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Effects of traffic lights for Manhattan-like urban traffic network in intelligent transportation systems

Bokui Chen^{a*}, David Z.W. Wang^{a*}, Yachun Gao^b, Kai Zhang^{c,d*}, Lixin Miao^{c,d} and Binghong Wang^e

^a*School of Civil and Environmental Engineering, Nanyang Technological University, Singapore;*

^b*School of Physical Electronics, University of Electronic Science and Technology of China,*

Chengdu, China; ^c*Division of logistics and transportation, Graduate school at Shenzhen,*

Tsinghua University, Shenzhen, China; ^d*Center of Environmental science and new energy*

technology, Tsinghua-Berkeley Shenzhen Institute, Shenzhen, China; ^e*Department of Modern*

Physics, University of Science and Technology of China, Hefei, China

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Traffic light is the core part of advanced transportation management system. Assuming travelers receive and follow the route guidance information designed by two specific route choice strategies, this paper investigates how the traffic lights rule, period and its quantity affect the traffic system performance on a Manhattan-like urban network. Firstly, the simulation results of the average flow against the traffic density and the vehicle distribution are studied under four traffic light rules. Then the relationship between the extremum of average speed and traffic light period are analyzed and the theoretical results have been proved basically in agreement with the simulation results. Lastly, the effects of the number of traffic lights on average flow and vehicle distribution are discussed. From these results, it is concluded that the traffic system performance can be improved if the anticlockwise rule combined with the congestion coefficient feedback strategy based route guidance is adopted and the number of traffic lights is reduced to its minimum requirement.

Keywords: Traffic light, Cellular automaton models, Intelligent transportation systems, Advanced transportation management system

1. Introduction

Traffic network capacity is an important index for assessing the transportation system performance. To alleviate traffic congestion, many major cities have increased the investment in transportation infrastructure in recent years. However, the infrastructure investment is usually prohibitively expensive but marginally effective, as the improved infrastructure leads to induced travel demand, gives rise to further growth of private cars population, and aggravates traffic jams. Therefore, in order to contain traffic congestion effectively, we must enhance the transport facilities condition on one hand and bring the scientific management and advanced technology to the traffic systems on the other hand. Many scientists from various research fields, such as mathematics, physics, engineering, computer science and economics, devote themselves to study and solve the traffic problems (Chow et al. 2015; Shepherd 2014; Sau et al. 2014; Zhao et al. 2014; Fu, Lam and

*First author. These authors contributed equally to this work.

*Corresponding author. Email: wangzhiwei@ntu.edu.sg

Meng 2014; Du and Wang 2014; Wang and Du 2013, 2016; Szeto 2013; Szeto and Wong 2011; Szeto, Wu and Ho 2011; Xie et al. 2012; Xie, Wang and Gao 2016).

Intelligent transportation systems (ITS), as a promising solution measure, has intrigued scientists for many years (Bazzan and Klügl 2014; Sussman 2000; Ge et al. 2004; Wang 2010; Crainic, Gendreau and Potvin 2009; Szeto and Wong 2011). It consists of many sub-systems, one of which is advanced transportation management systems (ATMS) (Bazzan and Klügl 2014; Sussman 2000). As is well known, traffic lights have a great impact on traffic systems (Han 1996; Lo 1999; Nagatani 2005; Yin 2008; Cai, Wong and Heydecker 2009; Zhu et al. 2011; Qin and Khan 2012; Lv and Zhang 2012; Ren et al. 2013). ATMS supervises the traffic network mainly by traffic light control.

In this study, we assume the advanced transportation information system (ATIS) is implemented, which acquires real-time traffic data, and then disseminates the information to travelers. ATIS will provide route guidance to travelers based on various route choice strategies and it is assumed that the travelers will follow the route guidance. As traffic light control has significant impacts on the traffic system performance, it is imperative for the traffic management agency to fully understand how to make rational and efficient controlling of traffic lights in presence of ATIS. In 2010, Scellato et al. (2010) investigated the traffic flow in urban networks based on local route choice strategy. But the effects of traffic lights were not taken into account. Afterwards, Li et al. (2011) studied the influence of one route choice strategy (i.e., mean velocity feedback strategy) on Manhattan-like urban network in ITS with consideration of traffic lights. Nevertheless, it should be noted that other than this mean velocity feedback route choice strategy, many other route choice strategies adopted by the ATIS have been proposed by the researchers (Wahle et al. 2000; Lee et al. 2001; Wang et al. 2005; Dong et al. 2009, 2010; Chen et al. 2011, 2012c,a,b; Dong and Ma 2010; Li et al. 2016). For example, Wang et al. (2005) and Chen et al. (2012a) proved that the congestion coefficient feedback strategy and the exponential function feedback strategy have greater advantages than the mean velocity feedback strategy. However, the effects of traffic lights on traffic system performance, when ATIS with these route choice strategies is presented, have not been addressed in the literature. Besides, the derivations of the expression of average speed as a function of traffic light period have not been fully investigated in previous researches. In view of this, based on two route choice strategies (i.e., congestion coefficient feedback strategy (CCFS) and the exponential function feedback strategy (EFS)) and a Manhattan-like urban traffic network, this paper makes a thorough investigation on how traffic light rules, traffic light period and traffic light number affect the traffic system, so that effective guideline for traffic management can be provided on how to set traffic lights in traffic network.

