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An MPC-based Position Controller for a Tilt-Rotor Tricopter VTOL UAV

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SUMMARY

This study presents a novel framework, fusion of a conventional controller and a linear model predictive controller, for the position control of a tilt-rotor tricopter. While the conventional controller in the outer loop is responsible for the position control, the inner-loop MPC controller handles the angular dynamics and vertical body velocity. Furthermore, a novel control allocation algorithm for the proposed controller is introduced. In addition, this study also covers mathematical modeling and trim analysis of the tilt-rotor tricopter dynamics. Evaluation of the designed control system is accomplished with a nonlinear 6DOF simulation model of the tilt-rotor tricopter in which realistic actuator limitations are considered. Efficiency of the proposed control algorithm are elaborated for a trajectory tracking problem where a basic surveillance operation is considered. The simulation results show that the proposed model predictive controller is able to provide a satisfactory trajectory tracking performance under the realistic actuator limits. Copyright © 2016 John Wiley & Sons, Ltd.

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KEY WORDS: Model predictive control, tilt-rotor tricopter, VTOL UAV.

1. INTRODUCTION

Rapid progress in embedded control systems, communication and miniaturization of sensors inspires research in design and manufacturing of the remotely controlled or fully autonomous unmanned aerial vehicles (UAVs) or drones. UAVs serve as cost-effective tools for various tasks in military and civil applications such as search and rescue [1], surveillance, target identification and localization [2], remote sensing [3], and delivery services [4].

Unlike the fixed-wing platforms, rotorcraft UAVs, such as multi-rotor platforms, are particularly preferable for a large number of applications due to their key features: ability to hover and vertical flight. Vertical take-off and landing (VTOL) capability eliminates the need for a runway and, therefore, allows to significantly broaden the application areas. A variety of commercially available multi-rotor UAVs includes tricopters, quadrotors and hexacopters and octacopters. The main advantage of a tricopter against other multi-rotor platforms is that less power is required to run three propellers. Tricopters are potentially more reliable since they have one less electronic speed controllers and motors that can fail. In addition, a tricopter inherits a better yaw authority compared to other multi-rotor UAVs since it tilts the propeller rather than varies motor torque, which essentially makes a tricopter more agile. When it comes to the surveillance applications, an advantage of a tricopter is that three rotors will provide a wider range of view for a body-mounted camera as compared to other multi-rotor platforms.

Fast growth of autonomous UAV applications rises problems related to flight safety and reliability, and thus, motivates more research in automatic flight control algorithms. A large number of control algorithms for UAVs are available in literature. In [5], a proportional-integral-derivative (PID) controller is designed and tested for attitude and altitude control of a quadrotor UAV, and resulted in a reasonable tracking of commanded attitude in the presence of small aerodynamic disturbances in an indoor flight. Another application of a conventional control algorithm for a micro quadrotor system is presented in [6]. A feedback linearization method-based controller for a quadrotor is presented in [7], and problems associated with the robustness of the control system are indicated. A stabilization control algorithm that uses a saturation control strategy is proposed in [8], where satisfactory experimental results are illustrated. In [9], a nonlinear controller for stabilization of a mini quadrotor is designed and experimentally tested. The proposed control algorithm is based on the nested saturations and allows to take into account the input amplitude constraints. Another popular control approach widely used for UAVs is the adaptive control. In [10] a controller, consisting of the nonlinear dynamic inversion position controller and the L_1 adaptive augmentation is designed and experimentally tested for a quadrotor UAV.

In real-time applications, operational envelope and safety of a UAV mission are directly related to the inherent physical limitations like available power, maximum speed and altitude, maximum RPM of the propellers, etc. For this reason, automatic aircraft control algorithms need to be able to cope with constraints in the aircraft states and inputs. One of the effective control methods that allows to solve such problems is model predictive control (MPC) [11, 12, 13, 14, 15]. MPC has been developed in early 1960s, and was originally proposed for industrial process control [16, 13, 17, 18]. MPC is a multi-input-multi-output (MIMO) control technique that provides optimal inputs for the plants with given prediction models under the consideration of state and input constraints. In the past, MPC applications were constrained by computational power of the processors and limited number of available solution tools, therefore, MPC was mostly suitable for the processes where the time constant of the system is low and the processes are operated around fixed operating points. Significant interest for MPC in aerospace control has been gained due to the recent advances in development of high performance processors and efficient tools for solving optimization problems. Hence, MPC method for attitude and translational control of a quadrotor is presented in [19]. The vehicle dynamics is given by a piecewise-affine linear model obtained from the linearization of the quadrotor dynamics at several operating points. Switching MPC approach provides optimal control for each region of the operating envelope and smooth transition of the control effort, as the vehicle moves from one subset of the flight envelope to the neighboring subset. The effectiveness of the controller is validated via experiments. In [20], an MPC controller is designed and tested for the Qball-X4 experimental quadrotor platform. In addition, a model reduction technique is introduced to minimize the computation burden and enhance effectiveness of the controller. The design of an MPC-based control system for quadrotor stabilization and trajectory tracking is presented in [21], where three MPC controllers are designed for attitude, altitude and position control. In [22], MPC is used to shape the reference commands for the hybrid controller for a hexacopter.

