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# Traffic managements for Household Travels in Congested Morning Commute

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**Abstract**

Due to the high car ownership cost or car ownership restrictions in many major cities, household travels, which include multiple trips for all the household members, become very common. One typical household travel can be observed as the consecutive school trip and work trip, which sends the traveler's children to school first and then drive to their workplaces. In this paper, we analyse the departure time choice of the household travels and the equilibrium trip scheduling, i.e., extending the standard Vickrey's bottleneck model from work commute with one single preferred arrival time (work start time) to household commute with two consecutive preferred arrival times (school start time and work start time). Then, we investigate one step toll in peak hour window to best manage the morning commute of household travels and analyse the impact of the school-work start time difference on individual cost, social cost and traffic managements, so that we can optimally set the school-work start time difference to minimize the total travel cost. In addition, an alternative tradable credit scheme is designed to manage the morning commute as a replacement of the road toll scheme.

*Keywords:* Household travels, Morning commute, Road toll scheme, Tradable credit scheme

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## 1. Introduction

Traffic congestion in the morning peak hours has been one of the most challenging urban problems faced by many major cities in the world. It is imperative for the urban traffic management agencies to understand the travelers' choice behavior and equilibrium travel patterns so that efficient management measures (e.g., pricing, tradable credit) can be devised to reduce the congestion in morning commute traffic. The analysis of morning commute can be originated from the seminal work of Vickrey (1969). After that, a large body of literature has emerged working on this problem (Hendrickson & Kocur 1981; Smith 1984; Daganzo 1985; Arnott et al. 1990b; Nie & Yin 2013; Liu & Nie 2011; Liu & Nie 2011; Liu et al. 2015; Liu et al. 2012; Xiao et al. 2010; Qian & Zhang 2011; Qian et al. 2011; Qian & Rajagopal 2013; Qian & Rajagopal 2014). Most of these studies are based on the assumption that travelers choose the departure time to minimize individual travel cost, which includes both travel time and scheduling delay. Analytical approaches are employed to obtain the travelers' equilibrium trip scheduling in the morning peak hours.

Most of the previous studies in the literature analyzed the morning commute for individual travelers. However, in many Asian cities, a large amount of morning commute trips are indeed household travels, i.e., multiple trips made for the household members, rather than only one person. For example, in Singapore, due to the high car ownership cost (certificate of entitlement), most of households have only one private car, which is used to make household travels in the morning. Indeed, many Singaporean families decide to buy a car mainly because they want to use the car to send their children or pick up their children to and from the school. It is very common in Singapore that the drivers firstly make school trips to send the children to the school and then they drive to their own workplaces. The fact that one can often observe congested traffic in the bottleneck transport facilities before schools in Singapore even if most of Singaporean public schools set very early school time (7:30am) necessitates the study on how traffic management measures should be proposed to contain traffic congestion for these household travels. The departure time choice for household travels is certainly different from that for only individual trip, as multiple household members are involved in the joint decision, i.e., the members' preference of arrival times and intra-household interaction have to be considered in a group decision-making manner. In this study, it is assumed that morning household travel is comprised of multiple household member trips, and the departure time choice is made to ensure the total household travel cost, i.e., the travel time cost and scheduling delay cost for all the household members, is minimized. It is necessary for the traffic managers to understand the travelers' choice behavior for household travels and thus the equilibrium trip scheduling of the morning commute traffic with household travels, so that efficient traffic management measures can be designed to reduce the traffic congestion for morning commute traffic.

Despite that there is a large body of literature studying the morning commute traffic equilibrium for individual traveling, the research works on household are much less. Zhang et al. (2009) developed a household-based discrete choice model by integrating different types of household choice models based on latent class modeling approach and the approach of random utility maximization was applied. Li et al. (2014) proposed day-long activity-travel scheduling model to simultaneously determine the departure times for both morning and evening commutes, along with allocations of time spent on travel and activities at home or at the workplace. One can find that the obvious different departure time choice behavior for the household travels, in which the travel costs of all the household members must be considered and balanced, will affect the equilibrium scheduling of the morning commute significantly. In this study, we will firstly analyze the equilibrium trip scheduling of household travels in the morning commute and determine the equilibrium travel pattern by applying the bottleneck model approach. Then, based on the equilibrium travel pattern, we seek to propose optimal management measures to contain the traffic congestion.

To alleviate the congestion in morning commute, many management measures have been proposed. Vickrey's analysis shows how individuals' choice of departure time might shape traffic congestion and reveals the promise of policy interventions in managing this type of traffic congestion. Among these policies, the Vickrey's time-varying toll is the one that is most studied, which could completely eliminate the travel congestion delay induced by the bottleneck. However , in principle, the Vickrey's toll is the Pigouvian toll and thus shares its limitations. Moreover, the time-varying feature of the Vickrey's toll is indeed impractical, as a continuously changing toll may emit more pricing signals than what commuters could effectively recognized and respond to ( Bonsall et al. 2007). Many studies in the literature then proposed some alternatives to the Vickreys toll. One typical example is the so-called step toll, which keeps toll rates constant in one or more predefined discrete time windows (Arnott et al. 1990b; Arnott et al. 1993; Laih 1994; Laih 2004; Lindsey et al. 2010). Nie (2013) proposes a new tradable credit scheme (TCS), which is a simple alternative to the toll scheme. Under this scheme, the managers set a peak time window, the individuals who pass the bottleneck during this window will be charged some credits; individuals who pass the bottleneck out of this window will be rewarded with some credits. And there is a market for users to buy or sell the credits. The advantage of this scheme are as follows: firstly, managers do not need to allocate the credits to the individuals initially (compared to Nie 2012; Xiao et al. 2013; Yang & Wang 2011); secondly, the scheme does not rely on the trading market to alleviate the congestion; lastly, the scheme is easy to be implemented and more likely to be accepted by the public.

In this study, we consider a typical household travel consisting of two trips, i.e., firstly send the children to school and then go to workplaces through a single road. It is assumed that there exists a single bottleneck located before the school destination. We analyse the departure time choice of the household travels and the resultant equilibrium trip scheduling of household travelers. We investigate how to design traffic management measures to best manage the morning commute traffic with household travels. In this paper, we only use one step toll scheme to manage the congestion, wherein traffic management designs a peak time window to charge individual household travelers who enter the bottleneck in this peak time window. We further analyse the impact of the school-work start time difference on individual cost, social cost and traffic management tolls, so that we can optimally set the school-work start time difference to minimize the total travel cost. Besides, we also analyze how to design an alternative tradable credit scheme to replace the toll scheme to manage the household travel morning commute traffic.

In summary, this study contributes to the literature of morning commute management in the following aspects: Firstly, this study extends the standard Vickrey's bottleneck model from work commute with one single preferred arrival time (work start time) to household commute with two consecutive preferred arrival times (school start time and work start time). Secondly, this study explicitly investigates the impact of the school-work start time difference on individual cost, social cost and traffic management tools, so that the optimal school-work start time difference can be designed. Thirdly, under the optimal setting of school-work start time difference, the optimal traffic managements, including the one step toll scheme and an alternative tradable credit scheme, are determined.

The remainder of this paper is organized as follows. Section 2 presents the departure time choice behavior of the household travels to obtain the resultant equilibrium travel pattern for a typical household travel with school and work trips. Based on the setting of school-work start time difference, two different situations are discussed. In section 3, we first present a single step toll model for one situation. Under the toll model, we deduce the optimal toll rate for different time intervals between two desired arrival times; Then, we discuss the impact of the desired time interval to the total travel time cost, which is essential for designing the optimal time interval between two desired arrival times; In section 4, we conduct the similar analysis as in section 3 for the other situation. Section 5 gives some policy implications to the traffic management agencies on the optimal setting of the interval between the two desired arrival times , i.e., the optimal setting of school-work start time difference, and the optimal toll to achieve minimum total system travel cost. In addition, we develop a tradable credit scheme to replace the optimal toll model under the optimal setting of school-work start time difference. Section 6 presents a numerical example. Finally, we give some conclusion in section 7.

