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2016

Hsieh, S.-C., Low, Y. M., & Chiew, Y.-M. (2017). Flow characteristics around a circular cylinder undergoing vortex-induced vibration in the initial branch. *Ocean Engineering*, 129, 265-278.

<https://hdl.handle.net/10356/87426>

<https://doi.org/10.1016/j.oceaneng.2016.11.019>

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**Flow characteristics around a circular cylinder undergoing
vortex-induced vibration in the initial branch**

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ABSTRACT: Vortex-induced vibration (VIV) has been studied extensively and much attention has been devoted to the “lock-in” phenomenon, which is primarily responsible for fatigue damage to structures. Nevertheless, the behavior of the surrounding flow can also be important. This paper investigates the flow characteristics of the wake in the initial branch for a vibrating cylinder using a new PIV technique capable of a high sampling rate of 200 Hz. The experiments were carried out over a wide range of reduced velocities to better understand the important trends. The mean velocities and turbulence characteristics were obtained by ensemble averaging repeated velocity measurements. This study reveals new insights on the flow characteristics of the wake in the initial branch. In particular, the cylinder vibration is found to lead to the formation of oblique jets, which have a profound influence on the mean flow velocity, turbulence intensity, formation and convection of the vortices, and variations to the stagnation and separation points of the cylinder during vibration.

CE Database Subject headings:

Vortex-induced vibration; Particle image velocimetry; Wake; Turbulence

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1. Introduction

A flow passing a circular cylinder mounted elastically is characterized by flow separation from the surface and the formation of a wake carrying shed vortices downstream. The formation of these vortices that results from the roll up of the unstable shear layers generated at the separation point induces an alternating pressure loading on the cylinder surface and a transverse force. In many situations, the force can cause the cylinder to vibrate, as observed in many engineering applications such as heat exchanger tubes, submarine cables, marine risers, electrical transmission lines and cable-stayed bridges.

This phenomenon, known as vortex-induced vibration (VIV), has been studied extensively by many researchers, and has been recognized as one of the dominant causes for fatigue damage to structures. Laboratory and numerical experiments on this issue have been conducted by Sarpkaya (1979, 2004), Griffin and Ramberg (1982), Bearman (1984, 2011), Parkinson (1989), Blevins (1990), Naudascher and Rockwell (1994), Sumer and Fredsøe (1997), Govardhan and Williamson (2000, 2001, and 2002), Williamson and Govardhan (2004, 2008), and Blevins and Coughran (2009). Most of the previous investigations focused on the paradigm of a freely vibrating circular cylinder. Khalak and Williamson (1999) suggested that the maximum amplitude response of VIV depends not only on the combined non-dimensional parameter—the mass-damping ratio, $m^*\zeta$ (where the mass ratio m^* represents the ratio of the cylinder mass to displaced fluid mass; and ζ is the damping ratio in the working fluid) but also on m^* and ζ individually. The normalized response amplitude A^* (where $A^* = A/D$, A = vibration amplitude, D = cylinder diameter) shows two different types of behaviors. In the classical low $m^*\zeta$ case, the response curve comprises three distinct branches, namely ‘initial’, ‘upper’ and ‘lower’ when plotted against the reduced velocity V_r (where $V_r = u_0/f_N D$, u_0 = free stream velocity, f_N = natural frequency in fluid). However, in the high $m^*\zeta$ case, the ‘upper’ branch does not exist. One of the fundamental features in the

