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# The Combination of Continuous Network Design and route guidance

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**Abstract**

In this study, a traffic management measure is presented by combining the route guidance of Advanced Traveler Information System (ATIS) and the continuous network design (CNDP) to alleviate increasing traffic congestion. The route guidance recommends the travelers to choose the shortest path based on marginal travel cost and user constraints. The problem is formulated into a bi-level programming problem. The most distinct property of this problem formulation is that the feasible path set of its lower-level problem is determined by the decision variable of upper-level problem, while in conventional transportation network design problems the feasible path set for lower-level traffic assignment problem is fixed to be all the viable paths between each specific origin-destination pair. The simulated annealing algorithm is improved to solve this bi-level problem. A path-based traffic algorithm is developed to calculate the lower-level traffic assignment problem under the route guidance. Compared to the results of conventional CNDP, the measure presented in this study can better improve the transportation network performance.

*Keywords:* continuous network design, route guidance, system optimal, traffic demand management

## 1. Introduction

In presence of rapid economic development, urbanization and population growth, almost all large cities worldwide in the world are facing the serious problem of traffic congestion. Traffic congestion has induced not only huge economic loss, but environmental not only. Based on the report of Texas Transportation

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Institute, the congestion bill in the United States alone was $67.5 billion in the year 2000, comprised of 3.6 billion hours of delay and 5.7 billion gallons of gas (Schrank, 2002). International Energy Agency said that 23% of global energy related carbon emission in 2004 are related to transportation (IEA, 2006). It is reasonable to believe that the world will soon have to confront high levels of air pollution and congestion problems caused principally by the unrestricted use of private cars, and have to deploy practical instruments to achieve transportation sustainability efficiently, effectively and in a politically feasible manner (Yang and Wang, 2011).

There are generally two ways to alleviate traffic congestion: increasing traffic supply (capacity) and reducing traffic demand (Yang and Wang, 2011). The former way is usually called network design problem (NDP) in transportation network, which determines the enhancement of existing link capacity or the addition of network candidate links. Generally, NDP can be classified into three classes: CNDP (determining the optimal capacity enhancement for a subset of the existing links and its deterministic variables are continuous), discrete network design problem (DNDP) (Wang et al., 2015; Riemann et al., 2015) (dealing with the optimal location of new links addition from a set of candidate links and its deterministic variables often are expressed by 0-1 integer), and mixed network design problem (MNDP) (mixture of the CNDP and DNDP) (Luathep et al., 2011).

However, disparate evidence indicates that the enhancement of road capacity induces a greater volume of traffic (Goodwin, 1996; Hansen and Huang, 1997). Besides, the limitation of land resources in cities cannot support the unlimited increase of link capacity to solve traffic congestion. Basically, more sustainability issues should be considered in NDP (Szeto et al., 2013). The other measure to reduce traffic congestion is demand-oriented strategies or demand management. Historically, congestion pricing as a demand management instrument has been paid much attention both theoretically and practically. However congestion pricing is perceived as a flat tax since it requires the travelers to pay more for using public urban infrastructure. Meanwhile, there are equity debates of the congestion pricing. So congestion pricing causes the general political resistance and is only applied on urban road in a few cities worldwide (Yang and Wang, 2011). Other than congestion pricing, some quantity controls methods to reduce traffic demand are also applied in practice. For example, rationing policies on vehicle usage are used in Mexico City (Davis, 2008), Beijing and Guangzhou, China (Hao et al., 2011). Under short-term ration of vehicle usage, observable congestion reduction and air quality improvement have been reported. But it may lose its effectiveness over time as car ownership increases(e.g., there is evidence that driving restrictions in Mexico City led to an increase in the total number of vehicles (Davis, 2008)).

Some researchers also study the combination of NDP and traffic demand management. For example, Wang et al. (2014) considered the combination of CNDP and a tradable credit scheme and proved its effectiveness to improve traffic congestion by numerical examples. In this paper, we will study the combination of CNDP and route guidance.

With the development of Intelligent Transport System and advanced techniques of information in the past decades, the advanced traveler information system (ATIS) can easily provide travel information or give travel recommendations. It is widely believed that route guidance information to the travelers is able to efficiently reduce traffic congestion and enhance the performance of traffic networks (Yang, 1998). Nowadays a large portion of the private cars have been equipped with ATIS devices. While the prices of those deices keep going down, many more travelers are likely to use them and rely on route guidance to achieve trips in the near future. Therefore, it is imperative for the transportation authority to understand how to incorporate the route guidance into the transportation network design so that the network performance is optimized. Traditional network design problems in the literature have not considered the route guidance, assuming that travelers follows user equilibrium (UE) principle to minimize their individual travel costs. In this study, we assume that when transport planners decide to improve the road network, they have to consider that route guidance information would be provided to the travelers and therefore the resultant network traffic flow pattern is different from the UE traffic assignment. Besides, noting that the simple and naive system-optimal based route guidance is subject to unfairness issue, we assume that route guidance strategy with certain user constraints is applied to reduce the unfairness. Indeed, the network traffic flow pattern achieved with this route guidance strategy is constrained system optimal (CSO). The problem studied in this paper, i.e., the combined continuous network design and route guidance, is then formulated into a bi-level programming. A modified simulated annealing algorithms is proposed to solve the problem. To summarize, the main contribution of this research work is to fill in the research gap in transportation network design problems by considering the route guidances of the traveler information system.

The paper is organized as follows: Section 2 presents a bi-level programming problem to model the combination of CNDP and route guidance. The algorithm to solve the bi-level programming problem is given in Section 3. Section 4 gives the numerical test and the conclusions of the study is presented in Section 5.

## 2. Problem formulation

In NDP, the traffic authorities make a decision on the link capacity enhancement or the addition of new link to optimize a specific network index (e.g. total travel time or generalized cost). Meanwhile, the route choice of travelers is considered in NDP. Therefore, NDP is naturally described by bi-level program. Abdulaal and LeBlanc (1979) is the first one who describe CNDP by bi-level programming, in which the lower-level is the user equilibrium (UE) assignment problem. In this study, the general CNDP is called UE-CNDP. There is also NDP in which the lower-level problem is a stochastic user equilibrium (Liu and Wang, 2015). In the following, we will present the notations used in our paper.

### 2.1. Notations

In this study, $G = (N, A)$ denotes a direct connected traffic network of a node set $N$, a link set $A$. $W$ is the set of origin-destination (OD) pairs, $R_w$ is denoted as the set of paths between OD pair $w$, and all paths in this network are denoted by $R$, $R = \bigcup_{w \in W} R_w$. $x_a$ and $c_a$ respectively denote the traffic flow and the capacity on link $a$. $t_a(x_a)$ is the travel cost of link $a$ and increases with $x_a$. x is the link flow vector. $f_r^w$, $c_r^w$ are the flow and travel cost of path $r$, $r \in R_w$, $\forall w \in W$,

$$c_r^w = \sum_{a \in A} t_a \delta_{ar}^w, r \in R_w, w \in W \tag{1}$$

where $\delta_{ar}^w = 1$, if link $a$ is used by path $r$, and $\delta_{ar}^w = 0$ otherwise. $q_w$ denotes the traffic demand between OD pair $w$. $y_a$ is the capacity enhancements in CNDP and y is the vector of $y_a$. $G_a(y_a)$ is the cost function of incremental capacity on link $a$, $\forall a \in A$. The lower and upper bounds of allowed capacity enhancement for link $a \in A$ are respectively denoted by $l_a$ and $u_a$. $\tau_r$ is the normal length of path $r \in R_w$, its definition is given in Section 2.3. $T_w$ is the minimum normal length for all paths between OD pair $w$ and $\varphi$ is a parameter. $\mathcal{P}_w$ is the feasible path set under route guidance between OD pair $w$. Based on the notations, the UE-CNDP model is given in Section 2.2.