In recent years, modeling and simulation of traffic flow have attracted much research attention, especially on mathematical study of traffic flow theory (Zhang, Wong and Dai 2009, 2012; Zhang, Wu and Wong 2012; Dong and Mahmassani 2013; Chiou, Zhang and Chen 2014; Wu, Zhang and Wong 2014; Ngoduy 2015). Rather than using more aggregated macroscopic approaches, we would apply a microscopic, cellular automaton model based simulation approach as a traffic flow mechanism in this paper. Cellular automaton models are also indispensable for study of traffic flow because it is easy to code and simulate nonlinear behavior of traffic system (Wolfram 1983; Nagel and Schreckenberg 1992; Fukui and Ishibashi 1996; Barlovic et al. 1998; Li, Wu and Jiang 2001).

The paper is organized as follows: Section 2 introduces the Manhattan-like urban traffic network, the traffic light rules and two route choice strategies. Section 3 discusses the traffic light rules, the expressions of relationship between the extremum of average speed and traffic light period and the effects of traffic light number on network. The last section draws a conclusion of this paper.

2. Transportation Systems

2.1. Manhattan-like urban traffic network

Manhattan urban traffic network is similar to a square regular network. Li et al. (2011) presented an exemplified model in which the network consists of $N \times N$ square lattice. Between any two neighboring intersections, there are opposed-roads and each road is divided into L cells (see Figure 1). The cars move along the right lane and are updated by Nagel-Schreckenberg (NS) model which is the most popular cellular automaton (CA) models (Nagel and Schreckenberg 1992). The NS model can be decomposed to the following four rules:

R1 Acceleration:

$$v_i(t) \rightarrow v_i(t + \frac{1}{3}) = \min\{v_i(t) + 1, v_{max}\}. \quad (1)$$

R2 Braking:

If $v_i(t + \frac{1}{3}) > d_i(t)$, then

$$v_i(t + \frac{2}{3}) = d_i(t) \quad (2)$$

else

$$v_i(t + \frac{2}{3}) = v_i(t + \frac{1}{3}) \quad (3)$$

R3 Randomization with probability p :

$$v_i(t + \frac{2}{3}) \rightarrow v_i(t + 1) = \max\{0, v_i(t + \frac{2}{3}) - 1\}. \quad (4)$$

R4 Moving:

$$x_i(t + 1) = x_i(t) + v_i(t + 1). \quad (5)$$

here $d_i(t)$ is the number of empty cells in the front of vehicle i , v_{max} is the maximum speed and $x_i(t)$ is the position of the i th vehicle at time t . It is noteworthy that the distance d of the leading vehicle requires special treatments (Li et al. 2011). Initially, each vehicle is randomly assigned with their origin and destination and then reassigned with another destination after it arrives at the previous one. Therefore, the number of vehicles in the network is a constant. In order to determine it, we just need to set the traffic density.

2.2. Traffic light rules

In order to avoid potential situation that several vehicles coming from different approaches compete for one intersection, Li et al. (2011) assumed that the traffic lights are synchronous and have fixed period. There are four incoming streets at each intersection. If a light stays green for one incoming streets for T time, it will stay red for the other three streets at the same time. When the green light is on for a street, the vehicles at the head of it could go straight, turn left or right, or make a U-turn (see Figure 2). In