One of the challenges in designing a flight control system for a multi-rotor UAV is the selection of the control allocation algorithm, or so called mixing. A control allocation algorithm provides a relation between the control inputs, which are analogous to a conventional helicopter (collective, longitudinal, lateral and cyclic), and the direct inputs to the system. For a tilt-rotor tricopter, the conventional controls are nonlinear functions of the direct control inputs that are revolutions per minute (PRM) of the rotors and the tilt angle. MPC algorithm is based on conventional controls, therefore, control allocation is required. Mixing algorithms are commonly used in control systems for rotorcraft UAVs. An example of the control allocation algorithm for a tilt-rotor tricopter controller is proposed in [23, 24]. In [25], the problem of control allocation has overcome by using a sequential control strategy, where the tricopter model is considered as three connected subsystems.

In this study, a novel position controller for a tilt-rotor tricopter is proposed. The control system combines a low-level MPC-based controller for attitude and vertical body velocity control, and a high-level position controller, which serves as the reference generator for the inner-loop controller. Separating the kinematic and dynamic equations of motion of the vehicle yields the controllers of

smaller dimensions as compared to a single-loop controller. Such structure allows to decrease the computational cost for MPC. Also, the proposed architecture ensures the stabilizability condition for the reduced-order model that is used in MPC design. Moreover, the outer-loop controller serves as the reference generator for the inner-loop MPC. Prior to the controller design, a trim analysis is performed in order to establish the values of the rotors speeds, tilt angle and roll angle, which provide an equilibrium hover condition. The linearization around the hover equilibrium yields a reduced-order model for the MPC design. Another contribution of this study is a novel control allocation algorithm, which provides an invertible mixing matrix, and allows to use the control inputs similar a conventional helicopter in the MPC design, and provides the corresponding mapping to the rotors' RPM and tilt angle. This work is organized as follows: Section 2 presents formulation of MPC control. Section 3 covers the equations of motion and trim analysis. The MPC controller design including the control allocation algorithm is given in Section 4, whereas numerical simulation results are given in Section 5. Finally, concluding remarks are given in Section 6.

2. FORMULATION OF MODEL PREDICTIVE CONTROL

MPC is an optimization-based method for the control of MIMO systems. It involves solving a finite-horizon optimal control problem subjected to the specified dynamics, and input and state constraints imposed on the system. In the receding or moving horizon strategy for MPC shown in Fig. 1, future outputs are predicted at each time t for a given prediction horizon using the model of the process or system. The control sequence, or predicted inputs, are obtained by solving an optimization problem with a given optimality criterion for a control horizon. Then the first input of this optimal sequence is fed to the process. Afterwards, the optimization problem is solved again, and the new input sequence are obtained.

Figure 1. MPC strategy

Consider formulation of MPC for a regulation problem of a discrete-time linear time-invariant (LTI) system given by

$$x(t+1) = Ax(t) + Bu(t), \quad (1)$$

$$y(t) = Cx(t), \quad (2)$$

and, for all $t \geq 0$, the following constraints hold

$$y_{\min} \leq y(t) \leq y_{\max}, \quad u_{\min} \leq u(t) \leq u_{\max}, \quad (3)$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, and $y(t) \in \mathbb{R}^p$ are the state, control, and output vector, respectively. The pair (A, B) is stabilizable.

Under the full-state feedback assumption, the receding horizon regulator is based on the solution of the following optimization problem for each time t

$$\begin{aligned} \min_{\mathbf{U} \triangleq (u_t, \dots, u_{t+N_u-1})} & \left\{ J(\mathbf{U}, x(t), N_p, N_u) = x_{t+N_p|t}^T P x_{t+N_p|t} \right. \\ & \left. + \sum_{k=0}^{N_p-1} [x_{t+k|t}^T Q x_{t+k|t}] + \sum_{k=0}^{N_u-1} [u_{t+k|t}^T R u_{t+k|t}] \right\}, \end{aligned} \quad (4)$$

subjected to

$$\begin{aligned} y_{\min} &\leq y_{t+k|t} \leq y_{\max}, \quad k = 1, \dots, N_c, \\ u_{\min} &\leq u_{t+k} \leq u_{\max}, \quad k = 0, 1, \dots, N_c, \\ x_{t|t} &= x(t), \\ x_{t+k+1|t} &= A x_{t+k|t} + B u_{t+k}, \quad k \geq 0, \\ y_{t+k|t} &= C x_{t+k|t}, \quad k \geq 0, \\ u_{t+k} &= K x_{t+k|t}, \quad N_u \leq k < N_p. \end{aligned}$$

where $x_{t+k|t}$ is the predicted state vector at time $t+k$ which is obtained by applying the input sequence $u_t, \dots, u_{t+k-1}$ to the model in (2), starting from the state $x(t)$. In (4), N_p is the length of the prediction (output) horizon, N_u is the length of the control (input) horizon, and N_c is the length of the constraint horizon, with $N_u \leq N_p$ and $N_c \leq N_p$. It is assumed that $Q \in \mathbb{R}^{n \times n}$, $P \in \mathbb{R}^{n \times n}$ are positive semidefinite, $R \in \mathbb{R}^{m \times m}$ is positive definite, and K is a feedback gain.