### 2.2. UE-CNDP Model

The upper-level of UE-CNDP Model:

$$\min \ z(\mathrm{x}, \mathrm{y}) = \sum_{a \in A} t_a(x_a, y_a) x_a(\mathrm{y}) + \alpha \sum_{a \in A} \mathrm{G}(y_a) \tag{2}$$

$$s.t. \qquad l_a \leq y_a \leq u_a, \forall a \in A \tag{3}$$

where, the parameter $\alpha$ is a scaling coefficient which converts cost of increase link capacity into the travel cost. $x_a$ is the solution of the lower UE assignment problem:

$$\min \sum_a \int_0^{x_a} t_a(\omega) d\omega \tag{4}$$

$$s.t. \qquad \sum_r f_r^w = q_w, \forall r \in R_w, \forall w \in W \tag{5}$$

$$f_r^w \geq 0, \forall r \in R_w, \forall w \in W \tag{6}$$

$$x_a = \sum_w \sum_r f_r^w \delta_{ar}^w, \forall a \in A \tag{7}$$

### 2.3. Travel pattern under Route Guidance

Roughgarden and Tardos (2002) have showed that the user optimal route guidance generally cannot improve the performance of traffic network. However, from the traffic authority's perspective, it is certainly desirable to explicitly minimize the total travel time. To alleviate traffic congestion, it is imperative to develop route guidance based on system optimal principle. However, in system optimal, despite that the total system travel time is minimized, some travelers are routed on unacceptably long paths so that shorter paths can be used for many other travelers (Jahn et al., 2005). So, directly implemented, the route guidance based on system optimal may be not accepted by travelers. Jahn et al. (2005) developed a route guidance method which recommends the shortest path based on the marginal travel cost to travelers at current traffic flow. And the recommended paths all satisfy user constraints. With this route guidance, traffic system will attain constrained system optimal (CSO). The user constraints are described by the normal length of path and the parameter $\varphi$. Specifically, for the path $r \in R_w$, its normal length is $\tau_r$. Let $T_w := \min_{r \in R_w \tau_r}$, if $\tau_k \leq \varphi T_w$, path $k$ belongs to the feasible path set $\mathcal{P}_w$ between OD $w$; otherwise, it is not in the $\mathcal{P}_w$. It is to say that the user constraints is the feasible path set determined by the normal length and $\varphi$ under route guidance. The normal length of a path can be its traversal time in the uncongested network, its traversal time in user equilibrium, its geographic distance, or any other appropriate measure. The only condition of the normal length of a path is that it may not depend on the actual flow on the path (Jahn et al., 2005).

Schulz and Stier-Moses (2006) have verified that the resulting traffic assignment is provably efficient and close to fair when the normal length is defined as the travel time in user equilibrium. In this study, we use the route guidance method developed by Jahn et al. (2005) and choose the travel time at UE to be the normal length. The traffic state caused by this route guidance can be described by the following CSO problem:

$$\min \sum_a t_a(x_a)x_a \tag{8}$$

$$s.t. \sum_r f_r^w = q_w, \forall r \in \mathcal{P}_w, \forall w \in W \tag{9}$$

$$f_r^w \geq 0, \forall r \in \mathcal{P}_w, \forall w \in W \tag{10}$$

$$x_a = \sum_w \sum_{r \in \mathcal{P}_w} f_r^w \delta_{ar}^w, \forall a \in A \tag{11}$$

### 2.4. CSO-CNDP Model

Replacing the lower-level problem in UE-CNDP with CSO, the combination of route guidance and CNDP can be descried as follows:

upper-level:

$$\min\ z(\mathrm{x},\mathrm{y}) = \sum_{a\in A} t_a(x_a, y_a)x_a(\mathrm{y}) + \alpha \sum_{a\in A} \mathrm{G}(y_a) \tag{12}$$

$$s.t. \qquad l_a \leq y_a \leq u_a, \forall a \in A \tag{13}$$

lower-level:

$$\min\ \sum_a t_a(x_a)x_a \tag{14}$$

$$s.t. \qquad \sum_r f_r^w = q_w, \forall r \in \mathcal{P}_w, \forall w \in W \tag{15}$$

$$f_r^w \geq 0, \forall r \in \mathcal{P}_w, \forall w \in W \tag{16}$$

$$x_a = \sum_w \sum_{r\in\mathcal{P}_w} f_r^w \delta_{ar}^w, \forall a \in A \tag{17}$$

In this paper, the combination of route guidance and CNDP is called as CSO-CNDP. It should be noted that, in the constraint (16) of the lower-level problem, the feasible path set, i.e., the paths that would be recommended to the travelers in the route guidance information, is constrained to the the set Pw. To determine this constrained set $P_w$, we follow the research work in Schulz and Stier-Moses (2006): firstly, we solve the user equilibrium traffic flow pattern achieved on this network and set the normal length as the travel time between one specific OD pair at this user equilibrium state; then, we select parameter $\varphi$ to determine the constrained path set by eliminating some paths with too long travel time. One can easily find that, the major difference between our model formulation with conventional NDP is on this constrained feasible path set. Indeed, this makes the model solution in this study more difficult and many exact solution methods developed for solving conventional network design problem in the literature are not applicable in this study, which motivates us to apply the simulated annealing algorithm to solve this problem, as is specified in next section.

## 3. Algorithm

In CSO-CNDP, the feasible path set $\mathcal{P}_w$ between OD pair $w$ ($\forall w \in W$) is determined by the normal length of the paths and tolerance factor $\varphi$. The normal length $\tau_r$ of path $r$ is its travel time at UE in this paper, which is determined by the link capacity enhancement y for a given transportation network design. Therefore the decision variable y of upper-level problem of CSO-CNDP dictates the constraints of the lower-level problem of CSO-CNDP. It is to say that the feasible domain of the lower-level problem of CSO-CNDP varies with the link capacity enhancement plan y. Some existing efficient deterministic methods to solve UE-CNDP are no longer proper to solve CSO-CNDP. For example, based on the concepts of gap function and penalty, the global optimization algorithm to solve UE-CNDP developed by Li et al. (2012) transfer UE-CDNP into a sequence of single level concave programs, which is amenable to a global solution. It is to say that the feasible domain of the single level concave programs is convex. But CSO-CNDP cannot guarantee the convexity. Wang and Lo (2010) formulated the UE-CDNP as a single level optimization problem with equilibrium constraints, and they transform the equilibrium constraints into a set of mixed integer constraints and linearize the travel time function. The mixed integer constraints of the equilibrium constraints of UE-CNDP cannot consider the variability of feasible path set with link capacity enhancement. So the method cannot be used to solve CSO-CNDP. In the algorithm of Meng et al. (2001), the link flow and capacity enhancement are both updated at the same time in the convex combination method to solve the sub problem. It assumed that the feasible domain is not variable with the decision variable of upper-level problem. Although there are also some existing deterministic methods that can be used to solve CSO-CNDP, most of them are not tested by large size example (e.g., Gao et al. (2007)), or their results are not better than Simulated annealing algorithm (e.g., Chiou (2005)).

Other than Simulated annealing algorithm (Friesz et al., 1992), there are some heuristic or nondeterministic algorithms to solve UE-CNDP. These heuristic algorithms include Hook-Jeeves heuristic algorithm (Abdulaal and LeBlanc, 1979), Iterative optimization-assignment algorithm (Allsop, 1974), the equilibrium decomposed optimization algorithm (Suwansirikul et al., 1987), Sensitivity analysis-based algorithm (Yang and Yagar, 1995), and the three nondeterministic and derivative-free global optimization methods for the CNDP of Hellman (2010): NFFM (a filled function method), EGO (a surrogate model method) and DIRECT (dividing the search space into rectangles and evaluating the midpoints of them). From the results of these nondeterministic algorithms to solve UE-CNDP presented in Tab.8-Tab.12, it can be found that the objective value is better than other nondeterministic algorithms except DIRECT in Tab.8, meanwhile the code of simulated annealing algorithm is very simple. Therefore, this study chooses simulated annealing algorithm to solve CSO-CNDP.

In this study, we improve the simulated annealing algorithm developed by Friesz et al. (1992) and use it to deal with CSO-CNDP. In the simulated annealing algorithm (Friesz et al., 1992), any new solution may be not accepted when temperature is lower, the matrix that controls the step size distribution cannot be gained in this situation. Our improvement aims to handle this issue. As the route guidance system recommends paths to travelers, a path-based algorithm for CSO is developed based on the study of Jayakrishnan et al. (1994) to solve the lower-level CSO traffic pattern. In the following, we present that the specific process of the improved simulated annealing algorithm and path-based traffic assignment algorithm.

