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A simplified model for reinforced concrete beam–column joints under seismic loads

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This paper presents a simplified model for reinforced concrete beam–column joints under seismic loads. In the model, the beams, columns and the joint zone are assumed to remain rigid, and flexural deformations of the beams and column are concentrated at the joint interfaces. The compatibility condition of the joint is established in accordance with the rigid-plastic assumption. To correlate the flexural deformations of the beams and columns to strains of tensile reinforcement, piecewise linear strain profiles of tensile reinforcement are assumed along the beams and columns and slips of embedded reinforcement in the joint are also considered. Thereafter, the plane-section assumption is employed to determine the strain profiles at the joint interfaces. In calculating the compression force in concrete, the stress–strain model for concrete is used instead of the equivalent rectangular concrete compressive stress block. The rigid-plastic model was calibrated against published experimental results of interior and exterior beam–column joints in terms of load–displacement curves. Furthermore, the yield displacement of beam–column joints was calculated and verified by test data. Through the analytical model, forces transferred to the joint zone by tensile, compressive reinforcing bars and concrete can be quantified, which enables further development of the joint model to evaluate the shear resistance of beam–column joints.

Notation

A_s	cross-sectional area of tensile reinforcement	l_c	length of columns between joint interface and pin support
a_{sb}	distance between the centroid of beam tensile reinforcement and the extreme tension concrete fibre	l_{em}	effective embedment length of bent reinforcement
a_{sc}	distance between the centroid of tensile reinforcement and the extreme tension concrete fibre of the columns	l_{pl}	length of plastic hinge of left-hand beam
b_b	beam width	l_{pr}	length of plastic hinge of right-hand beam
C_{cc}, C_{cs}, T_{cs}	force in compressive concrete, compressive reinforcement and tensile reinforcement at column–joint interface, respectively	M_{fb}	bending moment at end of right-hand beam
C_{lc}, C_{ls}, T_{ls}	force sustained by compressive concrete, compressive reinforcement and tensile reinforcement at the left-hand beam–joint interface, respectively	N_c	axial compression force in columns
C_{rc}, C_{rs}, T_{rs}	force sustained by compressive concrete, compressive reinforcement and tensile reinforcement at the right-hand beam–joint interface, respectively	P_b	vertical load at beam end
c_{br}	neutral-axis depth of right-hand beam at joint interface	P_c	horizontal load at column top
c_c	neutral-axis depth of columns	s_{br}	slip of reinforcement from joint
d_r	diameter of steel reinforcement	s_m	slip of beam reinforcement at centre of embedment length
f'_c	compressive strength of concrete cylinders	s_1, s_2, s_3	slips associated with maximum and frictional bond stresses
f_{rs}	tensile stress of reinforcement	V_{jh}	horizontal shear force in beam–column joint
f_s	tensile stress of reinforcement at joint interface	Δ_{yc}	calculated yield displacement
h_b	depth of beams	Δ_{ym}	measured yield displacement
h_c	depth of columns	δ	lateral displacement
l_b	length of beams between pin support and joint interface	δ_{bl}	deformation of tensile reinforcement in left-hand beam at beam–joint interface
l_{be}, l_{by}	length of elastic and yielded steel segments, respectively	δ_{br}	deformation of tensile reinforcement in right-hand beam at beam–joint interface
		δ_{bs}	total deformation of tensile reinforcement in right-hand beam
		δ_c	deformation of tensile reinforcement at column–joint interfaces
		δ_h	lateral displacement at column top
		δ_{rb}	vertical displacement of beam–joint interfaces due to joint rotation

δ_{rc}	horizontal displacement at column–joint interfaces resulting from joint rotation relative to columns
δ_v	vertical displacement of beam
ε_{bl}	strain of beam tension reinforcement at the left face of the joint
ε_{bm}	strain of tensile reinforcement at the centre of its embedment length
ε_{br}	strain of tensile reinforcement at beam–joint interfaces
ε_c	strain of tensile reinforcement at column–joint interface
ε_{rc}	strain of extreme compression concrete fibre
ε_{rs}	strain of tensile reinforcement
ε_y	yield strain of tensile reinforcement
θ_b	rotation of beams
θ_{bj}	rotation of beam end section with regard to the joint interface
θ_c	rotation of column relative to vertical position
θ'_c	rotation of columns about the vertical axis after considering joint rotation
θ_{cj}	angle between column end section and joint interface
θ_j	rigid-body rotation of the joint zone
θ_{rs}, θ_{rb}	rotation of beam relative to the joint resulting from slip of longitudinal reinforcement from joint and beam flexural deformation, respectively
σ_c	compressive stress of concrete
τ_e	bond stress of reinforcement in elastic stage
τ_y	post-yield bond stress of reinforcement
τ_1, τ_3	maximum and frictional bond stresses of steel reinforcement, respectively

Introduction

Rigid-plastic models have been widely used in the analysis of reinforced concrete (RC) one-way slabs subjected to compressive arch action (Keenan, 1969; Park, 1964; Welch *et al.*, 1999). In a rigid-plastic model, it is generally assumed that slab segments remain rigid under flexural action, whereas rotations of the segments are concentrated at pre-specified plastic hinges. Furthermore, it is assumed that the steel reinforcement in the plastic hinge regions has yielded, and compressive concrete has attained its ultimate strength. Thus, equilibrium of internal forces at the plastic hinges can be explicitly established by calculating the tension force in the steel reinforcement from its yield strength, and the compression force sustained by concrete through the equivalent compressive concrete stress block. However, these models are not applicable to structural members with compressive steel reinforcement, such as beams and columns, as the stress state of the reinforcement in compression cannot be determined from the model (Park and Gamble, 2000). Moreover, due to the rigid-plastic assumption, such models can only be used to predict the response of slabs at the post-yield stage of tensile reinforcement, and the response of slabs at the initial elastic stage cannot be estimated.