Define the optimal solution to the optimization problem in (4), at time t by

$$\mathbf{U}^*(t) \triangleq \{u_t^*, \dots, u_{t+N_u-1}^*\}. \quad (5)$$

Then, only the first input of the optimal command sequence (5) is applied to system (2), that is

$$u(t) = u_t^*. \quad (6)$$

3. EQUATIONS OF MOTION AND TRIM ANALYSIS OF THE TILT-ROTOR TRICOPTER

This section provides the dynamic and kinematic equations of motion of the tilt-rotor tricopter and covers the trim analysis in which the values of the control inputs and states are established for the hover condition. The tilt-rotor tricopter, shown in Fig. 2, consists of a frame with three propellers, three motor units and a servo mechanism. The servo mechanism tilts the first propeller compensating for the adverse yaw. Other two propellers rotate in opposite directions to compensate the torque.

3.1. Nonlinear Equations of Motion

The position of the tricopter in the inertial (navigation) frame is defined by its coordinates $[X \ Y \ Z]^T$. Moreover, the attitude is defined by the Euler angles that include roll, ϕ , pitch, θ , and yaw, ψ . The transformation from the navigation frame (X, Y, Z) to the body frame (X_b, Y_b, Z_b) is done using the direction cosine matrix C_{bn} as follows

$$\begin{bmatrix} X_b \\ Y_b \\ Z_b \end{bmatrix} = C_{bn} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}. \quad (7)$$

The direction cosine matrix is given by

$$C_{bn} = \begin{bmatrix} c\theta c\psi & c\theta s\psi & -s\theta \\ s\phi s\theta c\psi - c\phi s\psi & s\phi s\theta s\psi + c\phi s\psi & -s\phi c\theta \\ c\phi s\theta c\psi + s\phi s\psi & c\phi s\theta s\psi - s\phi c\psi & c\phi c\theta \end{bmatrix},$$

Figure 2. Tilt-rotor tricopter configuration

where ‘s’ and ‘c’ stand for sin and cos, respectively.

The relation between the Euler rates $[\dot{\phi} \ \dot{\theta} \ \dot{\psi}]^T$, which are measured in the inertial frame, and angular body rates $[p \ q \ r]^T$ is given by

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\ 0 & \cos \phi & -\sin \phi \\ 0 & \frac{\sin \phi}{\cos \theta} & \frac{\cos \phi}{\cos \theta} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}. \quad (8)$$

Under the rigid-body assumption, the tilt-rotor tricopter dynamics is described by the general nonlinear 6 degrees-of-freedom (DOF) equations of motion [26]. The translational and rotational equations of motion, derived in the body frame neglecting the gyroscopic moments due to the rotors’ inertia, drag forces and moments, and induced pitching moment by the tilted rotor, are given as follows

$$\dot{u} = rv - qw - g \sin \theta + \frac{F_x}{m}, \quad (9)$$

$$\dot{v} = -ru + pw + g \cos \theta \sin \phi + \frac{F_y}{m}, \quad (10)$$

$$\dot{w} = qu - pv + g \cos \theta \cos \phi + \frac{F_z}{m}, \quad (11)$$

$$\dot{p} = \frac{I_{yy} - I_{zz}}{I_{xx}}qr + \frac{M_x}{I_{xx}}, \quad (12)$$

$$\dot{q} = \frac{I_{zz} - I_{xx}}{I_{yy}}pr + \frac{M_y}{I_{yy}}, \quad (13)$$

$$\dot{r} = \frac{I_{yy} - I_{xx}}{I_{zz}}pq + \frac{M_z}{I_{zz}}, \quad (14)$$

where the F_x , F_y , F_z and M_x , M_y , M_z are the components of aerodynamic force and moment generated by the rotors in x , y and z -body directions, respectively. Assuming that the altitude change is limited, and therefore, the air density is constant, the aerodynamic force and moment produced by the i^{th} rotor represented using the aerodynamic force and moment constants are given by

$$F_i = K_F \Omega_i^2, \quad (15)$$

$$M_i = K_M \Omega_i^2, \quad (16)$$

where K_F and K_M are the aerodynamic force and moment constants, respectively. The components of the aerodynamic force are given by

$$F_x = 0, \quad (17)$$

$$F_y = F_1 \sin \mu = K_F \Omega_1^2 \sin \mu, \quad (18)$$

$$F_z = -(F_1 \cos \mu + F_2 + F_3) = -K_F(\Omega_1^2 \cos \mu + \Omega_2^2 + \Omega_3^2). \quad (19)$$

Considering that rotors 1 and 3 are spinning in a clockwise direction, and rotor 2 is spinning in a counterclockwise direction, components of the aerodynamic moment are given by, [23]

$$M_x = -l_3(F_2 - F_3) = -l_3 K_F(\Omega_2^2 - \Omega_3^2), \quad (20)$$

$$M_y = -l_2(F_2 + F_3) + l_1 F_1 \cos \mu = -l_2 K_F(\Omega_2^2 + \Omega_3^2) + l_1 K_F \Omega_1^2 \cos \mu, \quad (21)$$

$$M_z = l_1 F_1 \sin \mu - M_1 \cos \mu + M_2 - M_3 = l_1 K_F \Omega_1^2 \sin \mu - K_M \Omega_1^2 \cos \mu + K_M \Omega_2^2 - K_M \Omega_3^2, \quad (22)$$

where the distances l_1 , l_2 and l_3 are shown in Fig. 2. The parameters of the tilt-rotor tricopter considered in this study are given in Table I, [23].