1 ‘upper’ branch is that the vibration of the structure tends to synchronize with the vortex
2 shedding frequency. This ‘lock-in’ (or ‘synchronization’) phenomenon is characterized by an
3 amplification of the vibration amplitude, which may be of the order of the cylinder diameter.
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7 Most researches in VIV pertain to the response of the cylinder induced by an
8 approaching flow. In particular, emphasis is placed on the characteristic of the ‘lock-in’ stage
9 that may cause catastrophic failure of the structure. However, in some engineering
10 applications, the surrounding flow field (relating to the wake vortex, vortex convection,
11 turbulence characteristics, etc) also may be important. For example, the alteration of the flow
12 characteristics due to the VIV response of a marine riser can influence the behavior of
13 another riser in proximity and local scour (Li et al., 2013).
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24 Over the past two decades, researchers have studied the downstream wake
25 characteristics using flow visualization techniques. For example, Williamson and Roshko
26 (1988) investigated the wake flow induced by a forced-vibrating cylinder under different
27 vibrating modes. They found that the flow pattern can be categorized into two modes, namely
28 the 2S and 2P modes. The 2S mode is associated with two single vortices per cycle, while for
29 the 2P mode, two pairs of vortices are formed per cycle. Brika and Laneville (1993) also
30 observed the foregoing two wake modes induced by a freely vibrating long flexible pipe by
31 using flow visualization techniques.
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43 The early flow visualization studies were primarily qualitative, focusing on the flow
44 pattern and the categorization of the flow modes in the wake. In recent years, flow-field
45 measuring devices such as particle imaging velocimetry (PIV) have been used for
46 quantitative measurements. Govardhan and Williamson (2000) used PIV to measure the
47 velocity field of different vibrating modes of cylinders. They observed that for low reduced
48 velocities (often referred to as the ‘initial’ branch of the flow), the pattern of the wake
49 vortices typically appeared as the 2S mode. As the reduced velocity increases into the
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“lower” or “upper” branches, the amplitude of the vibrating cylinder increases sharply, and vortex shedding manifests as the 2P mode. However, recent studies have indicated that response branches are not unequivocally related to specific vortex modes; for example Shang et al. (2013) discovered the occurrence of the P+S mode at very high reduced velocities, whereas as Zhao et al. (2014) found the existence of the 2S mode in the upper branch. Wang et al. (2013) also used PIV to investigate the effect of a planar boundary on vortex formation.

This paper aims to conduct a detailed study on the flow phenomena downstream of a cylinder undergoing VIV. This work will not address the widely studied lock-in phenomenon; instead the focus will be on the initial branch, which has important implications in engineering application, since many structures are designed for this scenario to avoid the undesirable lock-in regime. The experiments will cover a range of reduced velocities in the initial branch to better understand the change of the flow characteristics with reduced velocity. In summary, the evolution of the mean flow velocity, turbulence intensity, formation and convection of the vortices will be revealed using flow visualization techniques coupled with a high resolution PIV system. Finally, it may be noted that this paper focusses mainly on understanding the flow characteristics. The implications of VIV are broad, for instance the apparent drag and lift forces are of practical interest (e.g. Konstantinidis, 2013), but these aspects are outside the scope of the present study.

2. Experimental Setup and PIV Measurements

2.1 *Water channel, test model and coordinate systems*

Laboratory experiments were performed in a re-circulating flume in the Hydraulics Modeling Laboratory at Nanyang Technological University. The test section, with length = 1100 cm, width = 60 cm, and depth = 60 cm, has glass walls on both sides and the bottom to allow visual observations and PIV measurements. A honeycomb structure of length 20 cm is located at the entrance of the flume to remove any large-scale irregularities in the inlet flows.