### *3.1. Path-based Traffic Assignment Algorithm for CSO*

The only difference between the path-based traffic assignment algorithm developed by Jayakrishnan et al. (1994) and our algorithm is the constrained shortest paths rather than regular shortest paths in direction finding subproblems. To find the constrained shortest paths, we implement the label-correcting algorithm presented by Aneja et al. (1983). The specific description of our traffic assignment algorithm for lower-level CSO is as follows:

step 1: Initialization. Using the label-correcting algorithm of Aneja et al. (1983), find constrained shortest path based on marginal travel cost $t'_a$ for $x_a = 0, \forall a \in A$ and implement all-or-nothing assignments. This results in path flow $f^w_{r,1}$ $\forall w \in W, r \in R_w$ and link flow $x^1_a$ $\forall a \in A$. Set iteration counter $n = 1$. Initialize path set $K_w$ with constrained shortest path for each OD pair $w$.

step 2: Update. Update the marginal travel cost $t'_a$ and path $r$ cost $c^w_{r,n}$ using corresponding link marginal travel cost, $\forall r \in R_w, w \in W$.

step 3: Direction finding. Find the constrained shortest path $\bar{k}^n_w$ between OD pair $w \in W$ based on marginal travel cost. If the $\bar{k}^n_w$ doesn't exist in the path set $K_w$ (just compare path cost), add it to $K_w$ and record $c_{\bar{k}^n_w}$. If not, tag the constrained shortest path in $K_w$ as $c_{\bar{k}^n_w}$.

step 4: Move. Set the new path flows.

$$f^w_{r,n+1} = max\{0, f^w_{r,n} - \frac{\alpha^n}{s^n_k}(c^w_{r,n} - c_{\bar{k}^n_w})\}, \forall w \in W, r \in K_w, r \neq \bar{k}^n_w$$

where

$$s^n_r = \sum_{a^*} \frac{\partial t'_{a^*}(x^n_a)}{\partial x^n_{a^*}}$$

$a^*$ denotes link that are on either $r$ or $\bar{k}^n_w$, but not on both, and $\alpha^n$ is a scalar step-size modifier which is valued 0.01 in this study.

$$f^{n+1}_{\bar{k}^n_w} = q_w - \sum_r f^{n+1}_k$$

The path flow $f^{n+1}_r$ then generate the link flow $x^{n+1}_a$.

step 5: Convergence test. If the criterion of convergence is satisfied, stop. Otherwise, set $n = n + 1$ and go to step 1.

### 3.2. Improved Simulated annealing algorithm

Simulated annealing algorithm is motivated by the analogy with the physical annealing process to find low energy states of a solid in heat bath (Metropolis et al., 1953). It is a stochastic algorithm which has the ability to avoid getting stuck in a local, non-global optimum, when searching for a global optimum(Meng and Yang, 2002). Simulated annealing can be readily applied to any arbitrary combinatorial optimization problem. Application of the approach to minimization of continuous optimization problem is also stated in many researches (Vanderbilt and Louie, 1984; Dekkers and Aarts, 1991; Romeijn and Smith, 1994). Friesz et al. (1992) applied the simulated annealing method proposed by Vanderbilt and Louie (1984) to the UE-CNDP.

In their method, a candidate optimal solution can be generated by the following formulation:

$$\mathrm{y}_{\mathrm{new}} = \mathrm{y}_{\mathrm{old}} + \Delta \mathrm{y} \tag{18}$$

where $\Delta \mathrm{y} = \mathbf{Qu}$. The $\mathbf{Q}$ is the step size control matrix. Vector $\mathbf{u}$ is random and its entry $u_i$ is randomly and independently chosen from the normalized interval $[-\sqrt{3}, \sqrt{3}]$. To efficiently implement simulated annealing algorithm, Friesz et al. (1992) used the a self-regulating mechanism suggested by Vanderbilt and Louie (1984) for the step size determination. By the mechanism, step size can be efficiently chosen. The step size control matrix $\mathbf{Q}$ is obtained by solving the following equation via matrix decomposition (e.g. Choleski decomposition):

$$\mathbf{s} = \mathbf{Q} \cdot \mathbf{Q}^{\mathbf{T}} \tag{19}$$

where $\mathbf{s}$ is a covariance matrix. At temperature $l+1$, the covariance matrix $s^{l+1}$ can be solved as follows:

$$A_i^{(l)} = \frac{1}{M} \sum_{m=1}^{M} y_i^{m;l} \quad \forall i \tag{20}$$

$$\mathbf{S}_{ij}^{(l)} = \frac{1}{M} \sum_{i=1}^{M} [y_i^{(m;l)} - A_i^{(l)}][y_{a_j}^{(m;l)} - A_j^{(l)}] \tag{21}$$

$$\mathbf{s}^{l+1} = \frac{\chi_s}{\beta M} \mathbf{S}^{(l)} \tag{22}$$

where $\mathbf{y}^{(m;l)}$ is the value of $\mathbf{y}$ on the $m$th step at the $l$ temperature stage and $M$ is the iteration number at each temperature stage. $A_i^{(l)}$ is the average of the capacity enhancement $y_i^{(l)}$ on link $i$ at the temperature stage $l$. Matrix $\mathbf{S}^{(l)}$ has an entry $\mathbf{S}_{ij}^{(l)}$ which is the covariance between $y_i^{(l)}$ and $y_j^{(l)}$. Parameter $\chi \geq 1$ is the growth factor. $\beta$ is equal to 0.11 as suggested by Vanderbilt and Louie (1984).

In Eq.19, covariance matrix $s$ should be positive definite for the requirement of Choleski decomposition. However, it is possible that no new solution is accepted at lower temperature stage $l$ so that $s^{(l+1)}$ is not positive definite. Indeed, this situation did occur in our numerical test of Sioux Falls network for CSO-CNDP. To avoid this case, we set $s^{(l+1)} = s^{(l)}$ when $s^{(l+1)}$ is not positive definite. Except for this improvement, the simulated annealing algorithm used in this study is the same as the one of Friesz et al. (1992).

By implementing the improved simulated annealing algorithm and the path-based traffic assignment for CSO, we would present the results of numerical experiments of CSO-CNDP in the next section.

## 4. Numerical Experiments

In this paper, we use two examples to test the effect of CSO-CNDP to alleviate traffic congestion. These two examples are commonly used in UE-CNDP's numerical experiments. The first example is a small traffic network,

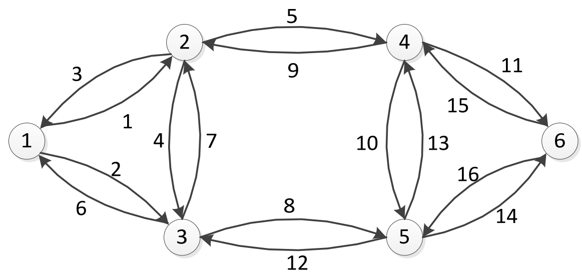



Figure 1: The first tested example

which has 6 nodes, 16 links and 2 OD pairs in Fig.1. The parameters of the example network are presented in Tab.1. The second example traffic network in this study is the aggregated network of the city Sioux Falls, South Dakota, which is shown in Fig.2. It has 24 nodes and 76 links. The link capacity enhancement are considered on the link 16, 17, 19, 20, 25, 26, 29, 39, 48, 74. It was first applied to NDP by Abdulaal and LeBlanc (1979). In this example, link travel time cost is described by BPR function (Bureau of Public Roads, BPR). The values of parameters of Sioux Falls network can be referred to the work of Suwansirikul et al. (1987). To illustrate improved transportation network performance of CSO-CNDP, we will present the results of UE-CNDP solved by some existing algorithms. Those algorithms are given in Tab.2. Meanwhile, it is well-known that the total travel cost or/and time of DUE is higher than that of SO. So the optimal value of the DUE-CNDP can be considered as an upper bound of the optimal value of CSO-CNDP of this study. Results of UE-CNDP can also be used to test the performance of our algorithm.

