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Online fringe projection profilometry based on scale-invariant feature transform

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Abstract. An online fringe projection profilometry (OFPP) based on scale-invariant feature transform (SIFT) is proposed. Both rotary and linear models are discussed. First, the captured images are enhanced by “retinex” theory for better contrast and an improved reprojection technique is carried out to rectify pixel size while keeping the right aspect ratio. Then the SIFT algorithm with random sample consensus algorithm is used to match feature points between frames. In this process, quick response code is innovatively adopted as a feature pattern as well as object modulation. The characteristic parameters, which include rotation angle in rotary OFPP and rectilinear displacement in linear OFPP, are calculated by a vector-based solution. Moreover, a statistical filter is applied to obtain more accurate values. The equivalent aligned fringe patterns are then extracted from each frame. The equal step algorithm, advanced iterative algorithm, and principal component analysis are eligible for phase retrieval according to whether the object moving direction accords with the fringe direction or not. The three-dimensional profile of the moving object can finally be reconstructed. Numerical simulations and experimental results verified the validity and feasibility of the proposed method. © 2016 Society of Photo-Optical Instrumentation Engineers (SPIE) [DOI: [10.1117/1.OE.55.8.084101](https://doi.org/10.1117/1.OE.55.8.084101)]

Keywords: fringe projection profilometry; online measurement; quick response code; object modulation; pixel matching; scale-invariant feature transform.

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1 Introduction

The rapid technological development in modern industrialization is accompanied by an increasing need to monitor the quality of products with high speed and accuracy at the same time. A static measurement, which is carried out one by one, is a time-consuming procedure and is not conducive regarding the production efficiency. Nowadays, an automatic pipeline has been widely used in manufacturing which helps integrate processes and save time. Therefore, online measurement has attracted much attention. The fringe projection technique has various applications due to its advantages in measuring speed and flexibility, which gives it a huge potential in online three-dimensional (3-D) measurements.

Many fringe projection techniques have been proposed to measure the 3-D shape of the object. There are two primary techniques named Fourier transform profilometry (FTP) and phase measuring profilometry (PMP), which have been extensively studied in recent years.^{1–6} FTP was initially proposed by Takeda and Mutoh.⁷ In this method, the 3-D profile is reconstructed with only one deformed fringe pattern. This method seems to be suitable for online measurement due to the varying object positions. Nevertheless, its precision is not very high due to the filtering operation in the Fourier domain, and it is not robust to high step samples. PMP was proposed to improve the performance.⁸ Despite the fact that it requires

more frames, PMP has shown a great promise in high precision measurement.

However, the traditional PMP algorithm cannot be implemented directly in both rotary and linear online fringe projection profilometry (OFPP), because the object positions in each frame are not the same. If the identical regions of interest (ROI) of the object in each frame are known, the fringe patterns can be realigned. Thus, those traditional fringe analysis methods can be available again. The characteristic parameters for realignment are usually obtained by pixel matching. Researchers have proposed many methods to conduct pixel matching when dealing with a moving object. Wu et al. proposed a pixel matching method with binarized modulation⁹ and delaminated modulation.¹⁰ Peng et al.¹¹ used low modulation parts to match pixels. Li et al.¹² proposed a layered modulation method to calculate object displacement based on a centroid concept with selected modulation layers. Chen et al.¹³ and Mao et al.¹⁴ determined moving pixels using extra markers composed of four symmetric angles. Wang et al.¹⁵ introduced a rotating model using round dots as the markers. A color-encoded model was presented by Wan et al.¹⁶ Yuan et al.¹⁷ used four corner points of a fringe pattern to calculate a reprojection matrix and the only rectilinear model discussed had no deviation of the moving direction.

In this paper, SIFT^{18,19} based rotary OFPP and linear OFPP are proposed. Considering the potential mismatching issue

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Fig. 1 Highlights of the proposed method.

brought by SIFT, the matching results will be filtered by random sample consensus (RANSAC)²⁰ algorithm to ensure correctness. To improve matching performance, the captured images are enhanced by "retinex" theory.^{21,22} Also, the perspective distortion of the image is rectified by an improved reprojection technique. Because of the advantages in feature recognition, quick response (QR) code²³ is innovatively adopted as feature patterns for pixel matching in OFPP. Additionally, object modulation can also be used as a feature pattern if no extra artificial markers are attached. A vector-based solution with a statistical filter is proposed to obtain accurate characteristic parameters, including rotation angle in rotary OFPP and rectilinear displacement in linear OFPP. The equivalent phase-shifting fringes can be extracted. Normally, in traditional OFPP, the moving direction is the same as the fringe direction. However, direction deviation occurs when the setup is not well aligned. AIA^{24,25} and PCA^{26,27} can be used in such cases for phase retrieval. After applying a phase to height mapping equation, the object profile on a moving stage can be reconstructed.

2 Principle

The major difference between OFPP and a static model is whether the object is moving. If the online dynamic process can be converted to an offline static process, the previous PMP knowledge and equations can be utilized again. Inspired by this idea, OFPP based on SIFT is proposed and the detailed principles will be presented in this section. The highlights of the proposed method are shown in Fig. 1. Rotary OFPP and linear OFPP are presented in Sec. 2.1. SIFT with RANSAC algorithm as well as criteria for sensitive parameters are introduced in Sec. 2.2. An introduction for image enhancement with retinex theory and an image reprojection technique used to rectify the image is presented in Sec. 2.3. The adopted feature patterns including QR code and object modulation are illustrated in Sec. 2.4. A vector-based solution with a statistical filter to obtain rotation angle and rectilinear displacement is proposed in Sec. 2.5. The phase retrieval methods with three different algorithms for

two moving scenarios, moving direction accords with fringe direction or deviates from fringe direction, are discussed in Sec. 2.6. The whole processing flow is illustrated in Fig. 2.

2.1 Online Fringe Projection Profilometry

There are two common types of pipeline in the industry: one is the rotary model and another is the linear model. They both are used to transport products to another place. Usually, the rotary pipeline is located at the joint where the product needs to change transport directions, while the linear pipeline is more often used for long distance transport. According to these two kinds of operation modes, rotary OFPP and linear OFPP are proposed. In this section, only structures and formulations are introduced. For the phase retrieval solutions, numerical simulations, and experimental results, please refer to the corresponding sections.

2.1.1 Rotary online fringe projection profilometry

The layout of rotary OFPP is shown in Fig. 3. The object is placed on a rotary pipeline, and the digital light processing (DLP) is located right above the platform rotation center. The projected patterns are concentric sinusoidal fringes, and the phase-shifting direction is the radial direction. CCD is used to record deformed fringe patterns while the platform is rotating.

