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Stabilizing Droop Variation of Converter-connected Generation in Autonomous Microgrids with Virtual Inertia Control

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Abstract—Droop scheduling plays an important role in the microgrid frequency regulation market, as the well-known proportional load sharing may be practically inapplicable in many cases of inverter-based microgrids. The drooping capability of each inverter unit in the microgrid is determined by the availability and response time of its associated energy storage. However, some renewable energy-based units may be chosen to operate at the maximum power point because of their zero fuel cost. This scenario demands some high capacity units to operate with a high or infinite droop coefficient, thereby restricting their contribution to the frequency regulation. In this work, we examine the influence of this droop variation on the small signal stability of the microgrid using the root locus technique. We then substitute some of the existing droop-controlled units with virtual inertia control at critical nodes to study the effect of coordinated droop and inertia control. Finally, the choice and size of energy storage necessary to emulate a certain level of inertia and droop are compared. The study shows that the stability of the system may be enhanced while saving on the economics of storage requirement if the inertia and droop control act in a coordinated manner.

Index Terms—Droop, oscillations, scheduling, small signal stability, virtual inertia.

I. INTRODUCTION

Global climatic concerns are gradually encouraging the growth of distributed renewable generation that in turn leads to an increasing trend of realizing small independently-operated power systems with converter-based generation units known as microgrids. Droop control is a very mature technique practised in large power systems where several grades of spinning reserves are purchased to serve as frequency support to the system. In [1], the authors have demonstrated that droop control can be effectively applied to operate inverter-based microgrids in the standalone mode. Droop control allows power sharing among the inverters without the requirement of a communication infrastructure. Although droop control has many variations, (1) and (2) represent the conventional Pf-Qv droop control in which the droop coefficient m_p establishes a linear relation between the active power and frequency (Pf) pair and n_p establishes the reactive power and voltage (QV) linearity. Other variations of droop control such as the inverse (Pv-Qf) droop and virtual resistance control techniques may be applied for power sharing in low-voltage networks [2].

$$\Delta\omega = -m_p \Delta P \quad (1)$$

$$\Delta V = -n_q \Delta Q \quad (2)$$

In conventional power systems, the generating units (GUs) operate according to the equitable load share rule, i.e. the proportional droop control as in (3), where G_s is the grid stiffness constant that describes the drooping capability of the grid as a whole. For the equitable share condition, the G_s , given by (4), determines the stability of the grid. The lower the m_p , the higher is the contribution to the frequency regulation. Thus, the largest GU is assigned the lowest m_p according to this rule. However, it is always wiser to operate a large solar photo-voltaic (PV) farm or a wind farm in the maximum power point (MPPT) mode and assign the task of frequency regulation to a dispatchable energy source equipped with storage facility. In [3], the authors develop a droop-scheduling algorithm by optimizing the production costs, while considering the reserve availability as constraints. Researches that have focused on the small signal stability analysis of the inverter-droop control, such as [4], [5] have shown that there exists a critical droop coefficient beyond which the increase of the coefficient leads to instability. The droop scheduling approach in [6] investigates this effect using the Hopf bifurcation theory, by which they conclude that not only does the stability of the system depend on the grid stiffness G_s , but also the individual unit droops m_{pi} and their location. The authors of [2] utilize this bifurcation-based index to compare the different variations of droop control.

$$m_{pi} P_i = G_s, i \in GU \quad (3)$$

$$G_s = \sum_{i \in GU} \frac{1}{m_{pi}} \quad (4)$$

In this work, we analyze the impact of droop variation and the location of the GUs that contribute to the droop using an eigenvalue analysis. The droop control analyzed throughout this work is of Pf-Qv type. The inverters in the microgrid may be of PQ type, i.e. those units that follow the MPPT algorithm, or of Vf type, i.e. the units that respond to the frequency and voltage variation. The Vf units are usually droop-controlled, however, we also show some case studies where some these units are equipped with virtual inertia

(VI) control that help enhance the stability of the system. The mathematical derivations that describe the energy storage requirement of the droop control and the VI control are established for application in the microgrid planning studies.

The remaining portion of the paper is organized as follows. The following section describes the comparison of the microgrid with droop and VI control and their respective state space models. Section III describes the effect of variation of droop and inertia parameters on the microgrid stability with the help of two case studies. Section IV establishes the mathematical equations for the energy storage requirements for the droop and VI control. Section V concludes the paper with some interpretations based on the case studies.