Table I. Tricopter Parameters

Parameter	Description	Value	Units
m	Tricopter mass	1.1	kg
I_{xx}	Moment of inertia around x -body axis	0.0239	kg-m ²
I_{yy}	Moment of inertia around y -body axis	0.01271	kg-m ²
I_{zz}	Moment of inertia around z -body axis	0.01273	kg-m ²
l_1	Moment arm	0.2483	m
l_2	Moment arm	0.1241	m
l_3	Moment arm	0.2150	m
K_F	Aerodynamic force constant	1.970×10^{-6}	
K_M	Aerodynamic moment constant	2.880×10^{-7}	

3.2. Trim Analysis

In the trim analysis, the aim is to find the values of the control inputs and the states that provide a trim condition. In particular, the interest is to establish the hover condition as an equilibrium point. In the trim analysis, the total force and moment acting on the vehicle are investigated. The total force F_{total} that acts on the tricopter is given by

$$F_{\text{total}} = [F_{\text{total}_x} \ F_{\text{total}_y} \ F_{\text{total}_z}]^T, \quad (23)$$

where

$$F_{\text{total}_x} = -mg \sin \theta,$$

$$F_{\text{total}_y} = mg \sin \phi \cos \theta + K_F \Omega_1^2 \sin \mu,$$

$$F_{\text{total}_z} = mg \cos \phi \cos \theta - K_F(\Omega_1^2 \cos \mu + \Omega_2^2 + \Omega_3^2).$$

The total moment M_{total} that acts on the tricopter is given by

$$M_{\text{total}} = [M_{\text{total}_x} \ M_{\text{total}_y} \ M_{\text{total}_z}]^T, \quad (24)$$

where $M_{\text{total}_x} = M_x$, $M_{\text{total}_y} = M_y$ and $M_{\text{total}_z} = M_z$ are defined by (20), (21) and (22), respectively.

In trim condition, the total force and the total moment acting on the vehicle are equal to zero. For a tilt-rotor tricopter the gravitational force must be compensated by the vertical component of the

thrust produced by three rotors. Moreover, the rotors' torques must be compensated by the tilt angle μ of the first rotor. A nonzero tilt angle leads to a nonzero side force that contributes to the total force and moment. In trim, the side force must be compensated by a nonzero roll angle. Thus, trim analysis aims to establish the values for the four control variables $\Omega_{1\text{trim}}$, $\Omega_{2\text{trim}}$, $\Omega_{3\text{trim}}$, μ_{trim} , and two states, roll angle ϕ_{trim} and pitch angle θ_{trim} , which provide the trim condition (hover) for the tricopter. In hover, the translational and rotational velocities are equal to zero, that is $[u \ v \ w]_{\text{trim}}^T = 0$ and $[p \ q \ r]_{\text{trim}}^T = 0$. Then, equating the left hand sides of the total force and total moment equations to zero, and solving for the unknown inputs and states, results in the following analytical expressions of the trim states and inputs,

$$\phi_{\text{trim}} = \tan^{-1} \left[-\frac{l_2 K_M}{l_1(l_1 + l_2) K_F} \right], \quad (25)$$

$$\theta_{\text{trim}} = 0, \quad (26)$$

$$\mu_{\text{trim}} = \tan^{-1} \left[\frac{K_M}{l_1 K_F} \right], \quad (27)$$

$$\Omega_{1\text{trim}} = \sqrt{\frac{l_2 g m}{(l_1 + l_2) K_F} \frac{\cos \phi_{\text{trim}}}{\cos \mu_{\text{trim}}}}, \quad (28)$$

$$\Omega_{2\text{trim}} = \sqrt{\frac{l_1 g m}{2(l_1 + l_2) K_F} \cos \phi_{\text{trim}}}, \quad (29)$$

$$\Omega_{3\text{trim}} = \Omega_{2\text{trim}}. \quad (30)$$

For the given tricopter configuration, numerical trim values are given in Table II.

Table II. Trim values

Parameter	Value	Units
ϕ_{trim}	-11.10	deg
θ_{trim}	0	deg
μ_{trim}	30.49	deg
$\Omega_{1\text{trim}}$	1441	RPM
$\Omega_{2\text{trim}}$	1338	RPM
$\Omega_{3\text{trim}}$	1338	RPM

4. CONTROL SYSTEM ARCHITECTURE AND DESIGN

The proposed control scheme for a tilt-rotor tricopter has a multi-loop structure. The outer loop is used for the position control, and consists of three parallel PID controllers that serve as reference generators for the inner loop. The inner-loop controller handles the angular position of the tricopter, and also serves for the stability augmentation. The desired position is set by the corresponding reference commands $[X \ Y \ Z]_{\text{ref}}^T \triangleq [X_{\text{ref}} \ Y_{\text{ref}} \ Z_{\text{ref}}]^T$, which are sent to the PID controllers that output references for the pitch angle, θ_{ref} , roll angle, ϕ_{ref} , and z -body velocity, w_{ref} for the low-level (inner-loop) controller. In the inner loop, a linear MPC controller ensures that the tricopter attains the desired attitude and z -body velocity. The inner-loop control is provided by the manipulated inputs to the tricopter that are RPM of the three rotors $\Omega_1, \Omega_2, \Omega_3$ and the tilt angle μ . The control allocation algorithm renders the relation between the conventional input Δu_δ and the manipulated input u_m . It is described in the subsection below. Block diagram of the proposed controller architecture is shown in Fig. 3.

