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Synthesis of multiple degrees-of-freedom spatial-motion compliant parallel mechanisms with desired stiffness and dynamics characteristics

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Abstract

This paper presents a new design method to synthesize multiple degrees-of-freedom (DOF) spatial-motion compliant parallel mechanisms (CPMs). Termed as the beam-based structural optimization approach, a novel curved-and-twisted (C-T) beam configuration is used as the basic design module to optimize the design parameters of the CPMs so as to achieve the targeted stiffness and dynamic characteristics. To derive well-defined fitness (objective) functions for the optimization algorithm, a new analytical approach is introduced to normalize the differences in the units, e.g., N/m or Nm/rad etc., for every component within the stiffness matrix. To evaluate the effectiveness of this design method, it was used to synthesize a 3-DOF spatial-motion ($\theta_x - \theta_y - Z$) CPM that delivers an optimized stiffness characteristics with a desired natural frequency of 100 Hz. A working prototype was developed and the experimental investigations show that the synthesized 3-DOF CPM can achieved a large workspace of $8^\circ \times 8^\circ \times 5.5\text{mm}$, high stiffness ratios, i.e., >200 for non-actuating over actuating stiffness, and a measured natural frequency of 84.4 Hz.

Key words: compliant parallel mechanism, structure optimization, beam-based flexure, spatial motions

1. Introduction

Compliant mechanism has been a popular solution for developing positioning stages in precise motion systems. Depending on the merits of elastic deformation such as zero backlash, frictionless as well as maintenance-free [1], the compliant mechanisms provide highly repeatable motions that the traditional ball-bearing based counterparts fail to deliver. Compliant mechanisms can be classified into two types, i.e., serial and parallel designs. For precise motion systems, the compliant parallel mechanisms (CPMs) are preferred because closed-loop parallel architecture offers symmetrical configuration that is less sensitive to external mechanical disturbances, higher payload but limited stroke. In addition, compliant joint/limb that de-

livers larger displacement exhibits poor off-axis (non-actuating direction) stiffness and such poor stiffness characteristic also leads to low dynamic behavior. As a result, developing a multiple degrees-of-freedom (DOF) spatial-motion CPM that offers high stiffness ratios, large workspace, and fast dynamic response, remains as one of the major challenges in the area of compliant mechanism.

Over the past one decade, many methods were proposed to design the CPMs and each has exhibited some benefits as well as limitations. The traditional method synthesized CPMs uses known parallel-kinematic configurations and articulated them via the flexure joints. Various CPMs have been synthesized by this method, ranging from simple 1-DOF [2, 3], 2-DOF [4, 5, 6, 7, 8] and 3-DOF CPMs [9, 10, 11, 12] to more complex 5-DOF [13] and 6-DOF CPMs [14, 15]. However, the traditional method depends heavily on human's intuition to synthesize a CPM. To balance the stiffness ratio, i.e., the non-actuating stiffness over the actuating stiffness, and desired workspace has proven to be a challenging

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process without even considering the desired dynamic property of the CPMs. Recently, the topology optimization method, which optimizes the stiffness and dynamic performance of a CPM using a novel mapping technique, was introduced [16]. It is a holistic design approach that accounts for the stiffness, mass distribution (affects the dynamic), workspace, and size of the CPM during the optimization process [17]. Nevertheless, CPMs with spatial motions cannot be synthesized using this method due to the restrictions of the mapping technique and the inherent 2D design domain. Furthermore, the fitness (objective) function for the stiffness optimization process was not well-defined because the difference in the units, e.g., N/m or Nm/rad etc., for every component within the stiffness matrix was not addressed.

To overcome the limitations of the existing methods, this paper presents a new structural optimization method, termed as the beam-based structural optimization approach. The proposed method is able to optimize both the stiffness and dynamic properties of the CPMs with spatial motions. In addition, a new analytical approach is introduced to normalize the difference in the units for every component within the stiffness matrix in order to derive well-defined fitness functions for the stiffness optimization process. Subsequently, the dynamic optimization process is carried out to achieve the desired resonant frequency while keeping the stiffness performance as high as possible. In this work, the effectiveness of the proposed method is demonstrated through the synthesis and experimental evaluation of a 3-DOF CPM with a out-of-plane motion. The remaining of the paper is organized as follows: the concept and modeling of the proposed method is described in details in Section 2 while the design and experimental investigations of the 3-DOF ($\theta_x - \theta_y - Z$) CPM are discussed in Section 3, and Section 4 respectively.

