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Electromagnetic retroreflection augmented by spherical and conical metasurfaces

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The focus of this paper is on phase gradient metasurfaces conformal to spherical and conical bodies of revolution, with an aim of engineering retroreflections and therefore augmenting backscattering cross-sections of those three-dimensional geometries under the illumination of a plane electromagnetic wave. Based on the conducting sphere and cone, the effect of the geometric revolution property on the selection of the unit inclusion of metasurfaces is considered. The procedure for using the selected unit inclusion to implement the proper reflection phase gradient onto the illuminated surfaces of those objects is formulated in detail. Retroreflections resembling conducting plates under normal incidence are observed for both the conducting sphere and cone coated with conformal metasurfaces. As a result, the redirection-induced retroreflection effectively contributes to the backscattering cross-section enhancement. A good agreement between full-wave simulations and measurements demonstrates the validity and effectiveness of backscattering cross-section enhancement using spherical and conical metasurfaces. *Published by AIP Publishing.*

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I. INTRODUCTION

Control of backscattering or radar cross-sections (RCS) of scatterers has been attracting continuous attention due to the wide applications of various radar systems. In the past, the emphasis of most reported work regarding RCS control was laid on its reduction. Many reported techniques, including shaping of the target geometry, impedance loading, radar absorbing materials, and active methods,¹ have been experimentally or practically applied to various simple or complex scatterers such as conducting plates,^{2,3} cylinders,^{4,5} antennas,^{6,7} frequency-selective surfaces,^{8–10} and military vehicles.¹¹ On the other hand, relatively less attention was concentrated on the RCS enhancement or augmentation. Besides shaping, impedance loading and active method were also reported for enhancing RCS; the design of passive enhancement devices has been the preferred solution,^{12–14} due to their advantages of low cost and structural simplicity.

As elementary shapes of many complex scatterers, spherical and conical conducting bodies of revolution have been treated as canonical objects in RCS prediction techniques^{15–26} and already considered within the scope of the backscattering cross-section reduction through the use of geometric shaping, radar absorbing materials, and metamaterials.^{27–35} However, limited reports were dedicated to the backscattering cross-section enhancement of spherical and conical bodies of revolution. During some studies investigating the backscattering reduction of the spherical conducting body of revolution, a certain potential for the backscattering enhancement was briefly mentioned. Examples are dielectric shell and plasma sheath,^{36–41} conducting post loading,^{42,43} replacement of solid spheres by spherical wire frames,^{44,45}

and wrapping of parallel wires around the spherical surface.⁴⁶ In addition, the replacement of a solid cone by wire frames,⁴⁷ the right angle reflecting step at the cone base,⁴⁸ and discontinuities on the cone body such as corrugation⁴⁹ and groove⁵⁰ were intentionally introduced to augment the backscattering cross-section from the conical conducting body of revolution.

Due to the physically ultra-thin profile and relatively convenient implementation, planar metasurfaces have been recently studied to control the reflection or transmission characteristics of electromagnetic waves.^{51–54} Curved metasurfaces^{13,14} were proposed for the purpose of enhancing the backscattering cross-section of cylindrical objects. In this paper, we focus on passive and conformal metasurfaces dedicated to augment backscattering cross-sections of spherical and conical bodies of revolution under the illumination of a uniform plane electromagnetic wave. It is illustrated that the polarization-insensitive inclusion can facilitate the design of metasurfaces conformal to three-dimensional objects with an axial symmetric property. Through full-wave simulations and experimental measurements, backscattering cross-sections close to circular conducting plates under normal incidence are observed for the conducting sphere and cone coated with conformal metasurfaces. This observation indicates that effective retroreflection is induced by scattering redirection. As a consequence, the backward scattering characteristics of spherical and conical bodies of revolution are decoupled from their geometric shapes and modified to resemble those of conducting plates.