Figure 3. Controller Block Diagram

4.1. Control Allocation

The control allocation algorithm provides the mapping from the control inputs that are similar to a conventional helicopter to the manipulated (direct) inputs of the tilt-rotor tricopter. The conventional control inputs include collective δ_{col} , longitudinal δ_{lon} , lateral δ_{lat} and pedal δ_{ped} inputs. Collective input δ_{col} is directly linked to the altitude control, longitudinal and lateral inputs, δ_{lon} and δ_{lat} are used for the pitch and roll control, which provide the setpoints for the longitudinal and lateral position control in the high-level controller, and finally, the pedal input δ_{ped} is directly linked to the yawing motion control.

For a tilt-rotor tricopter, let u_m denote the vector of manipulated inputs defined by

$$u_m = [\Omega_1^2 \ \Omega_2^2 \ \Omega_3^2 \ \mu]^T. \quad (31)$$

Considering the structure of aerodynamic forces and moments in the translational and rotational dynamics, the vector of conventional control inputs u_δ is defined as follows,

$$u_\delta = [\delta_{col} \ \delta_{lon} \ \delta_{lat} \ \delta_{ped}]^T, \quad (32)$$

where

$$\delta_{col} = -K_F \Omega_2^2 - K_F \Omega_3^2 - K_F \Omega_1^2 \cos \mu, \quad (33)$$

$$\delta_{lon} = -l_2 K_F (\Omega_2^2 + \Omega_3^2) + l_1 K_F \Omega_1^2 \cos \mu, \quad (34)$$

$$\delta_{lat} = -l_3 K_F (\Omega_2^2 - \Omega_3^2), \quad (35)$$

$$\delta_{ped} = l_1 K_F \Omega_1^2 \sin \mu - K_M \Omega_1^2 \cos \mu + K_M \Omega_2^2 - K_M \Omega_3^2. \quad (36)$$

Let u denote an intermediate control input given by

$$u = [u_1 \ u_2 \ u_3 \ u_4]^T = [\Omega_1^2 \sin \mu \ \Omega_1^2 \cos \mu \ \Omega_2^2 \ \Omega_3^2]^T. \quad (37)$$

The relation between u_δ and u is provided by the mixing matrix M as follows,

$$u_\delta = Mu, \quad (38)$$

where M is given by

$$M = \begin{bmatrix} 0 & -K_F & -K_F & -K_F \\ 0 & l_1 K_F & -l_2 K_F & -l_2 K_F \\ 0 & 0 & -l_3 K_F & l_3 K_F \\ l_1 K_F & -K_M & K_M & -K_M \end{bmatrix}.$$

An important condition for implementing a control allocation algorithm is invertability of the mixing matrix.

In the inner-loop controller, the control input Δu_δ , provided by MPC, represents a variation of the control input u_δ around its trim value $u_{\delta_{\text{trim}}}$, that is $\Delta u_\delta = u_\delta - u_{\delta_{\text{trim}}}$. Then, the following relation holds,

$$\Delta u = M^{-1} \Delta u_\delta, \quad (39)$$

where $\Delta u = u - u_{\text{trim}}$, and u_{trim} is given by

$$u_{\text{trim}} = [\Omega_{1\text{trim}}^2 \sin \mu_{\text{trim}} \quad \Omega_{1\text{trim}}^2 \cos \mu_{\text{trim}} \quad \Omega_{2\text{trim}}^2 \quad \Omega_{3\text{trim}}^2]^T. \quad (40)$$

Finally, the components of the manipulated control input u_m can be found as follows

$$\Omega_1^2 = \sqrt{u_1^2 + u_2^2}, \quad (41)$$

$$\Omega_2^2 = u_3, \quad (42)$$

$$\Omega_3^2 = u_4, \quad (43)$$

$$\mu = \text{atan2}\left(\frac{u_1}{u_2}\right). \quad (44)$$

A block diagram of the control allocation algorithm is shown in Fig. 4.

Figure 4. Control Allocation Algorithm

4.2. PID Position Controller

Control of the horizontal motion of the tilt-rotor tricopter is achieved by orienting the thrust vector towards the desired direction of motion, and in practice, is done by pitching or rolling the tricopter in response to the position error. Altitude is controlled using the vertical velocity of the tricopter, which in its turn is connected to the collective input. The outer-loop controller provides the corresponding references for the roll angle ϕ_{ref} , pitch angle θ_{ref} , and z -body velocity w_{ref} . Under the assumption of the small Euler angles, the outer-loop control scheme is given by

$$\phi_{\text{ref}} = K_{p_\phi}(Y_{\text{ref}} - Y) + K_{i_\phi} \int (Y_{\text{ref}} - Y) dt + K_{d_\phi} \frac{d}{dt} (Y_{\text{ref}} - Y), \quad (45)$$

$$\theta_{\text{ref}} = K_{p_\theta}(X_{\text{ref}} - X) + K_{i_\theta} \int (X_{\text{ref}} - X) dt + K_{d_\theta} \frac{d}{dt} (X_{\text{ref}} - X), \quad (46)$$

$$w_{\text{ref}} = K_{p_w}(Z_{\text{ref}} - Z) + K_{i_w} \int (Z_{\text{ref}} - Z) dt + K_{d_w} \frac{d}{dt} (Z_{\text{ref}} - Z). \quad (47)$$