II. DROOP AND VIRTUAL INERTIA CONTROL TOPOLOGIES

This section describes the working of the droop control and virtual inertia control and formulates the state space model of these control algorithms. In this modelling, we consider that the switching speed of the inverter switches are very fast compared to the change in the reference voltage given to the PWM generator. Thus, from here on we may assume that the inverter terminal voltage completely tracks the reference voltage sent to the PWM and the switching dynamics are ignored [7].

A. Droop control

Droop control is a phasor-domain control that implements a linear relationship between the measured active and reactive power with the frequency and voltage respectively. However, it may vary with the type of droop control as mentioned in the last section. Throughout this work we follow the Pf-Qv droop which is described by (1) and (2). Thus, the inverter terminal voltage reference and the frequency reference are given by the droop laws. The active and reactive power measurements P and Q , however, are not well defined in the real time and are computed from the measured inverter output filter current i_o and the bus voltage values v_b . The measured voltages and currents are represented as dq components (v_{od}, v_{oq}) and (i_{od}, i_{oq}) respectively. In order to avoid the harmonics in the measurements from propagating to the control, a low pass filter (LPF) with corner frequency ω_c is provided. Thus, this control works with the averaged values of P and Q , and makes the response sluggish compared to controls that work without the LPF [7].

The droop control may be implemented with the help of a droop block, a proportional integral (PI) voltage controller, and a PI current controller whose control schematics are presented in figure 1 [4]. The droop control block performs the measurement of the averaged values of real and reactive power given by (5) and (6) respectively. The frequency and the voltage references are then generated based on the droop laws (7) and (8) respectively, where ω_n and V_n represent the nominal frequency and voltage. The state equations of the inverter in this section are represented in the corresponding inverter's reference frame. Thus, the voltage reference generated V^* may be represented in the dq frame as The

Fig. 1. Control Schematic of Droop Control

voltage reference generated from the droop controller, $(v_{od}^*, 0)$. When considering a system of multiple inverters, δ , the angle reference to the inverter is calculated using (9). It is to be noted that ω_{com} refers to the frequency in the common reference frame. For an inverter-based microgrid, one of the inverter reference frame (inverter-1) may be considered as the common reference frame.

$$P = \frac{\omega_c}{s + \omega_c} (v_{od} i_{od} + v_{oq} i_{oq}) \quad (5)$$

$$Q = \frac{\omega_c}{s + \omega_c} (v_{od} i_{oq} - v_{oq} i_{od}) \quad (6)$$

$$\omega = \omega_n - m_p P \quad (7)$$

$$v_{od}^* = V_n - n_q Q \quad (8)$$

$$\delta = \int (\omega - \omega_{com}) \quad (9)$$

The inverter is connected to the bus through an LCL filter with components $(r_f, L_f), C_f, (r_c, L_c)$. It is to be noted that the voltage references generated at the droop control block corresponds to the bus voltage after the inverter LCL filter. The voltage reference generated from the droop controller, (v_{od}^*, v_{oq}^*) is fed to the inner voltage loop controller to generate the output current reference for the inverter given by (10)-(13). The corresponding PI control parameters are K_{pv} and K_{iv} .

$$\phi_d = \int (v_{od}^* - v_{od}) \quad (10)$$

$$\phi_q = \int (v_{oq}^* - v_{oq}) \quad (11)$$

$$i_{ld}^* = Fi_{od} - \omega_n C_f v_{oq} + K_{pv} (v_{od}^* - v_{od}) + K_{iv} \phi_d \quad (12)$$

$$i_{lq}^* = Fi_{oq} + \omega_n C_f v_{od} + K_{pv} (v_{oq}^* - v_{oq}) + K_{iv} \phi_q \quad (13)$$

The current reference generated from the PI current control loop, (i_{ld}^*, i_{lq}^*) , is fed to the inner voltage control loop to generate the inverter output voltage reference before the LCL filter as given by (14)-(17). The corresponding PI control parameters are K_{pc} and K_{ic} .

$$\nu_d = \int (i_{ld}^* - i_{ld}) \quad (14)$$

$$\nu_q = \int (i_{lq}^* - i_{lq}) \quad (15)$$

$$v_{id}^* = -\omega_n L_f i_{lq} + K_{pc} (i_{ld}^* - i_{ld}) + K_{ic} \nu_d \quad (16)$$

$$v_{iq}^* = \omega_n L_f i_{ld} + K_{pc} (i_{lq}^* - i_{lq}) + K_{ic} \nu_q \quad (17)$$