In N-step phase-shifting theory, the intensity of deformed pattern $I_n(x, y)$ for rotary model can be expressed as in Eq. (1):

$$I_n(x, y) = R(x, y) \left\{ A(x, y) + B(x, y) \cos \left[\frac{2\pi}{T} \sqrt{x^2 + y^2} + \varphi(x, y) + \frac{2\pi n}{N} \right] \right\},$$

$$n = 1, 2, \dots, N, \quad (1)$$

where $R(x, y)$ is the reflectivity of the object, $A(x, y)$ is the background intensity, $B(x, y)$ is the fringe contrast, T is the

Fig. 2 Processing flow of OFPP.

Fig. 3 Rotary OFPP setup: (a) rendering diagram and (b) schematic diagram.

period of fringes, $\varphi(x, y)$ is the phase to be measured which represents the height of the object, N is the total phase-shifting step, and n denotes the current step number. Therefore, there is a phase-shifting amount $2\pi/N$ between adjacent frames.

2.1.2 Linear online fringe projection profilometry

The layout of the linear OFPP is shown in Fig. 4. The object is placed on a linear pipeline, and the axis of DLP is vertical to the platform plane, which ensures a uniform period of fringes. The projected patterns are straight sinusoidal fringes, and the phase-shifting direction is perpendicular to the fringe direction. A CCD is used to record deformed fringe patterns while the platform is moving.

Similarly, in N -step phase-shifting theory, the intensity of deformed pattern $I_n(x, y)$ for a linear model can be expressed as in Eq. (2):

$$I_n(x, y) = R(x, y) \left\{ A(x, y) + B(x, y) \cos \left[\frac{2\pi}{T_x} x + \frac{2\pi}{T_y} y + \varphi(x, y) + \frac{2\pi n}{N} \right] \right\},$$

$$n = 1, 2, \dots, N, \quad (2)$$

where T_x and T_y are the fringe periods along the x and y directions, respectively, while the definitions of other symbols are the same as the rotary model.

2.2 Scale-Invariant Fourier Transform Algorithm

Scale-invariant feature transform (SIFT)^{18,19} was first proposed by David Lowe, whose characteristics are based on some local appearance of object interest points, and not affected by image scaling or rotation. Moreover, this method

Fig. 4 Linear OFPP setup: (a) rendering diagram and (b) schematic diagram.

has a quite high tolerance for illumination, noise and perspective change, and so on. It can be used for feature point matching in multiple transformations. Considering the mismatching problem brought by SIFT, the feature points will be filtered by the RANSAC²⁰ algorithm to ensure the high precision of the result. Some improved methods were also developed, such as PCA-SIFT and speeded up robust features (SURF) algorithms, but there is no superior method for every deformation.^{28,29} Hence, it is important to know the performance of most interest when choosing a feature detection method. The research shows that SIFT performs better compared to SURF in scale and rotation transformations under a multispectral environment. Therefore, SIFT is very suitable for pixel matching in this work. The basic theory of pixel matching with the SIFT algorithm is given below.

2.2.1 Initial points matching

SIFT can provide a vector with 128 dimensions for each feature point, denoted by v . For corresponding points (x_{iA}, y_{iA}) in image A and point (x_{jB}, y_{jB}) in image B, the vector v is subject to Eq. (3):

$$\frac{\langle v_{iA}, v_{kB} \rangle}{\langle v_{iA}, v_{jB} \rangle} < t, \quad (3)$$

where t is the threshold, i and j denote the i 'th and j 'th feature points in A and B respectively, $\langle \cdot \rangle$ means the Euclidean distance of vectors, and $\langle v_{iA}, v_{kB} \rangle$ is the second maximum value of $\langle v_{iA}, v_{jB} \rangle$ for all feature points in image B. For a different threshold t , the number of matches will be different. A large number of matches can be obtained by a large threshold, but there might be incorrect matches. With the decrease of the threshold, incorrect matches will be eliminated, but the number of matches will also decrease, which will result in a bad estimation of characteristic parameters because of fewer samples. Normally, t is assigned 0.5 in practice.

2.2.2 Points filtering with random sample consensus

Considering the trade-off between the number of matches and wrong matches, RANSAC will be used to filter the initial matches with a relatively larger threshold. Taking a linear model as an example, without losing generality, the moving direction of the object is assumed along the x axis. Thus, matches need to meet the requirements as follow:

$$\begin{cases} ||x_{iA} - x_{iB}| - |x_{kA} - x_{kB}|| < \varepsilon_x \\ |y_{iA} - y_{iB}| < \varepsilon_y \\ |y_{kA} - y_{kB}| < \varepsilon_y \end{cases}, \quad (4)$$

where $(x_{iA}, y_{iA}; x_{iB}, y_{iB})$ means the i 'th match, which is also called the corresponding point pair. ε_x and ε_y are the tolerances of displacement. To perform RANSAC, a seed match will be chosen randomly and then all the remaining matches will be added into seed matches by the rules in Eq. (5):

$$\begin{cases} |x_{jA} - x_{jB}| < \sqrt{\frac{1}{N_s} \sum |x_{iA} - x_{iB}|^2} + \varepsilon_x \\ |y_{jA} - y_{jB}| < \varepsilon_y \end{cases}, \quad (5)$$

where $(x_{jA}, y_{jA}; x_{jB}, y_{jB})$ is the j 'th match and N_s is the number of seed matches. After searching all matches, if the ratio between seed matches and total matches is larger than p , the seed matches are assumed to meet the requirement of Eq. (6). Otherwise, it will be discarded and repeated from choosing the first seed match.

$$N_s/N > p, \quad (6)$$

where N is the total number of matches. During an experiment, assigning a appreciate value to p is important, because p is the probability of accurate matches in total matches, which is unknown. On one hand, if p has a small value, very few matches will be obtained, which will affect the accuracy. Vice versa, if p is too large, there will be no matches either. In this work, p is assigned 0.1.

Figure 5 is the feature point matching the results of cameraman and coin images with themselves. It is noticed that feature points are matched correctly and accurately, and the matched points are mainly clustered in the place where the characteristic information is abundant, such as boundaries and edges.

2.3 Image Enhancement and Correction

The captured images should be enhanced and rectified before being using in pixel matching or modulation extraction. In this paper, captured images are enhanced by retinex theory for better contrast. Additionally, considering the perspective distortion of images caused by the triangulation architecture of the system, an image reprojection technique is proposed to rectify the pixel size while keeping the right aspect ratio.

Fig. 5 Feature point self-matching results with SIFT: (a) and (c) the original images of cameraman and coins, respectively; (b) and (d) the corresponding matching results.

2.3.1 Retinex theory

Retinex^{21,22} is short for retinal cortex, which is a representative color constancy theory. Land initially proposed it in the early 1970s. Retinex theory mainly includes two aspects: the object color is determined by the light reflection ability of the object on a long, medium, and short wave, rather than by the intensity of reflected light; object color has a consistency, which is not affected by nonhomogeneous illumination. The basic model of retinex can be expressed as in Eq. (7):

$$S(x, y) = R(x, y) \cdot L(x, y), \quad (7)$$

where $S(x, y)$ is the intensity distribution of the image, $R(x, y)$ is the reflectance, $L(x, y)$ is the illumination, and (x, y) denotes the pixel coordinate. $L(x, y)$ is determined by the light source, while $R(x, y)$ is determined by the characteristics of the object. The core of the retinex method is to estimate the illumination L . This component is estimated from image S , and then L component can be removed to get the original reflection component R .