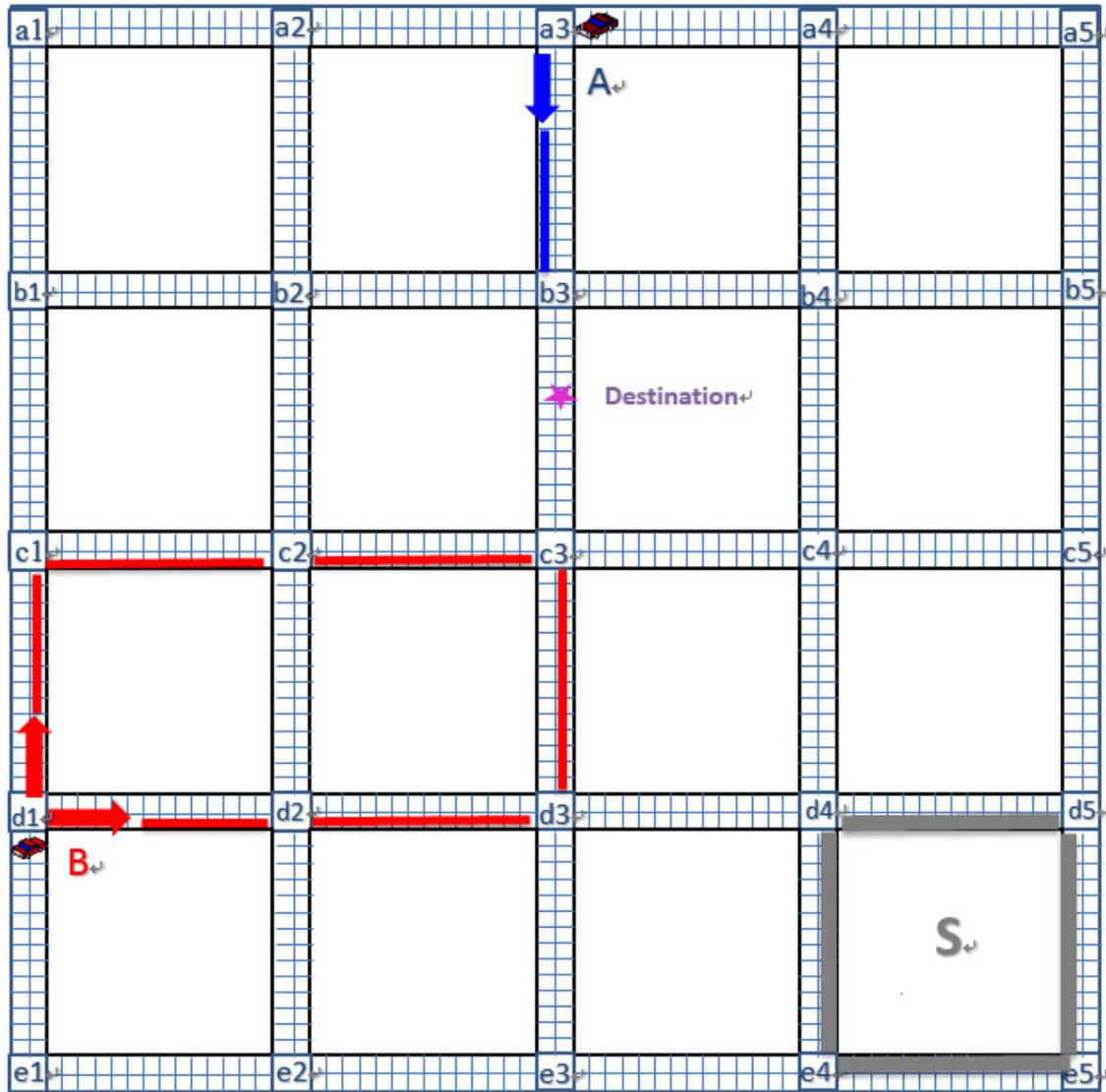


Figure 1. The Manhattan-like urban traffic network model.

this case, there are four change rules for the traffic light. We marked four roads with A,B,C,D,

- (a) Clockwise rule: the traffic lights turn green in a clockwise order A-B-C-D-A.
- (b) Anticlockwise rule: the traffic lights turn green in an anticlockwise order D-C-B-A-D.
- (c) 8-like rule: the traffic lights turn green in a 8-like order A-C-B-D-A.
- (d) Reverse 8-like rule: the traffic lights turn green in a reverse 8-like order A-B-D-C-A.

2.3. Route choice strategy

Route choice strategy is the core part of advanced travelers information systems (ATIS). Traffic control center firstly collects the traffic data and calculate the route guidance information based on a route choice strategy. The guidance information are then disseminated to drivers. In our model, vehicles move along the shortest path to destination. However, there are usually more than one shortest paths between one origin and desti-

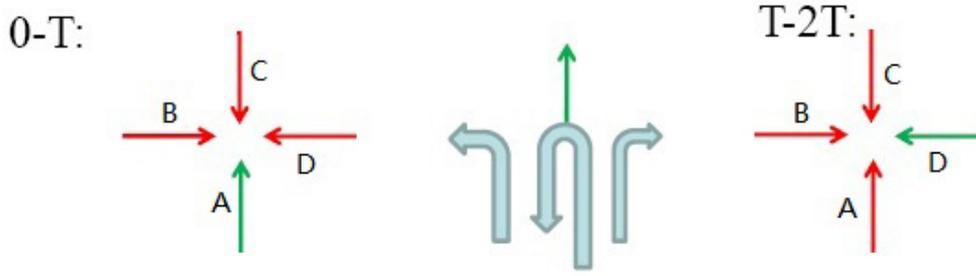


Figure 2. The traffic light rules and four optional direction of vehicles at a intersection.

nation pair. If there is only one path, the vehicle will move along with this path. If there are two paths available, the vehicle will choose one of them based on the route choice strategies.

Li et al. (2011) applied mean velocity feedback strategy in this model. However, Wang et al. (2005) and Chen et al. (2012a) proved that the congestion coefficient feedback strategy (CCFS) and the exponential function feedback strategy (EFFS) have greater advantages than mean velocity feedback strategy in previous studies. Thus, in this paper, we select them as the route choice strategy when providing route guidance information to the travelers. The congestion coefficient feedback strategy suggests vehicles to choose the road with a smaller congestion coefficient (Wang et al. 2005). The congestion coefficient is defined as follows:

$$C = \sum_{i=1}^m n_i^w, \quad (6)$$

where n_i is the vehicle number of the i th congestion cluster in which vehicles are close to each other without a gap between any two of them. Every cluster is evaluate data weight w , here $w = 2$. In the exponential function feedback strategy (EFFS)(Chen et al. 2012a), such as CCFS, vehicles choose the road with a smaller exponential congestion coefficient. The exponential congestion coefficient is defined as

$$C_e = \sum_{i=1}^m \exp(-y_i) \times n_i^w, \quad (7)$$

here n_i is the same as in CCFS and y_i stands for the middle position of the i th congestion cluster. These two strategies are applied in Manhattan-like urban traffic network as follows:

- (a) A vehicle at an intersection will move along the path with the shortest distance between the present intersection and the destination to the next intersection.
- (b) If there are more than one shortest paths, vehicles will choose one road out of the tied shortest paths with the smallest congestion coefficient in CCFS or the exponential congestion coefficient in EFFS, which will be followed by the vehicles to move from the present intersection to the next one.

For instance, if there is only one shortest path, such as vehicle A in Figure 1, it moves along the blue path to the destination. If there are more than one shortest paths, the driver will choose a road with smaller congestion coefficient to move to the next intersection. Using vehicle B in Figure 1 as another example, there are two shortest paths for it (green lines). ATIS will choose a road with a smaller congestion coefficient between the road d1-c1 and road d1-d2 to present to driver and then vehicle B moves to the next

intersection. When it arrives, ATIS will keep searching the next road in the same way until it reaches the destination.