The state space equations of the LCL filter are presented in (18)-(23). The LCL filter state space equations are also used in the virtual inertia control model.

$$\frac{di_{ld}}{dt} = -\frac{r_f}{L_f} i_{ld} + \omega i_{lq} + \frac{v_{id}}{L_f} - \frac{v_{od}}{L_f} \quad (18)$$

$$\frac{di_{lq}}{dt} = -\frac{r_f}{L_f} i_{lq} - \omega i_{ld} + \frac{v_{iq}}{L_f} - \frac{v_{oq}}{L_f} \quad (19)$$

$$\frac{dv_{od}}{dt} = \omega v_{oq} + \frac{i_{ld}}{C_f} - \frac{i_{od}}{C_f} \quad (20)$$

$$\frac{dv_{oq}}{dt} = -\omega v_{od} + \frac{i_{lq}}{C_f} - \frac{i_{oq}}{C_f} \quad (21)$$

$$\frac{di_{od}}{dt} = -\frac{r_c}{L_c} i_{od} + \omega i_{oq} + \frac{v_{od}}{L_c} - \frac{v_{bd}}{L_c} \quad (22)$$

$$\frac{di_{oq}}{dt} = -\frac{r_c}{L_c} i_{oq} - \omega i_{od} + \frac{v_{oq}}{L_c} - \frac{v_{bq}}{L_c} \quad (23)$$

The overall state variables of each individual inverter may be listed as shown in (24). The additional subscript i in (24) represents the state variables of a particular inverter i . The overall state space model of each inverter is given by (25)-(26), where the matrices $A_{INV}^i, B_{INV}^i, B_{\omega com}^i, C_{INV\omega i}^i$ and C_{INVci}^i may be computed using the state space equations (5)-(23).

$$\Delta x_{inv}^i = [\Delta \delta_i \quad \Delta P_i \quad \Delta Q_i \quad \Delta \phi_{di} \quad \Delta \phi_{qi} \quad \Delta \nu_{di} \quad \Delta \nu_{qi} \quad \Delta i_{ldi} \quad \Delta i_{lqi} \quad \Delta v_{odi} \quad \Delta v_{oqi} \quad \Delta i_{odi} \quad \Delta i_{oqi}] \quad (24)$$

$$[\Delta \dot{x}_{inv}^i] = A_{inv}^i [\Delta x_{inv}^i] + B_{inv}^i [\Delta v_{bDQ}^i] + B_{\omega com}^i [\Delta \omega_{com}] \quad (25)$$

$$\begin{bmatrix} \Delta \omega_i \\ \Delta i_{oDQi} \end{bmatrix} = \begin{bmatrix} C_{INV\omega i} \\ C_{INVci} \end{bmatrix} [\Delta x_{INV}^i] \quad (26)$$

The state space representation of the droop control model written above is with respect to the inverter reference frame. Considering a microgrid with more than one inverter connected in parallel, the state space model has to be represented in the common reference frame. The above equations may be written in the common reference frame using the δ computed earlier as given by (27) - (30). The subscripts d, q represent the individual inverter reference frame and the subscripts D, Q represent the common reference frame. represent

$$[\Delta i_{oDQi}] = T_S [i_{odqi}] + T_C [\Delta \delta_i] \quad (27)$$

$$[\Delta v_{bdq}] = T_S^{-1} [v_{bDQ}] + T_C^{-1} [\Delta \delta_i] \quad (28)$$

$$T_S = \begin{bmatrix} \cos(\delta) & -\sin(\delta) \\ \sin(\delta) & \cos(\delta) \end{bmatrix} \quad (29)$$

$$T_C = \begin{bmatrix} -V_{bD} \sin(\delta) & V_{bQ} \cos(\delta) \\ -V_{bD} \cos(\delta) & -V_{bQ} \sin(\delta) \end{bmatrix} \quad (30)$$