From the characteristics of this method, it can be concluded that the light source has a small influence on the relative brightness of each pixel. For low contrast images, such as underexposed or overexposed images, this technique can eliminate the uniformity influence caused by a light source, so it can be used to improve the contrast of the image and greatly improve the subjective quality of the image.

To verify the validity of this method on fringe patterns, the Frankle-McCann retinex algorithm³⁰ is applied to test the effect of enhancement with sample images. Figure 6 is the comparison before and after enhancing. The left column of Fig. 6 is the original image, while the right is the enhanced one by retinex. It is obvious that the contrast of fringe patterns is greatly enhanced. Furthermore, the nonuniform illumination of the background has also been eliminated.

Fig. 6 Images before and after retinex enhancing: (a) and (c) the original images; (b) and (d) the enhanced corresponding images.

2.3.2 Image reprojection

As the layouts are shown in Figs. 3 and 4, even if the perfect sinusoidal fringes are projected by DLP, the perspective distortion images would be recorded by the CCD. Apparently, the pixel size is no longer uniform in a frame, which results in failure of the traditional PMP solution. Figure 7 shows aerial views and oblique views of simulated frames from two models. Imaging from different view angles is actually a process of perspective transformation.³¹ The general representation of a perspective transformation is Eq. (8):

$$\begin{bmatrix} u \\ v \\ s \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}, \quad (8)$$

Fig. 7 Simulated perspective views of two frames in rotary and linear OFPP: (a) and (c) the aerial views of two frames in rotary OFPP; (b) and (d) the corresponding oblique views; (e) and (g) the aerial views of two frames in linear OFPP; (f) and (h) the corresponding oblique views.

where (x, y) and (u, v) denote the coordinates before and after transformation, respectively, h_{ij} ($i, j = 1, 2, 3$) is the parameter of the perspective transformation matrix $\mathbf{H}$, and s represents the scale factor. The transformed coordinates (x', y') in the homogeneous system can be calculated by $x' = u/s$ and $y' = v/s$. Thus, the new coordinates can be determined by forward mapping as in Eq. (9):

$$\begin{cases} x' = \frac{u}{s} = \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + 1} \\ y' = \frac{v}{s} = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + 1} \end{cases} \quad (9)$$

It should be noted that the perspective transformation matrix has eight degrees of freedom, so $h_{33} = 1$. Therefore, the matrix $\mathbf{H}$ can be solved by four point correspondences. Figure 8(a) is the calibration board pattern, which is projected by DLP onto the reference plane. Figure 8(b) is the assumed perspective distortion image, which is recorded by the CCD. The aim is to rectify Fig. 8(b) back to Fig. 8(a). To obtain $\mathbf{H}$, a very flexible reprojection method is proposed. One projected calibration board pattern is used to acquire transformation parameters. Normally, four corner coordinates are selected for calculation, because they enclose more areas. By utilizing a centroid recognition algorithm, the coordinates of all dots' centroids are obtained. To suppress the error, all these centroids are fitted to straight lines by column and row. The outermost line intersection points are considered to be the four initial points, as shown in Fig. 9.

Fig. 8 Calibration board patterns: (a) original pattern and (b) perspective distortion pattern.

Fig. 9 Centroids of the dots and their fitted grid lines. (Blue lines are the outermost fitted lines and red markers are the intersection points.)

The other four corresponding coordinates after rectifying are defined as in Eq. (10): x'_1

$$\begin{cases} (x'_1, y'_1) = (x_1, y_1) \\ (x'_2, y'_2) = (x_1 + l, y_1) \\ (x'_3, y'_3) = (x_1, y_1 + w) \\ (x'_4, y'_4) = (x_1 + l, y_1 + w) \end{cases}, \quad (10)$$

where l and w are the length and width of the rectified rectangle, respectively, $l = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, $w = \gamma l$, and γ is the aspect ratio of the width and length, which can be known by calibration board. The calibration pattern contains 19×13 circular dots, with the same interval distance between their centroids. Hence, the original ratio of width and length should be $\gamma = 2/3$. The perspective mapping is illustrated in Fig. 10. The four pairs of correspondence points yield an 8×8 system as in Eq. (11):

Fig. 10 Coordinates reprojection.

$$\begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ x'_3 \\ y'_3 \\ x'_4 \\ y'_4 \end{bmatrix} = \begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x'_1x_1 & -x'_1y_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -y'_1x_1 & -y'_1y_1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 & -x'_2x_2 & -x'_2y_2 \\ 0 & 0 & 0 & x_2 & y_2 & 1 & -y'_2x_2 & -y'_2y_2 \\ x_3 & y_3 & 1 & 0 & 0 & 0 & -x'_3x_3 & -x'_3y_3 \\ 0 & 0 & 0 & x_3 & y_3 & 1 & -y'_3x_3 & -y'_3y_3 \\ x_4 & y_4 & 1 & 0 & 0 & 0 & -x'_4x_4 & -x'_4y_4 \\ 0 & 0 & 0 & x_4 & y_4 & 1 & -y'_4x_4 & -y'_4y_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{bmatrix} \quad (11)$$

The simplified expression can be written as $\mathbf{b} = \mathbf{A}\mathbf{h}$. The least squares solution to the system $\mathbf{A}\mathbf{h} = \mathbf{b}$ is given by a so-called normal equation $\mathbf{A}^T\mathbf{A}\mathbf{x} = \mathbf{A}^T\mathbf{b}$.³² If the columns of $\mathbf{A}$ are linearly independent, the perspective transformation parameters can be obtained by Eq. (12):

$$\mathbf{h} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b}. \quad (12)$$

After obtaining $\mathbf{H}$, all points in the original image can be transformed to new coordinates. While most of the point coordinates are subpixel values, bicubic interpolation is implemented to compensate the discontinuities of the values. The rectified image is shown in Fig. 11. Obviously, the calibration board pattern has been successfully rectified with the correct aspect ratio. Although there is a scale difference compared to the original pattern, this will not affect the result of phase retrieval.

Theoretically, in a rectified image, the fitting lines of the dots' centroids per column or row should be parallel

Fig. 11 Rectified calibration board pattern.