Table 1: The parameters of the first tested example

$t_a(x_a, y_a) = A_a + B_a[x_a/(c_a + y_a)]^4$
$Z(\mathrm{x}, \mathrm{y}) = \sum_{a \in A}[t_a(x_a, y_a)x_a + d_a y_a]$

| link | $A_a$ | $B_a$ | $c_a$ | $d_a$ |
|---|---|---|---|---|
| 1 | 1 | 10 | 3 | 2 |
| 2 | 2 | 5 | 10 | 3 |
| 3 | 3 | 3 | 9 | 5 |
| 4 | 4 | 20 | 4 | 4 |
| 5 | 5 | 50 | 3 | 9 |
| 6 | 2 | 20 | 2 | 1 |
| 7 | 1 | 10 | 1 | 4 |
| 8 | 1 | 1 | 10 | 3 |
| 9 | 2 | 8 | 45 | 2 |
| 10 | 3 | 3 | 3 | 5 |
| 11 | 9 | 2 | 2 | 6 |
| 12 | 4 | 10 | 6 | 8 |
| 13 | 4 | 25 | 44 | 5 |
| 14 | 2 | 33 | 20 | 3 |
| 15 | 5 | 5 | 1 | 6 |
| 16 | 6 | 1 | 4.5 | 1 |

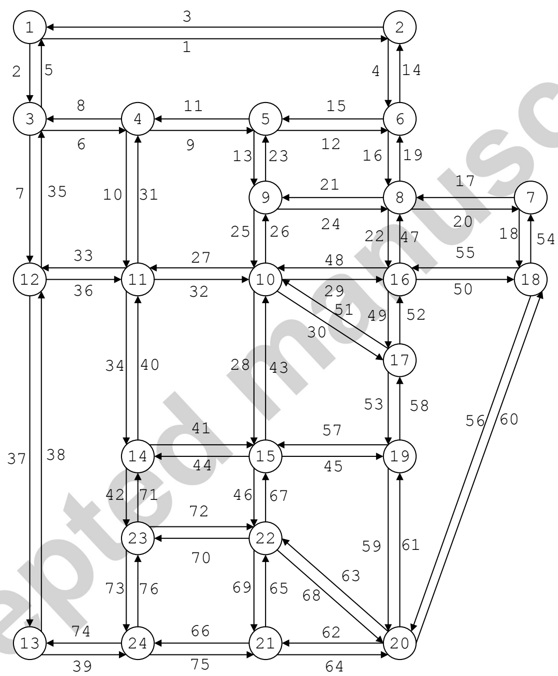



Figure 2: Sioux Falls network

Table 2: Abbreviations of algorithms to UE-CNDP

| Abbreviation | Name of the algorithm | Sourse |
|---|---|---|
| IOA | Iterative optimization-assignment algorithm | Allsop (1974) |
| HJ | Hooke-Jeeves algorithm | Abdulaal and LeBlanc (1979) |
| EDO | Equilibrium decomposed optimization | Suwansirikul et al. (1987) |
| MINOS | Modular in-core nonlinear system | Suwansirikul et al. (1987) |
| NFFN | Filled Function Method | Hellman (2010) |
| SA | Simulated annealing algorithm | Friesz et al. (1992) |
| SAB | Sensitivity analysis-based algorithm | Yang and Yagar (1995) |
| AL | Augmented lagrangian algorithm | Meng et al. (2001) |
| DIRECT | Dividing Rectangles | Hellman (2010) |
| EGO | Efficient Global Optimization | Hellman (2010) |
| GP | Gradient projection method | Chiou (2005) |
| CG | Conjugate gradient projection method | Chiou (2005) |
| QNEW | Quasi-Newton projection method | Chiou (2005) |
| PT | PARTAN version of gradient projection method | Chiou (2005) |
| PMILP | Path based mixed-integer linear program | Wang and Lo (2010) |
| LMILP | Link based mixed-integer linear program | Luathep et al. (2011) |
| PMC | Penalty with multicutting plane method | Li et al. (2012) |

Table 3: The traffic demand of different cases for the first tested example

| Case | traffic demand (1,6) | traffic demand (6,1) | total demand |
|---|---|---|---|
| I | 5 | 10 | 15 |
| II | 10 | 20 | 30 |

### *4.1. Comparison to the results of UE-CNDP*

The traffic demand of the first example has two cases shown in Tab.3. We use the same values of parameters of simulated annealing as those of Friesz et al. (1992) (Tab.4). In all numerical tests, the temperature of next stage is set to be 0.8 times of that at the current stage.

In demand case I of the first example, the results of CSO-CNDP with $\varphi \in [1.01, 4.00]$ and those of UE-CNDP obtained by some existing algorithms in Tab.2 are demonstrated in Tab.5 and Tab.6. In Tab.5 and Tab.6, TB is the total cost of the link capacity expansion, TT is total travel time and Z is the value of upper-level problem objective function.

It can be found from Tab.5 and Tab.6 that the value of objective function of CSO-CNDP and the total travel time are both better than those of UE-CNDP solved by algorithms in Tab.2. It is also found that the results of CSO-CNDP are all invariant with $\varphi \in [1.01, 4.00]$. These can be inferred from Tab.7, which

Table 4: The parameters of Simulated Annealing algorithm in different cases of first tested example

| Parameter | Case I | Case II |
|---|---|---|
| $\chi_s$:growth factor | 3 | 9 |
| $T_i$:initial temperature | 500 | 500 |
| $T_f$:final temperature | 200 | 100 |
| $M$: iterations at each temperature | 300 | 300 |
| upper bound on $y_a$: | 10 | 20 |
| lower bound on $y_a$: | 0 | 0 |

shows the total travel time and path set at UE and SO before implementing CSO-CNDP. In Tab.7, the path set of UE is equal with that of SO in the first example for case I. As the normal length of our CSO is assumed to be the travel time at UE state, the increase of $\varphi$ cannot change the feasible path recommended to travelers from the route guidance system and CSO is identical with SO in this case. For this reason, the results of CSO-CNDP with different $\varphi$ are all the same, and the value of TT and the Z of CSO-CNDP are less than those of UE-CNDP. But, the cost of link capacity expansion is more than the results of Luathep et al. (2011) and Wang and Lo (2010). They respectively developed a global optimization algorithm for UE-CNDP. It can be concluded from this result that the cost of UE-CNDP based on user optimal rule may be less than that with route guidance, but the social welfare of CSO-CNDP is better.

In case I of example 1, the less traffic demand leads to the constrained path set that is exactly the same with the solution of UE. Therefore, we further make comparison between the results of CSO-CNDP and UE-CNDP for case II of example 1. In case II, $\varphi \in [1.01, 2.00]$ is set for CSO-CNDP. The results are given in Tab.8, Tab.9 and Tab.10. It should be noted that the total travel time for case II is 5756.5917 at UE state before CSO-CNDP. It can be found that CSO-CNDP can more efficiently alleviate traffic congestion than that of UE-CNDP and its value of Z is less than all of known results of UE-CNDP. In those known results of UE-CNDP, Li et al. (2012) claimed that their result for case II of example 1 is the real global optimal one. From the result in Tab.10, it can be observed that the members of feasible path set increase as $\varphi$ goes up, and the difference between CSO and SO become smaller. One can envisage that when $\varphi$ goes to infinity, the CSO will reduce to SO. So, the value of Z will decrease with increasing $\varphi$. Because the feasible path set for $\varphi \in [1.01, 1.13]$ are all same, the results of CSO-CNDP do not change. This situation also occurs for $\varphi \in [1.14, 1.16]$ and $\varphi \in [1.17, 2]$. The interval of $\varphi \in [1.01, 2]$ is grounded into three parts in Tab.10.

The similar conclusions of case II of the first example can also be found in the numerical test for Sioux Falls network, as results is shown in Tab.11 and Tab.12.

### 4.2. The unfairness of CSO-CNDP

From the results in Section 4.1, one can observe that CSO-CNDP can efficiently improve the performance of transportation system as compared to the traditional UE-CNDP. However, the issue of unfairness for CSO-CNDP still remains, i.e., the route guidance system may recommend some individual travelers to choose a path that is not exactly the shortest path on this network. In this section, we would use two indices of unfairness to study the unfair problem of travelers suffering from CSO-CNDP. The two indices of unfairness are loaded unfairness and UE unfairness both presented by Jahn et al. (2005). The specific definitions of loaded unfairness and UE unfairness can be described as follows:

(i) Loaded unfairness is the ratio of the traveler experienced travel time to the experienced travel time of the fastest traveler between the same OD pair.

