# Multipartite polariton entanglement in semiconductor microcavities 

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#### Abstract

We study the entanglement of multiple polariton modes, which results in continuous variable cluster states suitable for quantum computation. Schemes are based on parametric scattering between spin-polarized lower and upper polariton branches in planar microcavities or spin-polarized orbital angular momentum states in mesa structures. Such systems are modeled by numerical solution of truncated density matrices and compared to the solution of the Heisenberg equations for the set of field correlators up to third order. Four-body entanglement is evidenced by violation of the van Loock-Furusawa quadripartite inequalities. We show that the entanglement is able to withstand a realistic strength of pure dephasing present in typical systems.


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## I. INTRODUCTION

The Bose-Einstein condensation of exciton polaritons in semiconductor microcavities [1-3], their superfluidity [4,5], and multistability [6-8] are examples of some of the stunning effects that arise from the strongly nonlinear interactions between polaritons [9-11]. In the quantum realm, squeezing [12] and quantum complementarity have been shown experimentally [13], while polariton pair entanglement [14-17] and nonclassical correlations [18-24] are expected theoretically.

Many works allude to the possibility of polaritons being used for quantum computation. However, it is difficult to imagine how polaritons could satisfy the DiVincenzo criteria [25] for conventional quantum computational schemes given their rather short lifetime, which hinders their isolation and manipulation without significant losses. Furthermore, even though nonlinearity at the single polariton level can be achieved through confinement [18], a proposal for the essential controlled-NOT (CNOT) quantum logic gate remains elusive.

Nevertheless, the crucial ingredient for quantum computation is entanglement. Advances in the field of quantum information science show that it is not necessary to have control of qubits; rather, one can work in a basis of continuous variables [26,27], such as the conjugate quadrature observables of a quantized field (which can be measured using homodyne detection [28]). Bipartite continuous variable entanglement has been measured between modes with both equal [28] and different energy [29].

Other developments have shown that, by taking advantage of quantum teleportation, one can replace the quantum circuit model with a scheme based on measurements. Starting with an initially highly entangled state of many qubits, known as a cluster state, a quantum circuit can be simulated using only local measurements on the qubits in a specific order [30]. In other words, if one has entanglement, one does not need to worry about moving particles around in a circuit or interacting them further to make quantum logic gates. The combination of the two ideas of working with continuous variables and with cluster-state quantum computation has appeared in Ref. [31] and systems for its implementation have been proposed [32]. The fundamental step of generating continuous variable multipartite entanglement has been considered based on coupled parametric processes in $\chi^{(2)}$ nonlinear crystals [33-35].

## II. SCHEME

Our interest lies in the creation of four-mode cluster states (the most basic cluster state) using exciton polaritons in semiconductor microcavities. We consider the simultaneous resonant excitation of polaritons at the bottom of both the lower and upper polariton branches by continuous wave linearly polarized laser beams, illustrated with black spots in Fig. 1(a). Parametric scattering can take place to the states marked by the gray spots [17], conserving the total energy and momentum. Accounting for $\sigma_{+}$and $\sigma_{-}$spin-polarized components [36], a total of four signal states are generated, labeled $a_{n}$. Interactions between polaritons with parallel spins entangle the modes $a_{0}$ and $a_{1}$ as well as $a_{2}$ and $a_{3}$; interactions between polaritons with antiparallel spins entangle the modes $a_{0}$ and $a_{3}$ as well as $a_{1}$ and $a_{2}$. This gives rise to the square-type cluster state illustrated in Fig. 1(b).

There are several advantages of this scheme. First, the signal states lie at a point on the lower polariton dispersion with both a decent photonic fraction and a high gradient. This is expected to suppress contributions due to excitonic noise and polariton-phonon scattering [37] as well as excitonic disorder, provided the exciton inhomogeneous broadening is sufficiently small [38]. Additionally, a high photonic fraction of the states is important for their efficient detection $[16,39]$. Second, because the signal states lie at a different energy to the pump states, polariton scattering mediated with disorder or phonons is strongly suppressed. This was likely a problem in previous experiments searching for two-mode entanglement [40,41] in planar microcavities. Additionally, surface scattering from the incoming pump does not pollute the signal states, allowing one to work in a reflection geometry. Third, the signal states are easily separable by exploiting their different wave vectors and polarizations. Fourth, we have chosen a scheme in which the signal states have the same energy (aiding measurements with homodyne detection), linewidth, and photonic fraction. Finally, although two excitation lasers are required, this represents an improvement on alternative schemes requiring four lasers [42].