2. Beam-based structural optimization approach

2.1. Principle

A CPM consists of a moving platform (end effector) that is connected by several compliant limbs as frictionless support bearings. To synthesize multi-DOF spatial-motion CPMs, a novel beam-based structural optimization approach is proposed to optimize the structure of each limb. By taking the illustrated 3-limb CPM in Fig. 1 as an example, the end effector is considered as a solid platform and the three compliant limbs are rotational symmetric about the center. The design space of each limb is a cube with one end being a moving link, which

is connected to the end effector, while the other is fixed to the base. Within the cubic design space, a curved-and-twisted (C-T) beam as shown in Fig. 2a is proposed as the initial architecture of each limb. As the proposed method is to be applicable for synthesizing CPMs with up to 6 DOF, each limb must provide 6 DOF, which are realized from the elastic deformation of the C-T beam.

Fig. 1: A generalized three-limb CPM with an end-effector fixed to the ground via three symmetrical cubic design spaces.

Here, Bezier-curve is used to generate the possible solutions for compliant limbs based on the desired motions. Although this technique has the ability to synthesize various topologies, such topologies are limited to planar motions [18]. In this work, a start twist angle and an end twist angle are added to this technique so that the orientation as well as the twist property of the C-T beam can be determined. Consequently, 6 DOF can be achieved at its free end. Subsequently, special geometries can be generated from such a C-T beam by sweeping a thin rectangular cross-sectional area through a cubic Bezier-curve together with a change in orientation at both ends to create the twist property.

In addition, Bezier-curve with twist angles can be changed into any form of flexure including the traditional straight beam-like shape, which is barrier for other curve architectures such as helix and spiral. As illustrated in Fig. 2a, the straight profile can be obtained when all control points of the Bezier-curve locate on a line and when the difference of twist angle between two ends is zero, the flat geometry can be obtained. Refer to Fig. 2b, a pair of symmetrical C-T beams about the $Y'Z'$ ($X' = 0$) plane are used to construct the structure of the compliant limb of the CPM. First, a cubic Bezier-curve is defined followed by another reflected curve mirrored from it. Subsequently, both C-T beams are obtained by sweeping a rectangular cross-sectional area through these curves. The sweep operation is carried out from the start point (A) to the end point (B) of the Bezier-curve. Referring to Fig. 2a, the local frame at any posi-

Fig. 2: Construction of (a) C-T beam and (b) compliant limb.

tion on the C-T beam is defined as follows: the Z' -axis of the local frame is coincident with the tangent vector of the Bezier-curve and the local X' -axis is defined by the projection of the global X -axis onto the local $X'Y'$ plane. The orientation of the cross-sectional area at a specific position on the C-T beam on the $X'Y'$ plane is defined by the twist angle. The twist angle is the angle between the X' -axis and the long edge of the rectangular cross-sectional area. For examples, the twist angle at point A and B are defined by α_A and α_B as shown in Fig. 2a.

Fig. 3 shows the flow of the structural optimization [17] that is employed in the proposed beam-based approach. The desired DOF and specifications such as the boundary dimensions and the initial cross-sectional area of the C-T beams etc., must be first specified. This is followed by defining the geometrical design variables of the compliant limb structure for the stiffness optimization. The stiffness optimization is carried out by finding the optimized fitness function, f , expressed in Eq. (10). In this process, the cross-sectional area of the C-T beam has been pre-defined and there is no additional mass since the purpose of this step is to find an

optimized C-T beam geometry that provides the optimized stiffness ratios for the CPM based on the targeted workspace. To achieve desired dynamic response, the design variables for the dynamic optimization are specified to determine the cross-sectional area of the C-T beam as well as the distribution and location of the additional mass. As high actuating compliance and the fast dynamic response are always conflictive, the fitness function of the dynamic optimization process is formulated by two sub-equations as expressed in Eq. (13) where one shifts the dynamic response of the CPM to the desired value and the other one maintains the stiffness property as high as possible by minimizing f . After the dynamic optimization process, the final design of a CPM having both desirable stiffness and dynamic characteristics will be obtained.

Fig. 3: Flow of the beam-based structural optimization approach.

2.2. Stiffness modeling

For any CPM, it is desirable to have high stiffness ratios, i.e., high non-actuating stiffness over low actuating stiffness. The aim of the stiffness optimization process is to find the optimized design parameters for the geometry of the beam so that the CPM can achieved the highest stiffness ratios based on the targeted workspace and the size constraint. The proposed C-T beam is first discretized into several segments and each segment is represented by a standard beam element. Finite element analysis (FEA) is employed to analyze the stiffness of the CPM. Assuming that all limbs are symmetrical and identical, fourteen geometrical parameters of the C-T beam as illustrated in Fig. 2a are used as the design variables for the stiffness optimization. Twelve of them are used to define the geometry of the Bezier curve based on the coordinates (x_i, y_i, z_i) of four control points $(i = 1, \dots, 4)$. Remaining two variables are

used to represent the amount of twist angle by specifying the start angle (α_A) and the end angle (α_B) of the beam. Note that the twist angle is assumed to change linearly through the length of beam.

Let K^e be the 12×12 stiffness matrix of each beam element. If the total number of elements in the CPM is N , then the $s \times s$ global stiffness matrix of the entire CPM, K^s , is given as

$$K^s = \sum_{k=1}^N K_k^e \quad (1)$$

where s denotes the dimension of K^s .