II. REDIRECTION-BASED OPERATING PRINCIPLE

For the sake of redirecting the deflected scattering toward the direction of the impinging wave, the phase

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gradient required by the generalized law of reflection⁵² is simply expressed as

$$d\Psi/dl = 2n_i k_0 \sin(|\varphi_i|), \quad (1)$$

where $d\Psi/dl$ denotes the reflection phase gradient along the interface where the conformal metasurface is situated, n_i is the refractive index of the surrounding medium, and $k_0 = 2\pi/\lambda_0$, with λ_0 being the free-space wavelength and φ_i the angle of incidence. It may be mentioned that the distance dl can be further developed and φ_i is not necessarily constant according to the curved geometry of the involved object. Relative to the plane wave incidence, the curved or inclined surface of a conducting object generally orients the intercepted incidence along the direction of specular reflection, which leads to a small backscattering cross-section, especially within the scattering regime of the optics region. Structurally composed of subwavelength conformal unit inclusions and electrically compensated with reflection phases, the studied conformal metasurface basically reorients the dispersed scattering toward the incident direction and thus produces local retroreflection for every sampled point within the illuminated surface of the three-dimensional object. By this way, an overall retroreflection is engineered to enhance the backscattering cross-section of the three-dimensional object.

III. SPHERICAL AND CONICAL METASURFACES

A. Spherical body of revolution

A conducting sphere illuminated by a θ -polarized plane electromagnetic wave coming from $(\varphi = 0^\circ, \theta = 0^\circ)$ is illustrated in Fig. 1. The radius of the sphere is denoted as $r_s = 112.5$ mm ($3.75 \lambda_0$, where $\lambda_0 \approx 30$ mm is the free space wavelength at 10 GHz). A dielectric dome with relative permittivity $\epsilon_r = 10.2$ and thickness $h = 0.635$ mm ($0.021 \lambda_0$) coats the sphere so as to support the conformal metasurface inclusions. It is seen that the illuminated surface corresponds to the upper half spherical surface whose angular range is $(0^\circ \leq \varphi \leq 360^\circ, 0^\circ \leq \theta \leq 90^\circ)$.

FIG. 1. Uniform plane electromagnetic wave propagating from the $+z$ to $-z$ direction illuminates a conducting sphere coated with a dielectric dome.

Then the determination of incidence at local points within the illuminated surface is conducted. The direction $\vec{o}$ represents the local outward-pointing normal at each considered point P . Since the cross-section view within each plane of incidence is actually a circle, it is known that the point located at θ possesses a local angle of incidence φ_i equal to θ .¹⁴ Moreover, due to the revolution property of the sphere, it is noted in Fig. 1 that all points on the same parallel exhibit the same local angle of incidence which is equal to the colatitude θ of the parallel. Based on the determined angles of incidence, the phase gradient employed to achieve retroreflection can be calculated with the unit of radian.¹⁴ It is therefore understandable that all points on the same parallel should also hold the same phase gradient. According to the axial symmetry induced by the revolution relative to the z axis, different meridians maintain identically varying angles of incidence and phase gradients when θ ranges from 0° to 90° , which is shown in Fig. 2 for the operating frequency of 10 GHz.

Furthermore, all points on the same meridian share the same local plane of incidence, although their directions of the local outward-pointing normals vary as θ . It can be found from Fig. 1 that the electric field vector lies within the local plane of incidence for the points on the meridian of $\varphi = 0^\circ$, which indicates that those points are illuminated with transverse magnetic (TM) polarization. On the other hand, the points on the meridian of $\varphi = 90^\circ$ are exposed to the illumination with transverse electric (TE) polarization, due to the electric field vector being perpendicular to the local plane of incidence. For the points on other meridians of φ not equal to the integer multiple of 90° , there is a polarization angle φ_p equal to neither 0° nor 90° existing between the electric field vector and the local plane of incidence. By stipulating φ_p an acute angle, it is exactly equal to φ for the points on the meridians of φ varying from 0° to 90° . In view of the fact that the local polarization of incidence changes for different meridians, the adoption of a unit inclusion with the capability of operating for arbitrarily linear polarization, can facilitate the conformal metasurface. Therefore, the modified conducting loop geometry¹⁴ proposed to stabilize TM and TE reflection phases for oblique incidence is used as the unit inclusion.