## 2. Equilibrium trip scheduling with single bottleneck before both trip destinations

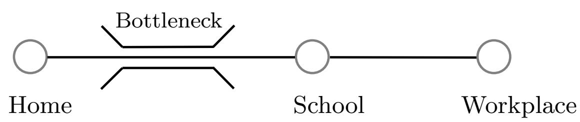


Figure 1: A network with a single bottleneck before both trip destinations

In this section, we assume there is a bottleneck located before both trip destinations, i.e., school and workplace. The network is schematically depicted in Figure 1. Suppose a fixed number of $N$ household travels depart from home to their children's school firstly and then to their workplaces every morning. We assume that the school/workplace desired arrival time is $t_1^*/t_2^*$, and early and/or late arrival penalty will be imposed. The bottleneck model is applied to analyze the equilibrium scheduling of the household travel. We assume that the capacity of the whole road is $s$, that is $s$ cars can pass the road per unit time. If the arrival rate at the bottleneck exceeds $s$, a queue is formed. It is assumed that the queue discipline is first-come, first-served (FIFO). As the capacity of the road is limited, individual household has to choose their departure time to make a trade-off among all the household members' travel time cost relating to queue length at the bottleneck and schedule delay cost of arriving early or later at school and workplaces. A Wardrop (1952) equilibrium is attained if no household could further reduce their cost by changing the departure time unilaterally. As was in Arnott et al. (1990a), the total travel time cost of individual household, who depart at time $t$ is defined as follows:

$$c(t) \quad = \quad \alpha \cdot T(t) + \beta \cdot \mathbf{max}\{0, t_1^* - t - T_1(t)\} + \gamma \cdot \mathbf{max}\{0, t + T_1(t) - t_1^*\}$$

$$+ \quad \beta \cdot \mathbf{max}\{0, t_2^* - t - T(t)\} + \gamma \cdot \mathbf{max}\{0, t + T(t) - t_2^*\}. \tag{1}$$

This cost includes the travel time cost and the schedule delay cost, where $T(t)$ is the total household travel time for departure time $t$, i.e., the total travel time from home to the workplace. $T_1(t)$ is the travel time for departure time $t$ when they arrive at school; as $t_1^*$ represents the preferred school arrival time, the second and third term in (1) calculate the school early and late arrival delays respectively; Similarly, $t_2^*$ is the preferred work arrival time, and the fourth and fifth term in (1) determine the work early arrival and late arrival delays. Indeed, the total household travel cost defined in (1) encapsulates the household travel time and the schedule delays for all the household members. Therefore, the household travelers make departure time decision to minimize the household travel cost in a group-decision manner, rather than only considering one household member. Hereby $\alpha$ is the shadow cost of travel time, $\beta$ is the schedule penalty for a unit time of early arrival, and $\gamma$ is that for a unit time of late arrival. We assumed that $\gamma > \alpha > 2\beta$. The travel time $T(t)$ and $T_1(t)$ are defined as follows:

$$T(t) = T^f + T^v(t),$$
$$T_1(t) = T_1^f + T^v(t),$$

where $t$ is the departure time from home; $T^f$ is the fixed travel time from home to workplace and $T_1^f$ is the fixed travel time from home to school; $T^v(t)$ is the variable travel delay time to pass the bottleneck. We only consider the situation of $t_2^* - t_1^* \geq T^f - T_1^f$, in which case, individual household who send their children to school on time will be able to arrive at their workplace on time or early, otherwise they must arrive late at their workplace no matter whether there is congestion or not before the bottleneck. Without affecting the results, we set $T^f = 0$, $T_1^f = 0$. So we have $t_2^* - t_1^* \geq 0$ correspondingly.

Let $D(t)$ be the queue length (the number of the cars at the bottleneck at time $t$). Then an individual household's queuing time equals to the queue length at the time the household enter the queue divided by the bottleneck capacity, i.e.,

$$T^v(t) = D(t)/s. \tag{2}$$

Let $\hat{t}$ denote the most recent time at which there was no queue and let r(t) denote the departure rate. Then the queue length at time $t$ can be written as

$$D(t) = \int_{\hat{t}}^{t} r(u)du - s(t - \hat{t}). \tag{3}$$

Each individual household needs to decide a departure time from home to minimize the total travel cost of the entire household. In doing so, he/she trade off household members' travel time cost and schedule delay cost. Equilibrium is achieved when no individual household can reduce their travel cost by altering the departure time, taking all other commuters' departure time as fixed. This is indeed a Nash equilibrium with departure time as the strategy variables.

For convenience of discussion below, we set $t_q$ to be the beginning of the rush hour and $t_{q'}$ be the end, let $\tilde{t}_1$ be the departure time for the exact on-time school arrival, i.e., $\tilde{t}_1 + T^v(\tilde{t}_1) = t_1^*$, and let $\tilde{t}_2$ be the departure time for the exact on-time workplace arrival, i.e., $\tilde{t}_2 + T^v(\tilde{t}_2) = t_2^*$.

At equilibrium, all the households will have equal travel cost. To calculate the total travel costs, we classify the departure time into three periods. Here we present the detailed analysis of the departure rate at equilibrium.

**Period 1:** For the household travel with school early arrival and workplace early arrival, i.e., $t \in [t_q, \tilde{t}_1)$, the travel cost is denoted as:

$$c_1(t) = \alpha \cdot T^v(t) + \beta \cdot [t_1^* - t - T^v(t)] + \beta \cdot [t_2^* - t - T^v(t)]. \tag{4}$$

Differentiation of (4) yields

$$\frac{dT^v(t)}{dt} = \frac{2\beta}{\alpha - 2\beta}. \tag{5}$$

From (3), we have

$$\frac{dD(t)}{dt} = r(t) - s. \tag{6}$$

Combining (2), (6) and (5), we have the departure rate of the individual household who depart home during $[t_q, \tilde{t}_1)$ is

$$r_1(t) = \frac{\alpha s}{\alpha - 2\beta} \ (> s). \tag{7}$$

**Period 2:** For the household travel with school late arrival and workplace early arrival, i.e., $t \in [\tilde{t}_1, \tilde{t}_2)$ the travel cost is denoted as:

$$c_2(t) = \alpha \cdot T^v(t) + \gamma \cdot [t + T^v(t) - t_1^*] + \beta \cdot [t_2^* - t - T^v(t)]. \tag{8}$$

Differentiation of (8) yields

$$\frac{dT^v(t)}{dt} = \frac{\beta - \gamma}{\alpha + \gamma - \beta}. \tag{9}$$

Combining (2), (6) and (9), we have the departure rate of the individual household who depart home during $[\tilde{t}_1, \tilde{t}_2)$ is

$$r_2(t) = \frac{\alpha s}{\alpha + \gamma - \beta} \ (< s). \tag{10}$$

**Period 3:** For the household travel with school late arrival and workplace late arrival, i.e., $t \in [\tilde{t}_2, t_{q'}]$ the travel cost is denoted as:

$$c_3(t) = \alpha \cdot T^v(t) + \gamma \cdot [t + T^v(t) - t_1^*] + \gamma \cdot [t + T^v(t) - t_2^*]. \tag{11}$$

Differentiation of (11) yields

$$\frac{dT^v(t)}{dt} = \frac{-2\gamma}{2\gamma + \alpha}. \tag{12}$$

Combining (2), (6) and (12), we have the departure rate of the individual household who depart home during $[\tilde{t}_2, t_{q'}]$ is

$$r_3(t) = \frac{\alpha s}{\alpha + 2\gamma} \ (< s). \tag{13}$$

Next, we will consider the practical situations that may occur in reality when travel equilibrium is achieved. Obviously, there will be some households arrive earlier than the desired school time at equilibrium, otherwise it will contradict to the equilibrium conditions. Specifically, if all households depart from their home in Period 2 and/or Period 3 at equilibrium, by observing the equations (10) and (13), one can find the flow in the road will not exceed $s$. All households travel delay time $T^v(t)$ will be 0, and one household can change his departure time to make him arrive earlier than the desired school time, meanwhile his total travel cost will be reduced due to the assumption $\gamma > \alpha > 2\beta$, which contradicts to the Wardrop equilibrium conditions. On the other hand, there will be some households who will arrive later than the desired school time at equilibrium, otherwise all households will be earlier than the desired school time, which is also contrary to the Wardrop equilibrium conditions. By observing the equation (7), we find that the flow of the road will always exceed $s$ if all the households travel starts in Period 1. One household can change his depart time just at $t_1^*$, he will not bear the travel delay time because all other household have arrived school in this situation, and his total travel cost will be reduced due to the assumption $\gamma > \alpha > 2\beta$, which is contrary to the Wardrop equilibrium. In all, based on the above analysis, there are only two possible situations that may occur in this model:

**Small school-work difference situation:** If the difference between the two desired arrival times (i.e., the school-work difference) is smaller than the specific value $\frac{2\beta}{\beta+\gamma}\frac{N}{s}$, i.e., $t_2^* - t_1^* < \frac{2\beta}{\beta+\gamma}\frac{N}{s}$ (the derivation of this value will be detailed in the following analysis), there will be some individual households who arrive later than the desired work time in the equilibrium, i.e., households may depart from home from Period 1, 2 and 3.