The stiffness optimization process requires a well-defined fitness function. Referring to previous works on the mutual potential energy [1], the characteristic stiffness [19] and the stiffness ratio [17], the fitness function can be formulated depending on the works done by the external loads. Here, the CPM is considered under effect of a general load vector, P , that contains three forces F_x, F_y, F_z components along the X, Y and Z axes respectively, and three moments M_x, M_y, M_z components about the X, Y and Z axes respectively. It is also assumed that the loading point is at the center of the end effector as illustrated in Fig. 1. Under the presence of P , the corresponding displacements ($\Delta_x, \Delta_y, \Delta_z, \theta_x, \theta_y, \theta_z$) at the loading point as shown in Fig. 4 are represented by the displacement vector, U .

Fig. 4: Corresponding displacements of the CPM under a general load at the end-effector.

The 6×6 stiffness matrix, K , of the CPM can be obtained by the condensation of K^s [19] and expressed as

$$K = K_{[6 \times 6]}^s - K_{[6 \times (s-6)]}^s \cdot (K_{[(s-6) \times (s-6)]}^s)^{-1} \cdot K_{[(s-6) \times 6]}^s \quad (2)$$

where $[a \times b]$ represents the dimension of the corresponding matrix (a denotes the row and b denotes the column). Eq. (2) is derived by the equilibrium equation of the entire CPM, given as

$$\underbrace{\begin{bmatrix} K_{[(s-6) \times (s-6)]}^s & K_{[(s-6) \times 6]}^s \\ K_{[6 \times (s-6)]}^s & K_{[6 \times 6]}^s \end{bmatrix}}_{K_{[6 \times 6]}^s} \cdot \underbrace{\begin{Bmatrix} U' \\ U_{[6 \times 1]} \end{Bmatrix}}_{U_{[6 \times 1]}^s} = \underbrace{\begin{Bmatrix} 0 \\ P_{[6 \times 1]} \end{Bmatrix}}_{P_{[6 \times 1]}^s} \quad (3)$$

where $U_{[s \times 1]}^s$ and $P_{[s \times 1]}^s$ are vectors that consist of the s components, which represent the displacements and loads over the entire CPM. As the CPM is only subjected to point loading, P , U^s and P^s are separated into two parts where U' indicates the displacements of the nodes under zero load and U indicates six corresponding displacements at the loading point subjected to P . Similarly, K^s is also partitioned into four corresponding parts as expressed in Eq. (3).

Based on the equilibrium equation, the relationship of U and P is re-expressed as

$$KU = P \quad (4)$$

Assuming that $\mathbb{D} = \{1, 2, \dots, 6\}$ is the set of DOF of the CPM, the 6 DOF of the CPM are represented by the members of $\mathbb{D}$ where 1, 2, 3 represent three translational motions along X, Y and Z axes respectively, and 4, 5, 6 represent three rotational motions about the X, Y and Z axes respectively. Let $\mathbb{N}$ be the set of desired DOF and $\mathbb{M}$ be the set of undesired DOF of the CPM, $\mathbb{N}$ and $\mathbb{M}$ are the subsets of $\mathbb{D}$ and $\mathbb{N} \cup \mathbb{M} = \mathbb{D}$. If the number of elements in $\mathbb{N}$ and $\mathbb{M}$ are η and μ respectively, the relationship of η and μ with 6 DOF is given as

$$\eta + \mu = 6 \quad (5)$$

Here, P_i and U_i represent the loads and displacements in the desired DOF, P_j and U_j represent the loads and displacements in the undesired DOF of the CPM where i and j denote the members in $\mathbb{N}$ and $\mathbb{M}$ respectively. Based on Eq. (4), the desired work done, W_i , and the undesired work done, W_j , are written as

$$W_i = P_i \cdot U_i = \frac{P_i^2}{K_i} \quad (6)$$

$$W_j = P_j \cdot U_j = \frac{P_j^2}{K_j} \quad (7)$$

where K_i (or K_j) denotes the i^{th} (or j^{th}) component along the diagonal of the stiffness matrix K .

