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Torsional frequency mixing and sensing in optomechanical resonators

J. G. Huang,^{1,2,3} H. Cai,³ Y. D. Gu,³ L. K. Chin,² J. H. Wu,¹ T. N. Chen,¹ Z. C. Yang,⁴
 Y. L. Hao,⁴ and A. Q. Liu^{2,4,a)}

¹*School of Mechanical Engineering, Xi'an Jiaotong University and State Key Laboratory for Strength and Vibration of Mechanical Structures, Xi'an 710049, China*

²*School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798*

³*Institute of Microelectronics, A*STAR (Agency for Science, Technology and Research), Singapore 138634*

⁴*School of Electronics Engineering and Computer Science, Peking University, Beijing 100871, China*

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In this letter, a torsional optomechanical resonator for torque sensing and torsional mechanical frequency mixing is experimentally demonstrated. The torsional mechanical resonator is embedded into a split optical racetrack resonator, which provides high sensitivity in measuring torsional mechanical motion. Using this high sensitivity, torsional mechanical frequency mixing is observed without regenerative mechanical motion. The displacement noise floor of the torsional mechanical resonator is 50 fm/Hz^{0.5}, which demonstrates a resonant torque sensitivity of 3.58×10^{-21} N m/Hz^{0.5}. This demonstration will benefit potential applications for on-chip RF signal modulation using optical mechanical resonators. *Published by AIP Publishing.* [<http://dx.doi.org/10.1063/1.4986811>]

Techniques for optical measurements and the control of mechanical resonators are important technological advances in physics and engineering. The optomechanical method, which is based on the interaction of an optical cavity with mechanical resonators, has recently attracted a lot of attention due to its great potential for improving optical sensing applications such as magnetometry,^{1–4} atomic force microscopy,^{5,6} gravitational wave detection,^{7,8} and high resolution accelerometry.^{9,10} Furthermore, mechanical frequency mixing in optomechanical resonators has great applications in mechanical RF conversion, RF mixing, and on-chip RF signal processing.^{11–13}

Previous investigations have focused on flexural or breathing modes of mechanical resonators.^{14–16} Torsional modes of mechanical resonators are less studied. For practical applications, it is desirable to detect the torsional modes of mechanical resonators and use the torsional frequency for frequency mixing because this could expand the RF signal processing toolbox. Although torsional optomechanics have been demonstrated in other studies,^{17–20} these approaches require either external tapered fiber coupling or complex alignment, which are time-consuming processes that cannot be integrated into silicon photonic chips. More importantly, conventional frequency mixing by flexural or breathing modes needs regenerative mechanical oscillations,²¹ which require a threshold power to excite mechanical resonators.

In this study, we design, fabricate, and characterize a split racetrack optomechanical resonator to demonstrate torsional mechanical frequency mixing and torque sensing. Unlike conventional frequency mixing that uses dispersive optomechanical coupling, this torsional mechanical frequency mixing is achieved by dissipative optomechanical coupling and does not require mechanical regenerative oscillations. Using the high sensitivity of the split racetrack design, we observe torsional mechanical frequency mixing, in which the pump frequency is mixed with the mechanical

frequency. At the same time, this device can be used to detect torque as small as 3.58×10^{-21} N m/Hz^{0.5}. This demonstration has implications for potential applications in on-chip physical sensors and RF signal modulation, such as super-heterodyne receivers using optical mechanical resonators.^{11–13}

Torsional optomechanical designs consist of a curved bus waveguide, a split racetrack optical resonator, and a suspended torsional mechanical resonator as shown in Fig. 1(a). The split optical cavity is specifically designed to measure torsional mechanical motion. The torsional mechanical resonator is formed by a long middle beam along the *z*-direction and a short beam along the *y*-direction. This mechanical resonator is suspended from the substrate and rotated along the *z*-direction, which corresponds to the torsional vibration of mechanical modes. Other mechanical modes, which are either flexural or at a much higher frequency range, are neglected in this discussion. The curved bus waveguide is coupled to the racetrack resonator with a gap of 200 nm. The torsional mechanical resonator is 25 μ m in the *z*-direction (cross-section 200×220 nm²) and 12 μ m in the *y*-direction (cross-section 450×220 nm²). Both are released from the substrate with a gap of 300 nm between the layers.