Before describing our model, it is worth noting that a completely analogous scheme can be constructed using the orbital angular momentum carrying modes [43] in mesa structures in semiconductor microcavities. For example, the modes with orbital angular momentum $l= \pm 1$ and circular


FIG. 1. (Color online) (a) Four-mode entanglement scheme: solid curves show the typical dispersion of lower and upper branch polaritons, formed by the strong coupling of exciton and photon modes (dashed curves). Parametric scattering to the modes marked by gray circles can realize a square cluster state illustrated in (b). (c) An alternative scheme for the creation of four-mode entanglement can be based on the orbital angular momentum carrying modes in a confined mesa structure. We have taken typical parameters for a GaAs/AlAs-based microcavity, with Rabi splitting 5 meV , cavity photon effective mass $10^{-5}$ of the free electron mass, and an exciton-photon detuning of -3 meV .
polarization $\sigma= \pm$ can be arranged such that $a_{0}=(1,+)$, $a_{1}=(-1,+), a_{2}=(1,-)$, and $a_{3}=(-1,-)$, as illustrated in Fig. 1(c). We will present a coupled-mode model in which the four modes of the cluster state are described in a fully quantum way. The model will be general enough such that its conclusions hold for both the interpolariton branch scattering in planar microcavities as well as interangular momentum mode scattering in mesas. In mesa structures, the pumps would be replaced with a single linearly polarized pump with no orbital angular momentum.

## III. THEORETICAL MODEL

The system is described by the Hamiltonian:

$$
\begin{align*}
\hat{\mathcal{H}}= & \sum_{n}\left[E_{n}+8\left(\alpha_{1}+\alpha_{2}\right)|\psi|^{2}\right] \hat{a}_{n}^{\dagger} \hat{a}_{n}+\alpha_{1} \hat{a}_{n}^{\dagger} \hat{a}_{n}^{\dagger} \hat{a}_{n} \hat{a}_{n} \\
& +\alpha_{1}\left[2 \psi^{2}\left(\hat{a}_{0}^{\dagger} \hat{a}_{1}^{\dagger}+\hat{a}_{2}^{\dagger} \hat{a}_{3}^{\dagger}\right)+2 \psi^{* 2}\left(\hat{a}_{0} \hat{a}_{1}+\hat{a}_{2} \hat{a}_{3}\right)\right. \\
& \left.+4\left(\hat{a}_{0}^{\dagger} \hat{a}_{1}^{\dagger} \hat{a}_{0} \hat{a}_{1}+\hat{a}_{2}^{\dagger} \hat{a}_{3}^{\dagger} \hat{a}_{2} \hat{a}_{3}\right)\right]+\alpha_{2}\left[8 | \psi | ^ { 2 } \left(\hat{a}_{0}^{\dagger} \hat{a}_{2}\right.\right. \\
& \left.+\hat{a}_{2}^{\dagger} \hat{a}_{0}+\hat{a}_{1}^{\dagger} \hat{a}_{3}+\hat{a}_{3}^{\dagger} \hat{a}_{1}\right)+8 \psi^{2}\left(\hat{a}_{0}^{\dagger} \hat{a}_{3}^{\dagger}+\hat{a}_{1}^{\dagger} \hat{a}_{2}^{\dagger}\right) \\
& +8 \psi^{* 2}\left(\hat{a}_{0} \hat{a}_{3}+\hat{a}_{1} \hat{a}_{2}\right)+4\left(\hat{a}_{0}^{\dagger} \hat{a}_{0}+\hat{a}_{1}^{\dagger} \hat{a}_{1}\right) \\
& \left.\times\left(\hat{a}_{2}^{\dagger} \hat{a}_{2}+\hat{a}_{3}^{\dagger} \hat{a}_{3}\right)+4\left(\hat{a}_{0}^{\dagger} \hat{a}_{3}^{\dagger} \hat{a}_{1} \hat{a}_{2}+\hat{a}_{1}^{\dagger} \hat{a}_{2}^{\dagger} \hat{a}_{0} \hat{a}_{3}\right)\right] \tag{1}
\end{align*}
$$