In this work, the ratio between an individual desired work done and the product of all undesired works done is used to represent the motion decoupling ability of the entire CPM in a specific DOF. E.g., the ratio of the work done in i^{th} DOF, R_i , and the product of all undesired

works done is expressed as

$$R_i = \prod_{j=1}^{\mu} \frac{W_{\mathbb{N}_i}}{W_{\mathbb{M}_j}} = \frac{W_{\mathbb{N}_i}^{\mu}}{\prod_{j=1}^{\eta} W_{\mathbb{M}_j}} \quad (8)$$

where $\mathbb{N}_i$ and $\mathbb{M}_j$ indicate the i^{th} and j^{th} components in $\mathbb{N}$ and $\mathbb{M}$ respectively. When considering all of the desired DOF, the total work done ratio, R , is given as

$$R = \prod_{i=1}^{\eta} R_i = \prod_{i=1}^{\eta} \frac{W_{\mathbb{N}_i}^{\mu}}{\prod_{j=1}^{\mu} W_{\mathbb{M}_j}} = \frac{\prod_{i=1}^{\eta} W_{\mathbb{N}_i}^{\mu}}{\prod_{j=1}^{\mu} W_{\mathbb{M}_j}^{\eta}} \quad (9)$$

The fitness function, f , can be derived by substituting Eqs. (6) and (7) to Eq. (9). The minimum value of f will give the best stiffness performance of the CPM written as

$$\begin{aligned} \text{minimize } f &= \frac{1}{R} = \frac{\prod_{j=1}^{\mu} (P_{\mathbb{M}_j}^2 / K_{\mathbb{M}_j})^{\eta}}{\prod_{i=1}^{\eta} (P_{\mathbb{N}_i}^2 / K_{\mathbb{N}_i})^{\mu}} = \kappa \cdot \frac{\prod_{j=1}^{\mu} K_{\mathbb{M}_j}^{-\eta}}{\prod_{i=1}^{\eta} K_{\mathbb{N}_i}^{-\mu}}, \\ \text{where } \kappa &= \frac{\prod_{j=1}^{\mu} P_{\mathbb{M}_j}^{2\eta}}{\prod_{i=1}^{\eta} P_{\mathbb{N}_i}^{2\mu}} \end{aligned} \quad (10)$$

In Eq. (10), κ is a coefficient factor. For unit wrench, $\kappa = 1$ because values of all loads ($P_{\mathbb{N}_i}$ and $P_{\mathbb{M}_j}$) are equal to 1 (Note: f is dimensionless). The proposed fitness function can be used to maximize the workspace of the CPM by maximizing the works done in the desired DOF, and to maximize the motion decoupling ability of the CPM by maximizing the stiffness ratios between the desired DOF and the undesired DOF. The unit issue is solved by using the ratios of the works done so that this function is useful for synthesizing every case of multi-DOF CPM.

2.3. Dynamic modeling

The aim of the dynamic optimization process is to achieve the targeted dynamic response, e.g., the first resonant mode. This is done by determining the cross-sectional area of the C-T beams and distributing material within the CPM. Fig. 5 shows the design variables used to represent the size and shape of the beam and the end effector of the CPM for the dynamic optimization process. Here, there are total of eight design variables: two variables (b_1, h_1) that define the cross-sectional area of the beam, two variables (b_2, h_2) that determine the cross-sectional area of the additional mass, and 2 variables (P, L) that represent the position and the length of

the additional mass. The last two variables (R, T) that define the mass of the end effector.

Fig. 5: Design variables of the dynamic optimization that are used to represent the geometry of (a) the C-T beam and (b) the end effector.

Let M^e be the 12×12 mass matrix of each beam element, the $s \times s$ global mass matrix of a CPM, M^s , is written as

$$M^s = \sum_{i=1}^N M_i^e \quad (11)$$

The relationship of the bandwidth vector, $\omega_{[s \times 1]}$, and the natural frequency vector, $F_{[s \times 1]}$, of the CPM is expressed as

$$F_{[s \times 1]} = \frac{\omega_{[s \times 1]}}{2\pi}, \quad (12)$$

where $|\omega^2 \cdot M_{[s \times s]}^s + K_{[s \times s]}^s| = 0$. In Eq. (12), the full-rank mass and stiffness matrix are used to synthesize the dynamic problem. As the C-T beams are used to create the skeleton of the CPM for meshing, the computational time and resource are not critical since the number of elements is low. Here, the component F_1 , which has the smallest value in $F_{[s \times 1]}$, is the first resonant frequency of the CPM.

The dynamic response of the CPM can be improved by increasing its stiffness. In this work, the objective of the dynamic optimization process is to synthesize the structure of the CPM so as to obtain a targeted first resonant frequency. If F_d is the desired value of frequency response, then the dynamic fitness function needs to find the best distribution of mass within the CPM so that $F_1 = F_d$ while keeping the stiffness ratios as high as possible. Consequently, the fitness function of the dynamic optimization process is expressed by a set of two equations

$$\begin{cases} \text{minimize } (|F_d - F_1|) \\ \text{minimize } (f) \end{cases} \quad (13)$$

3. Synthesis of 3-DOF ($\theta_x - \theta_y - Z$) CPM

3.1. Problem formulation

To demonstrate the effectiveness of the proposed design approach, it was used to synthesize a 3-DOF spatial-motion ($\theta_x - \theta_y - Z$) CPM. Refer to Fig. 3, the desired specifications and constraints were defined prior to the optimization process. In this work, the minimum workspace of the CPM was chosen to be $5^\circ \times 5^\circ \times 5\text{mm}$. The minimum stiffness ratio and the desired first resonant frequency were selected as 100, and 100 Hz respectively. Al7075-T6 was the selected material, which has Young's Modulus of 71.7 GPa, Poisson ratio of 0.33, density of 2.81 g/cm^3 and yield strength of 503 MPa. The design space of each compliant limb was assigned as $50 \times 50 \times 50\text{mm}^3$ and each C-T beam was meshed by 15 beam elements.