**Large school-work difference situation:** If the difference between the two desired arrival times (i.e., the school-work difference) is greater than the specific value $\frac{2\beta}{\beta+\gamma}\frac{N}{s}$, i.e., $t_2^* - t_1^* \geq \frac{2\beta}{\beta+\gamma}\frac{N}{s}$ (the derivation of this value will be detailed in the following analysis), all individual households arrive earlier than the desired work time in the equilibrium, i.e., households may depart from home only in Period 1 and 2.

### *2.1. Model result for Small school-work difference situation:*

Thus, the queue length evolves over the rush hour such that the equal travel cost condition is satisfied for all the households. The individual households who depart at the beginning and end of the rush hour incur only schedule delay cost, which must equal in equilibrium. Since arrivals are continuous over the rush hour, the length of the rush hour is $N/s$. The individual household who depart at time $\tilde{t}_1$ only incur the queue time and the early arrival penalty at workplace, and the individual who depart at time $\tilde{t}_2$ only incur the queue time and the late arrival penalty at school, whose travel time must be equal in equilibrium. Here we give the total travel time cost at some special departure time $t_q$, $\tilde{t}_1$, $\tilde{t}_2$, $t_{q'}$ respectively:

$$\begin{aligned}
c_1 &= \beta(t_1^* - t_q) + \beta(t_2^* - t_q), \\
c_2 &= \alpha\frac{D_1}{s} + \beta(t_2^* - \tilde{t}_1 - \frac{D_1}{s}), \\
c_3 &= \alpha\frac{D_1 + D_2}{s} + \gamma(\tilde{t}_2 + \frac{D_1 + D_2}{s} - t_1^*), \\
c_4 &= \gamma(t_{q'} - t_1^*) + \gamma(t_{q'} - t_2^*),
\end{aligned}$$

where $D_1 = \int_{t_q}^{\tilde{t}_1} r_1(t) - sdt$, $D_2 = \int_{\tilde{t}_1}^{\tilde{t}_2} r_2(t) - sdt$, and $t_{q'} - t_q = \frac{N}{s}$. These results together imply that

$$\begin{aligned}
t_q &= \frac{t_1^* + t_2^*}{2} - \frac{\gamma}{\beta + \gamma}\frac{N}{s}, \\
\tilde{t}_1 &= (1 - \frac{\beta}{\alpha})t_1^* + \frac{\beta}{\alpha}t_2^* - 2\frac{N\delta}{\alpha s}, \\
\tilde{t}_2 &= (1 + \frac{\gamma}{\alpha})t_2^* - \frac{\gamma}{\alpha}t_1^* - 2\frac{N\delta}{\alpha s}, \\
t_{q'} &= \frac{t_1^* + t_2^*}{2} + \frac{\beta}{\beta + \gamma}\frac{N}{s},
\end{aligned}$$

where $\delta = \frac{\beta\gamma}{\beta+\gamma}$, and we make this notation for illustration purpose. Indeed, the inequalities $\tilde{t}_2 < t_2^* < t_{q'}$ and $t_q < \tilde{t}_1 < t_1^*$ must be satisfied in this situation. Using the equations above, we deduce the following inequalities must be satisfied in this situation: $t_2^* - t_1^* < \frac{1}{\beta}\frac{2N\delta}{s}$ and $t_2^* - t_1^* < \frac{1}{\gamma}\frac{2N\delta}{s}$. Because $\gamma > \beta$ is satisfied, these two inequalities can be simplified as

$$t_2^* - t_1^* < \frac{1}{\gamma}\frac{2N\delta}{s}. \tag{14}$$

However, we can not determine the relationship between $t_1^*$ and $\tilde{t}_2$ yet. Using the expression of $\tilde{t}_2$ above, we deduce that, when $t_2^* - t_1^* \leq \frac{1}{\alpha+\gamma}\frac{2N\delta}{s}$, $\tilde{t}_2$ is no bigger than $t_1^*$, i.e., $\tilde{t}_2 \leq t_1^*$; when $\frac{1}{\alpha+\gamma}\frac{2N\delta}{s} < t_2^* - t_1^* < \frac{1}{\gamma}\frac{2N\delta}{s}$, $\tilde{t}_2$ is greater than $t_1^*$, i.e., $\tilde{t}_2 > t_1^*$. The relationship of all these time points in two cases are illustrated in Figure 2.

The solution of the equilibrium departure rate is depicted in Figure 2. It is easy to deduce that the equilibrium travel cost is identical for all households,

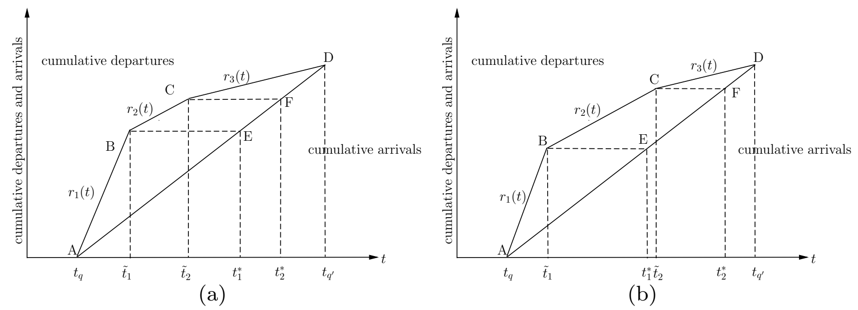


Figure 2: The departure rate in the equilibrium for small school-work difference situation, i.e., $t_2^* - t_1^* < \frac{1}{\gamma}\frac{2N\delta}{s}$, when there is a single bottleneck before both trip destinations. Figure (a) is the case: $\tilde{t}_2$ is earlier than $t_1^*$; Figure (b) is the case: $\tilde{t}_2$ is later than $t_1^*$.

which is given by

$$\bar{c} = 2\frac{N\delta}{s}, \tag{15}$$

and the total system cost at UE is

$$TC = 2\frac{N^2\delta}{s}. \tag{16}$$

One can observe an interesting finding from the above analysis that the equilibrium individual cost and total system cost are both independent of $t_2^* - t_1^*$. Besides, one can find that these results are similar to those in standard Vickrey's bottleneck model in which work commute with one single preferred arrival time (work start time) is considered. Referring to Nie (2013), the equilibrium individual travel cost in the standard Vickrey's model is $\bar{c} = \frac{N\delta}{s}$, and the total system cost at equilibrium is $TC = \frac{N^2\delta}{s}$. In our model, travel cost for one household is indeed total travel cost for both household members. Therefore, when $t_2^* - t_1^*$ is very small, more specifically, less than $\frac{1}{\gamma}\frac{2N\delta}{s}$, despite that the equilibrium travel pattern as illustrated in Figure 2 is obviously different from that for standard Vickey's model, the average equilibrium travel cost for one traveler is the same with that in standard Vickery's model. However, when $t_2^* - t_1^*$ is greater than $\frac{1}{\gamma}\frac{2N\delta}{s}$, the model result will be far different from that in standard Vickery's model, as is demonstrated in next sub-section.

*2.2. Model result for Large school-work difference situation:*

Following the same logic with the analysis in previous subsection, the queue length evolves over the rush hour and eventually the equal travel cost condition must be satisfied. The individual household who depart at the beginning and end of the rush hour incur only schedule delay cost, and the individual household who depart at time $\tilde{t}_1$ incurs the queue time and the early arrival penalty at workplace, all of these three costs must be equal in equilibrium.

Here we give the total travel time cost at some special departure time $t_q$, $\tilde{t}_1$, $t_{q'}$ respectively:

$$\begin{aligned} c_1 &= \beta(t_1^* - t_q) + \beta(t_2^* - t_q), \\ c_2 &= \alpha\frac{D_1}{s} + \beta(t_2^* - \tilde{t}_1 - \frac{D_1}{s}), \\ c_4 &= \gamma(t_{q'} - t_1^*) + \beta(t_2^* - t_{q'}), \end{aligned}$$

where $D_1 = \int_{t_q}^{\tilde{t}_1} r_1(t) - sdt$, $t_{q'} - t_q = \frac{N}{s}$. These results together imply that

$$\begin{aligned} t_q &= t_1^* - \frac{\gamma - \beta}{\beta + \gamma}\frac{N}{s}, \\ \tilde{t}_1 &= t_1^* - \frac{2\beta(\gamma - \beta)}{\alpha(\gamma + \beta)}\frac{N}{s}, \\ t_{q'} &= t_1^* + \frac{2N\delta}{s\gamma}, \end{aligned}$$

where $\delta = \frac{\beta\gamma}{\beta+\gamma}$, as is defined identically in the last subsection. Actually, the inequalities $t_{q'} \leq t_2^*$ and $t_q < \tilde{t}_1 < t_1^*$ must be satisfied in this situation. Using the equations above, we deduce the following inequality must be satisfied in this situation:

$$t_2^* - t_1^* \geq \frac{1}{\gamma}\frac{2N\delta}{s}. \tag{17}$$

The solution of the departure rate is depicted in Figure 3. It is easy to deduce that the equilibrium travel time cost is identical for all users, which is given by

$$\bar{c} = \frac{2N\delta}{s}\frac{\gamma - \beta}{\gamma} + \beta(t_2^* - t_1^*) \geq \frac{2N\delta}{s}. \tag{18}$$