where $E_{n}$ represents the bare energy levels of the signal states; $\alpha_{1}$ and $\alpha_{2}$ represent the strength of interactions between polaritons with parallel and antiparallel spins, respectively. $\hat{a}^{\dagger}$ and $\hat{a}$ are creation and annihilation operators, respectively, associated with the different modes labeled by $n . \psi$ represents
the mean field of the pump states. For simplicity, the two intracavity pump states are assumed to have equal amplitudes. Note that this would actually require external pumps with different intensities due to the differing photonic fractions of the upper and lower polariton branches. We also assume that the polarization is collinear such that the $\sigma_{+}$and $\sigma_{-}$ polarized pump amplitudes are equal. The choice of collinear polarization is important as it results in the strongest interaction between the two pumps. Scattering is expected to be suppressed in a cross-polarized setup [44].

The Hamiltonian can be derived from the general two-body interaction Hamiltonian [17] by separating out the eight spinpolarized modes of our scheme [Fig. 1(a)] from other unpopulated modes (and applying the mean-field approximation to the pumped modes). The first line of the Hamiltonian contains terms that arise from forward scattering processes, which cause a renormalization of the energy yet do not directly change the distribution of polaritons across the different modes. These forward scattering processes include processes in which polaritons in a given signal mode scatter with one of the pump modes, as well as processes in which polaritons in a given mode scatter with polaritons in the same mode (this latter process is described by the term $\alpha_{1} \hat{a}_{n}^{\dagger} \hat{a}_{n}^{\dagger} \hat{a}_{n} \hat{a}_{n}$ ). The second and third lines describe scattering processes involving polaritons with parallel spins. The second line contains parametric scattering processes where a pair of polaritons, one from each pump, scatter to the signal states. The reverse process is also accounted for. The third line contains processes in which polaritons in different signal modes interact. These are also forward scattering processes that do not change the distribution of polaritons in the different modes but rather contribute a renormalization of the energy. The remaining lines describe processes involving polaritons with antiparallel spins. The fourth line accounts for processes where a polariton interchanges its spin with a polariton in one of the pump states. The fifth line contains parametric scattering processes involving a pair of oppositely spin-polarized polaritons, one from each pump, scattering to the signal states. The sixth line contains forward scattering processes analogous to those in the third line, but involving polaritons with opposite spin polarization. The last line in the Hamiltonian accounts for a spin-flip process in which a pair of polaritons with opposite wave vectors interchanges their spins.

The evolution of the corresponding density matrix, $\rho$, is given by

$$
\begin{equation*}
i \hbar \frac{d \rho}{d t}=[\hat{\mathcal{H}}, \rho]+i \frac{\Gamma}{2} \sum_{n=1}^{2}\left(2 \hat{a}_{n} \rho \hat{a}_{n}^{\dagger}-\hat{a}_{n}^{\dagger} \hat{a}_{n} \rho-\rho \hat{a}_{n}^{\dagger} \hat{a}_{n}\right) \tag{2}
\end{equation*}
$$

where the last term represents the standard Lindblad dissipation characterized by decay rate $\Gamma$. Equation (2) can be solved by expanding the density matrix over a particle number basis in a similar way to that done in Ref. [18]; one truncates at a given particle number and propagates in time from the vacuum to the steady state.

An alternative method involves using the Heisenberg equation for the evolution of operators:
$i \hbar \frac{d \hat{\mathcal{O}}}{d t}=[\hat{\mathcal{O}}, \hat{\mathcal{H}}]+i \frac{\Gamma}{2} \sum_{n=1}^{2}\left(2 \hat{a}_{n}^{\dagger} \mathcal{O} \hat{a}_{n}-\hat{a}_{n}^{\dagger} \hat{a}_{n} \mathcal{O}-\mathcal{O} \hat{a}_{n}^{\dagger} \hat{a}_{n}\right)$.

This yields a large set of equations for the set of observables: $\left\langle\hat{a}_{n}\right\rangle,\left\langle\hat{a}_{n}^{\dagger}\right\rangle,\left\langle\hat{a}_{n} \hat{a}_{m}\right\rangle,\left\langle\hat{a}_{n}^{\dagger} \hat{a}_{m}\right\rangle,\left\langle\hat{a}_{n}^{\dagger} \hat{a}_{m}^{\dagger}\right\rangle,\left\langle\hat{a}_{n} \hat{a}_{m} \hat{a}_{l}\right\rangle$, etc. Due to the nonlinear interaction terms in the Hamiltonian, the set of equations cannot be closed since the evolution of a given observable depends on a correlator of higher order. However, in the regime of low occupancy of the modes, it is valid to truncate the set of equations at a given order. Below, we will show that the full result of Eq. (2), in the regime of considered pump intensities, is very well reproduced by Eq. (3) if one truncates after second-order correlators, $\left\langle\hat{a}_{n} \hat{a}_{m} \hat{a}_{l} \hat{a}_{k}\right\rangle$. While we still end up with a very large set of equations, the equations can be computer generated and the matrix of observables is much smaller than the density matrix in the particle-number basis, easing numerical solution.