3.2. Stiffness optimization

To develop a CPM that delivers deterministic and decoupled DOF, the magnitude of the diagonal components within the stiffness matrix must be much higher than those of the non-diagonal components. This means the stiffness matrix of the CPM obtained from the stiffness optimization process must be a diagonally dominant matrix. In this work, the desired 3-DOF spatial-motion are Z, θ_x, θ_y and thus directly correspond to the 3rd, 4th and 5th diagonal components within a diagonally dominant stiffness matrix. With the number of desired DOF being assigned as 3, $\mathbb{N} = \{4, 5, 3\}$ ($\eta = 3$) represent the DOF of θ_x, θ_y , and Z while $\mathbb{M} = \{1, 2, 6\}$ ($\mu = 3$) represent the undesired DOF of X, Y , and θ_z . Note that $\eta + \mu = 6$. Using Eq. (6), the desired work done, i.e., W_4 about the X -axis, W_5 about the Y -axis, and W_3 along the Z -axis, are defined and expressed as

$$W_4 = \frac{M_x^2}{K_4}, \quad W_5 = \frac{M_y^2}{K_5}, \quad W_3 = \frac{F_z^2}{K_3} \quad (14)$$

Using Eq. (7), the undesired work done, i.e., W_1 along the X -axis, W_2 along the Y -axis, W_6 about the Z -axis are defined and written as

$$W_1 = \frac{F_x^2}{K_1}, \quad W_2 = \frac{M_y^2}{K_2}, \quad W_6 = \frac{M_z^2}{K_6} \quad (15)$$

Using Eq. (9), the fitness function, f , is derived as

$$\text{minimize } f = \kappa \cdot \frac{K_4^3 \cdot K_5^3 \cdot K_3^3}{K_1^3 \cdot K_2^3 \cdot K_6^3} \quad (16)$$

where

$$\kappa = \frac{F_x^6 \cdot F_y^6 \cdot M_z^6}{M_x^6 \cdot M_y^6 \cdot F_z^6}$$

Consider P as a unit wrench with $\kappa = 1$, Eq. (16) can be simplified as

$$\text{minimize } f = \frac{K_4^3 \cdot K_5^3 \cdot K_3^3}{K_1^3 \cdot K_2^3 \cdot K_6^3} \quad (17)$$

As mentioned in Section 2.2, the design variables for the stiffness optimization process are defined in Fig. 2a. After defining the initial values and range values for these design variables as well as the fitness function expressed in Eq. (17), the stiffness optimization was carried out using the Matlab Genetic Algorithm (GA) solver. The skeleton of one limb synthesized by the stiffness optimization process is illustrated in Fig. 6a. It is observed that the beam became straight with zero twist angle. At this stage of research, a constraint, which states that the mirrored C-T beams cannot intersection between each other, is imposed to eliminate any undesired computational error that could occur during the synthesis process. In this design, the offset is chosen sufficiently large, namely 1mm, to avoid intersection between the beams as shown in Fig. 6a. Based on the optimized designed variables, the 3D CAD model of the synthesized CPM is shown in Fig. 6b. The compliant matrix of the CPM, C_s , obtained from the stiffness optimization process is expressed as

$$C_s = \begin{bmatrix} 1.45\text{e-}7 & -1.7\text{e-}19 & 1.45\text{e-}7 & & & & \\ 5.4\text{e-}16 & -2.3\text{e-}16 & 1.28\text{e-}3 & & & & \\ -1.61\text{e-}5 & -1.7\text{e-}16 & -4.2\text{e-}12 & 2.07\text{e-}1 & & & \\ 6.9\text{e-}16 & -1.61\text{e-}5 & 1.2\text{e-}13 & -8.8\text{e-}12 & 2.07\text{e-}1 & & \\ -9.1\text{e-}18 & 2.0\text{e-}17 & 9.53\text{e-}6 & -5.3\text{e-}14 & 1.8\text{e-}14 & 4.63\text{e-}4 & \\ & & & & & & \text{SYM} \end{bmatrix} \quad (18)$$

Fig. 6: Structure of (a) one limb synthesized by GA and (b) the entire CPM after the stiffness optimization process.

