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2013

New, T. H., Shi, S., & Liu, Y. (2013). Cylinder-wall interference effects on finite-length wavy cylinders at subcritical Reynolds number flows. *Experiments in Fluids*, (54), 1-24.

<https://hdl.handle.net/10356/85695>

<https://doi.org/10.1007/s00348-013-1601-8>

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Cylinder-wall interference effects on finite-length wavy cylinders at subcritical Reynolds number flows

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Abstract

An experimental study was conducted on aspect-ratio of six finite-length wavy cylinders immersed within a $Re_D=2700$ free-stream. Wavelengths of 2 and 4 diameters, as well as wave amplitude of 0.1, 0.2 and to 0.3 diameters were used for a comprehensive investigation. Time-resolved particle-image velocimetry measurements and Proper Orthogonal Decomposition analyses show that, for the present large wavelength wavy cylinders, vortex-shedding behaviour of high aspect-ratio wavy cylinders observed in past studies can be altered through variations in the aspect-ratio, exact geometric node and saddle locations, as well as the presence of end-walls. This is due to the persistent formation of recirculating regions close to the end-walls under certain wavy cylinder configurations, which affect the distributions of spanwise flows and vortex formation lengths. Vortex-shedding behaviour of smaller wavelength wavy cylinders has also been observed to be considerably less sensitive towards variations in the physical configurations, due to the formation of multiple streamwise vortices at the saddles. The presence of these coherent streamwise vortices is postulated to play a key role in significantly reducing flow-altering effects associated with the end-walls.

Keywords: wavy cylinders, finite-length cylinders, cylinder-wall interferences, time-resolved particle-image velocimetry

List of Symbols

a	Wave amplitude
D	Local cylinder diameter
D_m	Mean cylinder diameter
$D_{\max}$	Maximum cylinder diameter
$D_{\min}$	Minimum cylinder diameter
f	Vortex-shedding frequency
L	Cylinder length
L_{fc}	Vortex formation length based on wake closure
L_{fu}	Vortex formation length based on maximum streamwise velocity fluctuation
u	Streamwise velocity component
u_{rms}	Streamwise velocity fluctuation
U	Mean free stream velocity
w	Spanwise velocity component
x	Streamwise distance from cylinder origin
y	Cross-stream distance from cylinder origin
z	Spanwise distance from cylinder origin
Re_D	Reynolds number, $\rho U D_m / \mu$
St_D	Vortex-shedding Strouhal number, $f D_m / U$
λ	Wavelength
δ	Boundary layer thickness
ρ	Density
μ	Dynamic viscosity
ζ	Vorticity

1. Introduction

Flow behaviour associated with a circular cylinder immersed in a free-stream has been an extensively studied canonical flow scenario since more than forty years ago [For more detailed information, readers are referred to the excellent review by Williamson (1996)]. A classical flow phenomenon linked to a circular cylinder subjected to a free-stream at low Reynolds numbers is the formation of largely two-dimensional von Karman wake vortices, where they shed alternately from both sides of the cylinder and form a downstream vortex street [Unal and Rockwell (1988), Lin et al (1995), Rajagopalan and Antonia (2005)]. Depending on the exact Reynolds number, mode A and B three-dimensional instabilities may occur alongside with the wake vortices and hasten the transition from a two-dimensional to a three-dimensional wake behaviour [Wei and Smith (1986), Williamson (1988), Brede et al. (1996)]. Equally importantly, cylinder end boundary conditions was later observed to play an important role upon the vortex-shedding behaviour and frequency. In particular, oblique vortex-shedding (i.e. with respect to the cylinder) had been observed in studies where parallel vortex-shedding was expected instead.

Studies revealed that the vortex-shedding orientation is actually highly sensitive towards the end-walls configurations at both cylinder ends [Stansby (1974), Gerich and Eckelmann (1982), Eisenlohr and Eckelmann (1989), Williamson (1989), Hammache and Gharib (1991), Szepessy and Bearman (1992) and Miller and Williamson (1994)], where tilting them inwards sufficiently along the cylinder leading-edge could render an initially oblique wake to one which is parallel. Through these developments, it can be appreciated that flow influences exerted by cylinder end conditions cannot be ignored, especially for finite-length cylinders where these influences may propagate significantly along the cylinder spans. For these cylinders, aspect-ratio have also been ascertained to confer significant effects upon the wake formations and behaviour, through earlier investigations by Lee and Budwig (1991), Szepessy and Bearman (1992) and Norberg (1994). It is well-known that substantial alterations to the formation and behaviour of a bluff-body wake typically lead to

considerable changes to its surface pressure distribution and hence, mean and fluctuating lift and drag forces that it experiences. For studies seeking reductions in bluff-body drag or vortex-induced vibration levels, cylinder-wall interference and aspect-ratio effects need to be understood and taken into account.

Similarly, circular cylinders possessing sinusoidal variations in their diameters, also commonly known as wavy circular cylinders, have seen a significant number of experimental, numerical and theoretical studies over the past two decades to understand the effects of surface waviness on the subsequent cylindrical wake structures and behaviour. Due to the physically wavy surface, a wavy cylinder will possess a regular series of geometric nodes and saddles - locations whereby cylinder diameters are maximum and minimum respectively. Intuitively, the extents to which wake structures and behaviour are altered by these cylinders depend heavily not only upon parameters which dictates non-wavy cylinder flows such as Reynolds number, aspect-ratio, end-wall effects, free-stream turbulence level and surface roughness, but also wavelength and wave amplitude as well. Better understanding of how these nodes and saddles affect the canonical flow-past-circular-cylinder scenario has important implications in terms of flow stability analysis [Ling and Zhao (2009), Garbaruk and Crouch (2011)], bluff-body drag reductions and vortex-induced vibrations [Bearman and Owen (1998), Darekar and Sherwin (2001), Lam et al. (2004a, 2004b)]. Therefore, it should not be surprising that a large number of the above-mentioned studies tried to establish the critical relationships between the resultant wake behaviour, pressure distributions and lift/drag levels.

One of the earliest attempts to do so was by Ahmed and Bays-Muchmore (1992), where they experimentally investigated wavy cylinders of different wavelengths with a fixed wave amplitude. Their results showed that flow separations occurred earlier at the saddle locations than at the node locations, with the discrepancy growing as the wavelength was reduced. Furthermore, they observed the existence of spanwise flows along these cylinders, where reductions in the wavelength

accentuated their intensity. More interestingly, production of streamwise-aligned vortex structures was found to occur at node locations. These findings prompted several further investigations to explore wavy cylinders in greater detail. For instance, Lam et al. (2004a) observed drag reductions levels of up to 20% with significant lower vortex-induced vibrations in their wind tunnel tests. A follow-up study by Lam et al. (2004b) further found out that vortex formation lengths were longer for wavy cylinders than non-wavy cylinders, particularly along the saddle locations and thus indicated that formation of wake vortices had been delayed and/or reduced. Additionally, they observed spanwise flows where fluid moved towards the adjacent nodes from either sides of the saddle locations.