1 The test section is located at a distance 700 cm downstream from the flume entrance and the
2 water depth is 30 cm, which allows stable flows from 0.5 to 30 cm/s based on preliminary
3 tests. The free-stream flow is uniform and the turbulence intensity is less than 1.2% for the
4 largest velocity (18.76 cm/s) considered in this study.
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9 An acrylic circular cylinder with diameter = 0.02 m and length = 0.56 m was used as
10 the model in the experiments. The purpose of this study is to explore the characteristics of the
11 flow field around a two-dimensional rigid cylinder that is elastically mounted and moves
12 perpendicular (i.e. vertically) to the approaching flow. To this end, the cylinder was mounted
13 on an aluminum supporting frame, which, in turn, is connected by two springs to a frame
14 fixed to the flume, as illustrated in Fig. 1. The supporting frame is free to vibrate only along
15 the vertical direction due to the presence of four bearings. The cylinder is located at mid
16 depth. Accordingly, the distance between the stationary cylinder and boundaries (flume
17 bottom or water surface) is 14 cm. Previous studies (e.g. Wang et al., 2013) have shown that
18 the boundary has little effect on the VIV response characteristics and flow field if the gap-to-
19 diameter ratio g/D exceeds 1.5. Here, the maximum vibration amplitude is $\sim 0.61D$, giving
20 $g/D = 6.39$; thus the boundary should have negligible influence on the results. The
21 corresponding length to diameter ratio of the cylinder is 28, which is considered to be large
22 enough to ensure a two-dimensional flow in the near wake region of the cylinder. The weight
23 of the vibrating system, included the circular cylinder and the aluminum frame is 0.998 kg,
24 which corresponds to an equivalent mass ratio $m^* = 5.672$.
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48 Free decay tests were performed in still water to determine the natural frequency. Two
49 sets of tests were performed; in each set, there were three oscillations that can be used to
50 estimate the natural period from the time interval between two successive peaks. The natural
51 period is taken as the average of the six periods, amounting to $T_N = 0.71$ s, with a standard
52 deviation of 0.0025 s. The natural frequency is $f_N = 1/T_N = 1.41$ Hz. Following the procedure
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in Sumer and Fredsøe (1997), the damping ratio ζ is obtained from the decay test results as $\zeta = 0.106$, and correspondingly, $m^*\zeta = 0.601$. Since $m^*\zeta > 0.05$, the present setup belongs to the high- $m^*\zeta$ case and the vibrating modes of the cylinder only have ‘initial’ and ‘lower’ modes according to Khalak and Williamson (1999). Thus, the present experimental setup is able to fulfill the objective of studying the flow characteristics in the initial branch. Table 1 lists the thirteen values of u_0 (13.49 to 18.76 cm/s) used in the experiments, and the corresponding V_r , A^* , vibrating cylinder frequency f_p , and Reynold’s number Re (2698 to 3752), wherein $Re = u_0 D / \nu$, ν = kinematic viscosity. The coordinate system used in this study, with its origin located at the center of the cylinder in its stationary condition, is shown in Fig. 1. The x -axis is in the streamwise direction, while the y -axis is in the vertical direction.

2.2 Velocity measurement and analysis technique

The PIV measuring system comprises a 4W air-cooled laser with wavelength = 532 nm as the light source and a high speed camera (Olympus i-Speed 3). The beam emitted from the laser source is reflected by a mirror, resulting in a laser light fan with 1.5 mm thickness cast upward into the water through the bottom of the glass flume. Aluminum particles with diameter = 10 μm and specific density = 2.7 were used as seeding particles in the experiment. Using Stoke's law, the settling velocity of the aluminum particles was estimated to be less than 0.01 cm/s, which is much smaller than the free stream velocity u_0 .

The high speed camera with a ten-gigabyte memory storage, 1280×1024 pixel resolution and 10^5 maximum frame rate was used to capture images of the particle laden flow. A 105-mm focal lens was mounted on the high speed camera operating at 200 frames per second. It may be noted that conventional PIV techniques typically have sampling rates of about 15 to 30 frames per second (e.g. Govardhan and Williamson, 2000, Wang et al., 2013), although high sample rate PIV has also been used in previous studies (e.g. Cagney and

Balabani, 2013). Velocity fields were determined by the analysis of cross correlation of image pairs (Adrian, 1991). A multi-grid process was adopted to enhance the accuracy of the velocity calculation. The interrogation windows of the multi-grid process started from 32×32 pixels and ended with 16×8 pixels. In PIV measurements, there is usually a small region that is in the shadow of the cylinder body. To eliminate this shadow, the laser was projected from two angles, resulting in two different shadow regions, as depicted in Fig. 2. By combining the results using phase averaging, the shadows from the cylinder are eliminated.

Wavelet transform (e.g. Newland, 1993) is known to be a very useful technique for analyzing cylinder oscillation or vortex shedding (Lin et al. 2009). For a given displacement function of the cylinder, the wavelet transform may be defined as

$$W_{\psi} = \frac{1}{\sqrt{\alpha}} \int_{-\infty}^{\infty} \eta(t) \left[\psi \left(\frac{t-\beta}{\alpha} \right) \right]^* dt \quad (1)$$

where W_{ψ} is the wavelet coefficient; $\eta(t)$ is the cylinder displacement; α and β are the time dependent scale and translation parameters; “*” denotes the complex conjugate; and $\psi(\cdot)$ is the mother wavelet. Herein, the Morlet function is used as the mother wavelet and is given by $\psi_k(t) = e^{ikt} e^{-t^2/2}$, where $k = 2\pi$ is the wave number to satisfy the admissibility condition. Phase averaging is performed, and the instantaneous phase of the vibrating cylinder Φ may be obtained by $\Phi(\alpha, \beta) = \arg(W_{\psi})$. The displacement time history of the vibrating cylinder is used as the reference signal.