In recent years, rigid-plastic models have been modified by considering the effect of imperfect boundary conditions and the contribution of compressive reinforcement (Guice *et al.*, 1989; Yu and Tan, 2013). In these models, the extreme compression concrete fibre is assumed to have attained the ultimate strain, and the strain of steel reinforcement in the compression zone is determined through the plane-section assumption. Later, by assuming a linear strain profile along the beam length, the model was extended to the elastic stage prior to the formation of plastic hinges (Kang and Tan, 2015a). The compression force sustained by concrete was determined by integrating compressive stresses across the compression zone. Through this model, reasonably good predictions of load capacities were obtained for beam–column sub-assemblages under column-removal scenarios.

A similar concept can also be used for RC beam–column joints subjected to seismic loads. With sufficient stirrups in the plastic hinge regions at the ends of beams and columns and the beam–column joint, shear failure can be prevented and flexural behaviour of the beams and columns is dominant over shear distortion of the joint zone. Experimental tests indicated that shear distortion of the joint zone and shear deformations of the beams and columns only contribute a limited portion to the total deformations of beam–column joints (Lin *et al.*, 2000; Xin, 1992). Therefore, beam–column joints can be represented by plastic hinge mechanisms. However, in addition to flexural deformations in the plastic hinge regions, slip of the longitudinal reinforcement relative to the joint has to be considered, which requires a proper bond–slip model for steel reinforcement embedded in beam–column joints (Lowe and Altoontash, 2003; Lowe *et al.*, 2004).

This paper describes a rigid-plastic model for RC beam–column joints. In the model, the compatibility condition of the joint is considered by correlating the strains of the steel reinforcement and corresponding slips to flexural deformations of the beams and columns at the joint interfaces. A stress–strain model for concrete is employed rather than a rectangular concrete stress block. Comparisons of the experimental and analytical results indicate that the model yields reasonably good estimations of the load–displacement curves of interior and exterior joints under seismic loads. Furthermore, internal forces and slip of reinforcement at the joint interfaces were also investigated to shed light on the joint behaviour at the sectional level.

Rigid-plastic model for beam–column joints

When subjected to seismic loads, RC joints develop flexural deformations in beams and columns, as shown in Figures 1(a) and 1(b). Two types of loading conditions were considered in developing the joint model presented in this paper. In the first type, a horizontal load was applied to the column top of an interior joint, as shown in Figure 1(a). Alternatively, a vertical load can be applied to the beam, as shown in Figure 1(b),

Figure 1. Idealised beam–column joints: (a) interior joint; (b) exterior joint

which is widely used for exterior joints. Therefore, prior to derivation of the model, different compatibility conditions were considered under the two loading conditions.

Deformations of beams and columns

Based on the rigid-plastic assumption developed by Park (1964) for slab strips, a hinge mechanism was assumed for the joints, in which beams and columns remain rigid and their flexural deformations are concentrated at the joint interfaces. Figures 2(a) and 2(b) show the deformations of beams and columns in an interior joint. When a horizontal load is applied at the column top, compatibility between columns and the joint zone is established based on the rigid-plastic assumption. At a lateral displacement δ , rotation θ_c of the whole column relative to its original position can be determined as

$$1. \quad \theta_c = \frac{\delta_h}{2l_c + h_b}$$

where θ_c is the rotation of the columns relative to the vertical position, δ_h is the lateral displacement at the column top, l_c is the length of the columns between the joint interface and the pin support, and h_b is the depth of the beams.

However, due to the presence of adjoining beams, the joint zone develops a rotation θ_j with respect to the columns, as shown in Figure 2(a). This rotation generates a horizontal displacement δ_{rc} at the column–joint interfaces, as expressed in Equation 2. This displacement increases the rotation angle of the columns to θ'_c .

$$2. \quad \delta_{rc} = 0.5h_b(\theta_c - \theta_j)$$

$$3. \quad \theta'_c = \theta_c + \frac{\delta_{rc}}{l_c}$$

Thus, the angle of the column end section with respect to the joint interface can be calculated as the difference between θ'_c and θ_j

$$4. \quad \theta_{cj} = \theta'_c - \theta_j$$

in which θ_{cj} is the angle between the column end section and the joint interface.

In addition, compatibility between beams and the joint zone can also be established, as shown in Figure 2(b). With a rotation θ_j of the joint zone, the beam–joint interfaces develop a vertical displacement δ_{rb} , given by

$$5. \quad \delta_{rb} = 0.5h_c\theta_j$$

Correspondingly, the RC beams experience a rotation θ_b due to the joint rotation, expressed by

$$6. \quad \theta_b = \frac{\delta_{rb}}{l_b}$$

where l_b is the length of the beams between the pin support and the joint interface.

At the beam–joint interfaces, angle θ_{bj} of the beam end section with regard to the joint interface can be quantified using

$$7. \quad \theta_{bj} = \theta_b + \theta_j$$

If a vertical load is applied to the beam, deformations of the joint are different from that shown in Figure 2. Therefore, compatibility of the joint has to be re-established. Figure 3(a) shows the deformations of the beam–joint assembly. When the vertical displacement of the beam is δ_v , rotation of the joint

Figure 2. Compatibility of interior joint under horizontal load: (a) column–joint assembly; (b) beam–joint assembly

zone is assumed to be θ_j , and thus rotation of the beam with respect to its original position is

$$8. \quad \theta_b = \frac{\delta_v - 0.5h_c\theta_j}{l_b} + \theta_j$$

where h_c is the depth of columns.