Zhang et al. (2005) also investigated the mean near-wake behaviour associated with wavy cylinders and observed the formation of spot-like flow reversal regions along all saddles. Similar to Lam et al. (2004a, 2004b), they found out that the vortex formation lengths have increased due to a downstream shift of their formation regions. Pairs of counter-rotating vortices aligned in the streamwise direction were observed to be formed at the nodes, with a regular spacing of about one mean cylinder diameter between them. Their presence was deduced to suppress or delay the formation of spanwise vortices and thus reducing the turbulent kinetic energy levels in the immediate lee-side regions of the wavy cylinders. Many of these observations were also reported by Lee and Nguyen (2007) in their study on wavy cylinders, including wake region expansions and contractions along the saddles and nodes respectively. The latter was attributed to fluid entrainment and acceleration along these physical features correspondingly. Lam and Lin (2008, 2009) followed up on their earlier studies by conducting numerical simulations to narrow in on the characteristics of wavy cylinder near wakes. Observations of significant drag reductions led the authors to conclude that three-dimensional shear layers produced by wavy cylinder were more resilient towards vortex sheet rolling-up than nominally two-dimensional shear layers resulting from non-wavy cylinders. Wave amplitude was also deduced to be the key parameter influencing the

three-dimensional flow structures produced by wavy cylinder, as well as reducing vortex-induced vibrations.

Interestingly, the ability of wavy cylinders to reduce vortex-induced vibrations was used to explain how harbor seals can accurately detect their prey's wake trails through their vibrissae or whiskers (Dehnhardt, 1998). The overall geometry of a vibrissa resembles that of a wavy cylinder, albeit having cross-sections closer to an elliptic rather than a circular geometry (Ginter et al., 2012). Nevertheless, studies by Hanke et al. (2010), Beem et al. (2011) and Witte et al. (2012) have demonstrated experimentally and numerically that vortex-induced vibrations associated with harbor seal vibrissae are indeed significantly reduced. As compared to vibrissae without wavy surface features, the dynamic forces experienced by harbor seal vibrissae can be an order of magnitude lower. This translates to higher sensitivity towards detection of vortex trails by potential preys, with this capability incorporated into potential biomimetic sensors (Beem et al, 2012).

Almost all of the preceding studies on wavy cylinders will reveal a common but interesting trait – high aspect-ratio wavy cylinders were used experimentally and slip-conditions were imposed upon the wavy cylinder ends during numerical simulations. Take for instance, wavy cylinders investigated by Ahmed and Bays-Muchmore (1992) had an aspect-ratio of $AR=9.6$, while Lam et al. (2004a) studied wavy cylinders with aspect-ratios ranging from $AR=24.9$ to 27.2 . Lam et al. (2004b) made use of $AR=18.2$ wavy cylinders in the follow-up study while Zhang et al. (2005) used $AR=15$ in their investigation. Aspect-ratio information was not provided by Lee and Nguyen (2007) but based on the information given in their paper, their wavy cylinder aspect-ratio appeared to be between $AR=15$ to 20 . As for numerical simulations performed by Lam et al. (2008, 2009), slip surfaces based on symmetric conditions were used at the wavy cylinder ends. It is worthwhile to mention that when Ahmed and Bays-Muchmore (1992) conducted their study, they noted that their wavy cylinder was “more sensitive to the geometry at the cylinder-wall interface than to the local geometry of the

waviness”, despite the highly three-dimensional wake formations. It is worth noting that their $AR=9.6$ wavy cylinder was also the lowest aspect-ratio cylinder used among the studies listed here.

The assumption or use of high aspect-ratio wavy cylinders are necessary to suppress or reduce as much as possible to avoid cylinder-wall interferences or aspect-ratio effects, such that the unique flow characteristics of wavy cylinders can be properly discerned. However, in many practical engineering implementations, these two factors are important considerations in the areas of building design, aerodynamic noise control, heat exchangers and mixing of fluids, among others, and they should not be limited to the use of non-wavy cylinders. To the best knowledge of the authors, there have been no dedicated studies that specifically look at the effects of cylinder aspect-ratio and cylinder-wall interferences on wavy cylinder near wake structures and behaviour. This lack of information motivated the present study, where an experimental investigation on low aspect-ratio wavy cylinders enclosed by end-walls was conducted using time-resolved particle-image velocimetry (TR-PIV) at a fixed sub-critical Reynolds number. Low aspect-ratio wavy cylinders with different wavelengths and wave amplitudes were studied to further understand the impact upon different wavy cylinder geometries. Furthermore, effects arising from configuring the wavy cylinders such that their nodes or saddles were aligned along the symmetry plane of the flow fields would also be investigated. Such arrangements would result in different cylinder base geometries at the end-walls and provide additional insights. Effects of free-stream turbulence level and surface roughness were not explored here as they were expected to be less influential in wavy cylinder flows.

Lastly, note that only wavy cylinders of a fixed mean diameter and length (and hence aspect-ratio) would be investigated here. This was due to the the fact that the water tunnel test-section spanwise size was too limited to allow significant variations in the aspect-ratio. Increasing or decreasing the aspect-ratio beyond that of $AR=6$ used in the present study would lead to significant changes in the cylinder diameters (and associated vortex structures and behaviour), which would make TR-PIV

measurements more difficult and their accuracy levels more inconsistent. Even if the aspect-ratio were to be varied, using different cylinder lengths here would require several mounting frames with different spanwise dimensions which would in turn vary the initial conditions and may render the experiments inconclusive.

2. Experimental setup and procedures

(a) Water tunnel and test cylinders

The experiments were conducted in a recirculating water tunnel with a Plexiglas test section cross-section size of 250mm (H) x 150mm (W) x 1050mm (L). Driven by a centrifugal pump controlled by a frequency inverter, water was conditioned by a series of honeycombs, three layers of fine screens and a 4:1 ratio contraction section before entering the transparent test-section. After passing through the test-section and reaching the end tank, water was then channeled back to the centrifugal pump for recirculation. Turbulence intensity level at the test velocity of $U=0.13\text{m/s}$ was ascertained to be no higher than 2% in the experimental region of interest. Spanwise flow uniformity was also determined to be above 97% of the test velocity along the width of the test-section. For mounting purposes, the test cylinders were enclosed within a transparent Plexiglas frame immersed in the water tunnel test-section as shown in Fig. 1 (i.e. frame indicated as gray in Fig. 1(b)). The frame has internal dimensions of 120mm(W) x 200mm(H) x 500mm(L) and was constructed out of four 10mm thick Plexiglas panels with approximately 8° inclined entrances to ensure smooth transitions for the water entering it. The cylinders were mounted at a distance of 100mm away from the floor of the frame, as well as 400mm downstream of the test-section entrance. The distance between the frame entrance and test cylinder centers was 150mm and the end-wall boundary layer thickness where the cylinders were mounted was determined to be approximately $\delta/D_m=0.36$, where D_m is the mean cylinder diameter [i.e. defined as $D_m=(D_{\max}+D_{\min})/2$].

Fig. 1 Experimental apparatus and setup used in the present study

One reference non-wavy and six wavy cylinders were studied in the present investigation, with the designs of the wavy cylinders shown in Fig. 2 and Table 1. Spanwise variations of the cylinder diameter which produced the wavy surface profiles were designed according to

$$D = D_m + a \cos(2\pi z / \lambda), \quad (1)$$

where D is the local cylinder diameter, a is the wave amplitude, z is the spanwise distance from the cylinder origin and λ is the wavelength. Locations where the cylinder diameter was maximum and minimum were known as nodes and saddles hereafter. Mean cylinder diameter and length were maintained at $D_m=20\text{mm}$ and $L=120\text{mm}$ respectively, yielding a cylinder aspect-ratio of $L/D_m=6$.