Table 5: The comparison of results for example 1 in Case I, Part-I

| y | IOA | HJ | EDO | MINOS | SA | SAB | AL | GP | CG |
|---|---|---|---|---|---|---|---|---|---|
| y3 | | 1.2 | 0.13 | | | | 0.0062 | | |
| y6 | 6.95 | 3 | 6.26 | 6.58 | 3.1639 | 5.8352 | 5.2631 | 5.8302 | 6.1989 |
| y7 | | | | | | | 0.0032 | | |
| y12 | | | | | | | 0.0064 | | |
| y15 | 5.66 | 3 | 0.13 | 7.01 | | 0.9739 | 0.7171 | 0.87 | 0.0849 |
| y16 | 1.79 | 2.8 | 6.26 | 0.22 | 6.724 | 6.1762 | 6.7561 | 6.109 | 7.5888 |
| TB | 42.7 | 29.8 | 13.95 | 48.86 | 9.8879 | 17.8548 | 16.4168 | 17.1592 | 14.2971 |
| TT | 171.634 | 188.408 | 187.24 | 162.388 | 191.4401 | 186.3562 | 186.5862 | 186.6158 | 185.9309 |
| Z | 214.334 | 218.208 | 201.19 | 211.248 | 201.328 | 204.211 | 203.003 | 203.775 | 200.228 |

Table 6: The comparison of results for example 1 in Case I, Part-II

| y | QNEW | PT | PMILP | LMILP | $\varphi$ = 1.01 | $\varphi$ = 2.00 | $\varphi$ = 3.00 | $\varphi$ = 4.00 | SO |
|---|---|---|---|---|---|---|---|---|---|
| y3 | | | | | | | | | |
| y6 | 6.0021 | 5.9502 | 5.19 | 5.19 | 5.9536 | 5.9536 | 5.9536 | 5.9536 | 5.9536 |
| y7 | | | | | | | | | |
| y12 | | | | | | | | | |
| y15 | 0.1846 | 0.5798 | | 0.002 | | | | | |
| y16 | 7.5438 | 7.1064 | 7.5 | 7.585 | 7.7459 | 7.7459 | 7.7459 | 7.7459 | 7.7459 |
| TB | 14.6535 | 16.5354 | 12.69 | 12.837 | 13.6995 | 13.6995 | 13.6995 | 13.6995 | 13.6995 |
| TT | 185.9505 | 185.8846 | 186.94 | 186.785 | 179.273 | 179.273 | 179.273 | 179.273 | 179.273 |
| Z | 200.604 | 202.42 | 199.63 | 199.622 | 192.9725 | 192.9725 | 192.9725 | 192.9725 | 192.9725 |

Table 7: The path and system travel time of the first test example with case I at SO and UE

| paths at SO | path cost | total travel time at SO | paths at UE | path cost | total travel time at UE |
|---|---|---|---|---|---|
| 1→3→5→6 | 5.5039 | 334.60 | 1→3→5→6 | 5.5039 | 336.57 |
| 6→2→1 | 27.5668 | | 6→4→2→1 | 30.9052 | |
| 6→5→3→1 | 29.1668 | | 6→5→3→1 | 30.9052 | |
| 6→5→3→2→1 | 30.7668 | | 6→5→3→2→1 | 30.9052 | |
| 6→5→4→2→1 | 31.5668 | | 6→5→4→2→1 | 30.9052 | |

Table 8: The comparison of results of first tested example in case II Part-I

| y | IOA | HJ | EDO | MINOS | NFFN | SA | SAB | AL | DIRECT |
|---|---|---|---|---|---|---|---|---|---|
| y1 | | | | | | | 0.0189 | | |
| y2 | 4.55 | 5.40 | 4.88 | 4.61 | 0.3536 | | 2.2246 | 4.6153 | 4.6228 |
| y3 | 10.65 | 8.18 | 8.59 | 9.86 | 9.8808 | 10.1740 | 9.3394 | 9.8804 | 9.8720 |
| y6 | 6.43 | 8.10 | 7.48 | 7.71 | 7.4937 | 5.7769 | 9.0466 | 7.5995 | 7.4120 |
| y7 | | | 0.26 | | | | | 0.0016 | |
| y8 | 0.59 | 0.90 | 0.85 | 0.59 | 0.6173 | | 0.0175 | 0.6001 | 0.5898 |
| y9 | | | | | | | | 0.0010 | |
| y12 | | | | | | | 0.0816 | 0.1130 | |
| y14 | 1.32 | 3.90 | 1.54 | 1.32 | 1.3212 | | 0.0198 | 1.3184 | 1.3123 |
| y15 | 19.36 | 8.10 | 0.26 | 19.14 | | | 2.1429 | 2.7265 | |
| y16 | 0.78 | 8.40 | 12.52 | 0.85 | 20.0000 | 17.2786 | 18.9835 | 17.5774 | 19.9954 |
| TB | 196 | 136.6 | 87.36 | 191.41 | 83.774 | 73.9255 | 95.0608 | 111.4517 | 96.3421 |
| TT | 360.61 | 420.62 | 453.38 | 365.73 | 446.0389 | 454.5715 | 441.0232 | 421.2583 | 426.4987 |
| Z | 556.61 | 557.22 | 540.74 | 557.14 | 529.8129 | 528.497 | 536.084 | 532.710 | 522.8408 |

Table 9: The comparison of results of first tested example in case II Part-II

| y | EGO | GP | CG | QNEW | PT | PMILP | LMILP | PMC |
|---|---|---|---|---|---|---|---|---|
| y1 | 0.1013 | 0.1022 | 0.0916 | 0.101 | | | | |
| y2 | 4.3743 | 2.1818 | 2.1796 | 2.1521 | 2.1801 | 4.41 | 2.722 | 4.6905 |
| y3 | 7.3736 | 9.3423 | 9.3425 | 9.1408 | 9.3339 | 10 | 9.246 | 9.9778 |
| y6 | 17.0593 | 9.0443 | 9.0441 | 8.8503 | 9.0361 | 7.42 | 8.538 | 7.5554 |
| y7 | | | | | | | | |
| y8 | | 0.008 | 0.0074 | 0.0114 | 0.0079 | 0.54 | | 0.6333 |
| y9 | | | | | | | | |
| y12 | 2.8928 | 0.0375 | 0.0358 | 0.0377 | | | | |
| y14 | 0.4315 | 0.0089 | 0.0083 | 0.0129 | 0.0089 | 1.18 | | 1.7664 |
| y15 | 16.1513 | 1.9433 | 1.9483 | 1.9706 | 1.9429 | | | |
| y16 | 11.2042 | 18.9859 | 18.986 | 18.575 | 18.9687 | 19.5 | 20 | 19.6737 |
| TB | 199.8017 | 93.502 | 93.4879 | 73.4107 | 92.9224 | 95.31 | 82.934 | 98.3887 |
| TT | 371.8179 | 440.515 | 440.6211 | 460.6693 | 441.0976 | 428.317 | 443.554 | 424.3593 |
| Z | 571.6196 | 534.017 | 534.109 | 534.08 | 534.02 | 523.627 | 526.488 | 522.748 |

Table 10: The comparison of results of first tested example in case II Part-III

| y | $\varphi \in [1.01\ 1, 13]$ | $\varphi \in [1.14, 1.16]$ | $\varphi \in [1.17, 2]$ | SO |
|---|---|---|---|---|
| y1 | | | | |
| y2 | 2.7251 | 2.8896 | 2.9553 | 2.9553 |
| y3 | 10.0929 | 9.7580 | 8.4978 | 8.4978 |
| y6 | 7.5363 | 8.2976 | 10.7248 | 10.7248 |
| y7 | | | | |
| y8 | 0.5469 | 0.5467 | | |
| y9 | 0.0001 | | | |
| y12 | | | | |
| y14 | 1.3248 | 1.2420 | | |
| y15 | | 0.0012 | | |
| y16 | 20.0000 | 19.9993 | 20.0000 | 20.0000 |
| TB | 91.7914 | 91.129 | 82.0797 | 82.0797 |
| TT | 422.5464 | 423.1515 | 426.9082 | 426.9082 |
| Z | 514.3378 | 514.2805 | 508.9879 | 508.9879 |