As a result, rotation of the beam end section relative to the joint zone can be determined as

$$9. \quad \theta_{bj} = \theta_b - \theta_j$$

In addition, rotation of the joint generates a horizontal displacement at the column–joint interfaces, as shown in Figure 3(b), and the displacement can be calculated using

$$10. \quad \delta_{rc} = 0.5h_b\theta_j$$

Hence, rotation of the column end section about the joint interface satisfies

$$11. \quad \theta_{cj} = \frac{\delta_{rc}}{l_c} + \theta_j$$

Through the rigid-plastic assumption, rotations of the beams, columns and the joint zone can be correlated to each other. However, to calculate the internal forces acting at the joint interfaces, strains of steel reinforcement and concrete need to be quantified, which requires compatibility at the joint interfaces to be established.

Compatibility at joint interfaces

At the joint interfaces, elongation of tensile reinforcement in the columns can be obtained from the compatibility, expressed by

$$12. \quad \delta_c = (h_c - c_c - a_{sc}) \tan \theta_{cj}$$

where δ_c is the deformation of tensile reinforcement at the column–joint interfaces, c_c is the neutral-axis depth of the columns and a_{sc} is the distance between the centroid of tensile reinforcement and the extreme tension concrete fibre of the columns.

In RC beam–column joints, plastic hinges are only formed at the beam ends, whereas steel reinforcement in the columns remains in the elastic stage. In addition, column reinforcement passing through the joint is well-confined by horizontal hoops in the joint zone and slip of the reinforcement can be neglected. Therefore, to determine the strains of the steel reinforcement and concrete fibre, a linear strain profile of tensile reinforcement is assumed along the column height, with zero strain at the pin support and maximum value at the joint interfaces, as shown in Figure 2(a), and the sum of reinforcement tensile strain along the column height is equal to the elongation of tensile reinforcement at the joint interfaces, as expressed by Equation 13. Accordingly, the strain profile at the column–joint interfaces can be determined based on the plane-section assumption.

$$13. \quad \delta_c = 0.5l_c\epsilon_c$$

Here, ϵ_c is the strain of the tensile reinforcement at the column–joint interfaces.

Figure 3. Compatibility of exterior joints under vertical loads: (a) beam–joint assembly; (b) column–joint assembly

Likewise, total elongation of the tensile reinforcement at the beam–joint interface can be obtained from the rigid-plastic model, expressed by

$$14. \quad \delta_{br} = (h_b - c_{br} - a_{sb}) \tan \theta_{bj}$$

where δ_{br} is the deformation of the tensile reinforcement in the right-hand beam at the beam–joint interface, c_{br} is the neutral-axis depth of the right-hand beam at the joint interface and a_{sb} is the distance between the centroid of the beam tensile reinforcement and the extreme tension concrete fibre.

In most beam–column joints, plastic hinges form at the beam ends where the tensile reinforcement develops post-yield behaviour. At the post-yield stage of steel reinforcement, slip of the reinforcement relative to the joint zone contributes a significant portion to total joint deformation due to the reduced bond stress (Alsiwat and Saatcioglu, 1992; Sezen and Setzler, 2008). Therefore, slip of the reinforcement needs to be considered in establishing compatibility at the joint interfaces. The elongation of tensile reinforcement consists of two components – deformation of the reinforcement in the beams and slip of the reinforcement from the beam–column joint, expressed by

$$15. \quad \delta_{br} = \delta_{bs} + s_{br}$$

where δ_{bs} is the total deformation of the tensile reinforcement in the right-hand beam and s_{br} is the slip of the reinforcement from the joint.

To determine the elongation of the tensile reinforcement in the right-hand beam, different strain profiles are assumed along the beam length at the elastic and post-yield stages. When steel reinforcement remains elastic at the joint interface, the strain profile along the beam length is assumed to be linear between the pin support and the joint interface, with its maximum value at the interface and zero at the pin support, as shown in Figure 2(b). Hence, strain of the tensile reinforcement in the right-hand beam is correlated to the elongation of the reinforcement, as expressed by Equation 16. Once the tensile reinforcement develops its post-yield strain at the joint interface, a plastic hinge forms at the beam end. Thus, a bilinear strain profile is assumed along the plastic hinge region and the elastic beam segment (see Figure 2(b)). Correspondingly, the total deformation of tensile reinforcement in the beam can be determined from Equation 17.

$$16. \quad \delta_{bs} = 0.5 l_b \varepsilon_{br}$$

$$17. \quad \delta_{bs} = 0.5(l_b - l_{pr})\varepsilon_y + 0.5l_{pr}(\varepsilon_y + \varepsilon_{br})$$

Here, ε_{br} is the strain of the tensile reinforcement at the beam–joint interface, ε_y is the yield strain of the tensile reinforcement and l_{pr} is the length of plastic hinge, which can be taken as $0.5h_b$, as suggested by Paulay and Priestley (1992).

Slip of reinforcement

In addition to elongation of the tensile reinforcement in the beam, slip of reinforcement from the beam–column joint has

to be considered. As for interior joints, beam reinforcement passing through the joint sustains a tension force at one end and a compression force at the other. Push–pull tests of steel reinforcement anchored in concrete suggest that the bond stresses of steel reinforcement under compression and tension are identical and the steel stress at the centre of the embedment length is nearly zero (Viathanatepa *et al.*, 1979). Therefore, it was assumed in this work that steel stress is zero at the centre of the embedment length, whereas slip of the reinforcement and the associated bond stress are mobilised at the section. A similar bond stress profile to that postulated by Lowes *et al.* (2004) was used, in which uniform bond stresses are assumed along the elastic and post-yield steel segments, as shown in Figure 4.