Fig. 12 Standard deviation pixels along columns and rows.

or vertical to the image sides. To evaluate their parallelism and perpendicularity after rectifying, Eq. (13) is used to calculate standard deviation pixels of each column and row:

$$\begin{cases} \sigma_{\text{col},i} = \sqrt{\frac{1}{M} \sum_{j=1}^M (x_{ij} - \mu_{\text{col},i})^2} \\ \sigma_{\text{row},i} = \sqrt{\frac{1}{N} \sum_{j=1}^N (y_{ij} - \mu_{\text{row},i})^2} \end{cases}, \quad (13)$$

where $\sigma_{\text{col},i}$ and $\sigma_{\text{row},i}$ are the standard deviation pixels by column and row, respectively, and $\mu_{\text{col},i}$ and $\mu_{\text{row},i}$ are the means of corresponding groups. The result is shown in Fig. 12, which proves that this method is very reliable and accurate due to all deviations being much lower than one pixel.

2.4 Feature Pattern

Two types of feature patterns are proposed for pixel matching, in which the QR code is innovatively used to calculate the characteristic parameters. In previous work, more articles reported the pixel matching process is accomplished with object modulation⁹⁻¹² or special marker¹³⁻¹⁵ by image correlation method. QR code and object modulation are introduced below, respectively. Also, a discussion on spectrum choice in obtaining object modulation and a comparison with layered modulation method are given in this section.

2.4.1 Quick response code

QR code is a matrix two-dimensional code symbol, which was developed by Denso Wave Corporation in 1994.²³ QR code is composed of black square dots which are arranged in a grid on a white background. The contained information exists in horizontal and vertical components of the pattern. It has many advantages, such as large information capacity, high reliability, multiple characters support, and strong confidential security. QR code has been widely used in applications including product tracking, item identification, time tracking, document management, and general marketing.

In the proposed method, the QR code is not used for information transmission but as a feature pattern for pixel matching with the SIFT algorithm. Due to its structural characteristics, the QR code could have a better performance in

Fig. 13 QR code samples: (a) optical engineering, (b) Sichuan University, (c) Nanyang Technological University, and (d) Hongru Li.

the feature matching. On the macro, the coded black square dots are randomly arranged, which means surrounding patterns of the matched points are almost unique. There are usually no two identical patterns in a certain range, which greatly improves the matching accuracy. Moreover, QR code has clear boundaries and edges, which is more conducive to determine feature points. Four QR code samples are illustrated in Fig. 13. The first QR code pattern represents “Optical Engineering,” which will be used in multiple simulations in Sec. 3.

2.4.2 Object modulation

Based on Fourier analysis theory, a straight sinusoidal fringe pattern from linear OFPP model can be expressed as in Eq. (14) in the Fourier domain. It is noticed that the spectra could be divided into three parts. Please note that due to the formulation of Eq. (1), there is no similarity to Eq. (14) for the rotary OFPP model. Thus, the object modulation cannot be obtained with such a method in a rotary OFPP model:

$$S_n(f_x, f_y) = S_{n(0)}(f_x, f_y) + S_{n(1)}(f_x, f_y) + S_{n(-1)}(f_x, f_y), \quad (14)$$

where $S_{n(0)}(f_x, f_y)$, $S_{n(1)}(f_x, f_y)$, and $S_{n(-1)}(f_x, f_y)$ represent zero, first positive, and first negative order spectra, respectively. For Eq. (14), a proper band-pass filter is used to extract the specific order. Implementing an inverse Fourier transform on the selected spectrum, it will be converted back to a spatial domain with Eq. (15). Then, taking the absolute value of $Q_n(x, y)$, M_n is defined as the object modulation with Eq. (16).

$$Q_n(x, y) = \iint_{-\infty}^{+\infty} S_n(f_x, f_y) \exp[i2\pi(f_x x + f_y y)] df_x df_y, \quad n = 1, 2, \dots, 7, 8, \quad (15)$$

$$M_n = |Q_n(x, y)| \propto R_n(x, y) B_n(x, y). \quad (16)$$

It is indicated that the modulation is directly proportional to surface reflectivity $R_n(x, y)$ and fringe contrast $B_n(x, y)$. In addition, Eq. (16) has the same formulation as in Eq. (7), which confirms the relationship between the two.

Based on the above analysis, both the zero order spectrum and first order spectrum contain modulation information. However, in an actual experiment, the zero order spectrum is always in the center of the Fourier domain, which makes it very easy to extract. Furthermore, the modulation acquired by the zero order spectrum contains more details. Figures 14(a) and 14(b) are the modulations extracted from the zero order spectrum of first and eighth frames, respectively.

Fig. 14 Obtained sculpture modulations with different spectra: (a) and (b) the modulations of first frame and eighth frame with zero order spectrum; (c) and (d) the corresponding modulations with positive first order spectrum.

Figures 14(c) and 14(d) are derived from the first positive order spectrum. Obviously, the sense of modulation from the zero order spectrum is closer to a real object, and more detailed structure can be seen. On the contrary, the modulation from the first order spectrum provides a more intuitive object contour. There are many ghost waves of contours, which is caused by spectrum leakage in the Fourier transform. So, a zero order spectrum is adopted in the experiment. In the simulation section, the case with modulation recovered from the first order spectrum is also provided for reference.

The pixel matching in most previous articles depended on image correlation with the object modulation distribution. Without loss of generality, a brief comparison of this approach is discussed. Figure 15 shows binarized modulation of different layers with a layered modulation method.¹² Figures 15(a) and 15(e) are the masks of the first segmentation on Figs. 14(c) and 14(d). A repeated application of the Otsu method³³ enables to obtain the object's high, medium, and low modulation masks, which are shown in Figs. 15(b)–15(d) and Figs. 15(e)–15(h). Afterward, the displacement will be calculated by the related image correlation method.

It is observed that masks in each corresponding group are not the same. Thus, direct image correlation may not be accurate. If the object structure is simple or has a clear texture, binarized segmentation could be available. But for those objects with more complex textures, forcibly employing segmentation for image correlation may produce a large error. Under such circumstances, a feature matching method is very suitable.

Fig. 15 Segmentation process of modulations. (a) and (e) The first segmentation masks of Figs. 14(c) and 14(d) respectively, (b)–(d) and (f)–(h) the high, medium, and low modulation mask of the two frames, respectively.

2.5 Vector Conversion

As matched point pairs can be obtained by the SIFT algorithm, the next step is using matching information to obtain rotation angles and rectilinear displacements in rotary and linear OFPP models, respectively. A very effective and accurate vector-based solution with a statistical filter is proposed.

2.5.1 Rotation angle

Figure 16 is the diagram of a rotary vector in the rotary OFPP model. The following derivation procedure proves that the rotation angle could be obtained by calculating the vectors of the matched point pairs, while there is no need to consider where the rotation center actually is.