2.3 Verification of analysis techniques

In order to verify the velocity field and turbulence characteristics measured by using the PIV, the velocity profiles and dimensionless turbulence intensities profiles were compared with those measured with an acoustic Doppler velocity profiler (ADVP). Fig. 3 shows the results obtained from PIV and ADVP measurements were mutually consistent. The

1 maximum difference for the mean velocity fields is less than 3.2%, while the discrepancy for
2 the turbulence intensity is within 6%. Moreover, a continuity check method (Chang et al.
3 2001) was also performed to calculate the flux of each grid element in the mean velocity
4 field. The maximum error in the wake flow of the mean velocity field is about 4.6%, which
5 may be considered to be small, indicating that the PIV measurements were reliable. Details
6 concerning the verification can be found in Lin et al. (2009).
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14 Fig. 4 plots the normalized response amplitude A^* against V_r under high- $m^*\zeta$ conditions
15 ($m^*\zeta = 0.601$) based on the present experiments. The cylinder displacement history can be
16 measured directly from the PIV measurements. The cross-correlation method used to track
17 the particles is also applied to determine the motion of the cylinder from the images. The
18 amplitude A of the vibrating cylinder was obtained by taking the average of the 10% of the
19 highest peaks recorded for each case.
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28 The results obtained by Blevins and Coughran (2009) with $D = 6.35$ cm, $m^* = 6.39$ and
29 $m^*\zeta = 0.320, 0.639$ are superimposed in Fig. 4 for comparison. (It is noted that Blevins and
30 Coughran (2009) defined the mass ratio as $m^* = m/\rho D^2 = 5.02$; based on the definition in this
31 study, $m^* = m/\rho\pi (D/4)^2 = 6.39$). Both results by Blevins and Coughran (2009) exhibit two
32 distinct branches of response, namely the initial and lower branches, with $V_r \approx 5.5$ and 6.0
33 separating the two branches as can be observed by the abrupt drop in the vibration amplitude.
34 It is apparent from Fig. 4 that the two sets of data are in reasonably good agreement.
35 However, there is still some discrepancy between the data sets; in particular, all the present
36 data conform to the initial branch, and there is no transition to the lower branch. This is
37 because the V_r that demarcates the two branches is not unique, but depends on other factors.
38 Nevertheless, this is not a concern for the present study which is devoted to the investigation
39 of the initial branch.
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3. Results and discussions

3.1 *Flow pattern of the wake*

It is well known that vortex shedding represents the main characteristic for the wake of a stationary circular cylinder (Sumer and Fredsøe, 1997). For the case of flow passing through a stationary cylinder, the formation of vortex mainly occurs in the upper or lower shear layer behind the cylinder. The pair of vortices entrain the adjacent fluid into a rotational component to form regular shedding vortices that convect downstream of the cylinder. However, the characteristics of the wake and vortex shedding for a stationary and vibrating cylinder are completely different. For a better understanding of the shedding process in the initial branch for a vibrating cylinder, the flow field is observed qualitatively using visualization technique prior to PIV quantitative measurements.

Figs. 5(a)-(e) show the snapshots of the flow pattern downstream of a freely-vibrating cylinder at different time with $V_r = 6.66$. Since the displacement of the cylinder, η is approximately sinusoidal after phase averaging, for the purpose of the ensuing discussion, it is represented as $\eta(t) = A\cos(2\pi t/T)$, where $T = 0.608$ s is the vibrating period of the cylinder, and the time t varies from 0 to T . Figs. 5(a)-(b) show the flow pattern at $t = 0$ and $0.25T$ respectively, during which the cylinder moves from $\eta/A = 1$ to 0. Throughout this period, part of the incoming flow passes beneath the cylinder and forms a jet in the upward direction, impinging upon the upper shear layer and forces Vortex V1 (see Fig. 5a) to convect downstream. (The jet is essentially a narrow stream of flow with a high velocity; the presence of a localized region of high velocity will be discussed in Section 3.2.) At this moment, Vortex V2 which rotates in a counter-clockwise direction, forms in between the jet and the downstream (upper) side of the cylinder (see Fig. 5b). Meanwhile, Vortex V1, which moves further downstream of the jet, continues to receive energy therefrom.