3. Analysis and Simulation

In the simulation, the system area is fixed $N * N = 20 * 20$ and the road length is $L = 100$ cells. One cell is $7.5m$. The maximum velocity and the randomization probability are $v_{max} = 3$ and $p = 0.25$ respectively in NS model. The simulation results are obtained by discarding the first 10^5 time steps and averaged over the next 10^4 time steps. In addition, we set that the traffic light changes in accordance with the clockwise rule in subsection 3.2 and 3.3 and the traffic light period is $T = 20$ in subsection 3.1 and 3.3.

3.1. Traffic light rules

In section 2.2, four traffic light rules are introduced. Figure 3 illustrates the relationship between average flow and traffic density ρ by following these rules. When traffic density increases, traffic system shows four states, i.e., free-flow state, saturation state, metastable state and deadlock state, respectively. The red curve in Figure 3(b), for instance, displays all states. When $\rho < 0.05$, average flow increases almost linearly with traffic density. That means the growing traffic density does not bring about traffic jam. At this time, traffic system is in free-flow state. When $0.05 \leq \rho < 0.22$, the traffic density has negligible impact on average flow. That is the saturation state. In this state, the traffic flow increases slowly with respect to traffic density. When $0.22 \leq \rho < 0.26$, the average flow falls to shocks and the system is in metastable state. In this state, the system can either evolve into deadlock state or saturation state. When $\rho \geq 0.26$, the system is in the deadlock state, in which no vehicle can move. The threshold density $\rho = 0.26$ is called the critical density of deadlock state.

Simulation results show that, in presence of route guidance information based on exponential function feedback strategy, the critical density of deadlock state of 8-like and reverse 8-like traffic light rules are larger than those of another two rules (see Figure 3(a)). However, the exactly opposite results are obtained by using congestion coefficient feedback strategy. From Figure 3(b), it can be found that critical density of deadlock state of clockwise and anticlockwise rules are larger than those of other two ones. There are two reasons for this phenomenon. The first one is the distinction between two strategies. The essential distinction between congestion coefficient feedback strategy (CCFS) and exponential function feedback strategy (EFFS) is that CCFS gives a weight to each congestion cluster based on the length of it. For the congestion clusters with the same length, CCFS will give the same weight to them. However, in EFFS, the location of congestion cluster is also taken into account. Each congestion cluster is given a weight based on the length and location of it. For the same length congestion clusters, the closer the cluster is away from the entrance, the larger the weight is given. The second reason is that in fluid dynamics, if all individuals are equal, rotational flow can make larger flow than axisymmetric flow. CCFS equally give a weight to each cluster. So in Figure 3(b) critical density of deadlock state of clockwise and anticlockwise rules are larger than those of other two ones.

One can also observe that, no matter which rule is followed, the critical density of deadlock state of congestion coefficient feedback strategy is larger than that of exponential function feedback strategy. It is also caused by the distinction between two strategies. Chen et al. (2012a) pointed out that exponential function feedback strategy is better than congestion coefficient feedback strategy in one-dimensional semiopen traffic systems be-

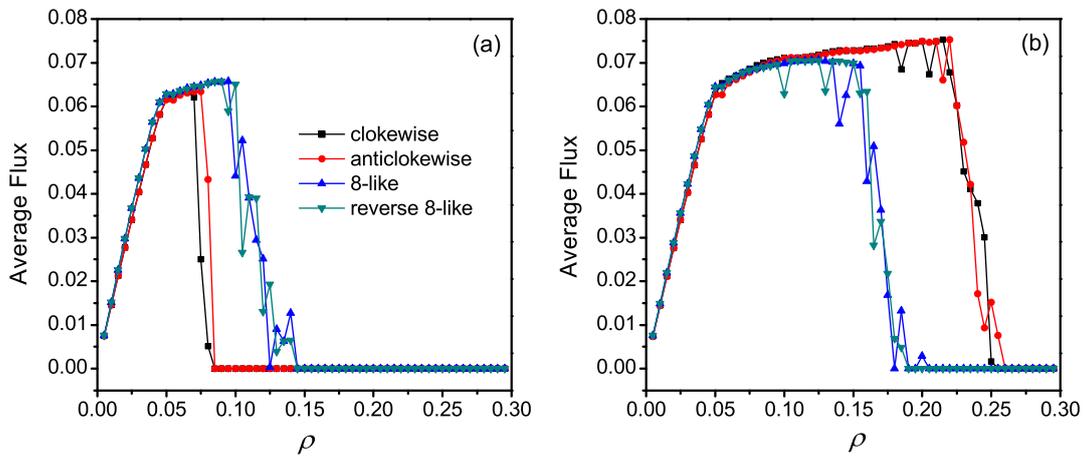


Figure 3. The average flow as a function of traffic density. (a) Exponential function feedback strategy. (b) Congestion coefficient feedback strategy.

cause the location of congestion cluster as a weight is taken into account. However, the Manhattan-like urban traffic network is a close and symmetrical system in which traffic information at any location are equally important. So congestion coefficient feedback strategy is more favorable for this type of traffic network.