Wave amplitudes and wavelengths studied here were $a/D_m=0.1, 0.2$ and 0.3 and $\lambda/D_m=2$ and 4 respectively. Two sets of wavy cylinders with similar wavelengths and wave amplitudes were used here – one set with one of their nodes aligned with the flow symmetry plane (i.e. node-configured), while the other had one of their saddles aligned similarly (i.e. saddle-configured). These test cylinders were machined from stainless steel rods and coated with a thin layer of matt black paint for particle-streak photography and TR-PIV experiments. Based on a free-stream velocity of $U=0.13\text{m/s}$ and above-mentioned mean cylinder diameter, the Reynolds number was approximately $Re_D=2700$.

Fig. 2 Geometries of the wavy test cylinders

Wavelength λ/D_m	Wave amplitude a/D_m	λ (mm)	a (mm)	D_{max} (mm)	D_{min} (mm)
2	0.1	40	2	22	18
	0.2		4	24	16
	0.3		6	26	14
4	0.1	80	2	22	18
	0.2		4	24	16
	0.3		6	26	14

Table 1 Geometrical details of the wavy cylinders

(b) Time-resolved particle-image velocimetry

For TR-PIV measurements, a system comprising of an 8W, 532nm, continuous-wave laser and a 1280px × 1024px, 506 frames-per-second capable CMOS camera with a 60mm, f2.8 macro lens was employed. TR-PIV measurements were taken along XY and XZ (i.e. streamwise and cross-stream respectively) planes. For streamwise and cross-stream measurements, the fields-of-view measured approximately 210.9mm × 168.7mm and 148.3mm × 118.7mm respectively. During the experiments,

the CMOS camera captured light scattered from 20micron hollow glass spheres at 250fps at an exposure time of 2.5millisecond. A total of 10,000 sequential particle images were captured for each test cylinder along either measurement planes. At the current working Reynolds number of $Re_D=2700$, expected Strouhal number of the vortex shedding behaviour is estimated to be close to $St=0.2$. Base on the mean cylinder diameter and test velocity, the actual vortex shedding frequency was estimated to be 1.2Hz. Hence, the CMOS camera capturing frame-rate could resolve the temporal evolutions of the flow fields in a satisfactory manner.

To post-process the particle images, they were interrogated using a two-pass multi-grid cross-correlation technique with a final interrogation window size of 32px by 32px respectively. The final interrogation window size was selected to ensure that there were approximately 5 to 10 particles within each interrogation window, and particle shift between two consecutive particle images was limited to 20% of the interrogation window. To further improve measurement accuracy and fidelity, 75% window overlaps in both horizontal and vertical directions were used, while spurious vectors were rejected via local and global validation rules. The ratios of rejected vectors in measurements obtained in the streamwise and cross-stream were typically about 2.3% and 3.5% of the total number of vectors per vector map respectively. These rejected vectors were then substituted through 3-point by 3-point neighbourhood interpolations to arrive at the final velocity vector fields. A total of 9999 instantaneous velocity fields were obtained from the 10,000 sequential images, where each velocity field consisted of 157 vectors x 125 vectors.

Based on the TR-PIV experimental procedures, uncertainty levels for the measured velocity components were estimated to be within $\pm 1\%$ of the actual values, though they were expected to increase closer to the downstream limit of the PIV measurement window due to heightened three-dimensionalities and hence, higher out-of-plane velocity component. On the other hand, for

vorticity and Reynolds shear stress results, their uncertainty levels were estimated to be approximately $\pm 3.4\%$ and $\pm 2.9\%$ respectively [Luff et al. (1999), Benedict and Gould (1996)].

(d) Proper orthogonal decomposition analysis

Limited Proper Orthogonal Decomposition (POD) analysis was performed here to better identify behaviour associated with vortex structures of different length-scales. It is a mathematic technique for approximating a high-dimensional process with a low dimensional description, particularly useful for analyzing unsteady flow structure and behaviour under a wide range of flow scenarios [Lumley (1967), Aubry et. al (1988), Arndt et. al (1997), and Kim and Rockwell (2005)]. Typical applications of POD in turbulent flow analysis include decomposing a large number of sequential velocity or vorticity data into a set of POD coefficients and its corresponding eigenfunctions or modes. In doing so, sequential velocity or vorticity fields will be decomposed into different length-scales associated with different flow structures. For the present study, POD procedures were applied as previously outlined in Sirovich (1987), Berkooz et. al (1993) and Chatterjee (2000), where all the streamwise velocity fields were decomposed into 9999 unique POD modes and reconstructed back with the first 30 modes. The number of POD modes to be included for velocity reconstructions were associated with the dominant 80% of the flow energy, where they were deemed sufficiently adequate to characterize the flow field of interest whilst suppressing irregular flow unsteadiness caused by small-scale turbulent flow structures.

3. Results and discussions

(a) Effects on vortex formation lengths

To evaluate the effects of the wavelengths, wave amplitudes and exact geometrical arrangements on the vortex formation lengths, Figs. 3 and 4 show the mean centerline u/U and u_{rms}/U distributions respectively of all the test cylinders. Both these quantities have been used in past studies to determine the vortex formation lengths successfully. For instance, vortex formation length may be

Fig. 3 Centerline velocity variations for (a) node- and (b) saddle-configured wavy cylinders

defined as the distance from the cylinder center to the point where centerline $u/U=0$ (i.e. wake closure location) or where centerline u_{rms}/U reaches a maximum, as demonstrated by Lam et al. (2004). Before looking at the exact vortex formation lengths in greater detail however, it will be informative to examine these figures for a first-hand appreciation.

In Fig. 3(a) where centerline u/U distributions for node-configured cylinders are presented, it can be observed that regardless of the wavelength, the primary effect of increasing the wave amplitude is to increase the vortex formation lengths at the center-located nodes. This is a widely reported phenomenon for wavy cylinders by many studies in the past [Lam et al. (2004b), Zhang et al. (2005), Lee and Nguyen (2007) and Lam and Lin (2008)] and hence unsurprising. For $\lambda/D_m=4$ cylinders, it can also be observed that the increased vortex formation lengths are relatively similar, regardless of the wave amplitude. Larger peak reversed flow reversal velocities are also produced, as compared to the reference cylinder. As for $\lambda/D_m=2$ cylinders, wave amplitude needs to reach $a/D_m=0.2$ and

Fig. 4 Centerline turbulence intensity variations for (a) node- and (b) saddle-configured wavy cylinders

beyond before the vortex formation length is significantly increased. Interestingly, peak reversed flow velocities are now smaller as compared to the reference cylinder. For saddle-configured cylinders, $\lambda/D_m=4$ cylinder vortex formation lengths remain relatively unchanged despite increases in the wave amplitude, thereby indicating their insensitivity towards amplitude variations in this particular configuration. However, their peak reversed flow velocities are now lower than the reference cylinder, which is in contrast to their node-configured counterparts. This is interesting because Lam et al. (2004b) and Zhang et al. (2005) had shown that peak reversed flow velocities are actually higher at the saddle locations of their wavy cylinders. Hence, a clear discrepancy from typical wavy cylinder behaviour exists for the $\lambda/D_m=4$ cylinders here. As for $\lambda/D_m=2$ cylinders, vortex formation lengths are increased significantly when the wave amplitude is increased. In this case though, their peak reversed flow velocities are now much larger than that of the reference cylinder, opposite to what is observed for their node-configured counterparts here but in-line with previous observations by Lam et al. (2004b) and Zhang et al. (2005).