B. Contribution of Droop control to System Inertia

In this section, the contribution of the droop control to the system inertia is estimated. If the active power output of the inverter is measured with the help of a low pass filter with time constant $T_f = 1/\omega_c$, the droop equation (31) may be re-arranged as (32) to compute the equivalent inertia and damping of the droop controller as given by (33). Thus the power measurement delay in the filter along with the droop coefficient m_p contributes to the virtual inertia J and the droop coefficient contributes towards the damping D . The virtual inertia is not fully controllable and the ratio J/D is constant for a system. Thus, droop controllers tend to have a poor transient response.

$$\omega_i^* = \omega_n - m_p \left(\frac{P}{(1 + T_f s)} - P_b \right) \quad (31)$$

$$P_b - P = \frac{T_f}{m_p} s(\omega_i^*) + \frac{\omega_i^* - \omega_n}{m_p} \quad (32)$$

$$J = \frac{T_f}{m_p}; D = \frac{1}{m_p} \quad (33)$$

where, P_b is the base power of the GU.

C. Virtual Inertia (VI) Control

In this section, the problem of inability to independently control inertia and damping of the GU is overcome by using a swing-equation based model for the VI control. The VI control, hence may be referred to as the virtual synchronous generator (VSG) control. The basic swing equation-based topology implemented in [8], is used for the stability analysis

Fig. 2. Swing equation-based control scheme

in this work. The schematic diagram of the swing equation-based control is shown in figure 2. The VI control block shown in the figure replaces the droop control block shown in figure 1. The grid frequency ω_{grid} is measured with the help of a phase locked loop (PLL) circuit and the active power P is measured the same way as in droop control using the voltage and current measurements and a LPF.

The virtual prime mover input power P_{in} is calculated by (35), modelling a governor as shown in the schematic. ω_n represents the nominal frequency of the unit and T_d is the time constant of the governor delay [8]. With the help of these inputs, the swing equation (34) is solved in every control cycle to obtain the angular velocity of the virtual rotor ω_m by numerical integration. The swing equation helps to emulate the action of a synchronous generator thereby contributing to the virtual inertia of the system. The control parameters J and K_d represent the virtual inertia and damping in the ideal swing equation and the contribution of the inverter unit to the microgrid inertia and damping may be varied independently by varying these parameters.

$$P_{in} - P = J\omega \frac{d\omega}{dt} + K_d(\omega - \omega_{grid}) \quad (34)$$

$$P_{in} = P + \frac{K(\omega_n - \omega)}{1 + T_d s} \quad (35)$$

The above VI equations replace the (1) in the droop controller, while the rest of the system remains the same. Apart from the active power-frequency response, the reactive power-voltage Qv droop equation may remain intact as in the droop control model.

The swing-based emulation can be implemented for grid-forming Vf controllers that participate in the frequency and

TABLE I
COMPARISON OF SELECTED VIE TOPOLOGIES

No-VI case: $f_{nadir} = 56.9\text{Hz}$, $RoCoF_{max} = 2\text{Hz/s}$, $t_{settle} = 9.2\text{s}$		
	Droop	Swing equation-based
$f_{nadir}(\text{Hz})$	57.3	58.4
$RoCoF_{max}(\text{Hz/s})$	1.7	1.4
$P_{peak}(\text{W})$	1560	1765
$t_{settle}(\text{s})$	10.1	12.8
Grid-forming	✓	✓
Numerical stability	✓	X
Noise susceptibility	✓	✓

Fig. 3. Schematic of the inverter-based microgrid

voltage regulation [9]. The controller does not rely on the frequency derivative measurement from PLL for influencing the rate-of-change-of-frequency (RoCoF) after a disturbance and is thus robust to noise. However, the problems of numerical instability is an issue along with the oscillatory stability issues that may arise due to the improper tuning of J and K_d parameters [10].

D. Comparison of Droop and VI control

The transient response of the swing-based topology in comparison to the droop controller is shown in table I. A simple 2-inverter system from [9] is simulated with MATLAB Simulink the two control topologies and the response characteristics of the controls to a 2kW step load change is compared in this table.

The results from the table show that the swing equation-based topology has a better transient response, while there is the issue of numerical instability in some cases, as the swing equation has to be solved for every control cycle. In the following section, the small signal stability of these controls are analyzed using eigenvalue analysis.

E. Overall System Model

The autonomous microgrid shown in figure 3 [5] is considered for the oscillatory stability study. It consists of 3 inverters fed from a DC bus by the combination of a DG source, which may be solar PV or wind, or an energy storage mix, and 3 loads of RL, RC and RL respectively. The system data may be obtained from [4], [5].