Figure 4 displays the typical patterns of the vehicle distribution in saturation state by employing four traffic light rules and congestion coefficient feedback strategy. The grid coarsening method is applied here (Li et al. 2011). Each subgraph in Figure 4 corresponds to the 25×25 Manhattan-like urban traffic network. Under the clockwise rule, the distribution of outer rings exhibit four-angle-star structure and the inner rings exhibit a clockwise windmill-like structure (see Figure 4(a)). Under the anticlockwise rule, the outer rings are also four-angle-star structure and the inner rings are anticlockwise windmill-like structure (see Figure 4(b)). Figure 4(c) and (d) show the vehicle distribution under the 8-like rule and the reverse 8-like rule. The inner rings display a Y-like structure and the outer rings display a triangle structure. Different colors show different the vehicle density in the area of network. For instance, in Figure 4 (a), blue indicates the vehicle density of less than 0.1. That means in the edge of urban, vehicle density is less than 0.1. Orange and red indicate the vehicle density on interval $[0.5, 0.6]$ and $[0.6, 0.7]$. That means in the center area of urban, vehicle density is more than 0.5. In this area, congestion is more likely to occur. One can see that the orange and red areas in Figure 4 (c) and (d) are larger than Figure 4 (a) and (b). Some areas of Figure 4 (c) and (d) show gray which indicates the vehicle density of more than 0.7. It indicates that the high-density areas of clockwise and anticlockwise rules are much less than those of another two rules. Moreover, the red area of Figure 4 (b) is much less than that of Figure 4 (a). Thus the anticlockwise rule is the best choice. It can make vehicles evenly distributed.

Taking all the above into consideration, we can safely draw the following conclusions:

- (a). No matter which traffic light rule is followed, the critical density of deadlock state of congestion coefficient feedback strategy is larger than that of exponential function feedback strategy.
- (b). The shape of vehicle distribution is highly related to traffic light rules.
- (c). The best choice for traffic system is a combination of the anticlockwise traffic light rule and the congestion coefficient feedback strategy based route guidance information provision.

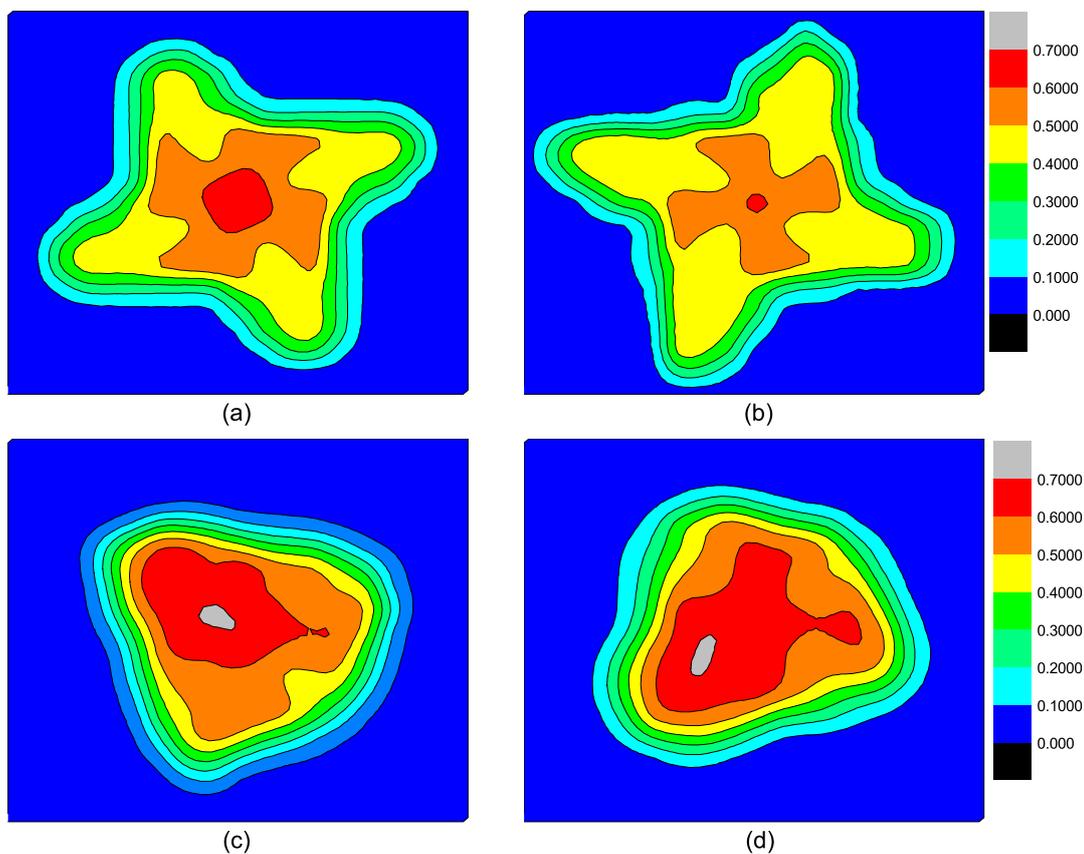


Figure 4. Typical patterns of the vehicle distribution in saturation state with congestion coefficient feedback strategy. (a) Clockwise rule with $\rho = 0.17$. (b) Anticlockwise rule with $\rho = 0.17$. (c) 8-like rule with $\rho = 0.15$. (d) Reverse 8-like rule with $\rho = 0.15$.

3.2. Traffic light period

Traffic light period significantly influences traffic systems. The longer period may cause vehicle queue accumulation on the roads and the shorter period may bring about congestion at the intersections because there is not enough time for vehicles to be dissipated. Thus, a thorough investigation on the effect of traffic light period is necessary.