3.3. Dynamic optimization

After the stiffness optimization, the dynamic optimization process was conducted to further optimize the mass distribution of the synthesized CPM (Fig. 6b) based on a targeted natural frequency of 100 Hz. As mentioned in Section 2.3, the design variables are shown in Fig. 5 while the fitness function is dependent on Eq. (13) where the value of F_d is substituted by 100. Multi-objectives optimization using MatLab GA solver was used to conduct the dynamic optimization and Fig. 7a shows the final design of the CPM, which was constructed based on the optimized design parameters. Refer to Fig. 7b, the FEA simulation via ANSYS has shown that the first resonant mode of the synthesized CPM is 100 Hz, which matches the targeted value. After the dynamic optimization process, the compliant matrix of the CPM becomes

$$C_d = \begin{bmatrix} 5.91e-8 & -3.8e-20 & 6.9e-18 & -6.49e-6 & -1.32e-7 & -6.7e-18 \\ 5.91e-8 & 1.6e-17 & 1.32e-7 & -6.49e-6 & -7.6e-18 & \\ & 2.59e-4 & 2.6e-14 & -1.3e-13 & 6.46e-6 & \\ & 6.78e-2 & -1.7e-13 & -1.8e-15 & -1.5e-14 & \\ & 6.78e-2 & -1.7e-13 & -1.8e-15 & -1.5e-14 & 2.54e-4 \\ & & & & & \end{bmatrix} \quad \begin{matrix} \\ \\ SYM \\ \\ \\ \end{matrix} \quad (19)$$

Fig. 7: (a) 3D model of the final design of the CPM and (b) the first resonant mode simulated via ANSYS.

Eq. (19) suggests the synthesized CPM has good motion decoupling. This is because the off-axis components are much smaller than the diagonal components. In addition, the two stiffness ratios can be obtained by taking the ratios of the corresponding compliance as given in the following

$$\begin{aligned} \frac{K_1}{K_3} = \frac{K_2}{K_3} = \frac{C_{d_3}}{C_{d_1}} = \frac{C_{d_3}}{C_{d_2}} &= 4382 \\ \frac{K_6}{K_4} = \frac{K_6}{K_5} = \frac{C_{d_4}}{C_{d_6}} = \frac{C_{d_5}}{C_{d_6}} &= 267 \end{aligned} \quad (20)$$

Refer to Eq. (20), it is observed that the stiffness ratios of the synthesized CPM are very high for large workspace CPM. Numerical simulation via ANSYS was used to analyze the workspace of the CPM with the yield strength of 503 MPa as the limit. The numerical results show that the synthesized CPM can reach a workspace of $8^\circ \times 8^\circ \times 5.5\text{mm}$, which is much better than the targeted workspace. Hence, the 3-DOF CPM synthesized by the proposed beam-based structural optimization approach not only deliver good stiffness and dynamic properties but also provided large workspace with good motion decoupling.

Lastly, having a moving platform connected to a fixed base via flexure beams generally leads to an over-constrained design if co-planar (out-of-plane) motion is desired. As shown in Fig. 7a, the synthesized CPM has an over-constrained design and will exhibit nonlinear stiffness characteristic especially along the Z-axis over a large displacement of few millimeters range. After obtaining the optimized structure from the beam-based approach, a secondary step was conducted to determine such nonlinear stiffness characteristic because the proposed approach is developed based on a FEA solver that considers just the linear deformation. This secondary step employed the Pseudo-Rigid-Body (PRB) model for an over-constrained compliant mechanism design [20] to obtain more accurate predictions on the translational stiffness along the Z-axis.

4. Experimental investigations and results

In this work, a prototype was developed to evaluate the accuracy of the proposed design method. The final design of the synthesized 3-DOF CPM required some post-processing before fabrication. The post-process was to separate the entire CPM into different components/parts for fabrication via CNC machining before assembling into a complete mechanism. In addition, the end effector (moving platform) was made bigger to accommodate for external actuators as shown in Fig. 8. As a result, the thickness of the end effector was reduced while supporting ribs were added so as to maintain its original mass, and stiffness respectively.

The experiments are conducted to explore the compliance of the proposed prototype over the full workspace of $8^\circ \times 8^\circ \times 5.5\text{mm}$. The experimental investigations were carried out by measuring the actuating forces at many different positions and conducted over five times.

The experimental setup for evaluating the compliance along the Z-axis is shown in Fig. 9. A micrometer with $10\mu\text{m}$ resolution was used to provide input displacement

Fig. 8: Prototype of the synthesized CPM.

Fig. 9: Experimental setup for evaluating the compliance along the Z-axis.

along the Z-axis while a 6-axis force/torque (F/T) sensor (ATI, MINI-40) was employed to measure the actuating force concurrently. In addition, a rigid rod with a sharp tip was fabricated and used to apply the force on the end-effector of the CPM at a specific point. For the compliance along the Z-axis, the predicted and FEA values are 2.59×10^{-4} m/N and 2.25×10^{-4} m/N respectively as shown in Fig. 10.