It should be noted, that in order for a tricopter to move forward in x -direction, which corresponds to the positive reference X_{ref} , a negative pitch angle is required. Therefore, K_{p_θ} , K_{i_θ} , K_{d_θ} are negative. On the other hand, positive Y_{ref} and Z_{ref} require positive roll angle and z -body velocity, respectively, therefore, K_{p_ϕ} , K_{i_ϕ} , K_{d_ϕ} , K_{p_w} , K_{i_w} and K_{d_w} are positive.

4.3. MPC Controller Structure and Design

The MPC controller provides trajectory tracking for given references of the Euler angles, angular rates and z -body velocity. Control of the z -body velocity is provided by the collective input δ_{col} , whereas control of the Euler angles and angular rates is provided by the lateral δ_{lat} , longitudinal δ_{lon} and pedal δ_{ped} inputs. Two important steps in MPC design include definition of the prediction model and the constraints.

4.3.1. Linear model for MPC. In this study, we consider a linear MPC controller. The prediction model for MPC design is based on the reduced-order linear model of the tricopter. It is obtained by linearizing the nonlinear dynamics around the hover trim condition, $w = 0$, $\phi = \phi_{\text{trim}}$, $\theta = \theta_{\text{trim}}$, $\psi = 0$, $p = 0$, $q = 0$, $r = 0$. The reduced-order model includes the rotational dynamics and translational dynamics in z -body axis, and is given by

$$\Delta \dot{x} = A \Delta x + B \Delta u_{\delta}, \quad (48)$$

where the state vector is defined as

$$\Delta x = [w \ \Delta \phi \ \Delta \theta \ \Delta \psi \ p \ q \ r]^T,$$

where $\Delta \phi = \phi - \phi_{\text{trim}}$, $\Delta \theta = \theta - \theta_{\text{trim}}$, $\Delta \psi = \psi - \psi_{\text{trim}}$. The state and control matrices A and B are respectively given by

$$A = \begin{bmatrix} 0 & -g \sin \phi_{\text{trim}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cos \phi_{\text{trim}} & -\sin \phi_{\text{trim}} \\ 0 & 0 & 0 & 0 & 0 & \sin \phi_{\text{trim}} & \cos \phi_{\text{trim}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} \frac{1}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{I_{xx}} & 0 \\ 0 & \frac{1}{I_{yy}} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{I_{zz}} \end{bmatrix}.$$

Remark: It should be noted that the pair (A, B) satisfies the controllability condition.

4.3.2. State and control constraints A great advantage of MPC is that it allows to take into account the constraints on both states and control inputs. For a tilt-rotor tricopter, the state and control constraints are defined by the physical limitations such as minimum/maximum rotor speed, minimum/maximum tilt angle, power available, working conditions such as operation altitude, and safety related restrictions. The altitude of the tricopter is constrained by the ground. Since the proposed MPC design only controls the vertical velocity, altitude limitation are not taken into account in this study. The pitch angle must be limited in order to avoid the singularities in the transformation matrix C_{bn} . The additional limitations on the pitch and roll angles as well as translational velocity can be dictated by the safety requirements. The angular rates of the rotors are restricted by the motor output and power limitations. In this study, it is assumed that the trim values for Ω_i represent 50% of the maximum achievable values. Thus, the following constraints are imposed on the tricopter states

$$-\pi/2 \text{ rad} < \theta, \phi < \pi/2 \text{ rad}, \quad (49)$$

$$-2\pi \text{ rad} < \psi < 2\pi \text{ rad}, \quad (50)$$

$$-2\pi \text{ rad/s} < p, q, r < 2\pi \text{ rad/s}. \quad (51)$$

The actuators constraints including the limitations for the rotors' RPM and tilt angle are defined as follows

$$0 < \Omega_i < 2\Omega_{i_{trim}}, \quad (52)$$

$$-\pi/2 < \mu < \pi/2, \quad (53)$$

It should be noted that the conventional control inputs (δ_{col} , δ_{lon} , δ_{lat} and δ_{ped}) are related to the direct actuator inputs (Ω_i , μ) via (33)–(36). Since there is no one-to-one relation between the actuator commands and the conventional control inputs, the constraints on the conventional controls cannot be found directly from (33)–(36). Therefore, in this study approximate input constraints are used in the MPC design. In the simulation model of the tilt-rotor tricopter, the actuators are modeled as first order transfer functions including amplitude and rate limits. It is important to note that in real applications, an anti-winding compensator must be added to prevent the integrator from saturating.

5. SIMULATION RESULTS

The efficiency and efficacy of the proposed control architecture is investigated via numerical simulations for the trajectory tracking problem. A desired trajectory for a tricopter is generated in terms of position vector components $[X, Y, Z]^T$. The outer-loop PID controller gains are selected by the trial and error to provide a satisfactory position tracking, and given in Table III. The design parameters for the MPC controller are selected to provide satisfactory attitude and z -body velocity tracking, and are given in Table IV.