The bond–slip model for concrete (CEB-FIP, 1991) was used in this work to calculate the slip at the joint interface. In this model, the maximum and frictional bond stresses and corresponding slips were defined from experimental results. However, the model is only applicable to elastic steel reinforcement. Once plasticity of the reinforcement is initiated along the embedment length, the post-yield bond stress is used over the inelastic steel segment (Kang and Tan, 2015b). The Poisson effect induced by the cross-section contraction of inelastic steel reinforcement reduces the frictional bond stress by 20–30% (Eligehausen *et al.*, 1983). Thus, the

post-yield bond stress can be taken as 70% of the friction bond stress.

$$\begin{aligned} \tau_e &= \tau_1 \left(\frac{s_m}{s_1} \right)^{0.4} & \text{if } s_m \leq s_1 \text{ and } \varepsilon_{br} \leq \varepsilon_y \\ 18. \quad \tau_e &= \tau_1 & \text{if } s_1 < s_m \leq s_2 \text{ and } \varepsilon_{br} \leq \varepsilon_y \\ \tau_e &= \tau_1 + \frac{\tau_3 - \tau_1}{s_3 - s_2} (s_m - s_2) & \text{if } s_2 < s_m \leq s_3 \text{ and } \varepsilon_{br} \leq \varepsilon_y \\ \tau_e &= \tau_3 & \text{if } s_3 < s_m \text{ and } \varepsilon_{br} \leq \varepsilon_y \end{aligned}$$

$$19. \quad \tau_y = 0.7\tau_3 \quad \text{if } \varepsilon_{br} > \varepsilon_y$$

Here, τ_e is the bond stress of the reinforcement at the elastic stage, τ_1 and τ_3 are the maximum and frictional bond stresses of steel reinforcement, expressed as $2.5\sqrt{f'_c}$ and $1.0\sqrt{f'_c}$, respectively, s_1 , s_2 and s_3 are the slips associated with the maximum and residual bond stresses, equal to 1.0 mm, 3.0 mm and the clear rib spacing, respectively, and τ_y is the post-yield bond stress of reinforcement.

For a given tensile strain ε_{br} of steel reinforcement at the joint interface, the associated tensile stress can be determined in accordance with a bilinear stress–strain model with hardening behaviour. In the meantime, slip of the embedded reinforcement at the centre of the joint has to be assumed, so that the sum of bond stresses over the embedment length between the joint centre and interface can be equal to the tension force of reinforcement at the joint interface, as expressed by Equations 20 and 21. Note that a debonding zone of reinforcement is assumed near the column face, with length equal to the distance between the centroid of the column longitudinal reinforcement to the nearest column face (see Figure 4). Over the debonding zone, the bond stress of the steel reinforcement is zero due to the formation of a failure cone near the beam–joint interface.

$$20. \quad \tau_e l_{be} \pi d_r = f_s A_s$$

$$21. \quad (\tau_e l_{be} + \tau_y l_{by}) \pi d_r = f_s A_s$$

Here, l_{be} and l_{by} are the lengths of the elastic and yielded steel segments, respectively, d_r is the diameter of the steel reinforcement, f_s is the tensile stress of the reinforcement at the joint interface and A_s is the cross-sectional area of the tensile reinforcement.

Therefore, slip of the beam longitudinal reinforcement at the joint face can be determined from Equations 22 and 23. The calculated slip must be equal to the value obtained from the compatibility at the joint interface.

$$22. \quad s_m + 0.5\varepsilon_{br}l_{be} + \varepsilon_{br}a_{sc} = s_{br}$$

Figure 4. Bond–slip of reinforcement in beam–column joints: (a) elastic stage; (b) post-yield stage

$$23. \quad s_m + 0.5\varepsilon_y l_{be} + 0.5(\varepsilon_y + \varepsilon_{br})l_{by} + \varepsilon_{br}a_{sc} = s_{br}$$

in which s_m is the slip of beam reinforcement at the centre of the embedment length.

In interior beam–column joints, the strain of the tensile reinforcement is assumed to be zero at the centre of the embedment length because the reinforcement is subjected to a tension force at one end and compression at the other. However, in exterior beam–column joints, the beam longitudinal reinforcement is typically bent into the joint and the strain and slip of reinforcement at the centre of the embedment length are not zero. Filippou *et al.* (1983) recommended that the effective embedment length of bent reinforcement can be taken as the sum of its straight portion and five times the rebar diameter. At the free end of the equivalent embedment length, the strain of steel reinforcement is zero, and thus the same bond–slip behaviour as that shown in Figure 4 can be used to determine the slip of the beam reinforcement relative to the joint.

Equilibrium

From the strain of the tensile reinforcement and the neutral-axis depth, the strain profile at the joint interface can be determined in accordance with the plane-section assumption, as shown in Figure 5. For the steel reinforcement, a bilinear stress–strain relationship with hardening was adopted and assumed to be identical in tension and compression. In calculating the internal forces sustained by concrete, the stress–strain model proposed by Mander *et al.* (1988) for concrete was used in the rigid-plastic model instead of the equivalent concrete compressive stress block. Accordingly, the concrete compression force at the joint interface was quantified through integration of the compressive stress over the compression zone (Equation 24). A similar method can also be used to calculate the bending moment contributed by compressive concrete (Equation 25).

$$24. \quad C_{rc} = b_b \int_0^{c_{br}} \sigma_c dx$$

$$25. \quad M_{rb} = b_b \int_0^{c_{br}} (0.5h_b - c_{br} + x)\sigma_c dx + C_{rs}(0.5h_b - a_{sb}) + T_{rs}(0.5h_b - a_{sb})$$

Here, C_{rc} is the compression force sustained by concrete at the right-hand beam–joint interface, C_{rs} and T_{rs} are the forces sustained by the compressive and tensile reinforcement at the right-hand beam–joint interface, respectively, b_b is the beam width, σ_c is the compressive stress of concrete and M_{rb} is the bending moment at the end of the right-hand beam.