Since the periodicity p of the unit inclusion is equal to 4 mm ($0.13 \lambda_0$), the illuminated range, angle of incidence,

FIG. 2. Calculated local angle of incidence (AoI), phase gradient (PG), and its normalization at 10 GHz for $0^\circ \leq \theta \leq 90^\circ$, which applies to all angles of φ . The normalization indicates that the shown PG is normalized with k_0 .

FIG. 3. Calculated reflection phase distribution at discretized angular positions to achieve the gradient.

and phase gradient are discretized with an angular interval of $\delta_\theta \approx 2.03^\circ$ between adjacent inclusions along the longitudinal direction, which is indicated in Fig. 2 by symbols and vertical grey lines. Determined by the discretized phase gradient, a reflection phase distribution pattern with the initial phase equal to -180° is shown in Fig. 3. It should be noted that the sampled positions on the same parallel assume the same reflection phase. According to the sampled angles of incidence and reflection phase distribution, the dimensions of each modified loop inclusion are determined through Computer Simulation Technology (CST) full-wave simulations of its planar periodic counterpart. For every angle of incidence, reflection phases of the loop inclusion for both TM and TE polarizations are tuned to be as close as possible to the calculation shown in Fig. 3. However, when the angle of incidence approaches 90° , the difference between TM and TE responses gradually becomes evident. Thus, a relatively increased deviation between the achieved reflection phase and the calculation is observed for near-grazing incidence.¹⁴ Because of the double curvature of the spherical surface, the conformal metasurface is constructed by the following steps so as to preserve as possible edge lengths of each conformal loop inclusion:

- (1) The meridian of $\varphi = 0^\circ$ and $0^\circ \leq \theta \leq 90^\circ$ is taken as the starting line. The center of each loop inclusion is identified on this starting line with an angular interval δ_θ , as indicated in Fig. 4(a).
- (2) Conformal loop inclusion n , where n is an integer ranging from 1 to $N = \text{RD}(90^\circ/\delta_\theta) + 1$, $\text{RD} = \text{round down}$, is constructed with respect to its corresponding center. For each loop inclusion, the lengths of its outer and inner edges along the θ (φ) direction are represented as $d_{o\theta}$ and $d_{i\theta}$ ($d_{o\varphi}$ and $d_{i\varphi}$), respectively. Those dimensions are plotted in Fig. 4(b).
- (3) Each conformal loop inclusion distributed along the starting line is duplicated along the latitudinal direction with a corresponding angular interval δ_φ . Consequently, an array of conformal loop inclusions covering the illuminated surface constitute the spherically conformal metasurface.

The duplication in step (3) is completed through axial rotation with respect to the z axis which coincides with the incident wave vector. As a result of the square lattice of the planar unit inclusion, δ_φ is initially calculated by $p/(r_s + h)/\sin\theta$. However, a certain margin surface exists between the last and first unit inclusions for every sampled parallel. This imposes an additional axial asymmetry upon the metasurface. Therefore, even distribution of the unit inclusions along the latitudinal direction is adopted in the following study and δ_φ is then slightly modified as $A^\circ/\text{RD} [(r_s + h) \times \sin\theta \times A^{\text{rad}}/p]$, where A° (rad) is the angular range, expressed in degree (radian), of even distribution along the latitudinal direction.

Figures 5 and 6 display the spherical metasurface with A equal to 180° and its simulated backscattering cross-section, respectively. For the sake of comparison, both the backscattering cross-sections of the conducting sphere without the metasurface coating and a circular conducting plate under normal incidence are also shown in Fig. 6. The dimension of the plate is determined by the intersection of the object and a plane passing through the object center and perpendicular to the incident wave vector. Thus, the area of the circular conducting plate coincides with the projection area of the sphere. The backscattering cross-sections of the conducting sphere, the circular conducting plate, and the metasurface-coated

FIG. 4. (a) Schematic drawing of conformal loop inclusions with their centers located at the discretized angular positions on the spherical surface, SL = starting line. (b) Dimensions of loop inclusions.