For centerline u_{rms}/U distributions for node-configured cylinders shown in Fig. 4(a), compared to the reference cylinder, the maximum u_{rms}/U magnitudes are reduced for $\lambda/D_m=4$ cylinders as the wave amplitude increases. This is related to the longer vortex formation lengths and stronger reversed flows observed previously, supporting the notion that wake formations will become increasingly reduced as the wave amplitude increases. In contrast, those for $\lambda/D_m=2$ cylinders exceed before falling below that of the reference cylinder. The heightened velocity fluctuation levels could be due to stronger reversed flow regions associated with the saddles adjacent to the center-located nodes here. For saddle-configured $\lambda/D_m=4$ cylinders, their u_{rms}/U distributions are not too drastically different from that of the reference cylinder when the wave amplitude is varied. This agrees well with Fig. 3(b), where their vortex formation lengths do not vary too much from that of the reference cylinder. On the other hand, $\lambda/D_m=2$ cylinders produce the largest reductions in u_{rms}/U levels seen in the present study when the wave amplitude is reduced to $a/D_m=0.3$. Similar to node-configured $\lambda/D_m=4$ cylinders discussed above, this can be linked to the increased vortex formation lengths and shows that saddle-configured $\lambda/D_m=2$ cylinders confer suppressive effects upon wake vortex formations. Under the present experimental conditions, it appears that only node-configured $\lambda/D_m=4$ and saddle-configured $\lambda/D_m=2$ cylinders managed to reduce wake formations substantially, and to a lesser extent, node-configured $\lambda/D_m=2$ cylinders.

Figure 5 shows the normalized vortex formation lengths, L_{fc}/D_m and L_{fu}/D_m , when the wave amplitude is varied. They do demonstrate a general tendency for the vortex formation length to increase with the wave amplitude, though the exact trends and extents differ. In agreement with Lam et al. (2004), L_{fu}/D_m tends to be smaller than L_{fc}/D_m at similar configurations. However, they also observed consistently larger formation lengths (i.e. both L_{fc}/D_m and L_{fu}/D_m) values along the saddle locations, for $\lambda/D_m=2.273$, $a/D_m=0.091$ wavy cylinder at $Re=3000$. The wavy cylinder which comes closest to their cylinder in the present study is the $\lambda/D_m=2$, $a/D_m=0.1$ test case. Results presented in Fig. 5 show that for that particular configuration, L_{fc}/D_m and L_{fu}/D_m are higher and lower

Fig. 5 Variations in vortex formation lengths due to changes in wave amplitude

along the saddle location respectively. However, if one inspects the figure more carefully, the general trend will be in good agreement with Lam et al. (2004) once the wave amplitude reaches $a/D_m=0.3$. Interestingly, increasing the wavelength of the present finite-length cylinders to $\lambda/D_m=4$ produces significantly larger vortex formation lengths (i.e. both L_{fc}/D_m and L_{fu}/D_m) at the node locations in a consistent manner. This is in direct contrast to the behaviour of a much higher aspect-ratio wavy cylinder studied by Lam et al. (2004) and shows that certain aspects of wavy cylinder flow behaviour could be reversed by reducing its aspect-ratio significantly. In fact, the less straightforward vortex formation length trends shown in Fig. 5 aptly demonstrates that cylinder-wall interference effects begin to introduce increasingly stronger flow effects at the mid-span location as the cylinder aspect-ratio decreases. Vortex behaviour and flow characteristics understood from results obtained from significant higher aspect-ratio wavy cylinders will not be representative of those of substantially lower aspect-ratio wavy cylinders. The underlying reasons will be elaborated later when the cross-stream results are discussed.

Fig. 6 Mean streamwise flow characteristics along the mid-span of the reference cylinder

(b) Effects on streamwise flow characteristics

Mean streamwise flow characteristics take along the mid-spans of the test cylinders will now be discussed in this section. For the sake of brevity, only results of the reference cylinder and selected wavy cylinder configurations will be presented. Figure 6 shows the mean u/U , U_{rms}/U and $\overline{u'v'}/U^2$ distributions associated with the reference cylinder, while Figs. 7 and 8 show corresponding results obtained for node- and saddle-configured $\lambda/D_m=4$ cylinders with $a/D_m=0.1$ and 0.3 wave amplitudes, as well as node- and saddle- configured $\lambda/D_m=2$ cylinders with $a/D_m=0.1$ and 0.3 wave amplitudes respectively.

Fig. 7 Mean streamwise flow characteristics along the mid-spans of node- and saddle-configured $\lambda/D_m=4$ cylinders with $a/D_m=0.1$ and 0.3 wave amplitudes.

Starting with the $\lambda/D_m=4$ cylinder results in Fig. 7, it can be discerned that regardless of the wave amplitude, widths of the recirculating wake regions immediately downstream of node-configured cylinders are always larger than that of the reference cylinder. Conversely, those of saddle-configured cylinders have smaller widths than that of the reference cylinder. This suggests that, as compared to the reference cylinder, flow separations occur earlier and later along the mid-spans of node- and saddle-configured cylinders respectively. This notion is further supported when the surface flow separation angles for the reference cylinder, as well as $\lambda/D_m=4$, $a/D_m=0.3$ node- and saddle-configured cylinders, were determined to be approximately 92° , 90° and 132° respectively from the mean PIV measurements. This is an interesting contrast to Ahmed and Bays-Muchmore (1992) and Lam et al. (2004), where they observed opposite behaviour. In addition, turbulence distributions for saddle-configured cylinders resemble more closely to that of the reference cylinder, where regions of peak turbulence tend to be located closer to the cylinder lee-side surface. On the other hand, turbulence distributions along the node locations are more elongated in the free-stream direction, consistent with the notion that flow separates earlier at these locations and produces larger recirculating wake regions. Note that some slight asymmetry in the turbulence distributions can be observed and likely to be due to minor bias caused by the higher-order flow statistics. Nevertheless, their global distributions are sufficiently dissimilar between different wavy cylinder configurations and more informative in terms of providing several key insights despite the slight asymmetry in the turbulence distributions, as described previously. Lastly, Reynolds shear stress distributions along either the node and saddle locations do not show significant differences at corresponding locations when the wave amplitude was varied. In contrast, between node and saddle locations and regardless of the exact wave amplitude, Reynolds shear stress distributions along the former location tend to be narrower but elongated in the streamwise direction.

Turning our attention to Fig. 8 where corresponding results for $\lambda/D_m=2$ cylinders are presented, it is clear that reducing the wavelength results in flow trends quite different from those observed in Fig.

Fig. 8 Mean streamwise flow characteristics along the mid-spans of node- and saddle-configured $\lambda/D_m=2$ cylinders with $a/D_m=0.1$ and 0.3 wave amplitudes.