Besides compatibility, force equilibrium at the joint interfaces has to be satisfied. The resulting internal forces acting on each joint face has to be equal to the net axial force in the beams or columns, as expressed in Equations 26–28. Thus, bending moments at the beam–joint and column–joint interfaces can be calculated from equilibrium. Correspondingly, the horizontal load on the columns and the vertical force on the beams can be determined.

$$26. \quad C_{rc} + C_{rs} - T_{rs} = 0$$

$$27. \quad C_{lc} + C_{ls} - T_{ls} = 0$$

$$28. \quad C_{cc} + C_{cs} - T_{cs} = N_c$$

Here, C_{lc} , C_{ls} and T_{ls} are the forces sustained by the compressive concrete, compressive reinforcement and tensile reinforcement at the left-hand beam–joint interface, respectively, C_{cc} , C_{cs} and T_{cs} are the forces in compressive concrete, compressive reinforcement and tensile reinforcement at the column–joint interfaces, respectively, and N_c is the axial compression force in the columns.

Figure 5. Force equilibrium of beam sections

Solution procedures

With compatibility, equilibrium and material models for steel reinforcement and concrete, a set of solution procedures was developed for the simplified model. For a beam–column joint subjected to a horizontal load at the column top, the following procedure can be utilised to determine the load–displacement curve.

- Step 1. Provide a lateral displacement δ_h at the column top and calculate the corresponding rotation θ_c via Equation 1.
- Step 2. Assume rotation θ_j of the joint and compute rotation θ_c of the column through Equations 2 and 3. Thus, rotation θ_{cj} of the column with respect to the joint interface can be determined from Equation 4.
- Step 3. Assume a neutral-axis depth c_c of the column and calculate the strain ε_c of the tensile reinforcement through Equations 12 and 13. Thereafter, the strain profile at the column–joint interface can be determined in accordance with the plane-section assumption. Associated forces in the steel reinforcement and concrete can be quantified based on respective constitutive models. Force equilibrium at the column–joint interface has to be satisfied, as expressed by Equation 28.
- Step 4. Calculate rotation θ_{bj} from Equations 5–7.
- Step 5. Assume a neutral-axis depth c_{br} and determine elongation δ_{br} of the tensile reinforcement at the beam–joint interface from Equation 14.

- Step 6. Assume a strain ε_{br} and calculate δ_{bs} and s_{br} until the value obtained from Equation 15 is equal to that from Equation 14.
- Step 7. Determine the strain profile at the beam–joint interface and calculate the internal forces sustained by the steel reinforcement and concrete. If equilibrium in Equation 26 is satisfied, the corresponding bending moment can be computed from Equation 25.
- Step 8. Calculate the horizontal force P_c applied to the column top in accordance with force equilibrium.

When a vertical load is applied to the beam, an associated vertical displacement δ_v has to be assumed and rotations θ_{bj} and θ_{cj} can be determined from Equations 9 and 11, respectively. The foregoing procedure can also be employed to calculate the vertical load.

Validation of the rigid-plastic model

The behaviour of interior and exterior beam–column joints under seismic loads was simulated using the rigid-plastic model. Table 1 summarises the geometric properties and reinforcement details of the interior and exterior joints. Figures 6 and 7 show the envelopes of load–displacement of interior joints tested by Ruitong and Park (1987) and Xin (1992) and the model results; horizontal load was applied at the column top during testing. A comparison of the analytical and experimental results suggests that the model yields reasonably good estimations of the load–displacement curves at the ascending and plateau stages. However, as buckling of compressive reinforcement is not

Table 1. Properties of interior and exterior beam–column joints and measured and calculated yield displacements

Joint	Beam			Column			Δ_{ym} : mm	Δ_{yc} : mm	$\frac{\Delta_{yc}}{\Delta_{ym}}$
	Length: mm	Cross- section: mm	Rebar details ^a	Height: mm	Cross- section: mm	Rebar details ^a			
Ruitong and Park (1987)									
Unit 1	1916	229 × 457	5D16; 2D16	1008	305 × 406	8HD16	15.1	17.0	1.13
Unit 2			2D28; 2D20			8HD20	19.6	19.0	0.97
Unit 3			5D16; 2D16			6HD16 + 2HD12	13.1	19.0	1.45
Unit 4			2D28; 2D20			6HD20 + 2HD16	18.0	19.0	1.06
Xin (1992)									
Unit 1	1525	250 × 500	7HD12; 2HD12	985	300 × 450	4HD28 + 2HD20	20.7	20.0	0.97
Unit 2			4HD16; 2HD16			4HD24 + 4HD16	14.4	17.5	1.22
Unit 3			4HD16; 4HD16			4HD32 + 4HD20	16.5	17.0	1.03
Unit 4			2HD20; 2HD16			4HD28 + 2HD20 + 2HD16	14.1	15.5	1.10
Unit 5			3HD20; 3HD20			4HD32 + 4HD20	12.7	17.0	1.34
Unit 6			2HD28; 2HD20				16.3	17.5	1.07
Megget (1974)									
Unit A	1400	255 × 460	2D25 + D29; 2D29	1270	330 × 380	8D22	18.0	18.0	1.00
Scarpas (1981)									
Unit 1	2210	355 × 610	6D20; 6D20	1372	457 × 457	12D20	—	16.0	—
Unit 2			6D20 + 2D24; 6D20 + 2D24				—	17.0	—
Unit 3			6D20; 6D20				—	15.0	—
								Mean	1.12
								Coefficient of variation: %	14