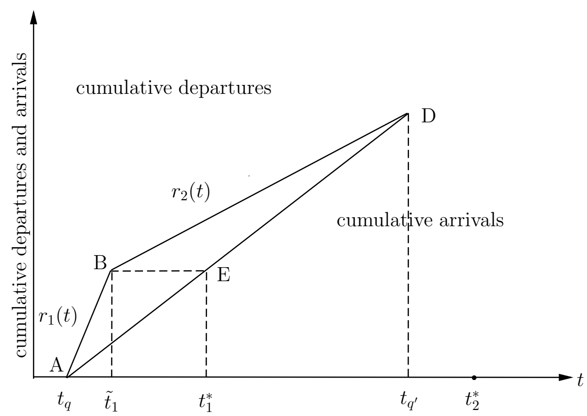


Figure 3: The departure rate in the equilibrium for large school-work difference situation, i.e., $t_2^* - t_1^* \geq \frac{1}{\gamma}\frac{2N\delta}{s}$, when there is a single bottleneck before both trip destinations.

The inequality is true because of $t_2^* - t_1^* \geq \frac{1}{\gamma}\frac{2N\delta}{s}$, and the equality holds only when $t_2^* - t_1^* = \frac{1}{\gamma}\frac{2N\delta}{s}$. So the total system cost at UE is

$$TC = \bar{c} \cdot N \geq \frac{2N^2\delta}{s}. \tag{19}$$

In large school-work difference situation, only when $t_2^* - t_1^* = \frac{1}{\gamma}\frac{2N\delta}{s}$, both the equilibrium travel time cost $\bar{c}$ and the total system cost $TC$ at UE can be the same as those two costs in small school-work difference situation, otherwise both costs will be larger than the costs in small school-work difference situation.

## 3. The optimal toll model for small school-work difference situation

To reduce the traffic congestion, the road toll charging is one management measure that is most studied. Vickrey's time-varying toll scheme could completely eliminate the travel congestion delay induced by the bottleneck, however, is too complicated to gain public acceptance. Therefore, in this study, we instigate how a one-step toll scheme should be designed, in which there is a constant toll $\rho$ only charged over an interval $[t_+, t_-] \in [t_q, t_{q'}]$, to manage the traffic congestion in morning commute traffic with household travels. Here, only the traffic management for normal vehicles is considered, while the extension of management for electric vehicles (Riemann et al. 2015; Kumar et al. 2014) could be addressed in future study. The most important issue in analyzing single step toll is on how to deal with discontinuity at the boundary of the peak-time window. Users, who leave the bottleneck at the boundary, have different travel time costs, depending on whether or not they need to pay the toll. Arnott et al. (1990b) argued that, because the first person who pays the toll must have a lower travel time cost compared to his/her immediate predecessor who escapes the toll, he/she arrives at the bottleneck later by $\rho/\alpha$, which implies that there will be a period of time during which the arrival rate at the bottleneck is zero (see Figure 4 and Figure 5). The discontinuity between the last person who pays the toll and his/her immediate successor leads to the following behavioral assumption:

**Separated waiting (SW) assumption** (Laih 1994; Laih 2004): commuters who arrive at the same time can use different waiting facilities, hence are allowed to have different travel delays.

It should be noted that other assumptions such as Mass Arrival (MA) ( Arnott et al. 1990b) can also be applied to handle the discontinuity between the last toll-payer and her immediate successor. In this study, we apply SW assumption for illustration, while there is no theoretical obstacle in applying other assumptions into the model, which could be addressed in future study.

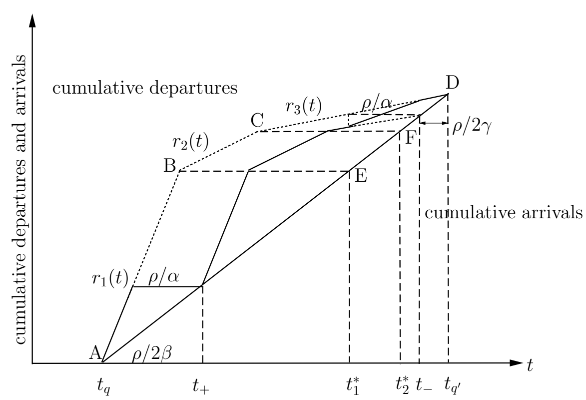


Figure 4: Equilibrium solutions for one-step toll model in small school-work difference situation, when the peak time window $[t_+, t_-]$ contains $[t_1^*, t_2^*]$.

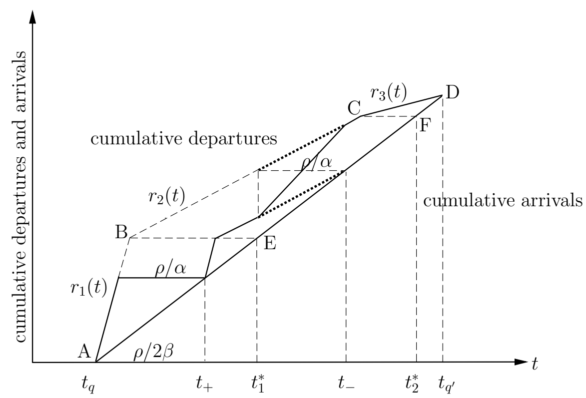


Figure 5: Equilibrium solutions for one-step toll model in small school-work difference situation, when the peak time window $[t_+, t_-]$ is before $t_2^*$.

The equilibrium solution with the behavioral assumption above is summarized in Figure 4 and Figure 5. The difference between these two figures is whether the peak time window with toll charging $[t_+, t_-]$ contains $t_2^*$ or not. In the following, we will investigate how the toll model works in our problem under these two types of peak time windows respectively.

*3.1. The peak time window for Case $t_- \geq t_2^*$ (see Figure 4)*

In this case, the peak time window contains $[t_1^*, t_2^*]$. When a toll $\rho$ is imposed, the difference between the total travel time costs of the individual household who depart from home at $t_q$ and the individual household who is the first one entering the tolling window will be $\rho$, so we have the following equation:

$$c_1 = \beta(t_1^* - t_q) + \beta(t_2^* - t_q) = \rho + \beta(t_1^* - t_+) + \beta(t_2^* - t_+),$$

which further deduces:

$$t_+ - t_q = \rho/2\beta.$$

Similarly, the difference between the total travel time costs of the individuals who depart from home at $t_{q'}$ and those who are the last ones in the tolling window will be $\rho$, so we have the following equation:

$$c_4 = \gamma(t_{q'} - t_1^*) + \gamma(t_{q'} - t_2^*) = \rho + \gamma(t_- - t_1^*) + \gamma(t_- - t_2^*),$$

which leads to:

$$t_{q'} - t_- = \rho/2\gamma.$$

Then the tolling peak time window can be given as follows:

$$t_+ = t_q + \rho/2\beta, \quad t_- = t_{q'} - \rho/2\gamma.$$

Therefore, we can represent the number of the individual household who need to pay the toll is

$$N_t = (t_- - t_+)s = N - \frac{\rho s}{2\delta}, \tag{20}$$

and the total system travel time cost excluding the toll is

$$\begin{aligned} TC_1 &= (2\frac{N\delta}{s}) \cdot \frac{\rho s}{2\delta} + (2\delta \frac{N - \frac{\rho s}{2\delta}}{s}) \cdot (N - \frac{\rho s}{2\delta}) \\ &= \frac{3}{2}\frac{N^2\delta}{s} + \frac{s}{2\delta}(\rho - \frac{N\delta}{s})^2. \end{aligned} \tag{21}$$

We note that the equilibrium travel cost with toll for all the households can be given by

$$\tilde{c} = 2\frac{N\delta}{s} = \bar{c}. \tag{22}$$

*3.1.1. The analysis of the optimal system travel time cost under toll model*

In this case, the peak time window contains $[t_1^*, t_2^*]$, so the length of the CF, as shown in Figure 4, must be larger than the value of $\rho/\alpha$, i.e., $CF \geq \rho/\alpha$; and the length of the BE must be strictly larger than the value of $\rho/\alpha$, i.e., $BE > \rho/\alpha$ (due to the rate of departure and arrival of the individual household). Here we have

$$CF = t_2^* - \tilde{t}_2 = \frac{2N\delta}{s\alpha} - \frac{\gamma}{\alpha}(t_2^* - t_1^*),$$

$$BE = t_1^* - \tilde{t}_1 = \frac{2N\delta}{s\alpha} - \frac{\beta}{\alpha}(t_2^* - t_1^*).$$