FIG. 5. Spherically conformal metasurface containing inclusions evenly distributed with A equal to 180° coats the illuminated surface of the sphere.

sphere at 10 GHz are -16 dBsm, 13.69 dBsm, and 11.99 dBsm, respectively. Compared with the backscattering cross-section area of the conducting plate, the backscattering cross-section area of the metasurface-coated sphere corresponds to an enhancement efficiency of 67.61% . The imperfect operation of the metasurface resulting from the reflection phase deviation, axial asymmetry, and skewed grid of the conformal loop inclusion array⁵⁵ accounts for the backscattering balance relative to the conducting plate. Although the enhancement efficiency is compromised, the backscattering cross-section is enhanced by 27.99 dB at the operating frequency, compared with the conducting sphere without any coating. It may be mentioned that similar backscattering results can be observed with other A , such as 360° and 90° .

B. Conical body of revolution

A right circular conducting cone at nose-on incidence is depicted in Fig. 7(a). The length and base radius of the conducting cone are $l_c = 159$ mm ($5.3 \lambda_0$) and $r_c = 113.22$ mm ($3.77 \lambda_0$), respectively. Analogically, the cone is covered by a dielectric shell with relative permittivity $\epsilon_r = 10.2$ and

FIG. 6. Simulated backscattering cross-sections of the conducting sphere, circular conducting plate, and metasurface-coated sphere under the θ -polarized incidence from ($\varphi = 0^\circ$ and $\theta = 0^\circ$).

thickness $h = 0.635$ mm to support the conformal metasurface inclusions. Different from the sphere, all points on this conical surface assume the same angle of incidence φ_i equal to $90^\circ - \alpha$, where α is the half cone angle. In such a case, the phase gradient to achieve retroreflection is calculated as a constant, which is 341.14 rad/m at the operating frequency of 10 GHz. Due to the revolution property of the cone, the polarization angle φ_p exists for the points on the same generator and it varies as φ , which is similar to the sphere. Thus, it is also favorable to adopt the modified conducting loop geometry as the conformal metasurface inclusion for the cone.

With a specified initial phase, a reflection phase pattern for discrete inclusions distributed along the generator of the conical surface can be calculated according to the phase gradient and the periodicity of unit inclusion. With the angle of incidence and reflection phase distribution, the dimensions of inclusions are determined through full-wave simulations. As indicated in Fig. 7, the generator of $\varphi = 0^\circ$ is considered as the starting line. First, the inclusions are located along this straight line connecting the cone tip and base. Second, the inclusions on the generator are duplicated along the φ direction with an angular interval of $\delta\varphi$. An even distribution of inclusions is also adopted with the modified $\delta\varphi = A^\circ / RD$ ($l_g \times \sin\alpha \times A^{\text{rad}}/p$), where l_g is the length of the generator segment from the cone tip to the involved inclusion center.

FIG. 7. (a) Conducting cone coated with a conical dielectric shell at nose-on incidence with θ -polarization. (b) Conically conformal metasurface containing inclusions evenly distributed with A equal to 180° coats the cone at nose-on incidence.

FIG. 8. Simulated backscattering cross-sections of the conducting cone, circular conducting plate, and metasurface-coated cone under the θ -polarized incidence from ($\varphi = 0^\circ$ and $\theta = 0^\circ$).

Figure 7(b) shows a conical metasurface with A equal to 180° . The backscattering cross-section of the cone with the conical metasurface coating is compared in Fig. 8 with the cone without any coating at nose-on incidence and a circular conducting plate with radius equal to r_c under normal incidence. The backscattering data of the conducting cone, plate, and metasurface-coated cone at 10 GHz are -10.46 dBsm, 13.75 dBsm, and 10.76 dBsm, respectively. The imperfect operation similar to the spherical metasurface leads to an enhancement efficiency calculated as 50.23%, relative to the backscattering cross-section area of the conducting plate. Compared with the conducting cone without any coating, the backscattering cross-section enhancement is 21.22 dB at the operating frequency.