As mentioned, the results of beam-based approach and FEA are meant for small displacement. It is seen that the deviation between the experiments and the predicted as well as FEA results is small ($< 10\%$) from 0 to 1mm as shown in Fig. 10. The measured compliance from 1mm onwards becomes nonlinear and the deviation is large. However, the nonlinear portion agrees well with the equivalent PRB model [20] with an average

Fig. 10: Compliance along the Z-axis with the experimental results plotted against the predicted, FEA and PRB model.

deviation of 4.2%. Consequently, suitable equivalent models are helpful for predicting the nonlinear compliance behaviors of the over-constrained CPM over the large displacement.

Fig. 11: Experimental setup for evaluating the compliance about the X- and Y-axis.

Fig. 11 shows the experimental setup for evaluating the compliance about the X-axis. A two-point support was placed below the moving platform to constrain the motion along the Z-axis so that the end-effector can deliver pure rotation motion about the X-axis. Similar concept was adopted when evaluating the compliance about the Y-axis. The actuating force and displacement were applied, and measured at one point respectively to calculate the bending moment as well as the rotation angle. To ensure a pure rotational motion was produced for each measurement, a digital gauge was used to measure the displacement at the opposite position; making sure it was similar to the input displacement. For the com-

pliance about the X - and Y -axis, the predicted and FEA values are 6.78×10^{-2} rad/Nm and 7.65×10^{-2} rad/Nm respectively as shown in Figs. 12 and 13.

Fig. 12: Experimental results compared with predicted and FEA compliance about the X -axis.

Fig. 13: Experimental results compared with predicted and FEA compliance about the Y -axis.

Refer to Figs. 12 and 13, the nonlinear characteristic is less significant for the rotational stiffness as compared to the translational stiffness, which is significantly affected by the over-constrained design. For the rotational compliance about the X - and Y -axis, the average deviations between the measured results and the predicted results as well as FEA results are presented in Table. 1. Note that for all actuating compliance, the deviations between the predicted and experimental results are lower as compared to the deviations between the FEA and experimental results. It is because the FEA results obtained from ANSYS were analyzed via 10-node tetrahedral structural solid elements while the predicted re-

sults calculated via Matlab were based on 2-node beam elements .

Fig. 14: Experimental setup for evaluating the dynamic response of the CPM.

Fig. 15: Experimental dynamic response of the CPM.

The experimental setup for measuring the frequency response of the CPM is shown in Fig. 14. An accelerometer was used to measure the acceleration of the end effector along the Z -axis, which was generated by an input force applied in the same direction of the impact hammer. Figure 15 shows the dynamic response of the CPM obtained from the output data of the modal analysis equipment. The first resonant frequency of the CPM was recorded as 84.4 Hz, which is 15.6% lower than the desired value of 100 Hz. Such variation can be due to the change in the material properties and the manufacturing errors. For example, the Young's Modulus and density of a 7075-series aluminum is ideally 71.7 GPa and 2.81 g/cm^3 respectively. However, these values will never be exactly similar to the actual aluminum used to develop the prototype. Even 7075-series aluminum has different variations. Furthermore, the part tolerance is approximately $\pm 0.02 \text{ mm}$. Therefore, 15.6%

variation of the dynamic response is reasonable. Based on the results obtained from the numerical simulation (Fig. 7b) and the experimental investigation, the proposed design method has proven to be able to predict the dynamic behavior of the CPM.

5. Discussion

Based on the experimental investigation, the synthesized 3-DOF ($\theta_x - \theta_y - Z$) CPM achieved large motion range of $8^\circ \times 8^\circ \times 5.5\text{mm}$ with high dynamic response of 84.4 Hz. By comparing with an existing 3-DOF ($\theta_x - \theta_y - Z$) CPM [21], which demonstrated a motion range of $5^\circ \times 5^\circ \times 5\text{mm}$, the workspace of the synthesized CPM is larger; especially for the rotational displacements. One major challenge of developing CPMs with large workspace is to maintain high stiffness ratios w.r.t. the non-actuating directions. In another word, it is a challenge to synthesize a large displacement CPM with good motion decoupling. This is demonstrated by the low stiffness ratios of the 3-DOF ($\theta_x - \theta_y - Z$) CPM presented in [21] where the translational and rotational stiffness ratios are just 29 and 13 respectively. On the other hand, the translational and rotational stiffness ratios of the CPM synthesized in this work are expected to reach >4000 and >200 respectively. More recently, another large workspace 3-DOF ($X - Y - \theta_z$) CPM with high translational stiffness ratio of ~ 130 has been developed [17]. By comparing against such 3-DOF ($X - Y - \theta_z$) CPM, the synthesized CPM performs better with higher translational stiffness ratios, i.e., > 4000 . This is also mainly due to the analytical approach presented in this work that normalizes the differences in the unit of every component within the stiffness matrix, i.e., N/m or Nm/rad, to establish a well-defined fitness function for the optimization process.