## IV. ANALYZING ENTANGLEMENT

In analogy to the Bell inequalities that were used to evidence bipartite qubit entanglement, the violation of inequalities for locally realistic theories can be used to evidence entanglement between continuous variable states. Bipartite entanglement between any pair of modes can be evidenced by violation of the inequality $[27,29]$ :

$$
\begin{align*}
1 \leqslant S_{n m}= & V\left(\hat{p}_{n}-\hat{p}_{m}\right)+V\left(\hat{q}_{n}+\hat{q}_{m}\right)  \tag{4}\\
= & 1+\left\langle\hat{a}_{a}^{\dagger} \hat{a}_{n}\right\rangle+\left\langle\hat{a}_{m}^{\dagger} \hat{a}_{m}\right\rangle-\left\langle\hat{a}_{n} \hat{a}_{m}\right\rangle-\left\langle\hat{a}_{n}^{\dagger} \hat{a}_{m}^{\dagger}\right\rangle \\
& +\left\langle\hat{a}_{n}^{\dagger}-\hat{a}_{m}\right\rangle\left\langle\hat{a}_{m}^{\dagger}-\hat{a}_{n}\right\rangle, \tag{5}
\end{align*}
$$

where the variance is defined as $V(\hat{\mathcal{O}})=\left\langle\hat{\mathcal{O}}^{2}\right\rangle-\langle\hat{\mathcal{O}}\rangle^{2}$ and $\hat{p}_{n}$ and $\hat{q}_{n}$ are conjugate quadrature operators, defined by $\hat{p}_{n}=\left(\hat{a}_{n}+\hat{a}_{n}^{\dagger}\right) / 2$ and $\hat{q}_{n}=\left(\hat{a}_{n}-\hat{a}_{n}^{\dagger}\right) /(2 i)$. It is necessary to optimize over the phase references of the modes in Eq. (5) (which can be changed by local operations), that is, $\hat{a}_{n} \mapsto$ $\hat{a}_{n} e^{-i \phi_{n}}$. The minimum value of $S_{n m}$ is

$$
\begin{align*}
S_{n m}^{\prime}= & 1+\left\langle\hat{a}_{n}^{\dagger} \hat{a}_{n}\right\rangle+\left\langle\hat{a}_{m}^{\dagger} \hat{a}_{m}\right\rangle-\left\langle\hat{a}_{n}^{\dagger}\right\rangle\left\langle\hat{a}_{n}\right\rangle-\left\langle\hat{a}_{m}^{\dagger}\right\rangle\left\langle\hat{a}_{m}\right\rangle \\
& -\sqrt{\left\langle\hat{a}_{n}^{\dagger} a_{m}^{\dagger}\right\rangle-\left\langle\hat{a}_{n}^{\dagger}\right\rangle\left\langle\hat{a}_{m}^{\dagger}\right\rangle} \sqrt{\left\langle\hat{a}_{n} \hat{a}_{m}\right\rangle-\left\langle\hat{a}_{n}\right\rangle\left\langle\hat{a}_{m}\right\rangle}, \tag{6}
\end{align*}
$$

and the violation of the inequality $1 \leqslant S_{n m}^{\prime}$ evidences a bipartite entangled state.