7. For instance, mean wake regions along the node and saddle locations are much narrower and wider than that of the reference cylinder. This is due to significantly earlier and later flow separations along the mid-spans of saddle- and node-configured cylinders respectively, as suggested by the turbulence intensity distributions. As a result, the recirculation behaviour is also weaker and stronger for node- and saddle-configured cylinders as well, as indicated by the significant size differences in the wake regions. In fact, these discrepancies between node- and saddle-configured cylinders are clearest at a maximum wave amplitude of $a/D_m=0.3$. Furthermore, high turbulence levels are concentrated closer along the node location, agreeing well with the notion that the wake region is comparatively narrower. Lastly, the Reynolds shear stress distributions resemble closely to those of the turbulence intensity and become elongated downstream when the wave amplitude was increased to $a/D_m=0.3$, regardless of the exact location.

The preceding observations for $\lambda/D_m=2$ cylinders are in-line with those associated with a large aspect-ratio wavy cylinder. Collating with observations made for $\lambda/D_m=4$ cylinders earlier, it can be deduced that flow behaviour of smaller wavelength wavy cylinders is less sensitive towards reductions in the aspect-ratio, particularly when the wave amplitude is high. It would appear that as the wavelength of the wavy cylinder become smaller, cylinder-wall interferences become more restricted towards the near-wall regions. One plausible explanation may be found in the studies by Gerrard (1978), Ahmed and Bays-Muchmore (1992), Lam et al. (2004), Zhang et al. (2005), as well as Lam and Lin (2008). In these studies, regular formations of streamwise-aligned counter-rotating vortices within the wake vortices had been observed. While these streamwise vortices formed at random spanwise locations along the wavy cylinders in Gerrard (1978), subsequent investigations indicate that their locations are less random and likely to manifest at/or near the saddle locations. Nonetheless, note that the $\lambda/D_m=2$, $a/D_m=0.1$ and 0.3 wavy cylinders used here are not too different from the $\lambda/D_m=2.273$, $a/D_m=0.091$ wavy cylinder used by Lam et al. (2004). Therefore, one plausible explanation is that a higher number of streamwise vortices produced along $\lambda/D_m=2$ cylinder saddle

Fig. 9 (a) POD mode coefficient correlation, (b) eigenvalue spectrum (c) Mode 1 and (d) Mode 2 distributions for the reference cylinder

locations tends to prevent significant propagation of cylinder-wall interferences towards the cylinder centerline. As a result, their wake behaviour will be fundamentally similar to that of a much higher aspect-ratio wavy cylinder at similar wave amplitudes.

(c) Phase-averaged vortex-shedding behaviour

To understand the vortex-shedding behaviour responsible for the preceding mean streamwise flow characteristics, phase-averaged flow fields were determined through POD analysis of the TR-PIV results. Before the results are presented however, Fig. 9 shows the mode coefficient correlation, eigenvalue spectrum and the 1st and 2nd POD Modes (plotted using u/U and v/U information) for the reference cylinder. For the sake of brevity, corresponding results for other cylinders are not shown here since they are quite similar. Figure 9(a) demonstrates that the flow fields were highly cyclical and well-suited for phase-averaging analysis (Oudheusden et al. 2005), while Fig. 9(b) shows energy

Fig. 10 Phase-averaged vortex-shedding patterns for the reference cylinder

spectra for the first 30 POD modes which reveals the dominance of the first two POD modes. On the other hand, Figs. 9(c) and 9(d) show that the vortex-shedding behaviour captured during the present study were highly symmetrical and repeatable. To arrive at the phase-averaged results, datasets at 60° phase intervals were identified and used to determine six different flow phases that define typical flow cycles for the selected cylinders. To illustrate, Fig. 10 shows a complete cycle of phase-averaged vortex-shedding behaviour for the reference cylinder. With the availability of approximately three-hundred instantaneously velocity fields to determine each flow phase (i.e. phase bin size used was $\pm 5^\circ$), distinct and highly symmetrical large-scale von Karman wake vortices can be seen to shed from both sides of the cylinder in a regular fashion, typical of circular cylinder vortex-shedding behaviour established previously.

Figure 11 shows the vortex-shedding patterns for node- and saddle-configured $\lambda/D_m=4$ cylinders with $a/D_m=0.3$ wave amplitude. In agreement with their mean flow characteristics presented earlier, the wake region along the node location shown in Fig. 11(a) is comparatively wider than that along the saddle region. Additionally, the wake vortex core size appears to be smaller along the saddle location as well, presumably due to the smaller diameter at that location. It can also be discerned

Fig. 11 Phase-averaged vortex-shedding patterns for node- and saddle-configured $\lambda/D_m=4$ cylinder with $a/D_m=0.3$ wave amplitude.

qualitatively that the wake vortices are located further downstream along the node location, which will be in good agreement with the vortex formation lengths obtained earlier. For $\lambda/D_m=4$ cylinders, it would appear that physically larger wake vortices and longer formation lengths result from the larger diameter at the node location, while the opposite is true at the saddle location. Interestingly though, Fig. 12 shows that corresponding vortex-shedding patterns for node- and saddle-configured $\lambda/D_m=2$ cylinders with $a/D_m=0.3$ wave amplitude demonstrate exact opposite trends. For instance, the wake region along the node location shown in Fig. 12(a) was significantly narrower than that

Fig. 12 Phase-averaged vortex-shedding patterns for node- and saddle-configured $\lambda/D_m=2$ cylinder with $a/D_m=0.3$ wave amplitude.

along the saddle location, even though the diameter was larger for the former. The distributions of the streamlines indicate that surface flow separations took place discernibly further downstream along the node location, as suggested by similar observations in the mean streamwise flow results presented earlier. Intriguingly, while large-scale wake vortices were shed and convect downstream along the node location, those formed along the saddle location appear to “break up” into smaller vortex structures in a persistent manner. This is supported by the vorticity distributions shown in Figs. 12(b)(iii) and 12(b)(vi). And in accordance to explanations covered earlier on, the breaking-up

of the large-scale vortices seen here could be evidence of their interactions with streamwise vortices forming at the saddle locations.

To shed more light upon the flow behaviour at the saddle locations, Fig. 13 shows the deconstructed flow fields taken along the centerlines of $\lambda/D_m=2$, $a/D_m=0.1$ and 0.3 saddle-configured cylinders for POD modes 1 to 4, illustrated in terms of the velocity vectors and vorticity levels to represent the different flow structures across the POD modes qualitatively. For consistency, the vorticity levels are set to be the same for all modes shown in the figure. It can be readily appreciated that the first three POD modes for the $a/D_m=0.1$ cylinder consist of coherent flow structures, whereas the $a/D_m=0.3$ cylinder produces coherent flow structures only for POD modes 1 and 2. Beyond these modes, the flow structures are considerably more incoherent. Thus, it would appear that increasing the wave amplitude for saddle-configured cylinders results in a reduction in flow coherence along the XY-planes at saddle locations and this supports the earlier conjecture of heightened levels of flow interactions between the streamwise vortices forming at the saddle locations and the cylinder wake-vortices. Note also that the POD coefficients support this postulation. For instance, POD modes 1 to 4 coefficients are 0.277, 0.235, 0.037 and 0.025 respectively for the $a/D_m=0.1$ cylinder, whereas they are 0.154, 0.128, 0.070 and 0.032 respectively for the $a/D_m=0.3$ cylinder. It can be readily discerned that for the $a/D_m=0.1$ cylinder, POD modes 1 and 2 constitute more than 50% of the total flow energy. On the other hand, these two POD modes constitute only 28% of the total flow energy for the $a/D_m=0.3$ cylinder. Since higher POD modes are associated with more incoherent flow structures, this reinforces the notion that an increase in the wave amplitude for saddle-configured cylinders hastens the breaking-down of the cylinder wake-vortices by the streamwise vortices and thus, rendering the wake region more incoherent.