Assuming A_1 and A_2 , B_1 and B_2 are the two pairs of matched points, O is the rotation center, the angle $\angle A_1OB_1$ for vector $\overrightarrow{OA_1}$ and $\overrightarrow{OB_1}$ is α , the rotation angle is β , and the angle between vector A_1B_1 and A_2B_2 is γ . It can be proved that the rotation angle β equals γ . According to vector theory, the angle γ follows the following equation in Eq. (17):

$$\cos \gamma = \frac{\overrightarrow{A_1B_1} \cdot \overrightarrow{A_2B_2}}{|\overrightarrow{A_1B_1}| |\overrightarrow{A_2B_2}|}, \quad (17)$$

where $\overrightarrow{A_1B_1} = \overrightarrow{OB_1} - \overrightarrow{OA_1}$, $\overrightarrow{A_2B_2} = \overrightarrow{OB_2} - \overrightarrow{OA_2}$. Substituting them into Eq. (17), we get

Fig. 16 Rotation angle in rotary model: (a) rotating vectors, (b) the angle between the vector and the x-axis positive direction, (c) the angle between the two vectors.

$$\cos \gamma = \frac{\overrightarrow{OB_1} \cdot \overrightarrow{OB_2} - \overrightarrow{OB_1} \cdot \overrightarrow{OA_2} - \overrightarrow{OA_1} \cdot \overrightarrow{OB_2} + \overrightarrow{OA_1} \cdot \overrightarrow{OA_2}}{\sqrt{(\overrightarrow{OB_1} - \overrightarrow{OA_1})^2} \sqrt{(\overrightarrow{OB_2} - \overrightarrow{OA_2})^2}}, \quad (18)$$

which defines $|\overrightarrow{OA_1}| = a$ and $|\overrightarrow{OB_1}| = b$. Apparently, $|\overrightarrow{OA_2}| = |\overrightarrow{OA_1}|$, $|\overrightarrow{OB_2}| = |\overrightarrow{OB_1}|$, and $|\overrightarrow{A_2B_2}| = |\overrightarrow{A_1B_1}|$ in the rotation. Hence, the numerator and denominator of Eq. (18) can be expanded to Eqs. (19) and (20), respectively.

$$\begin{aligned} & \overrightarrow{OB_1} \cdot \overrightarrow{OB_2} - \overrightarrow{OB_1} \cdot \overrightarrow{OA_2} - \overrightarrow{OA_1} \cdot \overrightarrow{OB_2} + \overrightarrow{OA_1} \cdot \overrightarrow{OA_2} \\ &= b^2 \cos \beta - ab \cos(\beta - \alpha) - ab \cos(\beta + \alpha) + a^2 \cos \beta, \end{aligned} \quad (19)$$

$$\begin{aligned} & \sqrt{(\overrightarrow{OB_1} - \overrightarrow{OA_1})^2} \sqrt{(\overrightarrow{OB_2} - \overrightarrow{OA_2})^2} \\ &= \overrightarrow{OB_1} \cdot \overrightarrow{OB_1} - \overrightarrow{OB_1} \cdot \overrightarrow{OA_1} - \overrightarrow{OA_1} \cdot \overrightarrow{OB_1} + \overrightarrow{OA_1} \cdot \overrightarrow{OA_1} \\ &= b^2 - 2ab \cos \alpha + a^2. \end{aligned} \quad (20)$$

Substituting the above two equations back into Eq. (18), it can be simplified as in Eq. (21):

$$\begin{aligned}\cos \gamma &= \frac{b^2 \cos \beta - ab \cos(\beta - \alpha) - ab \cos(\beta + \alpha) + a^2 \cos \beta}{b^2 - 2ab \cos \alpha + a^2} \\ &= \frac{b^2 \cos \beta - 2ab \cos \beta \cos \alpha + a^2 \cos \beta}{b^2 - 2ab \cos \alpha + a^2} = \cos \beta.\end{aligned}\quad (21)$$

Because γ and β are proportional in their variance, $\gamma \propto \beta$. Therefore, we obtain Eq. (22):

$$\gamma = \beta. \quad (22)$$

Hence the relationship is proved. Although the rotation angle can be figured out directly with the two matched vectors based on the conclusion above, the range of the calculated angle is $[0, \pi]$. To expand the scope of application to a full 2π range, the angle between the vector and the x -axis positive direction in the counterclockwise is calculated first. Assume $\mathbf{a}$ is one of the vectors, whose normalized coordinate is (x, y) , and $\mathbf{e}$ is the unit vector in the positive direction of the x -axis. The relationship between $\mathbf{a}$, $\mathbf{e}$, and θ can be expressed by Eq. (23). Due to the vector coordinate which could be in four different quadrants, the angle θ can be determined by Eq. (24), which guarantees the solved θ is in the 2π range:

$$\cos \theta = \frac{\mathbf{a} \cdot \mathbf{e}}{|\mathbf{a}|} = \frac{x}{\sqrt{x^2 + y^2}}, \quad (23)$$

$$\begin{cases} \theta = \arccos \cos \theta & x > 0, y \geq 0, \\ \theta = \arccos \cos \theta & x \leq 0, y > 0, \\ \theta = 2\pi - \arccos \cos \theta & x < 0, y \leq 0, \\ \theta = 2\pi - \arccos \cos \theta & x \geq 0, y < 0. \end{cases} \quad (24)$$

After obtaining both of the angles θ_1 and θ_2 from two frames, the angle of the two vectors can be calculated by Eq. (25). Please pay attention to the relationship between θ_1 and θ_2 :

$$\begin{cases} \gamma = \theta_2 - \theta_1 & \theta_2 - \theta_1 > 0, \\ \gamma = \theta_2 - \theta_1 + 2\pi & \theta_2 - \theta_1 \leq 0. \end{cases} \quad (25)$$

A vector can be generated between every two points, and all rotation angles in a vector structure can be obtained by every two matched vector pairs. The average of rotation angles is defined by Eq. (26), where N is the total number of angles, whose value is determined by $N = C_n^2 = n(n-1)/2$ and n is the number of matched point pairs:

$$\gamma_{\text{avg}} = \frac{1}{N} \sum_{i=1}^N \gamma_i. \quad (26)$$

2.5.2 Rectilinear displacement

Figure 17 is the diagram of a rectilinear vector in a linear OFPP model. Assume A_i and B_i are the matched feature points in frames A and B, respectively. Thus, the rectilinear displacement is determined by the vector $A_i B_i$. The displacement x_i and y_i in the x and y deflections can be calculated by Eq. (27), where A_{ij} and B_{ij} ($j \in x, y$) are the coordinates in the corresponding direction:

Fig. 17 Rectilinear displacement in linear model.

$$\begin{cases} x_i = B_{ix} - A_{ix} \\ y_i = B_{iy} - A_{iy} \end{cases}. \quad (27)$$

Similarly, the average of rectilinear displacement in the x and y directions is defined by Eq. (28), where N is the total number of vector pairs:

$$\begin{cases} x_{\text{avg}} = \frac{1}{N} \sum_{i=1}^N x_i \\ y_{\text{avg}} = \frac{1}{N} \sum_{i=1}^N y_i \end{cases}. \quad (28)$$

2.5.3 Statistical filter

Considering each vector pair could obtain one unique value for a rotation angle or rectilinear displacement, the results may have slight differences between. Although the final result is an average, some values with a large deviation will still affect the accuracy. A statistical filter is proposed to eliminate those values with large deviations.