The bipartite inequalities have been generalized to the multimode continuous variable case by van Loock and Furusawa [45]. For quadripartite entanglement, three inequalities must be simultaneously violated [42]:

$$
\begin{align*}
I_{1} & =V\left(\hat{p}_{0}-\hat{p}_{1}\right)+V\left(\hat{q}_{0}+\hat{q}_{1}+g_{2} \hat{q}_{2}+g_{3} \hat{q}_{3}\right) \geqslant 1,  \tag{7}\\
I_{2} & =V\left(\hat{p}_{1}-\hat{p}_{2}\right)+V\left(g_{0} \hat{q}_{0}+\hat{q}_{1}+\hat{q}_{2}+g_{3} \hat{q}_{3}\right) \geqslant 1,  \tag{8}\\
I_{3} & =V\left(\hat{p}_{2}-\hat{p}_{3}\right)+V\left(g_{0} \hat{q}_{0}+g_{1} \hat{q}_{1}+\hat{q}_{2}+\hat{q}_{3}\right) \geqslant 1 . \tag{9}
\end{align*}
$$

$g_{i}$ are arbitrary real parameters that should be chosen to optimize the violation of these inequalities. In Ref. [42], inequalities (7) and (9) are optimized separately over $\left(g_{2}, g_{3}\right)$ and ( $g_{0}, g_{1}$ ), respectively. While this can be done analytically, it does not guarantee that the $g_{i}$ are optimum for inequality (8). Instead, we prefer to make a numerical optimization of the parameters for the three inequalities simultaneously, where $g_{i}$ are chosen to minimize the largest value of the set of $I_{n}$, that is, $\tilde{I}=\max \left\{I_{1}, I_{2}, I_{3}\right\}$. As in the bipartite case, an additional optimization is made by varying the phase reference of each of the modes.


FIG. 2. (Color online) (a) Dependence of $\tilde{I}$ on the pump state population, $P_{0}=|\psi|^{2}$, calculated by truncated density matrices (spots) and Heisenberg equations up to second-order correlators (curves). $\tilde{I}<1$ characterizes a four-mode cluster state. The different sets of results correspond to different values of $\alpha_{2} / \alpha_{1}$. (b) Dependence of the two-mode entanglement parameter $\tilde{S}$ for the same parameters in (a). Parameters: $\hbar / \Gamma=10 \mathrm{ps}, \alpha_{1}=0.012 \mathrm{meV}$, and $E_{n}=0$.

## V. RESULTS

Figure 2(a) shows the variation of $\tilde{I}$ on the pump intensity for different values of $\alpha_{2} / \alpha_{1}$. We have chosen a realistic value of the polariton-polariton interaction constant, $\alpha_{1}$, in a GaAs/AlAs-based microcavity, which does not require any special conditions to be achieved. The spots indicate results from the solution of Eq. (2) over a truncated Fock basis, which can be matched by solving the Heisenberg equations [Eq. (3)] for the set of field correlators up to second order (shown by the solid curves). The relative difference between the two approaches is of the order of a few thousandths of a percent and shown in Fig. 3(a). The extremely small differences when solving the set of field correlators up to third order are also shown. To achieve multimode entanglement requires $\alpha_{2} \neq 0$.

It is well known that the exchange interaction between excitons provides the dominant mechanism [46,47] for polaritonpolariton interactions, which causes a strong repulsion between polaritons with parallel spins, represented by $\alpha_{1}>0$.


FIG. 3. (Color online) (a) Relative difference between $\tilde{I}$ calculated with truncated density matrices and $\tilde{I}$ calculated with the Heisenberg equations up to second- [green (light gray)] or third- [blue (dark gray)] order correlators. The parameters were the same as in Fig. 2. (b) Variation of $\tilde{I}$ [blue (dark gray)] and $\tilde{S}$ [green (light gray)] with $\alpha_{2}$ for a fixed pump intensity ( $P_{0}=10$ on the scale of Fig. 2). As in Fig. 2, the calculations using truncated density matrices are shown by spots and those with Heisenberg equations up to second-order correlators are shown by the curves. The other parameters were the same as in Fig. 2.

However, the mechanism is suppressed for polaritons with opposite spins, since it causes scattering to dark states, which are split in energy. For this reason, one expects $\left|\alpha_{2}\right|<\alpha_{1}$ [48]. The exact value of $\alpha_{2}$ seems to vary between samples and values ranging between $\alpha_{2}=-\alpha_{1}$ [49] and $\alpha_{2}=0.4 \alpha_{1}$ [8] have been reported. Multimode entanglement can be achieved for both positive and negative values of $\alpha_{2}$; however, it is more clearly evidenced for $\alpha_{2} / \alpha_{1} \approx \pm 0.25$. Figure 3(b) illustrates the variation of $\tilde{I}$ for a fixed pump intensity. The abrupt changes in the slope of the curve for some values of $\alpha_{2}$ correspond to crossovers in the inequality determining $\tilde{I}$ (that is, which of $I_{1}, I_{2}$, and $I_{3}$ is the largest after minimization of $\tilde{I}$ ).