Figure 5 illustrates the curves of the average speed against the period T by using two different route choice strategies. The corresponding parameters in the Figure 5 (a) and (b) are $L = 100, \rho = 0.015$ and $L = 1000, \rho = 0.03$ respectively, at which the network is in the free flow state. In Figure 5 (a), the average speed decreases with increasing T until $T = 20$ and then reduces with oscillation. Brockfeld et al. (2001) analyzed the oscillations by considering a minimal network with one single intersection. In Figure 5 (b), the amplitude of the average speed goes up with the traffic light period. As the traffic flow is defined as

$$F = \bar{V}\rho \tag{8}$$

and density ρ is a constant, the changes of the average speed are equivalent to that of the average flow.

In the free-flow network, there are almost no interaction among vehicles, the average velocity is determined by traffic light. Therefore, the travel-time of a vehicle on the road

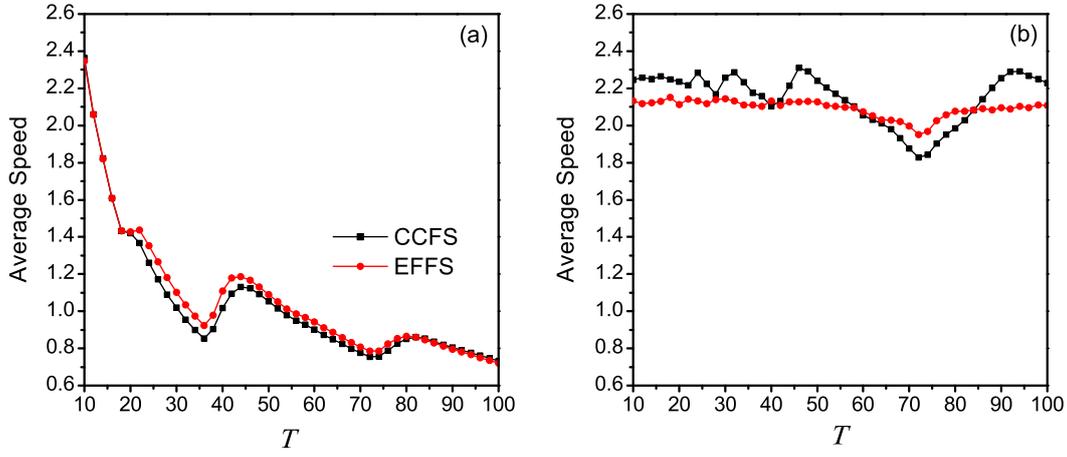


Figure 5. The relationship between the average speed and the traffic light period by adopting two kinds of route choice strategies. The parameters set in (a) are $L = 100$, $\rho = 0.015$ and the parameters in (b) are $L = 1000$, $\rho = 0.03$.

is equal to free travel-time

$$T_{free} = \frac{L}{v_{max} - p}, \quad (9)$$

here v_{max} and p are the same as defined in the NS model.

When $T \geq T_{free}$, it is supposed that there is a vehicle at the end of road A (see Figure 2) and the traffic light is green. This vehicle has gone n roads being the same direction as road A since the green light is on. So the remaining time for green light is

$$\Delta T = T - nT_{free}, \quad (10)$$

where $n = 0, 1, 2, \dots$ is determined by the inequality $0 < \Delta T \leq T_{free}$. The vehicle has four options next time. If it turns right into road D, this vehicle will cross over road without waiting, for the reason that the traffic light will be green after ΔT time steps based on the clockwise rule and $\Delta T \leq T_{free} < T$. If it goes straight into road C, the vehicle will wait for $T + \Delta T - T_{free}$ time steps at the exit of this road, as the traffic light will be green after $T + \Delta T$ time steps. If it turns left into road B, the vehicle will wait for $2T + \Delta T - T_{free}$ time steps, as the traffic light will be green after $2T + \Delta T$. If it makes a U-turn and moves to road A again, the vehicle will wait for $3T + \Delta T - T_{free}$ time steps, for the traffic light will be green after $3T + \Delta T$. Thus the average waiting time of the above four cases is

$$\begin{aligned} T_{wait} &= \frac{1}{4}(T + \Delta T - T_{free} + 2T + \Delta T - T_{free}) \\ &\quad + \frac{1}{4}(3T + \Delta T - T_{free}) \\ &= \frac{1}{4}(6T + 3\Delta T - 3T_{free}) \\ &= \frac{3}{4}(3T - (n + 1)T_{free}). \end{aligned} \quad (11)$$

Since $0 < \Delta T \leq T_{free}$, $nT_{free} \leq T \leq (n + 1)T_{free}$. That means T_{wait} gets its maximum

when $T = (n + 1)T_{free}$. For the average speed is

$$\bar{v} = \frac{L}{T_{wait} + T_{free}}, \quad (12)$$

the average speed reaches its local minimum at

$$T = (n + 1)T_{free} \quad (13)$$

here $n = 0, 1, 2 \dots$.

When $T < T_{free}$, we assume a vehicle lies at the exit of road A with green light on and nT time steps have already passed since the vehicle went into this road. Thus the remaining time of the green light is $T - \Delta T$, where $\Delta T = T_{free} - nT$ and n is determined by $0 \leq \Delta T < T$. At next time step, if the vehicle turns right into road D, the waiting time of it is $T - [T_{free} - (nT + \Delta T)]$, i.e. $2(T - \Delta T)$. Likewise, the waiting time of a vehicle at the end of road C, road B and reversed road A are $3(T - \Delta T)$, $4(T - \Delta T)$ and $5(T - \Delta T)$ respectively. Thus, the average waiting time is

$$\begin{aligned} T_{wait} &= \frac{14}{4}(T - \Delta T) \\ &= \frac{14}{4}((n + 1)T - T_{free}). \end{aligned} \quad (14)$$