Table III. PID Gains

$K_{p_\phi} = 0.5$	$K_{i_\phi} = 0.01$	$K_{d_\phi} = 0.1$
$K_{p_\theta} = -0.1$	$K_{i_\theta} = -0.01$	$K_{d_\theta} = -0.1$
$K_{p_w} = 5$	$K_{i_w} = 0.1$	$K_{d_w} = 0.1$

Table IV. MPC Controller Design Parameters

Parameter	Numerical value
Prediction horizon, N_p	10
Control horizon, N_u	5
Input weight, R_1	$\text{diag}(0.1, 0.1, 0.001, 0.001)$
Input rate weight, R_2	$0_{4 \times 4}$
Controlled variables weight, Q	$\text{diag}(10^2, 10^3, 10^3, 10^3, 10^4, 10^4, 10^4)$

Simulations are performed for a sampling time 0.01 sec. The desired and actual trajectories for the tilt-rotor tricopter are shown in Fig. 5, whereas the Euclidean position error is shown in Fig. 6. As can be seen from these figures, the outer-loop PID controller provides satisfactory position tracking with the maximum position error in transient less than 0.7 m. Considering the individual components of the position error in x , y and z axis, shown in Fig. 7, an altitude error of less than 0.1 m is achieved in the horizontal flight, which takes place from the 10th to the 50th second. Moreover, the altitude error does not exceed 0.2 m during the vertical flight. The responses for the Euler angles, angular rates and the translational z -body velocity are given in Fig. 8, 9 and 10, respectively. These figures show that the inner-loop MPC controller yields effective command following for the given references. The angular velocities of the rotors and the tilt angle are given in Fig. 11 and 12, respectively, which show that the MPC controller ensures abiding the hard constraints imposed on the control inputs. Soft constraints on the state and control are taken into account by the weighting matrices R_1 , R_2 and Q in MPC synthesis. The weighting matrices allow to adjust the penalties

imposed on the input, input rate and the state, providing additional flexibility in controller design. Note that zero input rate weighting matrix is used in the numerical tests in order to provide faster response, while the actuators dynamics including position and rate limitations is considered in the tilt-rotor tricopter simulation model. We implemented relatively small values of the control and prediction horizons. This is motivated by fast dynamics of the UAV, and also allows to reduce the computation time for solving the optimization problem. This is an essential aspect for a possible real-time implementation.

Figure 5. Reference trajectory and actual position of the tilt-rotor tricopter.

Figure 6. Euclidean position error of the tilt-rotor tricopter.

6. CONCLUSIONS

In this study we present a framework for designing a position controller for a tilt-rotor tricopter. The proposed controller has a multiple-loop structure that comprises a conventional PID control method and linear MPC. While the outer-loop PID controller handles the tricopter position, the inner-loop MPC controller handles the attitude and vertical body velocity of the tricopter. The control scheme provides effective tracking of the desired trajectory, and at the same time takes into account the constraints imposed on the states and control inputs. Since MPC is based on a linear prediction model, the optimization problem can be solved offline, reducing the computational power

Figure 7. X , Y and Z components of the position error.

Figure 8. Euler angles.

Figure 9. Components of the velocity.

requirements. Hence, the proposed approach has a large potential for the real-time applications where the computational power is restricted.

Figure 10. Translational z -body velocity.

Figure 11. Rotors angular velocities (RPM).

Figure 12. Tilt angle.

The proposed novel control mixing algorithm provides an effective allocation between the conventional control inputs and the direct control inputs to the tricopter. The control allocation allows to implement a linear control technique despite the input nonlinearities in the tilt-rotor tricopter dynamics. One of the challenges we faced during MPC design is the formulation of the input constraints due to the absence of a one-to-one mapping between the direct control inputs and MPC inputs. In this work we implement approximate input constraints, while the actuators limitations are taken into account in the simulation model. The trim analysis allows to establish the actuators' inputs such as PRM of three rotors and the tilt angle, and the values of the states for the hover trim condition. The control architecture has been validated via numerical simulations. The simulation results show that for a command following problem, the control algorithm provides satisfactory tracking for the given position references with acceptable errors. Moreover, the obedience of the state and input constraints in the attitude loop is illustrated.

The next step of this research will include development of the algorithm for the control constraints allocation. One of the possible ways to do it is using the search algorithm. The control constraints allocation algorithm will allow to incorporate the direct inputs constraints in MPC synthesis. In addition, we will focus on improving the inner-loop controller by applying a multiple MPC. In this approach, several local MPC controllers will be designed employing several linear prediction models obtained for the flight conditions such as hover and forward flight at given speed. A transition between the control actions of the local MPC controllers will be provided to attain the stability of the inner-loop.