The flow field corresponding to $t = 0.5T$, $0.75T$ and $1.0T$ are shown in Figs. 5(c)-(e), respectively. Similar to the previous discussion, part of the incoming flow passes through the upper part of the cylinder to form another jet, albeit in the downward direction, impinging onto the lower shear layer. Figs. 5(d)-(e) reveal how the downward oblique jet also stimulates the formation of Vortex V3, which is located in between the oblique jet and the downstream (lower) side of the cylinder. Vortex V3 rotates in the clockwise direction while Vortices V1 and V2 continue to move downstream.

The oblique jets caused by cylinder vibration evidently dominate the features of the wake flow, which is distinctly different from the stationary cylinder case. It is known that for a stationary cylinder, the vortices are formed from two stable shear layers downstream of the cylinder, and the formation zone is well confined by these two shear layers. In contrast, for the vibration case, the vortices are formed by the oblique jets, which intrude upon the formation zone and influences the wake flow downstream. A similar phenomenon was also observed by Konstantinidis et al. (2005) in the vortex shedding from a stationary cylinder induced by periodic inflow. The jets have important implications in many aspects (e.g. velocity fields, shedding processes), as will soon be apparent.

Experiments with different V_r (as listed in Table 1) were also performed, but due to space constraints they are not presented herein. In general, a similar vortex shedding behavior was also observed for the other V_r . The main distinction is that both the upward and downward jets move farther downstream and their strength increases with V_r .

3.2 Characteristics of the mean and fluctuating velocity fields

The purpose of this section is to study the mean velocity fields and turbulence measured by using the PIV to obtain a better insight into the wake characteristics of VIV. Fig. 6 compares the mean velocity vector (whose magnitude is $\sqrt{\bar{u}^2 + \bar{v}^2}$, where $\bar{u}$ and $\bar{v}$

denote the time-averaged flow velocity in the x and y directions, respectively) between the stationary and vibrating cases for two values of u_0 . For Figs. 6(a)-(b), $u_0 = 14.92$ cm/s ($V_r = 5.30$ and $A^* = 0.25$), whereas for Figs. 6(c)-(d), $u_0 = 18.76$ cm/s ($V_r = 6.66$ and $A^* = 0.61$). Figs. 6(a)-(b) plots the profiles of the normalized mean velocity $\bar{u}/u_0$ and $\bar{v}/u_0$ corresponding to Figs. 6(c) and 6(d), respectively.

For the stationary cylinder cases (Figs. 6a and 6c), a recirculation zone appears downstream, wherein vortices are formed. The vortices move downstream before they are dissipated when they move far away from the recirculation zone. In contrast to the stationary cylinder, no obvious recirculation zone can be found just downstream of the vibrating cylinder (Fig. 6b and 6d). However, in the near wake region ($x/D < 5.01$), the size of the affected region by the wake is much wider for a vibrating cylinder compared to that for a stationary cylinder with the same u_0 . For better clarity, Figs. 7(a)-(b) show the profiles of $\bar{u}$ and $\bar{v}$, both normalized by u_0 . In the vibrating case, Fig. 7(a) reveals an interesting feature in that the values of $\bar{u}$ near the x -axis are larger than or at least comparable to those away from the axis for $x/D \leq 4.0$. This clearly is different from that associated with a stationary cylinder. This phenomenon is predominantly caused by the oblique upward and downward jets associated with the vibrating cylinder (described earlier) whose magnitude increases with u_0 . Fig. 7(b) shows that the $\bar{v}$ -distribution for both the stationary and vibrating cases exhibit an S-shape downstream of the cylinder, albeit in the reverse order. This indicates that the flow is either directed downwards or upwards on either sides of the x -axis. The two localized large positive and negative values of $\bar{v}$ at $x/D = 0.55$ associated with the vibrating cylinder confirm the presence of the strong upward and downward oblique jets at these locations. This vertical motion eventually disappears when the vortex moves to locations where $x/D > 7.05$.

The typical turbulence characteristics for stationary and vibrating cylinders are shown in Figs. 8 and 9 for (the same) $u_0 = 14.92$ and 18.76 cm/s, respectively with the corresponding

$V_r = 5.30$ and 6.66 , $A^* = 0.25$ and 0.61 . The turbulence properties include detailed distributions of the velocity fluctuations in the horizontal and vertical directions ($\sqrt{u'^2}/u_0$ and $\sqrt{v'^2}/u_0$, where u' and v' are the fluctuations of u and v) and dimensionless Reynolds shear stress ($-\overline{u'v'}/u_0^2$).