According to central limit theorem, let $X_1, X_2, \dots, X_n$ be a sequence of independent and identically distributed random variables, whose expected values are μ and finite variances are σ^2 . Thus, the average of those variables can be expressed as in Eq. (29):

$$S_n = \frac{X_1 + X_2 + \dots + X_n}{n} = \frac{1}{n} \sum_{i=1}^n X_i. \quad (29)$$

Fig. 18 Probability density curve with normal distribution.

For a large enough n , the distribution of S_n is close to the normal distribution with mean μ and variance σ^2/n , which can be written as in Eq. (30):

$$S_n \sim N\left(\mu, \frac{\sigma^2}{n}\right). \tag{30}$$

A normal distribution probability density curve is illustrated in Fig. 18. Based on the statistical concept, those values which deviate with a large μ will be removed. In this paper, the selected range is $[\mu - 2\sigma, \mu + 2\sigma]$. This means about 95.5% values will be maintained for averaging calculation, and the other 4.5% values are considered to have

Fig. 19 The object moving direction deviates from the fringe direction: (a) deviation diagram in rotary OFPP and (b) deviation diagram in linear OFPP.

Table 1 Arrangement for numerical simulations.

Items	Case 1	Case 2	Case 3	Case 4
OFPP type	Rotary	Linear	Linear	Linear
Characteristic parameter	Angle	Displacement	Displacement	Displacement
Feature pattern	QR code	QR code	Object modulation	Object modulation
Moving direction	Accordance	Accordance	Accordance	Deviation

Fig. 20 Simulated fringe patterns with QR code in rotary OFPP: (a)–(h) the frames with rotation angles 0, 45, 90, 135, 180, 225, 270, and 315 deg, respectively.

relatively large deviations and need to be eliminated. This procedure can be expressed as in Eq. (31):

$$D \xrightarrow{D \in [\mu - 2\sigma, \mu + 2\sigma]} D_{\text{filtered}}. \tag{31}$$

2.6 Phase Retrieval

The equivalent phase-shifting patterns can be extracted based on the calculated phase characteristic parameters. Now the whole issue is converted into a static model. Thus, a traditional phase-shifting algorithm can be applied, such as the equal step algorithm (ESA). However, it is a strong assumption of the model that the object’s moving direction accords

Fig. 21 Feature point matching results with QR codes and their vector structures: (a), (c) and (e) the feature point matching results of QR codes between first frame and second, fifth, eighth frame, respectively, (b), (d) and (f) the corresponding vector structures.

Fig. 22 Calculated rotation angles based on vectors for each frame.

Table 2 Comparison of results on rotation angles for each frame.

Items	Frame 2	Frame 3	Frame 4	Frame 5	Frame 6	Frame 7	Frame 8
Theoretical angle (deg)	45	90	135	180	225	270	315
Average angle (deg)	45.0313	90.0011	134.9927	180.0000	224.9782	270.0000	314.9878
Filtered angle (deg)	45.0227	90.0000	134.9987	180.0000	224.9829	270.0000	314.9922

with the fringe direction. There may be some deviations in the experiment if the setup is not well aligned. Advanced iterative algorithm (AIA) and principal component analysis (PCA) can be applied in such cases to overcome this issue.

2.6.1 Moving direction accords with fringe direction

Because in this section the object moving direction is assumed to be in accord with the fringe direction, no extra additional phase is introduced. Thus, traditional ESA could be applied in such cases. After pixel matching, the

coordinates in each frame will be aligned; the intensity distribution $I_n(x, y)$ can be revised to $I'_n(x, y)$. For $N \geq 3$, the N-step phase-shifting algorithm can be applied and hence, the phase can be retrieved by the following Eq. (32):

$$\varphi(x, y) = \arctan \frac{\sum_{n=1}^N I'_n(x, y) \sin\left(\frac{2\pi n}{N}\right)}{\sum_{n=1}^N I'_n(x, y) \cos\left(\frac{2\pi n}{N}\right)}. \quad (32)$$

Please note that the $\varphi(x, y)$ here is truncated between $-\pi$ and π . Hence, phase unwrapping should be implemented to obtain a continuous phase. The height of the object $h(x, y)$ can be obtained by the relation in Eq. (33) between the phase and height:^{34,35}

$$\frac{1}{h(x, y)} = a(x, y) + b(x, y) \frac{1}{\varphi(x, y)} + c(x, y) \frac{1}{\varphi^2(x, y)}, \quad (33)$$

where $h(x, y)$ is the relative height from the reference plane, and $a(x, y)$, $b(x, y)$, and $c(x, y)$ are system parameters that can be obtained by calibration.

Fig. 23 The aligned fringe patterns of (a) first frame and (b) fifth frame.

Fig. 24 Reconstructed results of rotary OFPP with QR code: (a) theoretical profile, (b) reconstructed profile, (c) error distribution, and (d) error plot.

2.6.2 Moving direction deviates from fringe direction

Figure 19 illustrates the direction deviation diagrams of both OFPP models. In rotary OFPP, if the rotation axis is not in the center of the concentric circles, there will be a component along the radial direction. This would have an additional unknown phase $\Delta\delta_n$ for frames, while the component in the fringe tangent direction would not affect the phase. If the rotating angle is relatively small, Eq. (1) can be approximately revised to Eq. (34):

$$I_n(x, y) = R(x, y) \left\{ A(x, y) + B(x, y) \times \cos \left[\frac{2\pi}{T} \sqrt{x^2 + y^2} + \varphi(x, y) + \frac{2\pi n}{N} + \Delta\delta_n \right] \right\},$$

$$n = 1, 2, \dots, N. \quad (34)$$

Similarly, in the linear OFPP model, if there is an angle between the moving direction and the fringe direction, there also would have been an additional unknown phase $\Delta\delta_n$ for frames. Equation (2) can be strictly revised to Eq. (35):

$$I_n(x, y) = R(x, y) \left\{ A(x, y) + B(x, y) \times \cos \left[\frac{2\pi}{T_x} x + \frac{2\pi}{T_y} y + \varphi(x, y) + \frac{2\pi n}{N} + \Delta\delta_n \right] \right\},$$

$$n = 1, 2, \dots, N. \quad (35)$$

As the linear model is more common in practical usage, the following descriptions and simulations with randomly phase-shifting fringe patterns are for linear OFPP. Equation (35) can be written as Eq. (36) for simplicity:

$$I_n(x, y) = A_R(x, y) + B_R(x, y) \cos[\Phi(x, y) + \delta_n],$$

$$n = 1, 2, \dots, N, \quad (36)$$

where $\Phi(x, y) = \frac{2\pi}{T_x} x + \frac{2\pi}{T_y} y + \varphi(x, y)$, $\delta_n = \frac{2\pi n}{N} + \Delta\delta_n$, $A_R(x, y) = R(x, y)A(x, y)$, and $B_R(x, y) = R(x, y)B(x, y)$. For Eq. (36), the phase $\Phi(x, y)$ can be retrieved by AIA or PCA. AIA is an iterative least-squares approach, which could obtain stable results in high precision, but due to the iterative procedures, the processing time is relatively high compared to other algorithms.^{24,25} PCA is a noniterative spatial phase-shifting algorithm, which can directly extract the phase.^{26,27} Our previous work based on these algorithms has been applied on interferograms.³⁶ In this paper, projected randomly phase-shifting fringe patterns will be handled by all ESA, AIA, and PCA algorithms for comparison in the simulation section. Similarly, the height of the object can be reconstructed by Eq. (33) after obtaining the phase.