First, a signal light with constant power is coupled to the racetrack resonator through a grating coupler to measure the torsional motion of the mechanical resonator. The cross-section of the optomechanical resonator in the *xy* plane is shown in Fig. 1(b). When the torsional mechanical resonator vibrates, it induces an offset in the *x*-direction and results in a cavity resonance wavelength shift and a cavity linewidth change due to dissipative coupling.¹⁸ Considering the two effects, the intra-cavity mode amplitude *a* is described as^{18,25}

$$\dot{a}(t) = \left(-i(\Delta - g_{om}x) - \frac{1}{2}(\kappa + g_ex) \right) a(t) + i\sqrt{\kappa_e}A_{in}, \quad (1)$$

where $\Delta = \omega - \omega_c$ is the wavelength detuning, ω is the driving laser frequency, ω_c is the optical cavity resonance frequency, $g_{om} = d\omega/dx$ is the dispersive coupling, $g_e = d\kappa/dx$

^{a)}Author to whom correspondence should be addressed: eaqliu@ntu.edu.sg

FIG. 1. Schematic of torsional optomechanics. (a) Schematic of the torsional beam in an optical cavity. The torsional resonator can rotate along the middle beam and is released 300 nm from the substrate. (b) Schematic for the evanescent wave coupling. The inset is the simulation of the optical mode and torsional mode of the beam.

is the dissipative coupling, x is the displacement of the mechanical resonator, κ is the photon decay rate for the loaded cavity, κ_e is the photon escape rate associated with external coupling, and $P_{in} = |A_{in}|^2$ is the normalized incident power. By solving this equation, the total transmission is expressed as

$$T = \left| \frac{A_{out}}{A_{in}} \right|^2 = 1 + \frac{4(\kappa_e - 1/2(\kappa + g_e x))^2 - (\kappa + g_e x)^2}{4(\Delta - g_{om}x)^2 + (\kappa + g_e x)^2}, \quad (2)$$

where $A_{out} = A_{in} + i\sqrt{\kappa_e}a$ is the output amplitude. The total transmission of the input light is modulated by the mechanical motions through the combination of dispersive and dissipative coupling. The transmission variation can be deduced from Eq. (2) and is expressed as $dT = (g_{om}\partial T/\partial\Delta + g_e\partial T/\partial\kappa)dx$, where $\partial T/\partial\Delta$ and $\partial T/\partial\kappa$ are nonlinear transfer functions, demonstrating that both dispersive and dissipative optomechanical couplings contribute to the transduction mechanism. The modulated light induced by the mechanical perturbation is converted into electrical signals, and the resulting mechanical frequencies can be observed in the power spectrum analyzer. When mechanical resonator rotates along the z -direction, one side of the beam is far from substrate and the other side of the beam is close to the substrate, making total dispersive coupling to be insignificant.

The dispersive and dissipative coupling factors are estimated to be $g_{om}/2\pi = 2$ MHz/nm and $g_e/2\pi = 15$ MHz/nm based on simulation. As a result, the transduction mechanism is dominated by the dissipative coupling whereby the mechanical motions alter the transmission dip. The transmission spectrum of the split optical cavity at different offsets, a function of the normalized detuning (Δ/κ), is shown in Fig. 2(a). The normalized transmission efficiency dT/dx is $6 \times 10^{-4}/\text{nm}$.