For $0 \leq \Delta T < T$, T_{wait} achieves its minimum when $T_{free} = nT$, which means the average speed achieves local maximum when

$$T = \frac{T_{free}}{n} \quad (15)$$

To summarize, in free-flow state, if period is $T \geq T_{free}$, the average speed will reach the local minimum at $T = (n + 1)T_{free}$. If period is $T < T_{free}$, the local maximum will be achieved at $T = T_{free}/n$. The simulation results show the same laws. Since the maximum velocity is set $v_{max} = 3$ and the randomization probability is set $p = 0.25$, $T_{free} = 36.4$. In Figure 5(a), it is easy to find that the local minimum of speed appears at $T \approx T_{free}$ or $T \approx 2T_{free}$ and the local maximum appears at $T \approx T_{free}/2 = 18.2$. Similar results could be found in the long road network (see Figure 5(b)) in which $T_{free} = 363.6$.

3.3. Traffic light number

The number of traffic lights is also a factor affecting traffic systems. If there are many traffic lights, vehicles will frequently go through the intersections, which could increase the probability of congestion at crossroads. If there are few traffic lights, the probability of congestion on road and even traffic accidents will also increase and the network accessibility will be compromised. Therefore, the effects of traffic light number onto the network traffic performance is also investigated in this section.

In reality, the urban area is limited. Given the same area, four traffic networks with different sets of parameters are selected $N * N = 50 * 50$ & $L = 40$, $N * N = 25 * 25$ & $L = 80$, $N * N = 10 * 10$ & $L = 200$, $N * N = 5 * 5$ & $L = 400$. The number of traffic lights (NT) corresponding to the above four sets are $NT = 2500$, $NT = 625$, $NT = 100$, $NT = 25$ respectively. Figure 6 shows the average flow as a function of the traffic density in different traffic networks by using the exponential function feedback strategy (Figure 6(a)) and the congestion coefficient feedback strategy (Figure 6(b)). As the number of

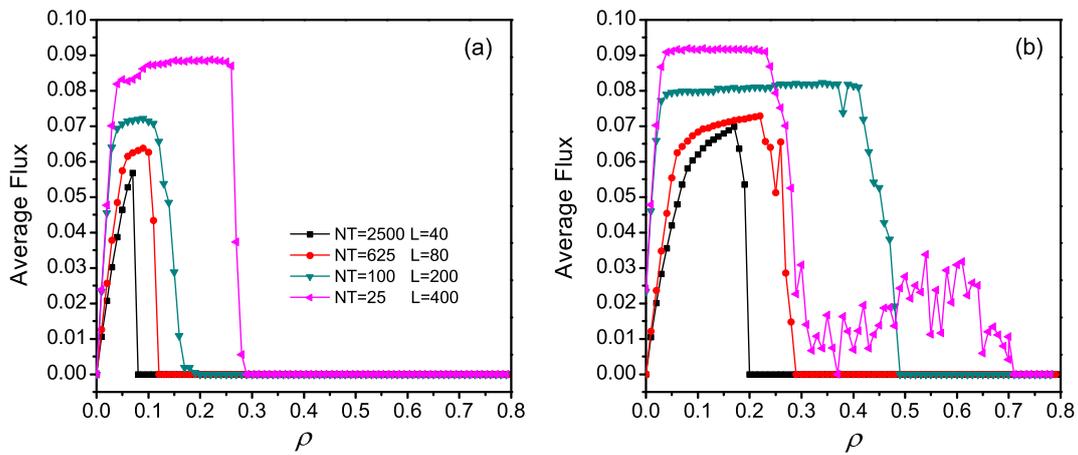


Figure 6. The average flow as a function of the traffic density in four different networks by using (a) the exponential function feedback strategy and (b) the congestion coefficient feedback strategy.

traffic lights decreases, the average flow of saturation state and the critical density of deadlock state are go up. The critical flow density of deadlock state of congestion coefficient feedback strategy is larger than that of exponential function feedback strategy in any network.

The typical patterns of vehicle distribution in the saturation state of different networks are displayed in Figure 7. Figure 7(a)-(d) are obtained by using exponential function feedback strategy. One can see that the central areas of Figure 7(b) and (c) observe a high density, especially in Figure 7(c). Compared with other ones, in the network with $NT = 25$ Figure 7(d), the vehicle distribution is more even. Similar results are also obtained by adopting congestion coefficient feedback strategy (see Figure 7(e)-(h)). The central areas of Figure 7(f) and (g) also have a high density and (g) is much higher. The most uniform distribution also appears in the same network, i.e. Figure 7(h). Thus the vehicle distribution is indeed independent of traffic light number. If a homogeneous vehicle distribution is required, the traffic light number must be determined by analysis and simulation. In traffic systems, high traffic density in the center area is more likely to lead to congestion, which could spread to the whole network as time evolves. Nevertheless, to provide a desired level of network accessibility for the transportation system, a certain traffic light number is required. Based on the above analysis, we can draw the following conclusions:

- (a) As the number of traffic light decreases, the average flow of saturation state and the critical density of deadlock state increase.
- (b) The critical density of deadlock state of congestion coefficient feedback strategy is larger than that of exponential function feedback strategy in five different networks.
- (c) The vehicle distribution is not relevant to the traffic light number.