From the observation of the equation (21), we need to divide the discussion in two cases:

**In the case when $0 \le t_2^* - t_1^* \le \frac{1}{\gamma}\frac{N\delta}{s}$:**

If $0 \le t_2^* - t_1^* \le \frac{1}{\gamma}\frac{N\delta}{s}$, which is equivalent with the following inequality

$$CF \ge \frac{N\delta}{\alpha s}, \tag{23}$$

we find that when we set

$$\rho^* = \frac{N\delta}{s} \quad (< \alpha \cdot BE), \tag{24}$$

$TC$ can attain the minimum

$$TC_1 = \frac{3}{2}\frac{N^2\delta}{s}. \tag{25}$$

Accordingly, the optimal tolling peak time window can be given as follows:

$$t_+ = \frac{t_1^* + t_2^*}{2} - \frac{N\delta}{2\beta s}, \quad t_- = \frac{t_1^* + t_2^*}{2} + \frac{N\delta}{2\gamma s}.$$

**In the case when $\frac{1}{\gamma}\frac{N\delta}{s} < t_2^* - t_1^* < \frac{2}{\gamma}\frac{N\delta}{s}$:**

If $\frac{1}{\gamma}\frac{N\delta}{s} < t_2^* - t_1^* < \frac{2}{\gamma}\frac{N\delta}{s}$, which is equivalent with the inequality

$$CF < \frac{N\delta}{\alpha s},$$

we find that when we set

$$\rho^* = \alpha \cdot CF = \frac{2N\delta}{s} - \gamma(t_2^* - t_1^*) \quad (< \alpha \cdot BE),$$

$TC$ can attain the minimum

$$TC_1 = \frac{3}{2}\frac{N^2\delta}{s} + \frac{\gamma^2 s}{2\delta}[(t_2^* - t_1^*) - \frac{1}{\gamma}\frac{N\delta}{s}]^2.$$

Considering the range of $t_2^* - t_1^*$, we find that

$$\frac{3}{2}\frac{N^2\delta}{s} < TC_1 < 2\frac{N^2\delta}{s}.$$

And the value of $TC_1$ will increase with $t_2^* - t_1^*$, in the range $\frac{1}{\gamma}\frac{N\delta}{s} < t_2^* - t_1^* < \frac{2}{\gamma}\frac{N\delta}{s}$.

Accordingly, the optimal tolling peak time window can be given as follows:

$$t_+ = \frac{t_1^* + t_2^*}{2} - \frac{\gamma(t_2^* - t_1^*)}{2\beta}, \quad t_- = t_2^*.$$

**Remark:** Based on the above analysis, if we want to obtain the minimum total system time cost $\frac{3}{2}\frac{N^2\delta}{s}$ under this type of peak time window, we should set $t_2^* - t_1^* \leq \frac{1}{\gamma}\frac{N\delta}{s}$ and the toll rate should be $\rho^* = \frac{N\delta}{s}$.

*3.2. The peak time window for Case $t_- \leq t_2^*$ (see Figure 5)*

In this case, the beginning of the peak time window is before $t_1^*$ and the end of the peak time window is between $t_1^*$ and $t_2^*$, so the inequality $t_2^* - t_1^* > 0$ must be satisfied. Similar to the analysis above, we will discuss the optimal one-step toll scheme in this case. For a given toll $\rho$, the difference between the total travel time cost of the individual household who depart from home at $t_q$ and the individual household who is the first one to enter the tolling window is $\rho$, which can be described by the following equation:

$$c_1 = \beta(t_1^* - t_q) + \beta(t_2^* - t_q) = \rho + \beta(t_1^* - t_+) + \beta(t_2^* - t_+),$$

and the difference between the total travel time cost of the person who depart from home at $t_{q'}$ and the person who is the last one to be charged in the tolling window is $\rho$, and we have the following equation:

$$c_4 = \gamma(t_{q'} - t_1^*) + \gamma(t_{q'} - t_2^*) = \rho + \gamma(t_- - t_1^*) + \beta(t_2^* - t_-).$$

Then the tolling window is given as follows:

$$t_+ = t_q + \rho/2\beta, \quad t_- = \frac{2\gamma t_{q'} - (\beta+\gamma)t_2^* - \rho}{\gamma - \beta}.$$

So we can represent the number of the individual households who are in the peak and off-peak time window respectively

$$N_t = (t_- - t_+)s, \tag{26}$$

$$N_u = N - N_t = \frac{\rho}{2\beta}s + \frac{(\gamma+\beta)(t_2^* - t_{q'}) + \rho}{\gamma - \beta}s. \tag{27}$$

The total system travel time cost excluding the toll is

$$\begin{aligned} TC_2 &= (2\frac{N\delta}{s})N_u + (2\frac{N\delta}{s} - \rho)N_t \\ &= 2\frac{N^2\delta}{s} - \frac{s(\beta+\gamma)}{2\beta(\gamma-\beta)}(\frac{N\delta}{s} - \frac{\beta(t_2^* - t_1^*)}{2})^2 \\ &\quad + \frac{s(\beta+\gamma)}{2\beta(\gamma-\beta)}[\rho - (\frac{N\delta}{s} - \frac{\beta(t_2^* - t_1^*)}{2})]^2. \end{aligned}$$

We note that the equilibrium travel cost can be still given by

$$\tilde{c} = 2\frac{N\delta}{s} = \bar{c}. \tag{28}$$

*3.2.1. The analysis of the optimal system travel time cost under toll model*

Under this type of peak time window, the length of the CF (as illustrated in Figure 5) must be smaller than the value of $\rho/\alpha$, i.e., $CF \leq \rho/\alpha$; and the length of the BE must be strictly larger than the value of $\rho/\alpha$, i.e., $BE > \rho/\alpha$ (due to the departure rate and the arrival rate of the individual household). Therefore, we have

$$CF = t_2^* - \tilde{t}_2 = \frac{2N\delta}{s\alpha} - \frac{\gamma}{\alpha}(t_2^* - t_1^*),$$

$$BE = t_1^* - \tilde{t}_1 = \frac{2N\delta}{s\alpha} - \frac{\beta}{\alpha}(t_2^* - t_1^*).$$

Based on the observation of the equation of $TC_2$ above, we need to further extend the discussion in two cases:

**In the case when $\frac{2}{2\gamma-\beta}\frac{N\delta}{s} \leq t_2^* - t_1^* < \frac{2}{\gamma}\frac{N\delta}{s}$:**

If $\frac{2}{2\gamma-\beta}\frac{N\delta}{s} \leq t_2^* - t_1^* < \frac{2}{\gamma}\frac{N\delta}{s}$, the following inequality is fulfilled:

$$CF \leq \frac{N\delta}{s\alpha} - \frac{\beta(t_2^* - t_1^*)}{2\alpha}.$$

It is obvious that when we set

$$\rho^* = \frac{N\delta}{s} - \frac{\beta(t_2^* - t_1^*)}{2} \quad (< \alpha \cdot BE),$$

$TC$ can achieve the minimum

$$TC_2 = 2\frac{N^2\delta}{s} - \frac{s(\beta+\gamma)}{2\beta(\gamma-\beta)}(\frac{N\delta}{s} - \frac{\beta(t_2^* - t_1^*)}{2})^2,$$

which is dependent on the value of $t_2^* - t_1^*$. Considering the range of $t_2^* - t_1^*$, we find that

$$\frac{3}{2}\frac{N^2\delta}{s} < \frac{3}{2}\frac{N^2\delta}{s} + \frac{\beta^2}{2(2\gamma-\beta)^2}\frac{N^2\delta}{s} \leq TC_2 \leq \frac{3}{2}\frac{N^2\delta}{s} + \frac{\beta}{2\gamma}\frac{N^2\delta}{s} < 2\frac{N^2\delta}{s},$$

where the inequality can be verified by inequality $\gamma > \beta$. And the value of $TC_2$ is increasing with respect to $t_2^* - t_1^*$, in the range of $\frac{2}{2\gamma-\beta}\frac{N\delta}{s} \le t_2^* - t_1^* < \frac{2}{\gamma}\frac{N\delta}{s}$. Accordingly, the optimal tolling peak time window can be given as follows:

$$t_+ = \frac{t_1^* + t_2^*}{2} - \frac{t_2^* - t_1^*}{4} - \frac{N\delta}{2\beta s}, \quad t_- = \frac{t_1^* + t_2^*}{2} - \frac{\gamma(t_2^* - t_1^*)}{2(\gamma-\beta)} + \frac{N\delta}{(\gamma-\beta)s}.$$