REFERENCES

1. Lin L, Roscheck M, Goodrich M, Morse B. Supporting wilderness search and rescue with integrated intelligence: autonomy and information at the right time and the right place. *Proc. 24th AAAI Conference on Artificial Intelligence*, Atlanta, GA, 2010; 1542–1547.
2. Darma S, Buessler JL, Hermann G, Urban JP, Kusumoputro B. Visual servoing quadrotor control in autonomous target search. *Proc. 2013 IEEE 3rd International Conference on System Engineering and Technology (ICSET)*, IEEE: Zurich, Switzerland, 2013; 319–324.
3. Pajares G. Overview and Current Status of Remote Sensing Applications Based on Unmanned Aerial Vehicles (UAVs). *Photogrammetric Engineering & Remote Sensing* 2015; **81**(4):281–329.
4. Raptopoulos A, Damm D, Santana P, Ling M, Baruchin I. Transportation using network of unmanned aerial vehicles Jan 30 2014. URL <http://www.google.com/patents/US20140032034>, uS Patent App. 13/890,165.
5. Hoffmann G, Huang H, Waslander S, Tomlin C. Quadrotor Helicopter Flight Dynamics and Control: Theory and Experiment. *Proc. AIAA Guidance Navigation and Control Conference*, Hilton Head, SC, 2007; 1–20.
6. Bouabdallah S, Noth A, Siegwart R. PID vs LQ Control Techniques Applied to an Indoor Micro Quadrotor. *Proc. IEEE/RSJ International Conference on Intelligent Robots and Systems*, vol. 3, 2004; 2451–2456.
7. Mokhtari A, Benallegue A. Dynamic Feedback Controller of Euler Angles and Wind Parameters Estimation for a Quadrotor Unmanned Aerial Vehicle. *Proc. IEEE International Conference on Robotics and Automation*, vol. 3, 2004; 2359–2366.
8. Castillo P, Lozano R, Dzul A. Stabilization of a Mini-Rotorcraft Having Four Rotors. *Proc. IEEE/RSJ International Conference on Intelligent Robots and Systems*, vol. 3, Sendai, Japan, 2004; 2693–2698.
9. Escareno J, Salazar-Cruz S, Lozano R. Embedded Control of a Four-Rotor UAV. *Proc. American Control Conference*, Minneapolis, MN, 2006; 3936–3941.
10. Wang J, Raffler T, Holzapfel F. High Velocity Path Control of Quadrotors. *Proc. AIAA Guidance, Navigation, and Control Conference*, Kissimmee, FL, 2015; 1–16.
11. Mayne DQ, Rawlings JB, Rao CV, Scokaert PO. Constrained model predictive control: Stability and optimality. *Automatica* 2000; **36**(6):789–814.
12. Maciejowski JM. *Predictive control: with constraints*. Pearson education, 2002.
13. Findeisen R, Allgöwer F, Biegler LT. *Assessment and future directions of nonlinear model predictive control*, vol. 358. Springer Berlin, Germany, 2007.
14. Camacho EF, Alba CB. *Model predictive control*. Springer, 2013.
15. Mayne DQ. Model predictive control: Recent developments and future promise. *Automatica* 2014; **50**(12):2967–2986.
16. Qin SJ, Badgwell TA. A survey of industrial model predictive control technology. *Control Engineering Practice* 2003; **11**(7):733–764.
17. Lee JH. Model predictive control: Review of the three decades of development. *International Journal of Control, Automation and Systems* 2011; **9**(3):415–424.
18. Eren U, Prach A, Koçer BB, Raković SV, Kayacan E, Açıkmüş B. Model predictive control in aerospace systems: Current state and opportunities. *Journal of Guidance, Control, and Dynamics* 2017; (Accepted for publication).
19. Alexis K, Papachristos C, Nikolakopoulos G, Tzes A. Model predictive quadrotor indoor position control. *Proc. 19th Mediterranean Conference on Control and Automation*, IEEE: Corfu, Greece, 2011; 1247–1252.
20. Abdolhosseini M, Zhang YM, Rabbath CA. An Efficient Model Predictive Control Scheme for an Unmanned Quadrotor Helicopter. *Journal of Intelligent & Robotic Systems* 2015; **70**(1):27–38.
21. Chipofya M, Lee DJ, Chong KT. Trajectory Tracking and Stabilization of a Quadrotor Using Model Predictive Control of Laguerre Functions. *Abstract and Applied Analysis* 2015; **2015**:1–11.
22. Muhlegg M, Niermeyer P, Holzapfel F. Reference command shaping for approximate dynamic inversion based model Reference Adaptive Control. *Proc. IEEE International Conference on Aerospace Electronics and Remote Sensing Technology (ICARES)*, Yogyakarta, 2014; 179–184.
23. Yoo DW, Oh HD, Won DY, Tahk MJ. Dynamic Modeling and Control System Design for Tri-Rotor UAV. *Proc. International Symposium on Systems and Control in Aeronautics and Astronautics*, Harbin, China, 2010; 762–767.
24. Yoo DW, Oh HD, Won DY, Tahk MJ. Dynamic Modeling and Stabilization Techniques for Tri-Rotor Unmanned Aerial Vehicles. *International Journal of Aeronautical and Space Science* 2010; **11**(3):167–174.
25. Salazar-Cruz S, Kendoul F, Lozano R, Fantoni I. Real-Time Stabilization of a Small Three-Rotor Aircraft. *IEEE Transactions on Aerospace and Electronic Systems* 2008; **44**:783–794.
26. Stengel RF. *Flight Dynamics*. Princeton University Press, 2004.