4. Conclusion

Traffic lights, as the core component of advanced transportation management systems, affect the traffic performance significantly. On the basis of Manhattan-like urban traffic network with two route choice strategies, this paper investigates the effects of the traffic light rules, the traffic light period and the traffic light number on traffic systems. Firstly, the effect of traffic light rules on traffic system is studied. From the results, it is concluded that adopting the anticlockwise rule together with the congestion coefficient feedback

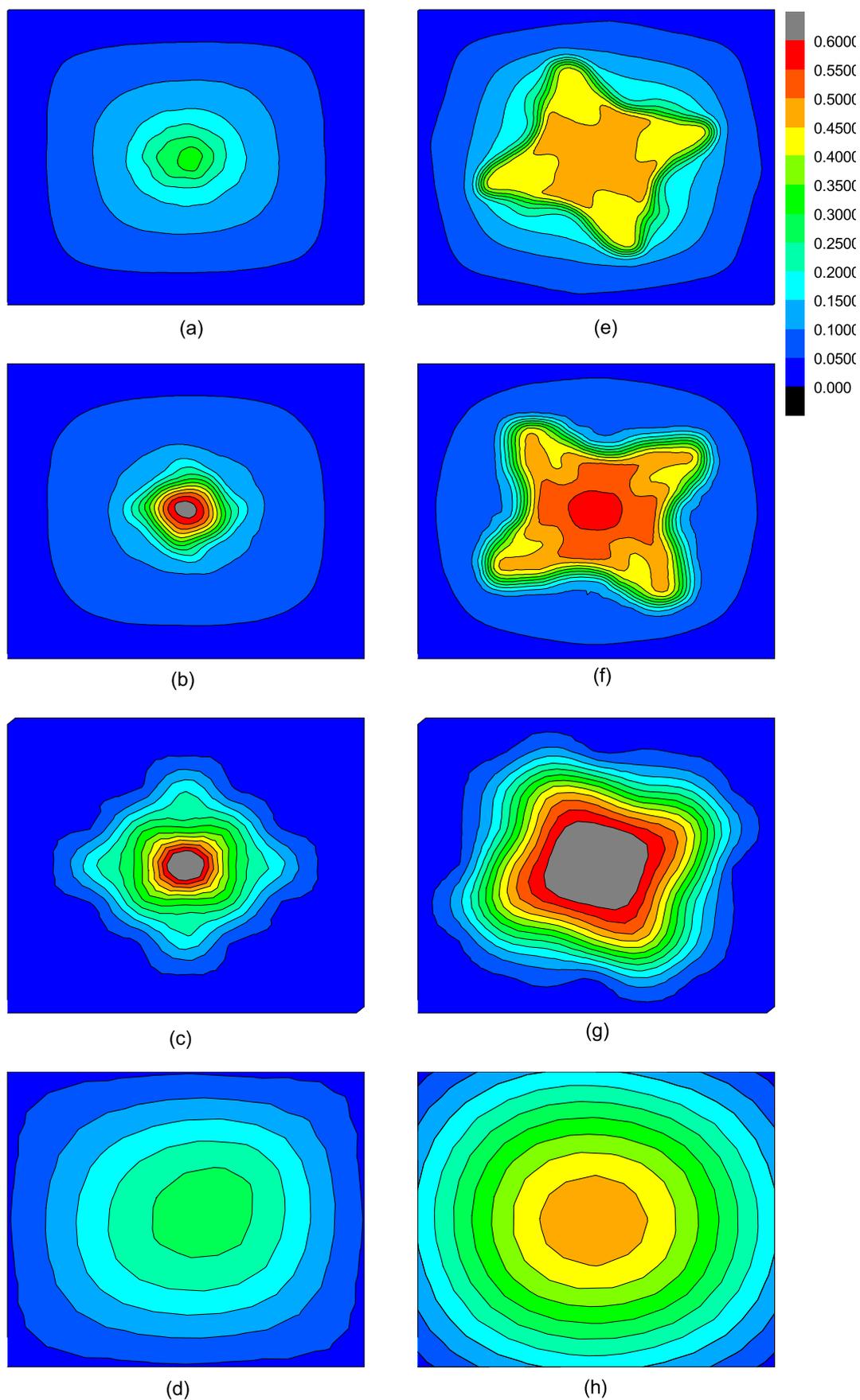


Figure 7. Typical patterns of the distribution of vehicles. In (a)-(d), ATIS uses exponential function feedback strategy and the traffic density is $\rho = 0.07$, at which the network is in the saturation state. (a) $NT = 2500$; (b) $NT = 625$, (c) $NT = 100$; (d) $NT = 25$. In (e)-(h), ATIS uses the congestion coefficient feedback strategy and the traffic density is $\rho = 0.15$, at which the network is also in the saturation state. (e) $NT = 2500$, (f) $NT = 625$, (g) $NT = 100$, (h) $NT = 25$.

strategy is beneficial for traffic system. Then we deduce the expressions of local maximum and local minimum of average velocity as a function of traffic light period. In free-flow state, if traffic light period is $T \geq T_{free}$, the average speed will reach the local minimum at $T = (n + 1)T_{free}$. If period is $T < T_{free}$, the local maximum will be achieved at $T = T_{free}/n$. According to two kinds of urban networks with different route lengths, the simulation results of relationship between average velocity and traffic light period are obtained, which are basically in agreement with the theoretical results. Lastly, the influence of the traffic light number is discussed based on four traffic networks with the same area. Simulation results indicate that as the number of traffic light decreases, the average flow of saturation state and the critical density of deadlock state are on the increase. The vehicle distribution is not relevant to the traffic light number.

In conclusion, it will improve the traffic systems performance if the anticlockwise rule combined with the congestion coefficient feedback strategy is adopted and the number of traffic lights is reduced to its minimum requirement.

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