices. Half-jet width results verify that conventional axis-switching behaviour in elliptic jets can be effectively suppressed by major-plane inclined nozzles. In contrast, it continues to exist in minor-plane inclined nozzles, though the behaviour is asymmetric. Lastly, momentum thickness results show that elliptic jet mixing layer characteristics can be significantly modified by the exact orientation and extent of the inclined exits.

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Fig. 3 Instantaneous streamwise LIF cross-sections of the reference [(a) and (d)], 30° [(b) and (e)] and 60° [(c) and (f)] minor-plane inclined nozzles

Fig. 4 Phase-averaged velocity and vorticity maps of the reference [(a) and (d)], 30° [(b) and (e)] and 60° [(c) and (f)] major-plane inclined nozzles

Fig. 5 Phase-averaged velocity and vorticity maps of the reference [(a) and (d)], 30° [(b) and (e)] and 60° [(c) and (f)] minor-plane inclined nozzles

Fig. 6 Phase-averaged Reynolds shear stress distributions of the reference [(a) and (d)], 30° [(b) and (e)] and 60° [(c) and (f)] major-plane inclined nozzles

Fig. 7 Phase-averaged Reynolds shear stress distributions of the reference [(a) and (d)], 30° [(b) and (e)] and 60° [(c) and (f)] minor-plane inclined nozzles

Fig. 8 Instantaneous cross-stream LIF cross-sections for the reference nozzle taken at $x/D_h=0.5$ to 3.0 locations

Fig. 9 Instantaneous cross-stream LIF cross-sections for the 30° major-plane inclined nozzle taken at $x/D_h=0.5$ to 3.0 locations

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