3 Simulation

According to the models and principles discussed above, four representative numerical simulations are carried out which cover most common cases. Table 1 is the arrangement for the simulations. In case 1 and case 2, the QR code is used as the feature pattern in both rotary and linear OFPP when the moving direction accords with the fringe direction. In case 3 and case 4, object modulation is adopted in linear

OFPP, but the moving direction deviates from the fringe direction in case 4.

3.1 Rotary Online Fringe Projection Profilometry with Quick Response Code

The object phase used in all simulations is a peaks function, whose maximum and minimum heights are 40.5285 and -32.7496 rad, respectively. The phase is modulated in sinusoidal concentric fringes. The QR code is located at the bottom of the object and is present outside the fringe region. The object and QR code revolve around the central axis of the platform in a counter clockwise manner. An eight-step phase-shifting model is used, and the rotation angle interval is set to 45 deg, then eight frames are captured. Figure 20 is the simulated fringe patterns in rotary OFPP with the QR

Fig. 25 More QR codes can be applied in rotary OFPP.

Fig. 26 Simulated fringe patterns with QR code in linear OFPP: (a)–(d) the frames with different rectilinear displacements.

code, whose rotation angles are from 0 to 315 deg. Using an image recognition method to locate the ROIs of the QR codes, all QR codes are extracted from the original images. Then feature point matching is implemented between the

Fig. 27 Feature point matching results of QR codes between first frame and third frame.

Fig. 28 The aligned fringe patterns of (a) first frame and (b) third frame.

first QR code and later ones. Figure 21 describes matching results and vector structures between the first frame and the second, fifth, and eighth frames, respectively. It can be seen that all matched point pairs are correctly found. Applying the vector-based solution, all rotation angles of all paired vectors are obtained. It is noticed from Fig. 22 that angles are concentrated around the theoretical value. Table 2 shows a comparison of direct averaging and filtered averaging results on rotation angles for each frame, which indicates that the filtered results have higher accuracy. After obtaining all frames in rotation angles, the aligned parts can be extracted from each frame. For example, Fig. 23 is the aligned fringe patterns from the first and fifth frames. In this case, the rotation center is just the concentric circles' center, so no additional phase is added in deformed patterns. The simulated phase can be reconstructed by ESA, and more comparison results are displayed. Figures 24(a) and 24(b) are the theoretical profile and reconstructed profile, respectively. The further analyses are for the next two figures. Figure 24(c) is the difference between the two; its mean absolute error is 0.0125 rad, and the standard deviation is 0.0399 rad. Figure 24(d) is a plot from a center column of Fig. 24(c), in which the errors can be seen more clearly. By the way, it is worth considering trying to use more QR codes to improve the accuracy of the rotation

Fig. 29 Reconstructed results of linear OFPP with QR code: (a) theoretical profile, (b) reconstructed profile, (c) error distribution, and (d) error plot.

angle such as in Fig. 25. Besides, multiple objects can be placed on a rotating platform because QR codes can also provide target tracking.

3.2 Linear Online Fringe Projection Profilometry with Quick Response Code

Similarly, in linear OFPP, four-step phase-shifting frames are generated, as shown in Fig. 26. The QR codes are located at the bottom of the parallel fringes. The pixel matching result for the first frame and third frame is shown in Fig. 27. After obtaining all frames' rectilinear displacements, the aligned parts can be extracted from those four frames. Figure 28 is the aligned fringe patterns from the first and third frames. In this case, due to the moving direction which accords with the fringe direction, there is no need to consider extra shifting phase. The final phase can be reconstructed by ESA. Figures 29(a) and 29(b) are the theoretical profile and reconstructed profile, respectively. Figure 29(c) is the difference

between the two profiles, its mean absolute error is 0.0021 rad, and the standard deviation is 0.0032 rad. Figure 29(d) is a plot from a center column of Fig. 29(c). More QR code patterns, as in Fig. 30, are also can be used in the linear OFPP model for higher accuracy of rectilinear displacement while simultaneously tracking more samples.

Fig. 30 More QR codes can be applied in linear OFPP.

Fig. 31 Simulated object modulation.

Fig. 32 Simulated fringe patterns with object modulation in linear OFPP: (a) and (b) the first frame and third frame; (c) and (d) the obtained modulations of corresponding frames (with positive first order spectrum).

Fig. 33 Feature point matching results of object modulations between first frame and third frame.

Fig. 34 The aligned fringe patterns of (a) first frame and (b) third frame.

Fig. 35 Reconstructed results of linear OFPP with object modulation: (a) theoretical profile, (b) reconstructed profile, (c) error distribution, and (d) error plot.

Fig. 36 Simulated fringe patterns with object modulation in linear OFPP. Please note the moving direction and the fringe direction is not the same in this case: (a) and (b) the first frame and third frame; (c) and (d) the obtained modulations of corresponding frames (with zero order spectrum).

Fig. 37 Feature point matching results of object modulations between first frame and third frame.

3.3 Linear Online Fringe Projection Profilometry with Object Modulation

Different from the last two cases, object modulation is used as the feature pattern for pixel matching in this case. An assumed object modulation is shown in Fig. 31, which is generated from the height of the peaks' profile. The virtual modulations are imposed on fringe patterns according to peaks' positions. Figures 32(a) and 32(b) are the composed

Fig. 38 The aligned fringe patterns of (a) first frame and (b) third frame.

images that represent the first and third frames, respectively. Figures 32(c) and 32(d) are the corresponding obtained modulations, which are extracted from the positive first order spectrum. To display the appearance of modulations extracted from a different spectrum, the zero order spectrum will be used in the next case. The pixel matching results on a modulation map for the first and third frames are shown in Fig. 33. After obtaining the displacement of each frame, the aligned parts can be extracted from those four frames. Figure 34 is the aligned fringe patterns from the first

Table 3 Comparison of the time consumption with different algorithms.