**In the case when** $0 < t_2^* - t_1^* < \frac{2}{2\gamma-\beta}\frac{N\delta}{s}$**:**

If $0 < t_2^* - t_1^* < \frac{2}{2\gamma-\beta}\frac{N\delta}{s}$, then we have the following inequality

$$CF > \frac{N\delta}{s\alpha} - \frac{\beta(t_2^* - t_1^*)}{2\alpha}.$$

Meanwhile, we find that when we set

$$\rho^* = \alpha \cdot CF = \frac{2N\delta}{s} - \gamma(t_2^* - t_1^*) \;\; (< \alpha \cdot BE), \tag{29}$$

$TC$ will achieve the minimum at

$$TC_2 = \frac{3}{2}\frac{N^2\delta}{s} + \frac{\gamma^2 s}{2\delta}[(t_2^* - t_1^*) - \frac{1}{\gamma}\frac{N\delta}{s}]^2,$$

which depends on the value of $t_2^* - t_1^*$, and $TC_2$ is a convex function with respect to $t_2^* - t_1^*$. Due to the inequality $\gamma > \beta$, we know that $0 < \frac{1}{\gamma}\frac{N\delta}{s} < \frac{2}{2\gamma-\beta}\frac{N\delta}{s}$. So in the range $0 < t_2^* - t_1^* < \frac{2}{2\gamma-\beta}\frac{N\delta}{s}$, $TC_2$ will be minimized, if we set the interval of the two desired time as

$$t_2^* - t_1^* = \frac{1}{\gamma}\frac{N\delta}{s}, \tag{30}$$

and the minimum is achieved as

$$TC_2 = \frac{3}{2}\frac{N^2\delta}{s}. \tag{31}$$

And the range of $TC_2$ is

$$\frac{3}{2}\frac{N^2\delta}{s} \le TC_2 < 2\frac{N^2\delta}{s}.$$

Accordingly, the optimal tolling peak time window can be given as follows:

$$t_+ = \frac{t_1^* + t_2^*}{2} - \frac{\gamma(t_2^* - t_1^*)}{2\beta}, \quad t_- = t_2^*.$$

**Remark:** Therefore, if we want to obtain the total system time cost $\frac{3}{2}\frac{N^2\delta}{s}$ under this peak time window scheme, we should set $t_2^* - t_1^* = \frac{1}{\gamma}\frac{N\delta}{s}$, and through (29) we can deduce the optimal toll

$$\rho^* = \frac{N\delta}{s}. \tag{32}$$

In section 5, we will design a tradable credit scheme to replace the toll scheme while this optimal situation can still be achieved. Indeed, a special case of the peak time window as in case $t_- \geq t_2^*$, as in this solution the end point of the peak time window $t_-$ is simply at $t_2^*$, i.e., $t_- = t_2^*$.

## 4. The optimal toll model for large school-work difference situation

As was done in previous section, the equilibrium trip scheduling solution with toll scheme can be summarized in Figure 6. The endpoint of the peak time window $t_-$ must be ahead of $t_2^*$. Next, we will consider how the toll model works in this situation.

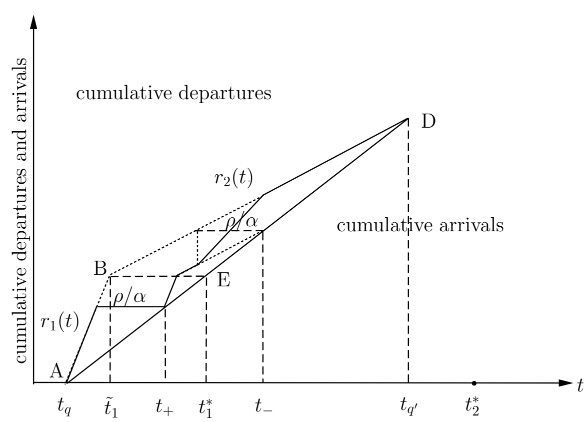


Figure 6: Equilibrium solutions for one-step toll model in large school-work difference situation.

### *4.1. The peak time window*

For a given toll $\rho$, the difference between the total travel time cost of the individual who depart from home at $t_q$ and the individual household who is the first to enter the tolling window is $\rho$, then we have the following equation:

$$\beta(t_1^* - t_q) + \beta(t_2^* - t_q) = \rho + \beta(t_1^* - t_+) + \beta(t_2^* - t_+),$$

Similarly, the difference between the total travel time cost of the person who depart from home at $t_{q'}$ and the person who is the last one that will be charged in the tolling window is $\rho$, and we have the following equation:

$$\gamma(t_{q'} - t_1^*) + \beta(t_2^* - t_{q'}) = \rho + \gamma(t_- - t_1^*) + \beta(t_2^* - t_-),$$

Then the tolling window can be determined as follows:

$$t_+ = t_q + \rho/2\beta, \quad t_- = t_{q'} - \frac{\rho}{\gamma - \beta}.$$

Thus, we can represent the number of the individual household who pay the toll is

$$N_t = (t_- - t_+)s = s(t_{q'} - t_q - \frac{\rho}{2\beta} - \frac{\rho}{\gamma - \beta}), \tag{33}$$

and the total system travel time cost excluding the toll is

$$\begin{aligned} TC &= [\frac{2N\delta}{s}\frac{\gamma - \beta}{\gamma} + \beta(t_2^* - t_1^*)] \cdot (N - N_t) + [\frac{2N\delta}{s}\frac{\gamma - \beta}{\gamma} + \beta(t_2^* - t_1^*) - \rho] \cdot N_t \\ &= [\frac{2N\delta}{s}\frac{\gamma - \beta}{\gamma} + \beta(t_2^* - t_1^*)] \cdot N + \frac{\beta + \gamma}{2\beta(\gamma - \beta)} s[\rho - \frac{N}{s}\frac{\beta(\gamma - \beta)}{\beta + \gamma}]^2 \\ &\quad - \frac{\beta + \gamma}{2\beta(\gamma - \beta)} s \cdot [\frac{N}{s}\frac{\beta(\gamma - \beta)}{\beta + \gamma}]^2. \end{aligned}$$

We note that the total travel cost at equilibrium with road toll in this situation will be:

$$\tilde{c} = \frac{2N\delta}{s}\frac{\gamma - \beta}{\gamma} + \beta(t_2^* - t_1^*) \geq 2\frac{N\delta}{s}. \tag{34}$$

*4.2. The analysis of the optimal system travel time cost under toll model*

In this case, as we set the endpoint of the peak time window $t_-$ before $t_2^*$, the value of the $t_1^* - \tilde{t}_1$ must be larger than the value of $\rho/\alpha$, i.e., $t_1^* - \tilde{t}_1 \geq \rho/\alpha$ (due to the rate of departure and arrival of the individual household). Therefore, we have

$$t_1^* - \tilde{t}_1 = \frac{2N}{\alpha s}\frac{\beta(\gamma - \beta)}{\beta + \gamma}.$$

So the optimal road toll rate would be

$$\rho^* = \frac{N}{s}\frac{\beta(\gamma-\beta)}{\beta+\gamma} \quad (\leq \alpha(t_1^* - \tilde{t}_1)), \tag{35}$$

so that $TC$ can attain the minimum

$$\begin{aligned} TC &= \frac{3}{2}\frac{N^2\delta}{s} - \frac{3\beta}{2\gamma}\frac{N^2\delta}{s} + N\beta(t_2^* - t_1^*) \\ &\geq \frac{3}{2}\frac{N^2\delta}{s} + \frac{\beta}{2\gamma}\frac{N^2\delta}{s} > \frac{3}{2}\frac{N^2\delta}{s}. \end{aligned} \tag{36}$$

Accordingly, the optimal tolling peak time window can be given as follows:

$$t_+ = t_1^*, \quad t_- = t_1^* + \frac{N\delta}{s\gamma}.$$

**Remark:** Under the peak time window tolling scheme, we achieve the minimum total system time cost $TC$ in large school-work difference situation if we set the toll to be $\rho^* = \frac{N}{s}\frac{\beta(\gamma-\beta)}{\beta+\gamma}$. Obviously, the minimum total system travel time in large school-work difference situation is not better than that in small school-work difference situation, as the value of TC in large school-work difference situation is larger than $\frac{3}{2}\frac{N^2\delta}{s}$, no matter how we adjust the value of the toll rate.