F. Network Model

The network lines are modelled as RL branches. The state space equations of the i_{th} line current flowing from node a to b may be written as in (36) and (37).

$$\frac{di_{lineDi}}{dt} = -\frac{r_{linei}}{L_{linei}}i_{lineDi} + \omega i_{lineQi} + \frac{1}{L_{linei}}(v_{bDa} - v_{bDb}) \quad (36)$$

$$\frac{di_{lineQi}}{dt} = -\frac{r_{linei}}{L_{linei}}i_{lineQi} - \omega i_{lineDi} + \frac{1}{L_{linei}}(v_{bQa} - v_{bQb}) \quad (37)$$

The state space model of the microgrid network may be represented by (38) where, the matrices A_{line} , B_{1line} , and B_{2line} may be computed from (36) and (37).

$$[\Delta \dot{i}_{lineDQ}] = A_{line} [\Delta i_{lineDQ}] + B_{1line} [\Delta v_{bDQ}] + B_{2line} \Delta \omega \quad (38)$$

G. Load Model

Loads 1 and 3 of the microgrid in figure 3 are modelled as inductive RL loads which are very common in the distribution level. Their load models are given by (39) and (40).

$$\frac{di_{loadDi}}{dt} = -\frac{r_{loadi}}{L_{loadi}}i_{loadDi} + \omega i_{loadQi} + \frac{1}{L_{loadi}}v_{bDi} \quad (39)$$

$$\frac{di_{loadQi}}{dt} = -\frac{r_{loadi}}{L_{loadi}}i_{loadQi} - \omega i_{loadDi} + \frac{1}{L_{loadi}}v_{bQi} \quad (40)$$

Load2 is modelled as a series RC load modelled by (41) and (42) that represent the effect of capacitor banks at the substation [6].

$$\frac{dv_{capDi}}{dt} = -\frac{1}{r_{loadi}C_{loadi}}v_{capDi} + \omega v_{capQi} + \frac{1}{r_{loadi}C_{loadi}}v_{bDi} \quad (41)$$

$$A_{MG} = \begin{bmatrix} A_{inv} + B_{inv}R_nM_{inv}C_{inv} & B_{inv}R_nM_{line} & B_{inv}R_nM_{load} \\ B_{1line}R_nM_{inv}C_{inv} + B_{2line}C_{inv\omega} & A_{line} + B_{1line}R_nM_{line} & B_{1line}R_nM_{load} \\ B_{1load}R_nM_{inv}C_{inv} + B_{2load}C_{inv\omega} & B_{1load}R_nM_{line} & A_{load} + B_{1load}R_nM_{load} \end{bmatrix} \quad (46)$$

III. IMPACT OF PARAMETER VARIATION ON MICROGRID STABILITY

The eigenvalue plot of the microgrid state matrix in the frequency or 's' domain, familiarly referred to as the root locus plot is used to analyze the oscillatory and small signal stability of the system. The real part of the eigenvalue indicates the power of the exponent term in the time response function and the imaginary value indicates the frequency of periodicity. As the real part of the eigenvalue moves towards the positive x-axis, the stability margin of the system reduces and it is unstable when one or more real parts of the eigenvalues are positive.

$$\frac{dv_{capQi}}{dt} = -\frac{1}{r_{loadi}C_{loadi}}v_{capQi} - \omega v_{capDi} + \frac{1}{r_{loadi}C_{loadi}}v_{bQi} \quad (42)$$

The combined state space model of the loads may be written as (43), where the matrices may be computed from (39)-(42). The state variables of the RL and RC loads are their load currents and the load voltages respectively.

$$[\Delta \dot{x}_{loadDQ}] = A_{load} [\Delta x_{loadDQ}] + B_{1load} [\Delta v_{bDQ}] + B_{2load} \Delta \omega + B_{3load} [\Delta i_{oDQ}] + B_{4load} [\Delta i_{lineDQ}] \quad (43)$$

H. Microgrid Model

The microgrid nodal voltage vector Δv_{bDQ} may be defined in terms of the state variables using the diagonal connectivity matrices M_{inv} , M_{load} , M_{line} , and a virtual high impedance diagonal matrix $R_n \approx 1000\Omega$ in (44) [4]. The diagonal elements of connectivity matrices are filled with ones for the inverter and negative ones for the loads. M_{line} is filled with 1 or -1 depending on whether the line current is entering or leaving a node respectively.