^aD16 denotes deformed grade 275 steel bar of 16 mm diameter and HD16 denotes deformed grade 380 steel bar of 16 mm diameter

Figure 6. Comparison of experimental (Ruitong and Park, 1987) and analytical load–displacement curves of interior joints: (a) unit 1; (b) unit 2; (c) unit 3; (d) unit 4

considered in the model, the decrease in horizontal load prior to failure cannot be accurately simulated. Regarding the exterior joints, vertical load was applied to the beam end. The envelopes of the load–displacement curves reported by Megget (1974) and Scarpas (1981) are compared with the model results in Figures 8 and 9. Once again, the agreement between the analytical and experimental results is reasonably good. The joint tested by Megget (1974), with different areas of top and bottom reinforcement in the beam, exhibited significantly different load–displacement responses when subjected to hogging and sagging moments at the beam end, and thus both loading conditions were considered by the model.

Components of internal forces

In addition to the overall load–displacement curves of beam–column joints, internal forces acting at the joint interfaces can

also be obtained through the rigid-plastic model. Unit 1, tested by Xin (1992), was analysed using the proposed model. Figure 10 shows the variations in forces sustained by the steel reinforcement and the concrete in the compression zones of columns and beams. A negative value represents compression force and a positive value denotes tension force. In the compression zone of the columns, the forces in the steel reinforcement and concrete increased at the initial stage and were then nearly constant with increasing horizontal displacement, as shown in Figure 10(a). The maximum compression force in the concrete was 2.36 times that in the steel reinforcement. However, compression forces in the beams displayed showed different variations, as shown in Figure 10(b). The compression force in the concrete increased to a maximum of 392.4 kN and thereafter decreased gradually due to crushing of the concrete. In the meantime, the compressive reinforcement started to

Figure 7. Comparison of experimental (Xin, 1992) and analytical load–displacement curves of interior joints: (a) unit 1; (b) unit 2; (c) unit 3; (d) unit 4; (e) unit 5; (f) unit 6

Figure 8. Comparison of experimental (Megget, 1974) and analytical load–displacement curves of exterior joints: (a) beam under hogging moment; (b) beam under sagging moment

sustain more compression force, and the maximum fraction of the reinforcement force in the total compression force was around 46%. Due to identical top and bottom reinforcement ratios in the beams, the left- and right-hand beams developed the same internal forces at the joint interfaces. By varying the top and bottom reinforcement ratios, the compression force sustained by the steel reinforcement can differ significantly.

Based on equilibrium of the beam–column joint, the horizontal shear force in the joint can be calculated (Paulay and Priestley, 1992) using

$$29. \quad V_{jh} = T_{rs} + C_{lc} + C_{ls} - P_c$$

in which P_c is the horizontal load at the column top and V_{jh} is the horizontal shear force in the beam–column joint.

Figure 11 shows the variation of horizontal shear force in the beam–column joint. At the initial stage, the joint shear force was nearly proportional to the horizontal displacement of the column, levelling off with increasing horizontal displacement at the column top. In addition to the total shear force, the portion of shear force transferred by the compressive concrete was calculated and is also shown in Figure 11. The contribution of the concrete compression force to the total shear force attained a maximum value of 65% at 40 mm horizontal displacement. With a further increase in horizontal displacement, the ratio decreased to around 34% due to crushing of the concrete in the compression zone.

Variations of neutral-axis depths and strains

To investigate the behaviour of beam–column joints at the sectional level, neutral-axis depths and strains of the steel

reinforcement and concrete at the joint interfaces were obtained from the rigid-plastic model. Figure 12 shows the variations of neutral-axis depths of beams and columns. It can be seen that the neutral-axis depth at the column–joint interface decreased when the horizontal displacement of the column was less than 15 mm. Thereafter, a nearly constant value of 136 mm was reached. However, the minimum neutral-axis depth of the beam was only 65 mm. It is noteworthy that with increasing horizontal displacement of the column, the beam neutral-axis depth increased slowly due to progressive crushing of the concrete in the compression zone.

Figure 13 shows the variations of strains of steel reinforcement and concrete with horizontal displacement of the column. At the column–joint interface, the maximum strain of tensile reinforcement was around 0.0013 (see Figure 13(a)), much less than its yield strain of 0.0023. In addition, the maximum strain of the extreme compression concrete fibre was only 0.0006. Thus, a plastic hinge did not form at the column–joint interface. However, at the beam–joint interface, the tensile reinforcing bars attained their yield strain at about 20.0 mm horizontal displacement, as shown in Figure 13(b), and then the strain increased rapidly due to post-yield behaviour. In the compression zone, the compressive strain of concrete increased slowly to 0.0172, whereas the strain of the compressive reinforcement was only 0.0012 – significantly smaller than that of the tensile reinforcement and concrete. These variations in strains agree well with the calculated forces in the compression zone of the beam, as shown in Figure 10(b).

Based on the calculated strain of the tensile reinforcement, the yield displacement of the beam–column joints can be determined from the analytical model; these are reported in Table 1. It should be noted that the yield displacement of the

Figure 9. Comparison of experimental (Scarpas, 1981) and analytical load–displacement curves of exterior joints: (a) unit 1; (b) unit 2; (c) unit 3

interior beam–column joints refers to the average value of horizontal displacement at which the top and bottom reinforcement of the beam attained the yield strength whereas, in exterior joints, it represents the displacement at which the beam top reinforcement yielded. The average ratio of calculated to measured yield displacements was 1.12, with a coefficient of variation of 14%. Thus, the yield displacement of beam–column joints is slightly overestimated through the model. However, the yield displacement of exterior joints tested by Scarpas (1981) was not provided and only analytical values are included in Table 1.