IV. DISCUSSION AND EXPERIMENTAL VERIFICATION

In order to further understand the backscattering cross-section enhancement through the engineered retroreflection, simulated electric field distributions within the xz and yz planes at 10 GHz are presented in Fig. 9 for the sphere. It is seen from Fig. 9(a) that the conducting sphere disperses most of the incident plane wave propagating from the $+z$ to $-z$ direction, due to the specular reflection by its conducting spherical surface. The circular conducting plate, with a radius equal to r_s as shown in Fig. 9(b), is situated at the xy plane and thus it is under normal incidence. It is obvious that the reflected wave by the conducting plate propagates back toward the $+z$ direction. As a result of interference between the incident and reflected waves, a standing wave pattern is built above the conducting plate. Figure 9(c) shows the conducting sphere coated with the conformal metasurface. Induced by the engineered retroreflection, the pattern above the coated sphere effectively mimics that above the conducting plate under normal incidence. Similar characteristics are observed for the metasurface-coated cone. In addition, since the metasurface-coated sphere and cone behave like conducting plates under normal incidence, the scattering lobe around the backward scattering direction is expected to be tilted toward the specular reflection direction when the direction of the illuminating plane wave is moderately changed to angles other than the designing one.

FIG. 9. Perspective view of simulated electric field (V/m) distributions within the xz and yz planes at 10 GHz. (a) Conducting sphere, (b) circular conducting plate with radius equal to r_s , and (c) metasurface-coated sphere; all the insets share a same scale indicated in (b).

This implies the relatively narrow angular range of backscattering cross-section enhancement.

In order to validate the predicted observations, the conformal metasurfaces are fabricated and their backscattering cross-sections are measured. Since the doubly curved spherical surface is non-developable, it would be convenient for the fabrication of the spherical metasurface if a direct-writing technology can be adopted to print the conducting inclusions onto a dielectric dome. However, fabrication based on printed circuit board (PCB) technology is conducted in this study based on our available facilities.

For the sake of achieving the PCB-based fabrication, the upper half spherical surface in Fig. 1, which is the illuminated surface of the sphere, is approximately developed by the polycylindric method. Twelve planar gores, which constitute the contour of the planar substrate to be fabricated, are obtained after the approximate development with a cutting angle of 30° . It may be mentioned that a circular central area with a radius of 10 mm is reserved to connect together the twelve gores, so that the assembly can be facilitated.

A commercial Taconic RF-10 laminate with a thickness of 0.635 mm and a copper cladding of $17.5\ \mu\text{m}$ is used as the substrate of the metasurface inclusions. Sequentially, the inclusions on the starting line in Fig. 4(a) are distributed along the radial midline of each gore. Moreover, the inclusions on the radial line are duplicated through bilateral translation so that twelve gores are identically filled with the metasurface inclusions. The conducting sphere is made of stainless steel. A photo of the fabricated approximate design is provided in Fig. 10. The fabricated substrate is fixed onto the sphere by plastic rivets. The angle φ_e existing between the electric field vector and x axis expresses the polarization of the incident plane wave. Using two identical horn antennas connected to a vector network analyzer for transmitting and receiving, the quasi-monostatic radar cross-section measurement procedure employing a square conducting plate for calibration is conducted within an anechoic chamber so as to retrieve the backscattering cross-section of the prototype.