$$[\Delta v_{bDQ}] = R_n(M_{inv} [\Delta i_{oDQ}] + M_{load} [\Delta x_{loadDQ}] + M_{line} [\Delta i_{lineDQ}]) \quad (44)$$

Combining the inverter, line, and the load state space models, the microgrid can be represented by (45) and (46) [4].

$$\begin{bmatrix} \Delta \dot{x}_{inv} \\ \Delta \dot{i}_{lineDQ} \\ \Delta \dot{x}_{loadDQ} \end{bmatrix} = A_{MG} \begin{bmatrix} \Delta x_{inv} \\ \Delta i_{lineDQ} \\ \Delta x_{loadDQ} \end{bmatrix} \quad (45)$$

To obtain the root locus plot for the small signal stability analysis, we require the initial steady state values for the system. In this work, we obtain the steady state values of the microgrid by executing a droop-based backward/forward sweep algorithm [11]. The parameters used for obtaining the steady state values are taken from [4].

A. Case Study: Droop variation

The root locus plot of the microgrid where all generators are equipped with droop control is shown in figure 4. The slanting lines present in the background of the root locus plot in all the following figures, represents the constant damping ratio lines. Figure 4 displays the entire eigen spectrum of the microgrid. However, it is to be noted that the state variables

Fig. 4. Effect of m_p variation in Case 1

introduced by the LCL filter, Voltage PI and current PI loop are far away from the imaginary axis and are not affected much by the droop variation. The state variables corresponding to the Droop control block are the ones sensitive to the droop variation. Based on sensitivity analysis we can identify the state variables that are most sensitive to each inverter's droop coefficient m_p . The pair of eigenvalues that move towards the real axis the fastest with increase in m_p are the ones that first result in instability. For this case, the sensitivity of the eigenvalues that undergo instability due to Pf droop variation are: $(m_{p1}, m_{p2}, m_{p3}) = (0.12, 0.3, 0.05)$. Hence, unit 2 is the most critical unit whose droop has to be scheduled mindful of the stability issue. The droop variation is also shown to have an almost linear relationship to shift of the real part of the critical eigenvalues, which is also shown in 4.

As m_p is increased, the microgrid state matrix is computed and the eigenvalues are plotted. High mp indicates that the inverter controls are less responsive to the frequency variations. The low frequency oscillatory modes shift to the right half, i.e. the unstable region as shown in figure 4. Critical mp is the value at which the oscillatory modes show a positive real part and the system stability margin reduces to zero. This value for the Pf droop variation is observed to be 7.7%, beyond which the system is oscillatory unstable.

Similarly, we can study the effect of variation of the reactive power-voltage droop by examining the sensitivity of the state variables to the Qv droop parameter n_q . It can be found that the same state variables that are introduced by the droop control block are the ones that are closest to the real axis. The sensitivity of this pair of eigenvalues to the n_q s are: $(n_{q1}, n_{q2}, n_{q3}) = (0.03, 0.4, 0.0006)$. Similar to the previous case, the generator unit 2 plays a critical role with respect to the Qv droop too. However, the effect of Qv droop variation does not cause instability as fast as the previous case. Figure 5 depicts the variation of the Qv droop and the critical droop.

As n_q is increased i.e. if the reactive power control of the inverter becomes less responsive to the node voltage variation at all, two of the real poles of the system break away into a pair of complex poles, i.e. it shows an oscillatory behaviour. When n_q is increased to a very high value 16.4% the oscillatory

Fig. 5. Effect of n_q variation in Case 1

modes break in on the real axis very close to the y axis, and moves in the opposite direction, where one real value approaches the y axis and becomes positive at $n_q = 18.8\%$, which is the critical n_q value. The QV droop variation effect is a localized phenomenon and is different for different inverters based on the load connected and the node location.

The case studies mentioned above show the local birth and death of periodicity with the variation of the droop parameters, which can be identified as the Sub-critical Homoclinic Hopf Bifurcation (HB) in the bifurcation theory that studies the change in the dynamic behavior of the system with respect to the change in the parameters [6]. This identification gives scope for quantification of the stability margin based on the mathematical representations of the HB hypersurface [12].