Figure 2(b) shows the corresponding value of the parameter $\tilde{S}=\max \left\{S_{01}^{\prime}, S_{12}^{\prime}, S_{23}^{\prime}, S_{30}^{\prime}\right\}$, which was calculated using Eq. (6). This parameter shows that ordinary bipartite entanglement is present between pairs of modes and exhibits a similar dependence on the parameters as $\tilde{I}$. The variation of $\tilde{S}$ with $\alpha_{2}$ for a fixed pump intensity is also shown in Fig. 3(b).

We would like to stress that the signal states are in the low occupation regime, which can be seen from Fig. 4(a). For this reason, additional parametric scattering processes such as those in Ref. [9] are not stimulated, since we are much below threshold. Even if these processes were to occur, energymomentum conservation would match them to a different state than our pump state at the bottom of the lower polariton branch [see Fig. 1(a)]. That is, our signal states are sufficiently far from the magic angle [9]. It is therefore also not possible for our pump state to trigger such parametric scattering processes.

An important question that should be asked when thinking of quantum effects in nanostructures is the significance of pure dephasing, which is associated with exciton-phonon scattering. Theoretically, pure dephasing can be accounted for by adding the term

$$
\begin{align*}
& \left.i \hbar \frac{d \boldsymbol{\rho}}{d t}\right|_{\text {pure deph }} \\
& \quad=\frac{i \Gamma_{P}}{2} \sum_{n}\left(2 \hat{a}_{n}^{\dagger} \hat{a}_{n} \rho \hat{a}_{n}^{\dagger} \hat{a}_{n}-\hat{a}_{n}^{\dagger} \hat{a}_{n} \hat{a}_{n}^{\dagger} \hat{a}_{n} \rho-\rho \hat{a}_{n}^{\dagger} \hat{a}_{n} \hat{a}_{n}^{\dagger} \hat{a}_{n}\right) \tag{10}
\end{align*}
$$

to the right-hand side of Eq. (2). The effect of this term on the parameters $\tilde{I}$ and $\tilde{S}$ is shown in Fig. 4(b). Following Ref. [50], a realistic estimate of the dephasing is $\Gamma_{P}=0.2 \mu \mathrm{eV}$. In our scheme, entanglement is very robust against this level of dephasing, requiring a dephasing strength over two orders of magnitude higher to be broken.

One may also consider the effects of nonlinear loss [17] or background photoluminescence (due to phonon-assisted scattering). We note that we operate at low intensities such that the former should not be a problem. Phonon-assisted scattering


FIG. 4. (Color online) (a) Dependence of the average signal occupation, $\left\langle\hat{a}_{n}^{\dagger} \hat{a}_{n}\right\rangle$, on the pump state population, $P_{0}=|\psi|^{2}$, calculated by truncated density matrices (spots) and Heisenberg equations up to second-order correlators (curves). (b) The effect of pure dephasing on $\tilde{I}$ [blue (dark gray)] and $\tilde{S}$ [green (light gray)], for $\alpha_{2}=-0.25 \alpha_{1}$ and a fixed pump intensity ( $P_{0}=10$ on the scale of Fig. 2). As in Fig. 2, the calculations using truncated density matrices are shown by spots and those with Heisenberg equations up to second-order correlators are shown by the curves. The parameters were the same as in Fig. 2.
is reduced for wave vectors at and below the bottleneck region in planar microcavities, where our signal states lie, and fully suppressed in mesa structures. In addition, background photoluminescence does not destroy entanglement [51] and could be further suppressed in a pulsed excitation scheme.

## VI. CONCLUSION

The generation of multimode cluster states is the challenging step toward the development of one-way quantum computers. We have considered the use of a semiconductor microcavity for the generation of a four-mode continuous variable cluster state, by exploiting the spin degree of freedom of polaritons together with either interbranch scattering in planar systems or scattering between orbital angular momentum carrying modes in mesas. The system can be described in a coupled quantum mode model using density matrices in a truncated Fock basis or the Heisenberg equations for a truncated set of field correlators. These two methods give matching results for low excitation density, which evidence four-mode entanglement by violation of the van LoockFurusawa quadripartite inequalities. Such entanglement survives a realistic strength of pure dephasing.

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