Table 11: The comparison of results of second test example part Part-I

| Y | IOA | HJ | EDO | SA | SAB | AL | GP | CG | QNEW | PT |
|---|---|---|---|---|---|---|---|---|---|---|
| y16 | 4.6875 | 4.8 | 4.59 | 5.38 | 5.7392 | 5.5728 | 5.4277 | 4.7691 | 5.3052 | 5.0237 |
| y17 | 3.9063 | 1.2 | 1.52 | 2.26 | 5.7182 | 1.6343 | 5.3235 | 4.8605 | 5.0541 | 5.2158 |
| y19 | 1.2695 | 4.8 | 5.45 | 5.5 | 4.9591 | 5.6228 | 1.6825 | 3.0706 | 2.4415 | 1.8298 |
| y20 | 1.6599 | 0.8 | 2.33 | 2.01 | 4.9612 | 1.6443 | 1.6761 | 2.6836 | 2.5442 | 1.5747 |
| y25 | 2.3331 | 2 | 1.27 | 2.64 | 5.5066 | 3.1437 | 2.8361 | 2.8397 | 3.9328 | 2.7947 |
| y26 | 2.3438 | 2.6 | 2.33 | 2.47 | 5.5199 | 3.2837 | 2.7288 | 2.9754 | 4.0927 | 2.639 |
| y29 | 5.5651 | 4.8 | 0.41 | 4.54 | 5.8024 | 7.6519 | 5.7501 | 5.6823 | 4.3454 | 6.1879 |
| y39 | 4.6862 | 4.4 | 4.59 | 4.45 | 5.5902 | 3.8035 | 4.9992 | 4.2726 | 5.2427 | 4.9624 |
| y48 | 5.4688 | 4.8 | 2.71 | 4.21 | 5.8439 | 7.382 | 4.4308 | 4.4026 | 4.7686 | 4.0674 |
| y74 | 6.25 | 4.4 | 2.71 | 4.67 | 5.8662 | 3.6935 | 4.3081 | 5.5183 | 4.0239 | 3.9199 |
| TT | 77.516 | 76.324 | 80.068 | 75.632 | 73.401 | 74.623 | 77.536 | 76.051 | 76.534 | 77.748 |
| TB | 6.604 | 5.079 | 3.132 | 5.487 | 10.796 | 8.743 | 6.483 | 6.629 | 6.456 | 6.295 |
| Z | 84.121 | 81.402 | 83.2 | 81.119 | 84.197 | 83.366 | 84.019 | 82.679 | 82.99 | 84.043 |

Table 12: The comparison of results of second test example part II

| y | SO | LMILP | $\varphi = 1.02$ | $\varphi = 1.03$ | $\varphi = 1.04$ | $\varphi = 1.05$ | $\varphi = 1.06$ |
|---|---|---|---|---|---|---|---|
| y16 | 4.9274 | 5.362 | 4.5349 | 5.2426 | 5.2415 | 5.3402 | 5.3316 |
| y17 | 5.0358 | 2.057 | 3.7646 | 2.6534 | 2.0729 | 2.1932 | 2.2950 |
| y19 | 2.6211 | 5.486 | 4.1521 | 4.7184 | 5.0467 | 5.4438 | 5.3290 |
| y20 | 1.9953 | 1.895 | 3.7536 | 2.6549 | 2.4117 | 2.4539 | 2.2899 |
| y25 | 3.4475 | 2.556 | 2.7186 | 2.3985 | 2.3840 | 2.4151 | 2.7086 |
| y26 | 3.1620 | 2.618 | 2.8957 | 2.3428 | 2.1979 | 2.5277 | 3.0817 |
| y29 | 8.1272 | 3.741 | 4.0447 | 4.3767 | 4.1909 | 3.6010 | 3.7972 |
| y39 | 6.7257 | 4.551 | 5.0055 | 4.0009 | 4.2731 | 4.5704 | 4.7062 |
| y48 | 4.2947 | 3.741 | 4.4592 | 4.0776 | 3.9351 | 3.9871 | 4.1511 |
| y74 | 5.0114 | 4.489 | 4.1783 | 4.1223 | 4.2126 | 4.4223 | 4.3687 |
| TT | 68.783 | 75.973 | 74.7467 | 74.1958 | 73.8781 | 73.4632 | 73.1839 |
| TB | 8.98 | 4.91 | 5.6930 | 4.9776 | 4.8544 | 5.0114 | 5.2398 |
| Z | 77.763 | 80.883 | 80.4396 | 79.1734 | 78.7325 | 78.4746 | 78.4238 |

(ii) UE unfairness is the ratio of the traveler experienced travel time to the travel time of a UE state between the same OD pair.

In the two unfairness definitions, the ‘experienced travel time’ means the travel time measured on condition of the current congestion level. For a traveler between OD pair $w$, he/she chooses path $r$ to make trip. Let $T_r$ be his/her experienced travel time, $T_{min}$ be the minimal travel time between his/her OD pair, and $T_{UE}$ be his/her travel time at UE state. Then his/her loaded unfairness and UE unfairness can be formulated as follows:

$$\text{loaded unfairness} = \frac{T_r}{T_{min}} \tag{23}$$

$$\text{UE unfairness} = \frac{T_r}{T_{UE}} \tag{24}$$

In case I of example 1, the path set used by travelers in UE is same as that in SO. Therefore, the constrained path set of CSO include that of SO as a subset, which is to say, the solution of CSO is indeed the SO solution. The unfairness experienced by travelers is unfairness of SO in case I of example 1. From Tab.7, one can observe that there is only one path used by travelers between OD pair (1,6), therefore, the loaded unfairness is equal to 1.00. The UE unfairness between OD pair (1,6) is also 1.00 as travelers' travel cost at CSO is exactly the same the travel cost at UE. The specific distributions of loaded unfairness and UE unfairness between OD pair (6,1) in case I are shown in Fig.3. It should be noted that the distribution of loaded unfairness or UE unfairness is given to measure how many portions of travelers experience a loaded unfairness or UE unfairness less than a specific value.

From Fig.3, about 7% travelers between OD (6,1) choose the shortest path, and 33% travelers suffer from a loaded unfairness of 1.14. The largest loaded unfairness is 1.33 and 60% travelers experience it. Compared to the travel time in UE, 60% travelers have a UE unfairness of 1.03. Others' UE fairness are all

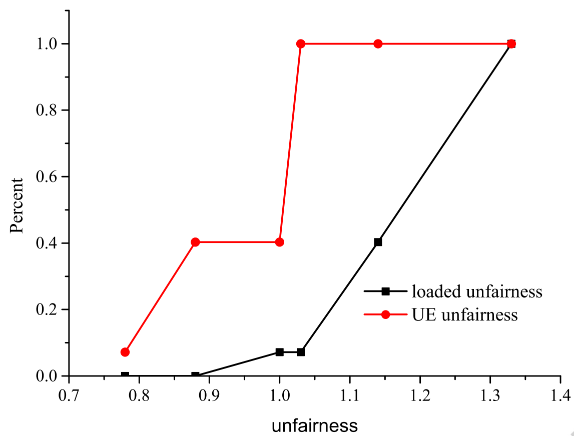



Figure 3: The unfairness distribution between OD (6,1) of the first test example in case I

Table 13: The results of CSO-CNDP when $\gamma = 0.8, 0.1$

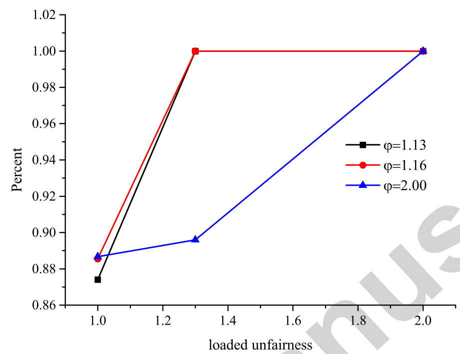
| OD \ $\varphi$ | 1.01-1.13 | 1.14-1.16 | 1.17-2.00 |
|---|---|---|---|
| (1,6) | 1→3→5→6<br>1→2→3→5→6 | 1→3→5→6<br>1→2→3→5→6 | 1→3→5→6<br>1→2→4→6<br>1→2→3→5→6 |
| (6,1) | 6→4→2→1<br>6→5→3→1<br>6→5→4→2→3→1 | 6→4→2→1<br>6→5→3→1<br>6→5→4→2→ 1<br>6→5→4→2→1 | 6→4→2→1<br>6→5→3→1<br>6→5→4→2→3→1<br>6→5→4→2→1 |

less than 1.00. Here the UE state is obtained based on the new link capacity after solving CSO-CNDP.