Torque sensing in this device is realized by the signal light with constant power. When the signal light is periodically modulated, the optomechanical resonator can be used for frequency mixing. Assuming the mechanical motion is $x(t) = x_0 \cos(\omega_m t)$ and $P_{in} = P_{in} \cos(\omega_p t)$, the output of the system is expressed as

$$P_{out} \propto |a_{out}(t)|^2 \propto x_0^2 \cos \omega_p t \cos \omega_m t \\ = \frac{1}{2} x_0^2 \cos(\omega_p t + \omega_m t) \cos(\omega_p t - \omega_m t). \quad (3)$$

The modulated light mixed with the mechanical frequency generates the mixing frequency $\omega_p \pm \omega_m$, and the intensity of the mixed frequency is correlated with the amplitude x_0 . The schematic results are shown in Fig. 2(b), in which the pump

FIG. 2. (a) Offset in the x -direction changes the transmission of the optical cavity, resulting in modulated transmission with ΔT . (b) The mechanical frequency and the pump frequency are mixed to generate more radio-frequencies.

frequency and mechanical frequency are mixed to generate $\omega_p \pm \omega_m$. It should be noted that higher-order frequencies are also generated but are too small to be observed in the experiment.

The torsional optomechanical device is fabricated using a nano-silicon photonic fabrication process. A scanning electron microscopy (SEM) image of the torsional optomechanical resonator is shown in Fig. 3. Our device is fabricated on an SOI wafer with a 2- μm box layer between the 200-nm silicon structure layer and substrate. The top silicon dioxide layer, which has a thickness of 1 μm above the silicon structure layer, is used as a cladding layer to reduce the optical loss of the waveguides. In our experiment, a broadband light (Amonics ALS-CL-13) with a bandwidth of 80 nm is first coupled into the waveguide through the grating coupler to characterize the transmission spectrum of the optical cavity. The grating period of the grating coupler is 630 nm and the etching depth is 130 nm for the 220-nm SOI wafer with a coupling efficiency of 50%.²⁴ Then, a signal light from a tunable laser (Santec TSL 510) is controlled to measure the torsional mechanical frequency using a fibre polarisation controller and a variable optical attenuator. To measure the frequency mixing, a signal light with 0 dBm modulated by an electrical-optical modulator and frequency controlled by a programmable function generator (PM 5193) is coupled into the device, which is under a high vacuum (2×10^{-6} Pa) environment. The transmission signal is then detected by a photodetector (FPD 510). The converted electrical signal is sent to an oscilloscope (MDO4104B-3) to measure the frequency domain response of the torsional frequency mixing, which is shown in Fig. 4(a).

The transmission spectrum of the optical cavity is first characterized as shown in Fig. 4(b). It should be noted that the quality factor of the split optical cavity is strongly degenerated due to the discontinuity in the torsional beam. In the fabrication process, we optimize the lithography and etching recipe to improve the optical quality factor. After optimization, the optical quality factor increases to $Q = 10\,548$ at

FIG. 3. SEM image of the fabricated device. The total device with a scale bar of 7 μm .

FIG. 4. (a) Schematic of the measurement setup. TL, tunable laser; EOM, electro-optical modulator; OC, 50:50 optical coupler; OSA, optical oscilloscope analyzer; PD, photon detector; and OS, oscilloscope. (b) Transmission spectrum of the optical cavity. The inset is the fitting of the optical resonance with an optical quality factor of 10 548.

1580.34 nm. The optimized optical quality enables the measurement of the torsional mechanical motion and the observation of frequency mixing.

Next, we characterize the torsional mechanical frequency using the signal light with a wavelength of 1580.34 nm. The thermal mechanical noise in the frequency domain is shown in Fig. 5(a). Several mechanical resonance peaks are observed in the experiment. To confirm the mechanical modes, the finite element method is used to simulate mechanical vibration modes, which are shown in the inset of Fig. 5(a). The measured mechanical frequencies are 2.30, 4.38, and 5.25 MHz, respectively. The most sensitive mode is at 4.38 MHz, which corresponds to -76 dBm in the power spectrum and can be used to sense the torsional mechanical motion.