Note that streamwise TR-PIV measurements were not conducted at other cylinder locations or near the cylinder-wall junction. Instead, spanwise variations in flow fields were captured through cross-

Fig. 13 Flow field deconstructions for the first four POD modes associated with $\lambda/D_m=2$, (a) $a/D_m=0.1$ and (b) $a/D_m=0.3$ cylinders.

stream TR-PIV measurements, which will be presented in the next section. However, based on the complex flow behaviour seen so far, future studies should explore the possibility of resolving the flow fields between a node and an adjacent saddle and those closer to the end walls in greater detail.

(d) Effects on cross-stream flow characteristics

To probe further into how the exact wavy cylinder geometry affects the resultant wake flow behaviour, Figs. 14 and 15 show the u/U distribution and velocity vector field at the lee-side regions of $\lambda/D_m=4$ cylinders along their XZ-planes respectively. For the reference cylinder result included in Fig. 14(a), the flow reversal region (i.e. where u/U values are negative) due to wake formation can be clearly discerned. This region is approximately parallel with the reference cylinder, though it is also clear that flow reversal behaviour gets progressively disrupted (and hence weaker) closer to the end-walls, as a result of cylinder-wall interferences.

For the node-configured $\lambda/D_m=4$ cylinders, Fig. 14(b) shows that distortions to the flow reversal region increase with the wave amplitude used, where they become less and more elongated along the Z- and X-axes respectively. Figure 15(a) shows that this is due to the recirculating regions forming at the saddles (i.e. indicated by red arrows), which incidentally are located adjacent to the end-walls. These recirculating regions can be observed to grow larger as the wave amplitude is increased and exert significant influences upon the nature of the spanwise flows along the wavy cylinders. Spanwise flows result from the regular variations in diameter along a wavy cylinder and it has been noted in the study by Ahmed and Bays-Muchmore (1992) that increasingly stronger spanwise flows occurred from the nodes towards the saddles as the wave amplitude was increased. On the other hand, Lam et al. (2004) and Zhang et al. (2005) observed spanwise flows from the saddles towards the nodes. This discrepancy has never been clarified. So, to investigate further, distributions of normalized spanwise flow component, w/U , for node- and saddle-configured $\lambda/D_m=2$

Fig. 14 Mean streamwise velocity distributions at the lee-side of the (a) reference cylinder, (b) node- and (c) saddle-configured $\lambda/D_m=4$ cylinders

and 4 cylinders with a wave amplitude of $a/D_m=0.3$ taken along $x/D_m=0.6$ downstream location are extracted from cross-stream measurements and presented in Fig. 16.

With reference to node-configured $\lambda/D_m=4$ cylinder results, it can be seen that spanwise flows exist and directed from the nodes towards the saddles, due to the rotational senses of the recirculating regions. This is in agreement with Ahmed and Bays-Muchmore (1992) and it is worth reminding that

Fig. 15 Velocity fields at the lee-side of (a) node- and (b) saddle-configured $\lambda/D_m=4$ wavy cylinders

the aspect-ratio of their wavy cylinders were significantly lower than those used by Lam et al. (2004) and Zhang et al. (2005), though not as low as that used here. It is worthwhile to note that the formations of the two recirculating regions lead to significantly non-uniform wake closure behaviour (and hence wake formation lengths) along the span of the wavy cylinders when $a/D_m=0.2$ and above. This can be discerned visually from the distributions of velocity vectors with $u/U=0$ components in Fig. 15(a). It is unknown at this point how horseshoe vortices, typically formed upstream of cylinder-

Fig. 16 Variations in cross-stream velocity component, w , along the wavy cylinder lengths determined along $x/D_m = 0.6$ cross-stream location

wall junctions due to wall boundary layer separations, will come into play here though it is possible that they contribute towards the observations made so far. However, note the “legs” of the horseshoe vortices at the lee-sides of the wavy cylinders will be displaced at significant distances away from the XZ-planes (i.e. at least half a cylinder diameter away and increasing with downstream distance) and will not be captured in the present TR-PIV measurements.

As for saddle-configured $\lambda/D_m = 4$ cylinders, opposite flow behaviour is essentially detected. Take for instance, Fig. 14(c) shows that while the flow reversal regions get increasingly distorted as the wave amplitude increases, they become stronger at the end-walls instead. At wave amplitudes of $a/D_m = 0.2$ and beyond, regions of strongest flow reversals are essentially concentrated at the end-walls. A check with Fig. 15(b) shows that this is due to the very different flow field resulting from the use of saddle-configured $\lambda/D_m = 4$ cylinders. Instead of two strong recirculating regions where fluid is moving from the cylinder center towards the end-walls as seen in node-configured $\lambda/D_m = 4$ cylinders,

opposite behaviour is observed here. As a result, two recirculating regions but of opposite rotational senses can be found at the nodes. Unlike those encountered in the node-configured cylinders however, these two recirculating regions are not very strong, even at the maximum wave amplitude of $a/D_m=0.3$. This is evident in Fig. 15(b)(iii) where they do not show up very well in the vector plot. Nonetheless, their behaviour and impact upon the spanwise flows can be inferred from Fig. 16. Correlating with node and saddle location, the figure shows that flow is moving from the half-saddle at the end walls towards the node. Thereafter however, the flow will move from the node towards the saddle at the cylinder center. No concrete evidence of recirculating regions can be found at the saddle however, due to the very low spanwise flow velocities there. In fact, Fig. 16 shows that spanwise flow velocities for all saddle-configured $\lambda/D_m=4$ cylinders are much lower than those for their node-configured counterparts. Lastly, because of the smaller and weaker recirculating regions, the wake formation length incur comparatively much smaller variations along the spans of saddle-configured cylinders, as can be seen from Fig. 15(b).

So far, results for $\lambda/D_m=4$ cylinders have been discussed, where the wavelength is significant compared to the length of the $L/D_m=6$ cylinders (i.e. one wavelength=2/3 cylinder length) used in the present study. On the other hand, results for $\lambda/D_m=2$ cylinders will now be presented, where the wavelength is comparative smaller now (i.e. one wavelength=1/3 cylinder length). Figure 17 shows u/U distributions of both node- and saddle-configured $\lambda/D_m=2$ cylinders, which indicate the flow reversals regions as well. Unlike the case for $\lambda/D_m=4$ cylinders, it appears that halving the wavelength results in less drastic differences in the flow reversal behaviour between node- and saddle-configured wavy cylinders. In particular, strong flow reversals occur only along the saddle locations for both configurations, in agreement with Lam et al. (2004) and Zhang et al. (2005). In particular, as the wave amplitude increases for either configuration, the flow reversal regions become more well-defined and segregated into distinct oval “spots” as observed by Zhang et al. (2005). Therefore, Fig. 17 demonstrates that the flow reversal regions are essentially similar and

Fig. 17 Mean streamwise velocity distributions at the lee-side of (a) node- and (b) saddle-configured $\lambda/D_m=2$ cylinders

displaced laterally between node- and saddle-configured $\lambda/D_m=2$ cylinders. However, it should also be noted that there are discrepancies between the two different cylinder configurations from $a/D_m=0.2$ onwards, due to differences in the cylinder-wall interferences.