Items	ESA	AIA	PCA
Test 1	0.012069 s	31.544580 s	0.114207 s
Test 2	0.020956 s	39.119602 s	0.095710 s
Test 3	0.007143 s	38.257603 s	0.112114 s

frame and third frame. Figures 35(a) and 35(b) are the theoretical profile and reconstructed profile, respectively. Figure 35(c) is the difference between the two profiles, its mean absolute error is 0.0021 rad, and the standard deviation is 0.0034 rad. Figure 35(d) is a plot from a center column of Fig. 35(c).

3.4 Linear Online Fringe Projection Profilometry Under Direction Deviation

When the object moving direction deviates from the fringe direction, there will be an extra phase in the deformed

Fig. 39 Reconstructed results with three different algorithms: (a)–(c) the results with equal step algorithm, (d)–(f) the results with AIA, (g)–(i) the results with PCA, while first column is the reconstructed profile, second column is error distribution, and third column is error plot.

Fig. 40 Experimental setup: (a) overview of the setup, (b) close-up of the translation stage, and (c) stone sculpture.

pattern. The simulated fringe patterns for the first and third frames are shown in Figs. 36(a) and 36(b). Figures 36(c) and 36(d) are the corresponding obtained modulations, which are extracted from the zero order spectrum. The pixel matching results on the modulation map for the first and third frames are shown in Fig. 37. Similarly, the aligned parts can be extracted from corresponding frames. Figure 38 is the aligned fringe patterns from the first and third frames. There are three different algorithms, ESA, AIA, and PCA, which were discussed in the previous section. To compare their performances, Figs. 39(a), 39(d), and 39(g) are the reconstruction results with those three algorithms, respectively. Figures 39(b), 39(e), and 39(h) are the corresponding residual distributions compared to the theoretical profile. Figures 39(c), 39(f), and 39(i) are the plots from the corresponding center columns of the residual maps. The mean absolute errors for ESA, AIA, and PCA algorithms are 0.5187, 0.0011, and 0.0016 rad, respectively, while the standard deviations are 0.2764, 0.0027, and 0.0032 rad. Apparently, the traditional ESA cannot get the correct result, while both the AIA and PCA algorithms get the correct phase with a high precision. A further study on time consumption is given in Table 3. All calculations are run on an Intel i7-4500U CPU laptop with MATLAB codes, closing all the figures output. Three tests were carried out to record the time consumption of each algorithm. The difference of each test is the frequency of the fringe pattern. It is noticed that the time cost of AIA is significantly higher than that of the other two algorithms, and the processing speed of PCA is only a little slower than ESA. The PCA algorithm shows great promise in online dynamic measurement field.

4 Experiment

As the linear model is more common in practical usage, the experiment is based on a linear OFPP. The experimental setup is shown in Fig. 40(a). IMAGINGSOURCE DMK 41BU02 is the CCD in use, with a resolution of 1280×960 pixels. The DLP is LightCrafter 3000, with a resolution of 684×608 pixels. The stone sculpture to be measured is placed on a manual translation stage with a high precision micrometer knob, which could simulate the uniform motion of a linear pipeline. The moving direction of the stage is parallel to the fringe direction. With the movement of the translation stage, the sculpture moves from left to right. The displacement between adjacent frames is 1 mm. A close-up of the object with the translation stage is shown in Fig. 40(b). Figure 40(c) illustrates the stone sculpture with a dragon pattern on it.

Fig. 41 Image preprocessing: (a) and (b) the original captured first frame and eighth frame; (c) and (d) the corresponding rectified images with proposed methods.

Fig. 42 Feature point matching results of object modulations between first frame and eighth frame.

Fig. 43 Matched ROI of two frames: (a) and (b) the first frame and eighth frame, where red dashed boxes indicate the matched region (c) and (d), respectively.

Fig. 44 Reconstructed results: (a) wrapped phase (b) reconstructed profile, and (c) aerial view of the profile.

Put all the devices in place. First, one calibration broad pattern is projected by DLP onto the platform to obtain the reprojection matrix. The camera and projector are static during the experiment. Thus, this matrix is valid for rectifying all fringe patterns. Using an eight-step phase-shifting model, eight frames of objects in different positions are recorded. Figures 41(a) and 41(b) are the first and eighth frames of the original fringe patterns captured in the experiment. Figures 41(c) and 41(d) are the corresponding rectified images with the proposed methods. It is noticed that the fringe patterns become brighter, illumination on the background also appears more uniform, and the contrast is greatly enhanced. Then object modulations of all frames can be figured out. Feature point matching results of object modulations between the first frame and the eighth frame are shown in Fig. 42. After obtaining the displacement, the matched regions of the first and eighth frames are illustrated in Figs. 43(a) and 43(b), respectively. Figures 43(c) and 43(d) are the extracted parts from the red dashed boxes.

The recovered wrapped phase with the PCA algorithm is shown in Fig. 44(a), and the reconstructed profile is Fig. 44(b), while its aerial view is Fig. 44(c). Figure 45 shows the center cross-section plot. The results are in very good accordance with the manual measurement. It indicates that the proposed method successfully performs pixel matching and enables an accurate reconstruction of an objects' profile.

Fig. 45 Sculpture profile along the red line of the thumbnail image.

Table 4 Measurement results of a 10-mm height standard cube with different algorithms.

Items	Static	ESA	AIA	PCA
$\bar{h}$ (mm)	9.884	9.826	9.863	9.877
MAE (mm)	0.119	0.175	0.140	0.126
RMS (mm)	0.081	0.115	0.099	0.092

Result comparisons between the static measurement and proposed methods are shown in Table 4. The measured sample is a 10-mm height standard cube. Please note that ESA can be applied theoretically when the moving direction accords with the fringe direction. $\bar{h}$ is the average height of the measurement result in the effective area. MAE is the mean absolute errors and RMS is the root of mean square errors. It can be seen that the results of PCA and AIA are very close to static results, while the deviation for ESA is relatively larger. Taking processing speed into account, the PCA algorithm has advantages in this case. Based on the experimental results, the proposed method has shown its validity and feasibility.

5 Conclusion

SIFT-based OFPPs, including rotary and linear models, are proposed in this paper. The contrast of captured fringe patterns can be improved significantly by the use of a retinex theory for better image detail and modulation distribution. In addition, an improved reprojection technique can rectify perspective distortion of the image while keeping the right aspect ratio. QR code and object modulation as feature patterns have good performances in pixel matching between frames by SIFT with the RANSAC algorithm. Then the rotation angle in rotary OFPP and rectilinear displacement in linear OFPP can be calculated by a vector-based solution with the matched point pairs. A statistical filter further improves the accuracy of the values. According to the actual situation, ESA, AIA, and PCA are proposed to handle phase retrieval issues. The latter two algorithms can be applied when the object moving direction deviates from the fringe direction. The profile of a moving object can be reconstructed with aligned images. The numerical simulations and experimental results verified the validity and feasibility of the proposed

method, which provides new ideas for achieving high accuracy online 3-D measurement.

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