Components of joint deformations

Under cyclic loading conditions, RC beam–column joints experience significant deformations. As shear distortion of the joint zone and shear deformations of beams and columns

are neglected in the rigid-plastic model, total deformations comprise two components – the deformations of beams and columns. Figure 14 shows the rotations of beams and columns relative to the joint zone. Note that the total rotation of the beams includes flexural deformation and the rotation induced by slip of the beam longitudinal reinforcement embedded in the joint zone. With increasing horizontal displacement at the column top, the rotation of the beams kept increasing, whereas rotation of the columns increased to a value of 0.0022 rad at about 20 mm horizontal displacement and then remained nearly constant. In terms of the ratio of beam rotation to total rotation of the joint, the ratio increased from 0.76 to 0.96 when the horizontal displacement at the column top varied from 10 mm to 125 mm, indicating that rotation of the joint started to concentrate at the beam–joint interfaces.

Figure 10. Forces in the compression zones of beams in unit 1 (Xin, 1992): (a) column; (b) beam

Figure 11. Horizontal shear force in the beam–column joint of unit 1 (Xin, 1992)

Figure 12. Variation of neutral-axis depths at joint interfaces of unit 1 (Xin, 1992)

Slip of steel reinforcement

Slips of the beam longitudinal reinforcement at the centre of the embedment length and the joint interface were calculated using the bond–slip model. Figure 15 shows the variations in reinforcement slip in relation to horizontal displacement at the column top. At the elastic stage of steel reinforcement, the slip at the centre of the embedment length was close to that at the joint interface, as shown in Figure 15. When the top reinforcement yielded at the beam–joint interface at 20 mm horizontal displacement of column, the slip at the joint centre was only 0.41 mm. Once post-yield behaviour of the steel reinforcement commenced, the slip at the joint interface increased rapidly to 7.63 mm due to inelastic elongation of the reinforcement, whereas the slip at the joint centre remained limited to only 0.79 mm – less than the slip associated with the maximum

bond stress. Therefore, pull-out failure of the beam reinforcement did occur in the joint zone, as the maximum bond stress between the steel reinforcement and concrete had not been attained along the elastic steel segment.

With slip of the tensile reinforcement at the joint interface, the total rotation of the beams with regard to the joint can be decomposed into two components – reinforcement slip and flexural deformation of the beams. These two components can be calculated using

$$30. \quad \theta_{rs} = \frac{s_{br}}{h_b - c_{br} - a_{sb}}$$

Figure 13. Strains of steel reinforcement in beams and columns: (a) column; (b) beam

Figure 14. Components of interior joint deformations

$$31. \quad \theta_{rb} = \frac{\delta_{bs}}{h_b - c_{br} - a_{sb}}$$

where θ_{rs} and θ_{rb} are the rotations of beams relative to the joint resulting from, respectively, slip of longitudinal reinforcement from the joint and beam flexural deformation.

Figure 16 shows the variation in beam rotations due to reinforcement slip and flexural deformation. At the beam–joint interface, the components of beam rotation induced by flexural deformation and reinforcement slip increased with increasing horizontal displacement at the column top. The maximum rotation caused by beam flexural deformation was 0.035 rad. In addition, the figure shows that slip of the longitudinal reinforcement in the beam contributed an increasingly greater portion to total rotation of the beam relative to the joint. The maximum ratio of beam rotation induced by reinforcement slip

Figure 15. Slips of beam reinforcement at the centre and interface of the joint

to the total rotation was around 37%. Therefore, if slip of the reinforcement embedded in the beam–column joint is not considered in the model, much stiffer behaviour is predicted.

Discussion of the joint model

The rigid-plastic model proposed by Park (1964) assumes that the steel reinforcement has yielded and the concrete has; thus, only the plastic behaviour of beams or slabs could be predicted. Furthermore, the slip of the longitudinal reinforcement relative to the joint was not considered and therefore the estimated load–displacement response was fairly stiff. In the proposed joint model, a bilinear stress–strain curve is assumed for the steel reinforcement, and the stress–strain model for concrete proposed by Mander *et al.* (1988) is utilised instead of an

Figure 16. Contributions of reinforcement slip and flexural deformation to beam rotation

equivalent rectangular compressive stress block to calculate the compression force sustained by the concrete. Therefore, the load–displacement curve of the beam–column joint can be determined at the elastic stage. In addition to the concrete model, the proposed model also considers slip of the beam longitudinal reinforcement embedded in the joint. Elastic and post-yield bond stresses determined by Kang and Tan (2015b) are used to determine the slip of longitudinal reinforcement at different stages. Accordingly, good agreement between experimental and analytical results can be obtained.

However, the simplified model has several limitations. During analysis, the beams, columns and joint zone are assumed to remain rigid and shear deformations are neglected. Therefore, the model is only applicable to beam–column joints with adequate stirrups, as shear failure of the joints cannot be predicted. In addition, only the force–displacement envelope can be obtained for interior and exterior beam–column joints subjected to lateral loads, whereas the whole response under cyclic loadings cannot be simulated. As a result, the accumulation of damage under cyclic loading conditions is not considered, which may result in overestimation of the applied load.