Figs. 8(a) and 8(d) show the horizontal turbulence intensity $\sqrt{u'^2}/u_0$ for the same $u_0 = 14.92$ cm/s, associated with the stationary and vibrating cases, respectively. In the stationary case, two symmetrical but with a higher turbulence intensity zones are found downstream of the recirculation zone. However, in the vibrating case, the symmetrical zones appear on the top and bottom of the vibrating cylinder and the magnitude of the horizontal turbulence intensity are distinctly larger than the stationary case with the same u_0 . This is because cylinder vibrations accentuate the level of velocity fluctuations in the wake, causing significantly higher turbulence intensity for the stationary cylinder. A similar feature is also observed in Figs. 9(a) and 9(d), which show the distribution of $\sqrt{u'^2}/u_0$ at $u_0 = 18.76$ cm/s. Moreover, the locations of the larger turbulence intensity zones not only are farther away from the centerline of the wake, its intensity increases with A^* .

The distributions of the vertical turbulence intensity $\sqrt{v'^2}/u_0$ under the stationary and vibrating conditions are shown in Figs. 8(b), 8(e), 9(b) and 9(e) for the same two u_0 -values. The value of $\sqrt{v'^2}/u_0$ is distinctly larger than $\sqrt{u'^2}/u_0$ for both the stationary and vibrating conditions. Additionally, the size of the high vertical turbulence intensity zone associated with the vibrating case is significantly larger than the stationary counterpart. This is primarily because of the presence of the two upward and downward oblique jets and their ensuing higher vertical velocities (see Figs. 5-7).

In addition, Figs. 8(c), 8(f), 9(c) and 9(f) show the dimensionless Reynolds stress $(-\overline{u'v'}/u_0^2)$, in the x - y plane for the stationary and vibrating conditions with the same two velocities. For the stationary case, the larger absolute values of $-\overline{u'v'}/u_0^2$ are mainly found at the upper and lower sides of the recirculation region, where vortex shedding originates. However, for the vibrating case, these larger values are mainly found downstream of the point of maximum displacement of the vibrating cylinder. The positive value of the Reynolds stress denotes a clockwise rotation of the vortex and vice-versa. The same phenomenon also can be deduced from Fig. 5, in which the maximum Reynolds stress is shown to occur at the place where the vortices form, indicating that the magnitude of the Reynolds stress is highly correlated with the presence of the vortices.

For more detail comparison of turbulent properties between stationary and vibration case, the distribution profiles of $\sqrt{u'^2}/u_0$, $\sqrt{v'^2}/u_0$ and $-\overline{u'v'}/u_0^2$ at different x/D with the same u_0 are depicted in Figs. 10(a)-(c), respectively. The figures clearly reveal that the primary influenced region of turbulence induced by a vibrating cylinder is about $x/D \leq 5.01$. At $x/D \geq 5.01$, downstream of the influenced region, the distribution of $\sqrt{u'^2}/u_0$ and $-\overline{u'v'}/u_0^2$ becomes almost uniform and the difference between the stationary and vibrating conditions becomes insignificant. However, the strength of $\sqrt{v'^2}/u_0$ with the vibrating case is distinctly larger than the stationary case (see Fig. 10b), even at $x/D > 5.01$ where the vortices of the wake have shed completely. The reason is the different vorticity strength of the shedding vortices and the variation of geometric arrangement of the von Kaman Vortex Street between the stationary and vibrating cases. This hypothesis will be verified later when considering the vorticity flow field data.