In order to clearly show and discuss how the results derived in Section 3 and Section 4 change with $t_2^* - t_1^*$, we summarize some (untolled and tolled) results in Table 1. From Table 1 one can easily observe that the minimum total travel cost in large school-work difference situation is higher than that in small school-work difference situation, and the minimum total cost that can be achieved is $\frac{3}{2}\frac{N^2\delta}{s}$.

Table 1: The summary of the (untolled and tolled) results

| Situation | | $t_2^* - t_1^*$ | $(t_+, t_-, \rho)$ | Total Cost |
|---|---|---|---|---|
| $t_2^* - t_1^* \in (0, \frac{2N\delta}{\gamma s})$ | No Toll | $[0, \frac{2}{\gamma}\frac{N\delta}{s})$ | N.A. | $2\frac{N^2\delta}{s}$ |
| | Case $t_- \geq t_2^*$ | $[0, \frac{1}{\gamma}\frac{N\delta}{s}]$ | $(\frac{t_1^*+t_2^*}{2} - \frac{N\delta}{2\beta s}, \frac{t_1^*+t_2^*}{2} + \frac{N\delta}{2\gamma s}, \frac{N\delta}{s})$ | $\frac{3}{2}\frac{N^2\delta}{s}$ |
| | | $(\frac{1}{\gamma}\frac{N\delta}{s}, \frac{2}{\gamma}\frac{N\delta}{s})$ | $(\frac{t_1^*+t_2^*}{2} - \frac{\gamma(t_2^*-t_1^*)}{2\beta}, t_2^*, \frac{2N\delta}{s} - \gamma(t_2^* - t_1^*))$ | $(\frac{3}{2}\frac{N^2\delta}{s}, 2\frac{N^2\delta}{s})$ |
| | Case $t_- \leq t_2^*$ | $[\frac{2}{2\gamma-\beta}\frac{N\delta}{s}, \frac{2}{\gamma}\frac{N\delta}{s})$ | $(\frac{t_1^*+t_2^*}{2} - \frac{t_2^*-t_1^*}{4} - \frac{N\delta}{2\beta s}, \frac{t_1^*+t_2^*}{2} - \frac{\gamma(t_2^*-t_1^*)}{2(\gamma-\beta)} + \frac{N\delta}{(\gamma-\beta)s}, \frac{N\delta}{s} - \frac{\beta(t_2^*-t_1^*)}{2})$ | $(\frac{3}{2}\frac{N^2\delta}{s}, 2\frac{N^2\delta}{s})$ |
| | | $(0, \frac{2}{2\gamma-\beta}\frac{N\delta}{s})$ | $(\frac{t_1^*+t_2^*}{2} - \frac{\gamma(t_2^*-t_1^*)}{2\beta}, t_2^*, \frac{2N\delta}{s} - \gamma(t_2^* - t_1^*))$ | $[\frac{3}{2}\frac{N^2\delta}{s}, 2\frac{N^2\delta}{s})$ |
| $t_2^* - t_1^* \in [\frac{2N\delta}{\gamma s}, +\infty)$ | No Toll | $[\frac{2}{\gamma}\frac{N\delta}{s}, +\infty)$ | N.A. | $(2\frac{N^2\delta}{s}, +\infty)$ |
| | $t_- \leq t_2^*$ | $[\frac{2}{\gamma}\frac{N\delta}{s}, +\infty)$ | $(t_1^*, t_1^* + \frac{N\delta}{s\gamma}, \frac{N}{s}\frac{\beta(\gamma-\beta)}{\beta+\gamma})$ | $(\frac{3}{2}\frac{N^2\delta}{s}, +\infty)$ |

## 5. The optimal setting of the desired arrival time and the alternative tradable credit scheme

In the previous two sections, we investigate optimal design of one step toll in peak hour window to best manage the morning commute of household travels, and analyse the impact of the time interval between desired arrival times (i.e., school-work start time difference) on travel cost by assuming given time interval. If we relax the assumption that time interval between the desired or preferred arrival times is given and fixed, we may want to ask the question that what is the optimal setting of this time interval so that the minimum total system travel cost can be achieved. Indeed, we have made some preliminary analysis on this issue in the remarks of previous two sections. Next, we will compare these two situations in details and try to derive some policy indications that are useful for traffic management agencies about the optimal setting of the interval between two desired arrival time (i.e., $t_2^* - t_1^*$) and the choice of the peak time window. We have discussed in section 4 that large school-work difference situation is not better than small school-work difference situation if total system travel time cost is to be minimized or we can see this result through Table 1 directly. Therefore, the management agencies should set this time interval between the two desired arrival times so that small school-work difference situation would occur in practice. Next we will make some comparisons between the two kinds of peak time windows as discussed in the small school-work difference situation.

### *5.1. Comparison of the two peak time windows in small school-work difference situation*

From Table 1, we find that in small school-work difference situation if we set the peak time window as case $t_- \geq t_2^*$, the optimal total travel time cost $TC_1 \geq \frac{3}{2}\frac{N^2\delta}{s}$, with varying values of the interval of two desired time. To attain the minimum value of $\frac{3}{2}\frac{N^2\delta}{s}$, the interval between these two desired time needs to satisfy:

$$0 \leq t_2^* - t_1^* \leq \frac{1}{\gamma}\frac{N\delta}{s}, \tag{37}$$

and the optimal total travel time cost is not dependent on the specific value of $t_2^* - t_1^*$, as long as it satisfies (37), and the toll rate is set as $\rho^* = \frac{N\delta}{s}$ at the same time.

From Table 1, if we set the peak time window as case $t_- \leq t_2^*$, the optimal total travel time cost $TC_2 \geq \frac{3}{2}\frac{N^2\delta}{s}$. To obtain the minimum $\frac{3}{2}\frac{N^2\delta}{s}$ under this type of peak time window, the interval between two desired time must fulfill the following condition:

$$t_2^* - t_1^* = \frac{1}{\gamma}\frac{N\delta}{s}. \tag{38}$$

and the toll should be set as $\rho^* = \frac{N\delta}{s}$, which is obviously a special case under the peak time window of case $t_- \geq t_2^*$. As in this case, the end point of the peak time window $t_-$ is just at $t_2^*$, i.e., $t_- = t_2^*$.

**Remark:** It should be noted that, if the time interval between the desired arrival time is set to satisfy (37), and the peak time window is designed as specified in section 3.1, the total travel time cost of travel demands will attain minimum $\frac{3}{2}\frac{N^2\delta}{s}$ under the toll model, in which the toll rate is set to fulfill (24).

*5.2. The tradable credit scheme for the optimal situation*

As is well known, road pricing is susceptible to public objection and hard to be practically implemented. Many researchers have been promoting the tradable credit scheme as the travel demand management measure (Yang & Wang 2011; Wang et al. 2012; Wang et al. 2013a;Wang & Yang 2012;Wang et al. 2013b; Nie 2012; Tian et al. 2013; Xiao et al. 2013; Nie & Yin 2013; Nie 2013). In this section, we will present how to use the tradable credit scheme to replace the the optimal toll scheme (24) under the optimal time interval arrangement as in (37), and the peak time window is designed for case $t_- \geq t_2^*$.

As is in the TCS proposed by Nie (2013), the authority sets a peak-time window $[t_+, t_-]$, and requires the person who departs from his home within that window to either pay $\kappa$ units of credits, or a much higher toll $\rho_g$ (in the monetary form). On the other hands, the authority rewards $r$ units of credits to those travellers who depart their home during the off-peak periods $[t_q, t_+]$ and $[t_-, t_{q'}]$. A market is created so that the users can trade the mobility credits with each other. Based on the assumption of credit conservation, i.e., the total number of credits earned by the off-peak users equal to the number of credits used by the peak-time users, we have:

$$(N - N_t)r = N_t\kappa.$$

We deduce that

$$\frac{N_t}{N} = \frac{r}{\kappa + r} = \frac{1}{2}, \tag{39}$$

where the second equality is obtained by replacing $N_t$ with (20), while the optimal step toll is given in (24). As the difference between the total cost of each person in off-peak and peak-time in the TCS must be equal to the toll charged in the one-step toll scheme, we have

$$\rho^* = P(\kappa + r),$$

where $P$ is the market clearing price of the credit, which can be derived from the above equation as long as $r$ is given. To ensure that the user has an incentive to earn and trade credits, the authority must set the alternative toll $\rho_g$ much higher than what one must pay for a credit, i.e., $\rho_g > P$.