The simulated and measured backscattering cross-section results are presented in Fig. 11. Compared with the simulations, the backscattering cross-section peak of the coated sphere is shifted to 9.66 GHz. The factors causing this frequency shift and other slight differences between the simulations and measurements include the radius and circularity tolerances of the sphere, relative permittivity tolerance of the used substrate, and the geometry variation introduced during the PCB fabrication and measurement processes. Despite those factors, the measured characteristics of backscattering cross-sections are generally compatible with the simulated ones. For the θ -polarized incidence with $\varphi_{e1} = 0^\circ$, the backscattering cross-sections of the sphere, circular plate, and coated sphere at 9.66 GHz are $-12.74\ \text{dBsm}$, $12.77\ \text{dBsm}$, and $10.91\ \text{dBsm}$, respectively. The enhancement efficiency is calculated as 65.16% relative to the circular plate. Compared with the conducting sphere, a 23.65 dB backscattering cross-section enhancement is obtained with the coated

FIG. 11. Simulated and measured backscattering cross-sections of the stainless steel sphere, circular plate under normal incidence, and the sphere coated with the approximate design.

sphere. When the incident polarization becomes $\varphi_{e2} = 15^\circ$, the electric field vector coincides with an axis of the approximate dielectric dome. In such a case, a backscattering peak very similar to $\varphi_{e1} = 0^\circ$ is noted from Fig. 11, which illustrates the arbitrary polarization capability of the designed metasurface.

In the interest of the PCB-based conical metasurface fabrication, the conical surface of the cone in Fig. 7(a) is developed as a planar sector. Similarly, the inclusions on the starting line are distributed along the radial midline of the obtained planar sector. Those inclusions are then bilaterally duplicated with the corresponding angles of rotation. The angles of rotation are slightly modified so that the inclusions are evenly distributed along the circumferential direction between the midline and straight edges of the planar sector. Therefore, this distribution after wrapping around the cone is different from but very close to the design in Fig. 7(b), due to the fact that there are no inclusions here across the two straight edges of the planar sector. The same Taconic RF-10 laminate is used for the metasurface and a conducting cone is fabricated with aluminum. Figure 12 presents a photo of the metasurface fixed onto the cone by plastic screws.

Comparisons of the backscattering cross-section results are shown in Fig. 13. Besides the aforementioned tolerances, the substrate around the cone tip slightly chaps during assembly, because of the small dimension of this portion. However, the similar backscattering cross-section peak is recorded. It is seen that the backscattering cross-section peak of the coated cone is shifted to 9.69 GHz. For the θ -polarized incidence with $\varphi_{e1} = 0^\circ$, the backscattering cross-sections of the aluminum cone, circular plate, and coated aluminum cone at 9.69 GHz are $-12.81\ \text{dBsm}$, $12.65\ \text{dBsm}$, and $10.4\ \text{dBsm}$, respectively. The results indicate an enhancement efficiency of 59.57% relative to the circular plate. Compared with the conducting cone, the backscattering cross-section is augmented by 23.21 dB through the conformal metasurface-coated cone. It may be mentioned that similar backscattering enhancement is obtained for other incident polarizations.

FIG. 10. Photo of the stainless steel sphere coated with the PCB-based approximate design. The prototype is placed on a stainless steel base. The inset on the upper right shows the top view of the prototype.

FIG. 12. Photo of the aluminum cone coated with the PCB-based metasurface. The inset on the upper right shows the top view of the prototype.

FIG. 13. Simulated and measured backscattering cross-sections of the aluminum cone, circular plate under normal incidence, and the cone coated with the PCB-based metasurface. $\phi_{e1} = 0^\circ$, $\phi_{e2} = 45^\circ$, and $\phi_{e3} = 90^\circ$.

V. CONCLUSION

Conformal, low-profile, and passive metasurfaces suitable for spherical and conical bodies of revolution have been presented to enhance their backscattering cross-sections. Through coating them with conformal metasurfaces, it has been demonstrated that the scattering behaviors of spherical and conical bodies of revolution can be turned to resemble those of conducting plates under normal incidence. Basically, the conformal metasurfaces generate effective retroreflections by redirecting the dispersed scattering, which in turn contributes to the augmentation of backscattering cross-sections. Simulations are validated through experimental measurements. It may be mentioned that spherical and conical dielectric bodies of revolution can also reside below the presented metasurface coating. The spherical and conical bodies of revolution coated with conformal metasurfaces may function as

the passive marker or identifier of special targets. Besides, the narrow beam of the backward scattering may find applications in passive collimation.

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