The power spectral density of the mechanical resonators at the resonance^{22,23} is given by $S_{th} = 4k_B T_r Q / m_{eff} \omega_m^3$, where k_B is the Boltzmann constant, T_r is the absolute environment temperature, Q is the mechanical quality factor, ω_m is the angular mechanical resonance frequency, and m_{eff} is the effective mass of the mechanical resonator. In the experiment, the mechanical quality factor for 4.38 MHz is estimated to be 560 by fitting the experimental results with the effective mass of 0.44 pg. The calibrated displacement noise floor gives 50 fm/Hz^{0.5}, corresponding to the angular displacement noise of 8.3 nrad/Hz^{0.5} as shown in Fig. 5(b). Off-resonant torque sensitivity S_t is obtained as 1×10^{-19} N m/Hz^{0.5}, with the estimated torsional constant J of 1.2×10^{-11} N m/rad (the effective moment of inertia is estimated to be 15.8 pg· μm^2) by finite element simulation through $S_t = J \cdot \sqrt{S_{ex}}$. The resonant torque sensitivity is 3.58×10^{-21} N m/Hz^{0.5}, which is obtained by $S_{rt} = \sqrt{\frac{4k_B T_r \omega_m I_{eff}}{Q}}$ enhanced by a factor of mechanical quality factor $Q^{0.5}$.

After characterization, the modulated light with a frequency of 7 MHz is coupled to the torsional optomechanical

FIG. 5. Experimental results for torsional sensing and frequency mixing. (a) Power spectral density for sensing at 4.38 MHz and (b) detailed calibrated displacement and angular noise. (c) Torsional frequency mixing between pump and mechanical frequency with a span of 4.38 MHz.

device to induce the mechanical frequency mixing. The power spectral density is shown in Fig. 5(c), in which frequency mixing is observed with an up-converted frequency of 11.38 MHz and a down-converted frequency of 2.62 MHz. Traditional mechanisms for frequency mixing using breathing or flexural modes rely on the dispersive optomechanical coupling and require a threshold power to excite the mechanical resonators into self-oscillation.¹⁹ In this split racetrack resonator design, the mechanism for the torsional frequency mixing is obtained by the dissipative optomechanical

coupling and no regenerative mechanical oscillation is required. For practical photonic RF receivers, this approach enables low-power photonic RF mixing without a threshold power.^{11,12} This mechanical torsional frequency mixing does not suffer from saturation, nonlinear effects, or high loss, which are normally found in conventional electrical frequency mixing.

Finally, the resonant torque sensitivity demonstrated in this design at 3.58×10^{-21} N m/Hz^{0.5} is modest in comparison with other state-of-the-art torque sensors (2.9×10^{-24} – 7×10^{-22} N m/Hz^{0.5}).^{18,20} Our sensitivity can be improved by structural design. For example, the torsional constant 1.2×10^{-11} N m/rad can be softened to 6.0×10^{-12} N m/rad by increasing the length of the beam in the y-direction from 12 to 24 μ m, which gives a twofold improvement in sensitivity. At the same time, the mechanical quality factor can also be improved by optimizing the anchor nodal design and reducing the mass of the mechanical resonators.²⁶ For instance, the resonant torque sensitivity will be threefold improved if the mechanical quality factor is increased to $Q = 5600$.

In conclusion, a torsional optomechanical resonator in a split optical racetrack resonator for torque sensing and torsional mechanical frequency mixing is designed, fabricated, and demonstrated. Unlike conventional frequency mixing, torsional frequency mixing using dissipative optomechanical coupling is demonstrated, which does not require mechanical self-oscillations. The torque sensing achieved a displacement noise floor of 50 fm/Hz^{0.5}, which can be used to detect torque as small as 3.58×10^{-21} N m/Hz^{0.5}. This demonstration has implications for applications in on-chip torque sensors and RF signal modulation, such as super-heterodyne receivers using optical mechanical resonators.

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