So the optimal situation is attained when TCS is given as follows:

$$\kappa = r, \quad P = \frac{N\delta}{(\kappa + r)s}. \tag{40}$$

For the peak and off-peak time users, their travel time cost (including travel delay and schedule delay costs) under TCS are $\bar{c} - \rho^* = \frac{1}{2}\bar{c}$ and $\bar{c}$, and the profit from the credit trading is $-\kappa P$ and $rP$, respectively. Thus the equilibrium cost under the optimal TCS is

$$c^* = \frac{1}{2}\bar{c} + \kappa P = \bar{c} - rP = \frac{3}{4}\bar{c}. \tag{41}$$

This cost is 25% lower than the equilibrium cost (22) under the optimal one-step toll scheme. The reason is straightforward, and one can easily observe that in TCS (40), the benefits are indeed redistributed to the commuters directly. But in the one-step toll scheme, the same amount of benefits will be collected by the transport authorities as toll revenues, and thus the members of individual household will not benefit until these revenues are redistributed. The optimal total system travel time cost is

$$TC_1 = \frac{3}{2}\frac{N^2\delta}{s}. \tag{42}$$

*5.3. The design of the interval between two desired times*

In the model analysis presented in previous sections, we set $T^f = 0$, $T_1^f = 0$ for convenience ( i.e., an individual household arrives at the bottleneck as soon as she leaves home, and arrives at school and workplace immediately upon leaving the bottleneck ). However, in practice, the equation $t_2^* - t_1^* = 0$ means that the actual interval between two desired time is exactly $\tilde{t}_2^* - \tilde{t}_1^* = T^f - T_1^f$, which is just the travel time we need to go from school to workplace with no congestion, and the rest equation about $t_2^* - t_1^*$ can be understood in the same manner.

So we suggest the managers setting the peak time window like case $t_- \geq t_2^*$, the tradable credit scheme like (40), and the value of $t_2^* - t_1^*$ satisfied (37), which means the actual interval between two desired time satisfies

$$T^f - T_1^f \le \tilde{t}_2^* - \tilde{t}_1^* \le T^f - T_1^f + \frac{1}{\gamma}\frac{N\delta}{s}. \tag{43}$$

so that we can attain the optimal total system travel time cost $\frac{3}{2}\frac{N^2\delta}{s}$.

## 6. Numerical example

In this section, we conduct a numerical example to illustrate the implementation of the model analysis results. In the morning, we assume there is a bottleneck located before both trip destinations of household travels, i.e., school and workplace, which is depicted in Figure 1. Suppose a fixed number of $N = 8000$ household travels depart from home to their children's school firstly and then to their workplaces every morning. The capacity of the whole road is $s = 200$, i.e., 200 cars can pass the road per minute. Here we set the the shadow cost of travel time $\alpha = 0.3$, the schedule penalty for one minute of early arrival $\beta = 0.1$ and the schedule penalty for one minute of late arrival $\gamma = 0.4$, which is reasonable in practice. In this example, we assume the fixed travel time from home to school is $T_1^f = 10$min and the fixed travel time from home to workplace is $T^f = 30$min, so the actual school-work start time difference $\tilde{t}_2^* - \tilde{t}_1^* = (t_2^* - t_1^*) + (T^f - T_1^f)$ should be no less than $T^f - T_1^f = 20$min. Here we set $t_1^* = \tilde{t}_1^* = 7:30$ am and $t_2^* = \tilde{t}_2^* - (T^f - T_1^f) = \tilde{t}_2^* - 20$min in the model we analyse above. We will give the equilibrium trip scheduling of the household travels first. Then, the optimal school-work start time difference and toll window to manage the bottleneck in the morning will be determined. At last, we give a tradable credit scheme to replace the toll scheme to manage the household travel morning commute traffic.

According to Section 2, when $t_2^* - t_1^* < \frac{1}{\gamma}\frac{2N\delta}{s} = 16$min, the departure rate is depicted as Figure 2; when $t_2^* - t_1^* \ge \frac{1}{\gamma}\frac{2N\delta}{s} = 16$min, the departure rate is depicted as Figure 3. From Section 3, Section 4 and Table 1, we find that if we set the school-work start time difference in the following interval

$$t_2^* - t_1^* \in [0, \frac{1}{\gamma}\frac{N\delta}{s}] = [0, 8]\text{min}, \tag{44}$$

and the toll scheme

$$\begin{aligned}(t_+, t_-, \rho) &= (\frac{t_1^* + t_2^*}{2} - \frac{N\delta}{2\beta s}, \frac{t_1^* + t_2^*}{2} + \frac{N\delta}{2\gamma s}, \frac{N\delta}{s}) \\ &= (\frac{t_1^* + t_2^*}{2} - 16\text{min}, \frac{t_1^* + t_2^*}{2} + 4\text{min}, 3.2), \qquad (45)\end{aligned}$$

the total travel time cost will be minimized to $TC = \frac{3}{2}\frac{N^2\delta}{s} = 38400$ (the fixed travel time cost on the road, which is $T^f$ for all household travels, is excluded here), while the total travel time cost will be $TC = 2\frac{N^2\delta}{s} = 51200$ if no toll scheme is implemented. The equilibrium travel cost with toll for each household will be $\tilde{c} = 2\frac{N\delta}{s} = 6.4$, which is the equal to the equilibrium travel cost for each household in the untolled scheme $\bar{c} = 2\frac{N\delta}{s} = 6.4$.

Therefore, traffic management can set the $t_2^* - t_1^*$ be any value between $[0, 8]$, here we just take $t_2^* - t_1^* = 4$min for illustration. Accordingly, the optimal toll scheme is $(t_+, t_-, \rho) = (7:16 \text{ am}, 7:36 \text{ am}, 3.2)$. The first one who enter the bottleneck is $t_q = 7:00$ am, and the last one who enter the bottleneck is $t_{q'} = 7:40$ am. Considering $T_1^f = 10$min, every one should spend 10min on the road before arriving the bottleneck, so $\tilde{t_q}$ and $\tilde{t_{q'}}$ will be ten minutes earlier than $t_q$ and $t_{q'}$ respectively. As $T^f$ equals to 30min, we need to set the actual school-work start time as $\tilde{t}_1^* = 7:30$ am and $\tilde{t}_2^* = 7:54$ am.

At last the optimal tradable credit scheme can be given as follows according to Section 5 to replace the toll scheme mentioned above. Instead of paying $\rho = 3.2$ in the toll window, the household need to pay $\kappa$ units of credits, and the household who enter the bottleneck between $[t_q, t_+]$ and $[t_-, t_{q'}]$ will be rewarded $\kappa$ units of credits. Here we just set $\kappa = 1$, and the market clearing price of the credit is $P = \frac{N\delta}{2s} = 1.6$. In this scheme, the total travel time cost will be $TC = \frac{3}{2}\frac{N^2\delta}{s} = 38400$, while the equilibrium travel cost for each household will be $c^* = \frac{3}{4}\tilde{c} = 4.8$. In Table 2, we summarize the total travel time cost $TC$ and the equilibrium travel cost for each household under different schemes.

Table 2: The travel cost under different schemes

| Schemes | Total Travel Time Cost | Equilibrium Travel Cost |
|---|---|---|
| Untolled Scheme | $TC = 2\frac{N^2\delta}{s} = 51200$ | $\bar{c} = 2\frac{N\delta}{s} = 6.4$ |
| Toll Scheme | $TC = \frac{3}{2}\frac{N^2\delta}{s} = 38400$ | $\tilde{c} = 2\frac{N\delta}{s} = 6.4$ |
| Tradable Credit Scheme | $TC = \frac{3}{2}\frac{N^2\delta}{s} = 38400$ | $c^* = \frac{3}{4}\frac{N\delta}{s} = 4.8$ |

## 7. Conclusion

In this study, we consider the equilibrium trip scheduling and the traffic managements for household travels in the morning commute. Different from individual travels, household travels meet the travel demands for all household members. As multiple household members are involved in the joint decision, the members' preference of arrival times and intra-household interaction have to be considered in a group decision-making manner. In this study, we firstly present the equilibrium trip scheduling for household travels, i.e., extending the standard Vickrey's bottleneck model from work commute with one single preferred arrival time to household commute with two consecutive preferred arrival time. Then we use one-step toll scheme to reduce the traffic congestion and analyse the impact of the school-work start time difference on individual cost, social cost and traffic management tolls, so that we can optimally set the school-work start time difference for the household members to make the total travel cost minimized. A tradable credit scheme is then devised to replace the optimal tolling scheme. This study seeks to fill in the research gap in the morning commute traffic management by investigating the management for the household travels. In this study, we assume that the preferred arrival times are specific time points rather than a preferred arrival time interval. To relax this assumption and consider the case with preferred arrival time window will be addressed in the future study. Besides, in the future study, we would also study on how to best manage the morning commute traffic with mixed individual and household travels to consider a more general and realistic scenario of traffic management.

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