6. Conclusion

This paper presents a new method to design any multi-DOF spatial-motion CPM with optimized stiffness and dynamic properties. Termed as the beam-based structure optimization approach, it generalizes a CPM that consists of an end-effector with limbs defined within a cubic design space. Within the design space, each pair of compliant limbs is articulated by a novel C-T beam structure. Design parameters (variables) of such C-T beam structure will be optimized via a stiffness optimization process and a dynamic optimization process so as to achieve the optimal stiffness, and targeted dynamic characteristics. To derive well-defined fitness

(objective) functions for the optimization algorithm, a new analytical approach is introduced to normalize the differences in the units, e.g., N/m or Nm/rad etc., for every component within the stiffness matrix. To evaluate the effectiveness of the proposed method, it was used to synthesize a 3-DOF spatial-motion ($\theta_x - \theta_y - Z$) CPM that delivers an optimized stiffness characteristics with a desired natural frequency of 100 Hz. The synthesized CPM with straight-and-flat flexures is obtained after the random optimization process based on GA. Starting from the very general and complex structure of the C-T beams, the resulting V-shape straight-and-flat flexures are able to provide the maximum motion decoupling ability as well as the largest workspace in desired directions for the CPM. Here, the obtained result demonstrates a special evolvement of the C-T beam due to the selected design example. Moreover, the straight-and-flat flexure also matches the traditional beam model which popularly used to synthesize 3-DOF ($\theta_x - \theta_y - Z$) compliant mechanisms. A working prototype was developed and the experimental investigations show that the synthesized 3-DOF CPM can achieved a large workspace of $8^\circ \times 8^\circ \times 5.5\text{mm}$, high stiffness ratios, i.e., >200 for translational and >4000 for rotational motions, and a measured natural frequency of 84.4 Hz. Experimental investigations also show that the proposed method can be properly used to predict the stiffness of the CPM with a maximum deviation less than 9% found between the predicted and measured values. The nonlinear characteristic of the translational compliance over the large workspace due to the over-constrained design is well predicted by the PRB model with a deviation of 4.2% as compared to the experimental result. Lastly, a deviation of 15.6% between the actual and predicted natural frequency demonstrated that the proposed method is able to predict the dynamic behavior accurately. For future work, the beam-based structure optimization approach will be improved to cover all intersectional cases and it will be used to synthesize more higher DOF spatial CPMs, e.g., a 6-DOF CPM. The dynamic optimization formulation will be enhanced to shape the first six modes of the 6-DOF CPM. In addition, 3D printing technology will be explored for realizing complex geometries of the synthesized CPMs.

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Table 1: Deviations between the experimental results compared against the predicted, FEA and PRB model.

Property	Deviation between experiment and Predicted		
	FEA	PRB model	
Compliance along the Z-axis	---	---	4.2%
Compliance about the X-axis	3.9%	15.7%	---
Compliance about the Y-axis	8.2%	11.9%	---
The first resonant frequency	15.6%	15.6%	---

Appendix A. The equivalent PRB model for over-constrained CPMs

The compliance along the Z-axis which exhibits the nonlinear characteristic as shown in Fig. 10 is caused by the large deformation of the over-constrained three-limb CPM. In this case, each limb can be considered as a fixed-clamped beam as illustrated in Fig. 16. The equivalent fixed-clamped beam can be represented by some key parameters as follows

Fig. 16: Equivalent PRB model of the compliant limb

- The equivalent length, l , is the distance from the fixed end to the loading point.
- The equivalent axial force per unit strain, EA , and bending flexural rigidity, EI , are derived respectively based on the first and the third components along the diagonal of the stiffness matrix of the compliant limb.

The PRB model for large deformation of the linear spring is used to analyze the nonlinear stiffness characteristics of the over-constrained CPM. The S-shape of the deformed equivalent beam can be modeled by two torsion springs at both ends connected by a linear

spring as illustrated in Fig. 16. Then, the force (F_z)-displacement (δ_z) relationship along the Z-axis can be expressed as [20]

$$F_z = 3 \left[K_A \Delta l \sin(\beta) + 4K_T \frac{\beta}{\gamma l \cos(\beta)} \right],$$

where

$$\Delta l = \frac{\gamma l}{\cos(\beta)} - \gamma l$$

$$K_T = \frac{\gamma}{l} K_\theta (EI)$$

$$K_A = \frac{(EA)}{\gamma l + \Delta l}$$

$$\beta = \tan^{-1} \left(\frac{\delta_z}{\gamma l} \right)$$
(21)

where α represents the length change in between two torsional springs, i.e., $\gamma l + \Delta l$ and β represents the deflection angle of the deformed beam. K_T represents the stiffness of the torsion spring and K_A represents the stiffness of the linear spring. Based on past literatures, the spring constant, K_θ , is selected as 2 [1] and γ is derived as 2/3 [22]. The resulting compliance characteristic along the Z-axis of the equivalent-PRB-model CPM is shown in Fig. 10.

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