To take a closer look, Fig. 18 presents the velocity vector plots obtained for $\lambda/D_m=2$ cylinders. It can be observed that for node-configured $\lambda/D_m=2$ cylinders shown in Fig. 18(a), pairs of weak recirculating regions are formed only at the nodes with none at the end-walls. As a result, Fig. 16 shows that spanwise flows consist of flow movements from the saddles to the nodes and would agree with Lam et al. (2004) and Zhang et al. (2005). Thus, it can be deduced that the half-saddles at the end-walls are not conducive towards supporting the formation of recirculating regions such as those seen for node-configured $\lambda/D_m=4$ cylinders. Instead, regular high-velocity regions associated with streamwise vortices can be found at the lee-side of all the saddles. In contrast, only saddle-

Fig. 18 Velocity fields at the lee-side of (a) node- and (b) saddle-configured $\lambda/D_m=2$ wavy cylinders

configured $\lambda/D_m=2$ cylinders produce significant recirculating regions at the end-walls. This is similar to what is observed for node-configured $\lambda/D_m=4$ cylinders earlier, where incomplete but sufficiently large saddles at the end-walls produce recirculating regions as well. In this case, full saddles adjacent to the end-walls essentially lead to similar flow behaviour, which is unsurprising. Note that

half-saddles in node-configured $\lambda/D_m=2$ cylinders do not produce such a behaviour.

The nature of spanwise flows in saddle-configured $\lambda/D_m=2$ cylinders can be discerned from Fig. 16, where it is not as straightforward as that for node-configured $\lambda/D_m=2$ cylinders. In this case, for the saddle located at the cylinder center, spanwise flows comprise of fluid motions from the saddle towards the adjacent nodes. Note that a small region of very weak recirculating flow appears to exist at each of these two nodes as well. Further away, fluid will then travel from the nodes towards the saddle close to the end-walls. The presence of the recirculating regions there means that the spanwise flow will continue to move towards the half-nodes at the end-walls. Hence, it can be deduced that spanwise flows for saddle-configured $\lambda/D_m=2$ cylinders are significantly more convoluted and complex than for the other wavy cylinders. The strong presence of two recirculating regions at the end-walls for saddle-configured $\lambda/D_m=2$ cylinders also causes the adjacent streamwise vortices to be displaced inwards and towards the $z/D_m=0$ location. This may have disrupted the recirculating flows at the nodes here (see Fig. 18(b)(iii)), such that they are not as coherent or strong as those seen in their node-configured counterparts (see Fig. 18(a)(iii)). Lastly, it can be discerned visually that the vortex formation lengths vary significantly along the spans of the $\lambda/D_m=2$ cylinders once the wave amplitude reaches $a/D_m=0.3$.

(e) Relationships between cylinder-wall interferences and vortex formation lengths

Earlier on, velocity vector plots have suggested that significant variations in the vortex formation lengths occur along the spans of the wavy cylinders. For a closer inspection, information on the locations downstream of the wavy cylinders where $u/U=0$ are extracted from Figs. 15 and 18 and presented in Fig. 19. Since cross-stream PIV measurements are taken along XZ-plane, these locations can be reasonably assumed to be very close to those associated with wake formation lengths based on wake closure, L_{fc} , defined by $u/U=0$ locations along the cylinder centerline as determined by streamwise PIV measurements earlier. To use the reference cylinder as an

Fig. 19 Variations in vortex formation length as determined from cross-stream measurements ($x/D_m|_{fc}$) along the wavy cylinder spans

illustration (i.e. indicated by red crosses), Fig. 19 shows that its vortex formation length at $z/D_m=0$, as determined from these cross-stream measurements, is approximately $x/D_m|_{fc}=2.87$, while $L_{fc}=2.71$ based on earlier streamwise flow information. Considering that these vortex formation lengths are determined along different 2D-PIV measurement planes where different out-of-plane flow components are involved, the +5.9% discrepancy remains reasonable.

By comparing the spanwise variations in the vortex formation lengths and collating them with earlier results, light will be shed upon the flow mechanisms in which cylinder-wall interferences influence the vortex formation lengths in low aspect-ratio, finite-length wavy cylinder. In contrast, high aspect-ratio wavy cylinders investigated in past studies would mean that these interferences would be limited to relatively much smaller regions as compared to the cylinder length. In addition, the presence of additional physical wave instances along the span for high aspect-ratio wavy cylinders further isolate spanwise regions close to the cylinder mid-points from those closer to the end-walls.

Interestingly, this also hints the possibility of a critical cylinder length to wavelength (i.e. L/a) ratio for a fixed cylinder aspect-ratio, above which the end-walls would have negligible effects. Note that comparisons will only be carried out between specific results presented in Fig. 19, rather than with results determined from streamwise PIV measurements, to isolate the effects of node/saddle configurations, wave length and wave amplitudes for the sake of consistency. For ease of comparisons, node and saddle locations for $\lambda/D_m=2$ and 4 cylinders are indicated in the figure.

For node-configured wavy cylinders shown in Fig. 19(a), apart for some end-wall effects, their vortex formation lengths are always larger than that of the reference cylinder. For node-configured $\lambda/D_m=4$ cylinders, increasing the wave amplitude from $a/D_m=0.1$ to 0.2 does not produce any practical difference in the spanwise distribution of wake formation lengths. Increasing the wave amplitude to $a/D_m=0.3$ will see a significant increase in the overall wake formation lengths however, though the latter reduces drastically along both saddle locations. Correlating with Fig. 15(a), this behaviour is a result of the recirculating regions residing within the saddles. Additionally, vortex formation lengths can also be noticed to be slightly smaller at the nodes. On the other hand, the trends for saddle-configured $\lambda/D_m=4$ cylinders are very different. For example, vortex formation lengths are significantly smaller than that of the reference cylinder at $a/D_m=0.1$ and 0.2 wave amplitudes. Increasing the wavelength to $a/D_m=0.3$ will see them increasing almost to the levels corresponding to the reference cylinder. However, note that the vortex formation lengths for $a/D_m=0.2$ and 0.3 wave amplitudes tend to increase closer to the end-walls and linked to the stronger flow reversal behaviour seen in Fig. 14(c) earlier.