The strength and the geometric arrangement of the shedding vorticity are closely related to the strength of the oblique jets and response amplitude of the vibrating cylinder. The characteristics also result in the mean velocity fields downstream of the formation region. Fig. 11(a) shows the value of $\bar{u}$ along the x -axis, referred to as $\bar{u}_{y=0}$, at different locations downstream of the cylinder for the stationary and free-vibrating conditions. Three representative values of u_0 , namely 14.92, 17.22, 18.76 cm/s, are chosen for inclusion in the plot because the results for both the stationary and vibrating cases are available. These u_0 values correspond with $V_r = 5.30, 6.11$ and 6.66 and $A^* = 0.25, 0.51$ and 0.61 , respectively. Fig. 11(a) reveals that for the region downstream of the recirculation zone ($x/D > 4.0$), the ratio $\bar{u}_{y=0}/u_0 \approx \text{constant}$ with respect to x/D (for convenience denote this constant $\bar{u}_{y=0}/u_0$ as $\bar{u}_c/u_0$). Fig. 11(b) depicts the variation of $\bar{u}_c/u_0$ with A^* . For the stationary conditions, the values of $\bar{u}_c/u_0$ are more or less similar, ranging from 0.65 to 0.70 for different u_0 . However, $\bar{u}_c/u_0$ for the vibrating condition deviates considerably from the stationary case. This reason may be due to the geometric configuration of the vortex street, as well as the fact that the magnitude of the vortices is influenced by A^* .

For the vibrating condition, the behavior of $\bar{u}_c/u_0$ as a function of A^* can be separated into two regions. For the region where $A^* > 0.274$, which should apply to the majority of practical applications, $\bar{u}_c/u_0$ increases with A^* . Moreover, the data appears to be fairly linear, and linear regression yields

$$\frac{\bar{u}_c}{u_0} = 0.673A^* + 0.233, \quad \text{for } 0.262 < A^* \leq 0.61 \quad (2)$$

with a goodness-of-fit R^2 value of 0.905. On the other hand, when A^* is smaller than 0.274, $\bar{u}_c/u_0$ diminishes with increasing A^* . As the behavior is noticeably nonlinear, the data is fitted to the following nonlinear form

$$\frac{\bar{u}_c}{u_0} = 0.663 - 14.3(A^*)^3, \quad \text{for } 0 \leq A^* \leq 0.262 \quad (3)$$

where $R^2 = 0.999$. However, one should exercise caution in applying Eq. (3) as it is based on limited data points, and there is a range of A^* in the middle where there are no observations.

3.3 Vortex structure under shedding and convection process

In order to study the characteristics of vortex shedding, distributions, convections and evolution of the wake in the initial branch under different vibrating conditions (amplitudes), the techniques of wavelet transform and phase averaging are applied. The phase is defined as $2\pi/T$. This study considers 72 different discrete phases in a sinusoid. As measurements are recorded at discrete time intervals, the phase computed by the wavelet transform will in general not correspond exactly to the predetermined 72 desired phases. Therefore, the velocity field at a desired phase is obtained by linear interpolation between the two closest phases in each cycle.

However, if the time interval between two measurements is too large, significant errors may arise from the interpolation process, especially at critical locations such as the separation points on the cylinder surface and vortex shedding center. Thus, it is important to understand how the sampling rate affects the quality of the PIV measurements. To this end, Fig. 12 compares the PIV results of a vibrating cylinder with $u_0 = 18.76$ cm/s ($V_r = 6.66$) under different sampling rates (10, 33.3, 100 and 200 Hz). The sampling rate for the original PIV measurements was 200 Hz; the results for the lower sampling rates were derived from 200 Hz simply by omitting intermediate data points.

Each velocity field was phase averaged by 200 interpolated velocity fields, and the phase-averaged dimensionless vorticity contour is the span-wise vorticity defined as $\omega_z D/u_0$, where $\omega_z (= (\Delta \bar{v}/\Delta x) - (\Delta \bar{u}/\Delta y))$. The red contours signify positive vorticity in the

counter-clockwise direction, (originating from the lower shear layer), while the blue counterpart represents negative vorticity in the clockwise direction (originating from the upper shear layer). A comparison of Figs. 12(a)-(d) reveals apparent differences between the vorticity contours obtained from different sampling rates. The results from the lower sampling rates (10 and 33.3 Hz) have lower strength around the vortex center ($\omega_z D/u_0 = 3.72$ and 5.57 , respectively) due to the smoothing (scattering) effect. The interpolation between large time intervals is incapable of accurately locating the moving vortex center. Conversely, for the higher sampling rates of 100 and 200 Hz, the vorticity fields and the strengths at the vortex center ($\omega_z D/u_0 = 6.28$ and 6.32 , respectively) are in close agreement, indicating a converged result. Moreover, Fig. 13 compares the dimensionless circulation $|\Gamma|/u_0 D$ of the moving vortex for different sampling rates. The circulation Γ was obtained by applying Stokes' theorem and integrating the phase-averaged vorticity contours. Note that there are fewer data points for lower sampling rates, because the spatial and temporal resolutions Δx and Δt are related by $u = \Delta x/\Delta t$. It is observed that $|\Gamma|/u_0 D$ converges only when the sampling rate is at least 100 Hz. From these results, one may conclude that low sampling rates associated with conventional PIV techniques (typically 10 – 30 Hz) are inadequate for the present purpose. The foregoing results highlight the importance of using high speed PIV (200 Hz in this study) to provide the necessary temporal resolution to study the variation of vortex shedding and the convecting process.