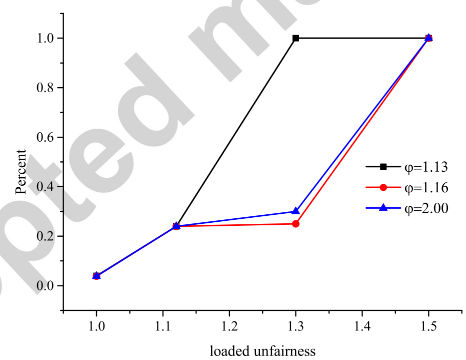
The distributions of loaded unfairness and UE unfairness for case II of example 1 are respectively presented in Fig.4 and Fig.5. Fig.4(a) and Fig.5(a) show the results of OD pair (1,6), while Fig.4(b) and Fig.5(b) are for OD pair (6,1). As mentioned in Section 4.2, $\varphi \in [1.01, 2.00]$ can be grouped into three parts based on the path set used in CSO-CNDP. The specific paths of every sub-interval of $\varphi$ are given in Tab.13.

In Fig.4, it can be found that, between the same OD pair, the largest loaded unfairness increases as $\varphi$ goes up. For $\varphi \in [1.01, 1.13]$, between OD pair (1,6), about 13% travelers suffer the largest loaded unfairness 1.30, and the other about 87% travelers use the shortest path. Travelers between OD pair (6,1), 76% travelers have the largest loaded unfairness at 1.30, only 4% travelers use the shortest path, while travelers loaded unfairness of other 20% travelers is 1.12.

When $\varphi \in [1.14, 1.16]$, 11% traveler experience the largest loaded unfairness 1.30 between OD pair (1,6), and other 89% travelers are on the shortest path. The largest loaded unfairness of travelers between OD (6,1) is 1.50, and about 1% travelers experience it. 3% travelers use the shortest path. 75% travelers

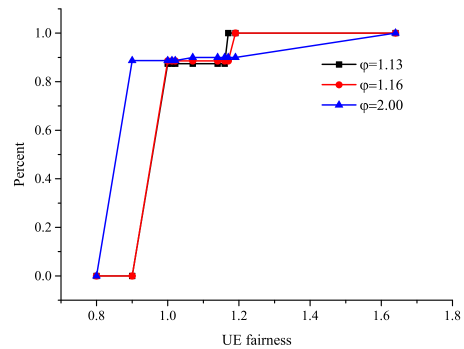



(a) OD (1,6)

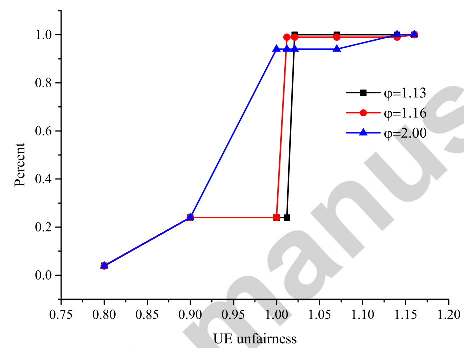



(b) OD (6,1)

Figure 4: The UE unfairness of first tested example for case II's CSO-CNDP with different $\varphi$




(a) OD (1,6)




(b) OD (6,1)

Figure 5: The UE unfairness of first tested example for case II's CSO-CNDP with different $\varphi$

have the loaded unfairness 1.30, 20% travelers' loaded unfairness is 1.12.

For $\varphi \in [1.17, 2.00]$, between OD pair (1,6), the largest loaded unfairness is 2.00. About 1% travelers suffer the largest loaded unfairness, and 1% travelers have the loaded unfairness 1.30. Other 87% travelers are on the shortest path. The largest loaded unfairness of travelers between OD pair (6,1) is 1.50, and about 6% travelers experience it. 70% travelers have loaded unfairness at the value of 1.30, and 20% travelers bear loaded unfairness 1.12. Only 4% travelers use the shortest path.

In Fig.5, the largest UE unfairness between the same OD pair also increases with the increase of $\varphi$. For $\varphi \in [1.01, 1.13]$, between OD pair (1,6), the largest UE fairness is 1.17, about 13% travelers suffer this UE fairness. The travel time of other 87% travelers is less than their experienced travel time at UE state. Between OD pair (6,1), about 76% travelers suffer the largest UE fairness at 1.02. Other 24% travelers' travel time is less than that at UE.

When $\varphi \in [1.14, 1.16]$, between OD pair (1,6), about 12% travelers suffer the largest UE unfairness which is 1.19. 88% travelers' travel time is less than that at UE state. Between OD pair (6,1), 1% travelers' UE fairness is 1.16. 75%

Table 14: The results of CSO-CNDP when $\gamma = 0.8, 0.1$

| y | $\gamma = 0.8$ | $\gamma = 0.1$ |
|---|---|---|
| y2 | 2.9742 | 3.1401 |
| y3 | 8.5068 | 7.9819 |
| y6 | 10.8408 | 12.1705 |
| y16 | 19.9926 | 19.9968 |
| TB | 82.2907 | 81.4980 |
| TT | 426.8441 | 434.1245 |
| Z | 509.1347 | 515.6225 |

travelers experience the UE unfairness 1.016. Other 24% travelers' travel time is less than that at UE.

For $\varphi \in [1.17, 2.00]$, between OD pair (1,6), about 10% travelers have the largest UE unfairness 1.60, and 1% travelers suffer the UE unfairness 1.06. Other 89% travelers' travel time is less than that at UE. Between OD pair (6,1), the largest UE unfairness is 1.14, and about 6% travelers suffer it. Other 94% travelers' travel time is smaller than that at UE.

### *4.3. Results for partial travelers following the route guidance*

One can note that the loaded unfairness and UE unfairness can be improved by the choice of parameter $\varphi$ based on the results in Section 4.2. However, there are still some travelers whose travel time is more than that experienced in UE. So, some travelers may not accept the route guidance due to the unfairness problem. To study this issue, let $\chi \in [0, 1]$ denote the percent of travelers accepting the route guidance, and other travelers follow user optimal rule to choose paths.

Tab.14 shows the results of case II of example 1 when $\chi = 0.8, 0.1$. It can be found that the objective values of CSO-CNDP are better than the global optimal value of UE-CNDP given by Li et al. (2012), even only part of travelers following the route guidance based on CSO.

### *4.4. Performance of the modified simulated algorithm*

The modification of simulated annealing algorithm of Friesz et al. (1992) only takes effect at stage when the temperature at stage is very low so that there is no new solution accepted. This means that, at stage , the probability to accept a solution of bad objective value is very small. Therefore, the algorithm can be terminated at stage . In this case, implementing our modification on the simulated annealing algorithm of Friesz et al. (1992) at stage with a lower temperature than that at stage , if a new solution is accepted, then it is more likely to obtain a better solution than that acquired at stage . Therefore, the modification on the algorithm in this paper can improve the performance of the simulated annealing algorithm of Friesz et al. (1992). To this end, we contend that our modification does not change the probability of the algorithm to converge to the optimal solution, if not better than the algorithm in Friesz et al. (1992).

To our best knowledge, there is no comparison between simulated annealing algorithm and genetic algorithm to solve the two tested examples presented in our study. Here, we choose the genetic algorithm to solve CSO-CNDP and compare its results with the modified simulated annealing algorithm of this study. The values of parameters of genetic algorithm are given by Xu et al. (2009) which is shown as follows:

- Population size is 10.
- Probability of reproduction is 0.2.
- Probability of crossover probability is 0.2.
- Probability of mutation probability is 0.2.
- Step in mutation operation is 0.5.

To compare the computational speed of the two algorithms, the number of lower-level problems solved should be the same. So, we set the maximal number of generation as follows:

- the maximal number of generation is 270 for the case II of the first example.
- the maximal number of generation is 720 for the Sioux Falls network.