For node-configured $\lambda/D_m=2$ wavy cylinders, its spanwise distribution of vortex formation lengths at $a/D_m=0.1$ wavelength resembles those of node-configured $\lambda/D_m=4$, $a/D_m=0.1$ and 0.2 cylinders. This is likely due to the use of a small wave amplitude, where it is incapable of exerting significant flow influences. As the wave amplitude increases to $a/D_m=0.2$ and 0.3 however, overall vortex formation

lengths increase abruptly and exhibit clear variations along the cylinder spans. In particular, vortex formation lengths are consistently larger at the saddles than at the nodes, even if the saddles are exactly at the end-walls. Furthermore, while vortex formation lengths increase as the wave amplitude increases from $a/D_m=0.2$ to 0.3 , they grow at faster rates along the saddle locations as well. This suggests that the streamwise vortices at the saddles become discernibly stronger at $a/D_m=0.2$ and beyond, resulting in significant suppression of wake vortex formations. As for their saddle-configured counterparts, relatively similar situation exists whereby a low wave amplitude of $a/D_m=0.1$ does not lead to significant deviations in the spanwise vortex formation length distribution, as compared to the reference cylinder.

It is only at wavelengths of $a/D_m=0.2$ and 0.3 that the cylinder nodes and saddles start to exert considerable flow effects. In this case, spanwise variations of vortex formation lengths can be observed to retain characteristics similar to their node-configured counterparts, where larger vortex formation lengths occur along saddle locations. This agrees well with Figs. 17 and 18 where the flow reversal regions are deduced to be laterally displaced between node- and saddle-configured $\lambda/D_m=0.2$ cylinders here. It should be noted however, that vortex formation lengths of saddle-configured cylinders are substantially smaller than those of node-configured cylinders. Based on the present experimental evidence, the explanation can be found in the two persistent recirculating regions at the end-walls, where they tend to draw fluid towards the cylinder lee-side and result in earlier closures of the wake vortices.

(f) Formation of streamwise vortices

Experimental evidence have so far indicated that the wakes of $\lambda/D_m=0.2$ cylinders here are less sensitive towards whether the latter are node- or saddle-configured. While the vortex formation lengths have been shown to be different between node- and saddle-configured $\lambda/D_m=0.2$ cylinders in the preceding discussions, the major flow field features remain resilient, despite the formation of

Fig. 20 Vorticity distributions at the lee-side of the (a) $\lambda/D_m=4$ and (b) $\lambda/D_m=2$ wavy cylinders with $a/D_m=0.3$ wave amplitude

the recirculating regions in the latter. Results presented earlier have suggested that this could be due to the higher number of coherent and parallel streamwise vortices. These characteristics have been postulated to have the abilities to better deter flow effects arising from the cylinder-wall interferences. To understand further, Fig. 20 shows mean cross- stream vorticity distributions determined for the $\lambda/D_m=2$ and 4 cylinder lee-side regions. For the sake of brevity, only results associated with the largest wave amplitude of $a/D_m=0.3$ are presented.

Two major observations can be made from Fig. 20: Firstly, wake formations are significantly more coherent for $\lambda/D_m=2$ cylinders, where coherent pairs of streamwise vortices are produced and arranged approximately parallel to one another, regardless of whether the wavy cylinders are node- or saddle-configured. In contrast, while there are some evidence of coherent vortex structures in $\lambda/D_m=4$ cylinders, they are associated only with the recirculating regions within the saddles of node-configured cylinders and flow structures adjacent to the end-walls. These observations further support the notion that the increased number and higher coherence of streamwise vortices may explain the smaller effects that cylinder-wall interferences have on $\lambda/D_m=2$ cylinders. Secondly, sustained recirculating flows can be found only within the saddles of node-configured $\lambda/D_m=4$ and saddle-configured $\lambda/D_m=2$ cylinders, which is consistent with the results presented earlier.

The present experimental observations support the postulation that low aspect-ratio, finite-length wavy cylinders enclosed by end-walls are more likely to exhibit behaviour different from those associated with large aspect-ratio wavy cylinders, particularly when the wave amplitude is low. The Reynolds number used here is slightly lower but nonetheless reasonably close to those used in Lam et al. (2004), Zhang et al. (2005) and Lam et al. (2009). Hence, Reynolds number differences should not be the main driving factor for the present observations. Instead, results presented so far suggest that they are due to the low aspect-ratio cylinders, wavelengths and cylinder-wall interferences imposed here. It should be noted here that differentiating exactly the effects due to the end-wall boundary layer and finite-length wavy cylinder geometries remains a challenge, especially when one considers the number of geometrical parameters for the wavy cylinders. However, more extensive surveys of the three-dimensional flow behaviour at the cylinder-wall junctions through additional TR-PIV or tomographic PIV measurements should be able to shed more light upon their respective influences.

4. Conclusions

The behaviour of low aspect-ratio $AR=6$ wavy cylinders with different wavelengths and wave amplitudes enclosed by end-walls has been investigated and compared experimentally in the present study. Furthermore, flow influences arising from whether the wavy cylinder is configured to have a node or saddles at its mid-span have also been studied. Results show that the near-wake behaviour of longer wavelength cylinders is more sensitive towards the presence of end-walls and whether they are node- or saddle-configured. For instance, saddle-configured $\lambda/D_m=4$ cylinders produce significantly smaller centerline vortex formation length increments as compared to their node-configured counterparts. In contrast, centerline vortex formation lengths of node- and saddle-configured $\lambda/D_m=2$ cylinders have smaller differences. In addition, for node- and saddle-configured $\lambda/D_m=4$ cylinders, flow separations occur earlier and later along their mid-spans respectively, which are opposite to observations made in earlier studies. On the other hand, corresponding results for $\lambda/D_m=2$ cylinders show good agreements with historical observations.

Cross-stream measurements reveal that these differences can be attributed to the very different flow behaviour at the near-wake regions of the various wavy cylinders, particularly for $\lambda/D_m=4$ cylinders. Under node-configured arrangements, large-scale recirculating regions will be produced at the end-walls, due to fluid moving from the node towards the saddle and the end-walls. On the other hand, their saddle-configured counterparts produce weaker recirculating regions with opposite rotational sense, due to fluid moving from the nodes into the saddle. For $\lambda/D_m=2$ cylinders, their near-wake flow behaviour are more identical to each other, though they are not without their differences. Results indicate that pairs of streamwise vortices are formed downstream of the saddles once the wave amplitude reaches $a/D_m=0.2$ and beyond. This is regardless of whether they are node- or saddle-configured. However, closer inspections reveal that only saddle-configured $\lambda/D_m=2$ cylinders produce recirculating regions at the end-walls, which serve to displace the

streamwise vortices slightly further away from the end-walls. As a result, less coherent recirculating flows exist behind the nodes of saddle-configured $\lambda/D_m=2$ cylinders.

To summarize, results from the present study essentially demonstrate that flow behaviour of shorter wavelength $\lambda/D_m=2$ cylinders is less sensitive towards variations in the geometrical arrangement, aspect-ratio and the use of end-walls. This is primarily due to the regular formations of streamwise vortices at their saddles under both node- and saddle-configured arrangements, which reduce external effects associated with cylinder-wall interferences. In contrast, with little or no formation of streamwise vortices observed in the longer wavelength $\lambda/D_m=4$ cylinders, they are more susceptible towards end-effects from cylinder-wall interferences.

Acknowledgements

The authors gratefully acknowledge the support for the present study by Nanyang Technological University under the Tan Chin Tuan Exchange Fellowship in Engineering programme and the National Natural Science Foundation of China (NSFC 51106096).

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