B. Case Study: Inertia and Damping Variation

In the sensitivity analysis done in the previous section, it was observed that the generator unit 2 was the critical node that played a crucial role in determining the system stability. In this case, we replace the droop control of the GU-2 with the swing equation-based VSG control. Thus, the control parameters for node-2 are J and K_d , while the other nodes are droop-controlled. The modes of the inverter droop control (IDC) are referred to as inverter power modes and the modes of the swing equation-based control are referred to as the electro-mechanical modes, since the control is based on a virtual rotor and prime-mover input power [5].

Increase in inertia J shifts the inverter power modes towards the left-half of the s-plane and thereby stabilizing the system as shown in figure 6. However, the increase in J , with constant m_p as 0.05, shifts the electro-mechanical modes of the swing-based controller to the right, making the stability margin narrower. Figure 7 presents the effect of increase in m_p when J is constant, 0.03. It can be observed that m_p has the reverse effect of J i.e. as the m_p is increased the electro-mechanical modes are damped and the inverter power modes are destabilized. The critical m_p at which the oscillatory modes become purely imaginary and the stability margin reduces to zero is computed as 16%.

Fig. 6. Effect of increase of Inertia Constant in Case 2

Fig. 7. Effect of increase of Droop Coefficient in Case 2

Interestingly, there is a mention of an optimal J/D ratio in [13], to ensure that the system stability is maintained. From these plots, we can infer that if the J/D ratio is increased the frequency damping increases while the active power damping reduces. Thus, it is possible to arrive at an optimal trade-off point where both the frequency and the active power response are well-damped while minimizing the requirement of energy storage.

For the droop control, we have already seen that in (33), the droop along with the corner frequency of the LPF ω_c determines the inertia and the damping is directly given by the droop. Thus, in case of the droop control, the J/D has the constant value of $1/\omega_c$ or T_f . Hence, if the ratio is less than ω_c , the frequency response is over-damped and the power response is under-damped in case of droop control, and if the ratio is greater than ω_c , the converse is true.

IV. ENERGY STORAGE REQUIREMENT

For a particular inertia and damping level in case of a swing-based VSG or a particular droop level in case of a droop control, there is a particular amount of associated energy storage required to realize these controls in a smooth manner. This section briefly computes the maximum energy storage capacity necessary to realize these controls. The energy associated with the inertia and damping coefficients may be written as (47). Thus, the maximum energy storage capacity may be calculated using (47) where ω_{max} and δ_{max} are the maximum frequency and angle deviations in the microgrid. The maximum frequency deviation is a characteristic of the load variations in the system.

$$E = \frac{J}{2} \Delta \omega^2 + K_d \delta \quad (47)$$

Hence, for the droop control to work with a droop parameter m_p , the E_{max} is given by (48) and for the swing based virtual

inertia control to work with inertia J and damping parameter K_d , the maximum energy reserve capacity is given by (49).

$$E_{max\ droop} \approx \frac{1}{m_p} \delta_{max} \quad (48)$$

$$E_{max\ VI} = \frac{J}{2} \Delta \omega_{max}^2 + K_d \delta_{max} \quad (49)$$

Clearly, the reserve requirement of the VSG is higher than the reserve requirement of the droop control, due to (33), as it provides additional inertia. For case study 1, the reserve requirement is given by $3 * E_{max\ droop}$, if $m_{p1} = m_{p2} = m_{p3} = m_p$, which is definitely less than the reserve requirement of case study 2.

Thus, replacing the droop controlled GUs with VSG enhances the stability of the system with more expense on the energy storage reserve capacity. However, the identification of the critical node saves the extra reserve cost by avoiding the inertia emulation at the other nodes. Thus, virtual inertia control may be optimally placed while ensuring that the frequency stability is maintained in the event of a contingency. The contingency and the frequency stability study is beyond the scope of this work as we have focused merely on the small signal analysis.

V. CONCLUSION AND FUTURE SCOPE

In this work, we have studied the detailed topology, modeling, and comparison of the droop control and the swing equation-based virtual inertia control. The importance of tuning the droop and inertia control parameters in a systematic way is stressed in this study. The study shows patterns of effects that the droop and inertia parameters can produce on variation. The results show that there is a possibility for one particular node to play a crucial role in the stability of the system. By identifying the critical bus, it is possible to explore options for increasing the energy storage reserve requirement at the critical nodes, so that they are capable of a faster frequency response, which is the inertia response. Hence, this work has potential applicability to microgrid planning and resource management.

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