It is well known that simulated annealing and genetic algorithms are both stochastic method. So, the optimal solution may be not obtained by only implementing them once. To compare simulated annealing algorithm and genetic algorithm, for the case II of first tested example, we respectively implement both algorithms 100 times; and the two algorithms are respectively implemented 10 times for Sioux Falls network. We choose the solution with minimal objective value to be the best solution. We also analyze the mean objective value and the mean variation of the objective value of two algorithms. The mean time to obtain a solution of two algorithms is also compared. In the experiments, a personal laptop computer with Intel Core 2 Duo 2.53 GHz CPU, 6 GB RAM, and Windows 7 64bit operating system was used for all tests. The comparison between the algorithm proposed in this study and the genetic algorithm is shown in Tab.15, 16, 17 and 18:

From Tab.15, 16, one can find that, although the solution of simulated annealing algorithm has longer mean time to obtain a solution than those of genetic algorithm, simulated annealing algorithm has better mean objective value than that of genetic algorithm with the same value of $\varphi$ . Meanwhile, it can also be observed from Tab.17 and 18 that the best solution of the simulated annealing algorithm has better objective value than that of genetic algorithm.

## 5. Conclusion

In this study, we present model formulation to combine a route guidance and CNDP to alleviate traffic congestion. The route guidance recommends shortest

Table 15: The comparison between genetic algorithm and simulated annealing algorithm for the case II of the first example part I

| Genetic algorithm | | | Simulated Annealing algorithm | | |
|---|---|---|---|---|---|
| $\varphi \in [1.01\ 1, 13]$ | $\varphi \in [1.14, 1.16]$ | $\varphi \in [1.17, 2]$ | $\varphi \in [1.01\ 1, 13]$ | $\varphi \in [1.14, 1.16]$ | $\varphi \in [1.17, 2]$ |
| mean objective value | | | | | |
| 533.15 | 533.18 | 533.79 | 518.46 | 516.86 | 514.39 |
| mean variation of objective value | | | | | |
| 7.4227 | 8.4221 | 8.2348 | 44.3848 | 28.9599 | 64.3579 |
| mean time (min) | | | | | |
| 5.4118 | 5.3488 | 5.3259 | 17.1137 | 19.4689 | 19.3818 |

Table 16: The comparison between genetic algorithm and simulated annealing algorithm for the second example part I

| Genetic algorithm | | | | | Simulated Annealing algorithm | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\varphi = 1.02$ | $\varphi = 1.03$ | $\varphi = 1.04$ | $\varphi = 1.05$ | $\varphi = 1.06$ | $\varphi = 1.02$ | $\varphi = 1.03$ | $\varphi = 1.04$ | $\varphi = 1.05$ | $\varphi = 1.06$ |
| mean objective value | | | | | | | | | |
| 82.8522 | 80.9372 | 79.4546 | 78.8875 | 78.8757 | 81.5276 | 79.5681 | 78.8103 | 78.5624 | 78.5042 |
| mean variation of objective value | | | | | | | | | |
| 0.1926 | 0.0098 | 0.0160 | 0.0070 | 0.0100 | 0.3174 | 0.0158 | 0.0121 | 0.0054 | 0.0055 |
| mean time (min) | | | | | | | | | |
| 867.82 | 942.07 | 914.71 | 995.52 | 1092.66 | 1247.59 | 1327.54 | 1426.41 | 1405.00 | 1612.72 |

Table 17: The comparison between genetic algorithm and simulated annealing algorithm for the case II of the first example part II

| | Genetic algorithm | | | Simulated annealing algorithm | | |
|---|---|---|---|---|---|---|
| y | $\varphi \in [1.01\ 1, 13]$ | $\varphi \in [1.14, 1.16]$ | $\varphi \in [1.17, 2]$ | $\varphi \in [1.01\ 1, 13]$ | $\varphi \in [1.14, 1.16]$ | $\varphi \in [1.17, 2]$ |
| y1 | 0.4266 | 0.1551 | | | | |
| y2 | 3.9694 | 1.6152 | 2.3397 | 2.7251 | 2.8896 | 2.9553 |
| y3 | 8.4223 | 10.1963 | 10.0711 | 10.0929 | 9.7580 | 8.4978 |
| y4 | | 0.0554 | | | | |
| y6 | 8.9572 | 6.8979 | 8.7017 | 7.5363 | 8.2976 | 10.7248 |
| y7 | | | | | | |
| y8 | 0.4927 | | 0.3624 | 0.5469 | 0.5467 | |
| y9 | 0.2078 | | | 0.0001 | | |
| y10 | 0.0098 | | | | | |
| y13 | | | 0.1595 | | | |
| y14 | 1.6343 | 0.5649 | 1.8720 | 1.3248 | 1.2420 | |
| y15 | 2.7138 | 3.5981 | 0.6280 | | 0.0012 | |
| y16 | 13.2753 | 14.6842 | 16.6834 | 20.0000 | 19.9993 | 20.0000 |
| TT | 420.3146 | 415.0387 | 423.6571 | 91.7914 | 91.129 | 82.0797 |
| TB | 101.2334 | 101.8859 | 94.0281 | 422.5464 | 423.1515 | 426.9082 |
| Z | 520.5480 | 516.9246 | 517.6852 | 514.3378 | 514.2805 | 508.9879 |

Table 18: The comparison between genetic algorithm and simulated annealing algorithm for the second example part II

| | Genetic algorithm | | | | | Simulated annealing algorithm | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| y | $\varphi=1.02$ | $\varphi=1.03$ | $\varphi=1.04$ | $\varphi=1.05$ | $\varphi=1.06$ | $\varphi=1.02$ | $\varphi=1.03$ | $\varphi=1.04$ | $\varphi=1.05$ | $\varphi=1.06$ |
| y16 | 4.4107 | 5.1475 | 4.8594 | 5.7379 | 5.4878 | 4.5349 | 5.2426 | 5.2415 | 5.3402 | 5.3316 |
| y17 | 2.6753 | 2.5522 | 2.9169 | 2.0644 | 2.3627 | 3.7646 | 2.6534 | 2.0729 | 2.1932 | 2.2950 |
| y19 | 4.9066 | 4.4087 | 4.5476 | 5.1892 | 5.1731 | 4.1521 | 4.7184 | 5.0467 | 5.4438 | 5.3290 |
| y20 | 2.3422 | 2.8603 | 2.5190 | 2.4433 | 2.6198 | 3.7536 | 2.6549 | 2.4117 | 2.4539 | 2.2899 |
| y25 | 2.9141 | 2.6502 | 2.3287 | 2.2359 | 2.5724 | 2.7186 | 2.3985 | 2.3840 | 2.4151 | 2.7086 |
| y26 | 2.2228 | 1.9172 | 1.8715 | 1.8586 | 2.0972 | 2.8957 | 2.3428 | 2.1979 | 2.5277 | 3.0817 |
| y29 | 3.5090 | 4.4740 | 3.2419 | 4.2019 | 3.2611 | 4.0447 | 4.3767 | 4.1909 | 3.6010 | 3.7972 |
| y39 | 2.4046 | 3.4971 | 4.0957 | 4.1193 | 4.4156 | 5.0055 | 4.0009 | 4.2731 | 4.5704 | 4.7062 |
| y48 | 2.3617 | 4.2079 | 4.1415 | 4.1349 | 3.8448 | 4.4592 | 4.0776 | 3.9351 | 3.9871 | 4.1511 |
| y74 | 2.3291 | 3.9911 | 3.5269 | 4.3836 | 4.7761 | 4.1783 | 4.1223 | 4.2126 | 4.4223 | 4.3687 |
| TT | 78.6891 | 76.0224 | 75.0042 | 73.6898 | 73.6214 | 74.7467 | 74.1958 | 73.8781 | 73.4632 | 73.1839 |
| TB | 3.5363 | 4.8176 | 4.2899 | 5.0752 | 4.9105 | 5.6930 | 4.9776 | 4.8544 | 5.0114 | 5.2398 |
| Z | 82.2254 | 80.8400 | 79.2942 | 78.7650 | 78.5319 | 80.4396 | 79.1734 | 78.7325 | 78.4746 | 78.4238 |

paths to travelers. These shortest paths are calculated by the marginal travel cost and satisfy the user constraints. Compared to known results of UE-CNDP, CSO-CNDP can significantly improve the performance of traffic network and reduce congestion. The conclusion is also verified when only part of travelers follow the route guidance due to the unfairness issue. In future study, we want to study the value parameter $\varphi$ to simultaneously improve the unfairness problem of CSO and the how to set the traffic network performance.

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