Conclusions

An analytical model for RC beam–column joints under seismic loads has been described. In the model, the beams, columns and the joint are assumed to remain rigid and flexural deformations are concentrated at the joint interfaces. Strains of steel reinforcement and concrete are determined from the compatibility condition at the joint interfaces, and slips of beam longitudinal reinforcement embedded in the joint are taken into consideration. Furthermore, forces acting at the joint interfaces are calculated by means of stress–strain models of steel bars and concrete. The model was validated using previously obtained experimental results of interior and exterior beam–column joints under seismic loads. In addition to the load–displacement

envelopes, neutral-axis depths and strains at the joint interfaces can also be obtained through the model to gain insight into the joint behaviour at the sectional level. The model can also be used to estimate the yield displacement of beam–column joints and to quantify the forces transferred through the steel reinforcement and the concrete into the joint zone. The model therefore enables further analytical study of the behaviour of a joint zone subjected to horizontal shear forces.

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REFERENCES

- Alsiwat JM and Saatcioglu M (1992) Reinforcement anchorage slip under monotonic loading. *Journal of Structural Engineering* **118**(9): 2421–2438.
- CEB-FIP (Comité Euro-International du Béton and Fédération International de la Précontrainte) (1991) CEB-FIP Model Code 1990. CEB, Lausanne, Switzerland.
- Eligehausen R, Popov EP and Bertero VV (1983) *Local Bond Stress–Slip Relationships of Deformed Bars under Generalized Excitations*. College of Engineering, University of California, Berkeley, CA, USA, Report UCB/EERC-83/23.
- Filippou FC, Popov EP and Bertero VV (1983) *Effects of Bond Deterioration on Hysteretic Behavior of Reinforced Concrete Joints*. Earthquake Engineering Research Center, University of California, Berkeley, CA, USA, Report UCB/EERC-83/19.
- Guice LK, Slawson TR and Rhomberg EJ (1989) Membrane analysis of flat plate slabs. *ACI Structural Journal* **86**(1): 83–92.
- Kang SB and Tan KH (2015a) Analytical model for compressive arch action in horizontally-restrained beam–column sub-assemblages. *ACI Structural Journal* **113**(4): 813–826.
- Kang SB and Tan KH (2015b) Bond–slip behaviour of deformed reinforcing bars embedded in well-confined concrete. *Magazine of Concrete Research* **68**(10): 515–529, <http://dx.doi.org/10.1680/jmacr.15.00245>.
- Keenan WA (1969) *Strength and Behavior of Restrained Reinforced Concrete Slabs Under Static and Dynamic Loading*. Naval Civil Engineering Laboratory, Port Hueneme, CA, USA, Technical Report R-621.
- Lin CM, Restrepo JJ and Park R (2000) *Seismic Behaviour and Design of Reinforced Concrete Interior Beam Column Joints*. Department of Civil Engineering, University of Canterbury, Christchurch, New Zealand, Report 2001-1.
- Lowes LN and Altoontash A (2003) Modeling reinforced-concrete beam–column joints subjected to cyclic loading. *Journal of Structural Engineering* **129**(12): 1686–1697.
- Lowes LN, Mitra N and Altoontash A (2004) *A Beam–Column Joint Model for Simulating the Earthquake Response of Reinforced Concrete Frames*. Pacific Earthquake Engineering Research Center, University of California, Berkeley, CA, USA, PEER Report 2003/10.
- Mander JB, Priestley MJN and Park R (1988) Theoretical stress–strain model for confined concrete. *Journal of Structural Engineering* **114**(8): 1804–1826.

- Megget LM (1974) Cyclic behaviour of exterior reinforced concrete beam–column joints. *Bulletin of the New Zealand National Society for Earthquake Engineering* **70(1)**: 27–47.
- Park R (1964) The ultimate strength and long-term behaviour of uniformly loaded, two-way concrete slabs with partial lateral restraint at all edges. *Magazine of Concrete Research* **16(48)**: 139–152, <http://dx.doi.org/10.1680/mac.1964.16.48.139>.
- Park R and Gamble WL (2000) *Reinforced Concrete Slabs*. Wiley, New York, NY, USA.
- Paulay T and Priestley MJN (1992) *Seismic Design of Reinforced Concrete and Masonry Buildings*. Wiley, New York, NY, USA.
- Ruitong D and Park R (1987) *A Comparison of the Behaviour of Reinforced Concrete Beam–column Joints Designed for Ductility and Limited Ductility*. Department of Civil Engineering, University of Canterbury, Christchurch, New Zealand, Report 87-4.
- Scarpas A (1981) *The Inelastic Behaviour of Earthquake Resistant Reinforced Concrete Exterior Beam–column Joints*. Department of Civil Engineering, University of Canterbury, Christchurch, New Zealand, Report 81-2.
- Sezen H and Setzler EJ (2008) Reinforcement slip in reinforced concrete columns. *ACI Structural Journal* **105(3)**: 280–289.
- Viwathanatepa S, Popov EP and Bertero VV (1979) *Effects of Generalized Loadings on Bond of Reinforcing Bars Embedded in Confined Concrete Blocks*. College of Engineering, University of California, Berkeley, CA, USA, Report UCB/EERC-79/22.
- Welch RW, Hall WJ and Gamble WL (1999) *Compressive Membrane Capacity Estimates in Laterally Edge Restrained Reinforced Concrete One-way Slabs*. Department of Civil and Environmental Engineering, University of Illinois at Urbana-Champaign, Champaign, IL, USA, Report UILU-ENG-99-2009.
- Xin XZ (1992) *Behaviour of Reinforced Concrete Interior Beam–column Joints Designed Using High Strength Concrete and Steel*. Department of Civil Engineering, University of Canterbury, Christchurch, New Zealand, Report 92-3.
- Yu J and Tan KH (2013) Analytical model for the capacity of compressive arch action of reinforced concrete beam–column sub-assemblages. *Magazine of Concrete Research* **66(3)**: 109–126, <http://dx.doi.org/10.1680/mac.13.00217>.

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