Figs. 14 and 15 show the phase-averaged velocity fields for the vibrating cylinder with $u_0 = 14.92$ and 18.76 cm/s ($V_r = 5.30$ and 6.66), respectively. Moreover, Video 1 and 2 show the detail process of Figs. 14 and 15. Fig. 14 and Video 1 reveal that the vortices in the wake zone are convecting more or less along the centerline, and moreover they alternate between positive and negative vorticity; this pattern corresponds to the classical 2S mode (Williamson

and Roshko, 1988). However, the vortex shedding characteristics change as V_r increases. An additional small positive vortex is visible in Fig. 15(a, h) around $x/D = 2$, $y/D = -1.5$ on the lower side, and similarly a negative vortex appears in Fig. 15(d, e) around $x/D = 2$, $y/D = 1.5$ on the upper side. These small vortices could be generated by the strong velocity gradient of the turbulence shear layers between the wake zone and free stream (see Fig. 6(d)). As the vortex patterns can be sensitive to the displacement profile since it affects the relative velocity between the free stream and the vibrating cylinder (Konstantinidis and Bouris, 2016), the shedding of additional vortices are promoted in the higher V_r case.

The figures reveal that when the cylinder is under the falling phase, an oblique jet is formed beneath the cylinder, which shoots upward to generate an anti-clockwise vortex located in between the cylinder and jet. Additionally, this upward-shooting jet also forces the previously generated vortex downstream of the jet to move farther away. It also continuously supplies energy to the two vortices that rotates in the opposite direction during the falling phase of the cylinder. On the other hand, when the cylinder reverses direction (see Figs. 14e-h and 15e-h), the downward-shooting jet impinges from the upper shear layer to similarly enhances the degree of turbulence and vorticity in its vicinity. It must be stated that this phenomenon is also supported by the observation discussed in association with Fig. 3.

Based on the above observation, one may surmise that the process of vortex formation and shedding under vortex-induced vibration may be divided into two sections: the falling and rising phases of the cylinder. In the vortex formation process, for instance, the generation and evolution of the vortex are depicted in Figs. 14(a)-(d) and Figs. 15(a)-(d), respectively. The vortex is generated beneath the cylinder initially; however, as the cylinder falls, this vortex is pushed upward by the ensuing upward jet, causing the center of the vortex to be above the x -axis ($y/D > 0$). Combining the flow components induced by the approaching flow and falling cylinder, the upward jet and accompanying recirculation zone are formed below

the cylinder. Likewise, Figs. 14(e)-(h) and 15(e)-(h) show the formation of the downward and associated recirculation zone during the rising phase of the vibrating cylinder. After the vortex is shed from the cylinder, it is forced to move downstream by the opposite jet as its magnitude increases (see Figs. 14 and 15). As it moves farther away from the cylinder, it begins to diffuse as it loses contact with the energy source allied with the oblique jets.

In problems involving flow separation, the stagnation point and separation points are of interest. The stagnation point is the location on the cylinder with zero velocity components, while the separation point is the intersection between forward and backward flow, in which shear stress is zero. As the flow field is measured experimentally, the spatial resolution is not sufficiently fine to allow the formal definitions to be utilized in locating the points. Hence, the stagnation and separation points are estimated visually according to the streamlines that delineate the flow pattern, as exemplified in Fig. 16. Fig. 17 shows the locations of one stagnation point and two separation points on the vibrating cylinder surface for $u_0 = 14.92$ and 18.76 cm/s. The angle θ is defined as clockwise positive, measured from the frontal face of the cylinder. Together with Fig. 15(a)-(c), it is found that the stagnation point for the case $u_0 = 18.76$ cm/s has moved upward due to the increment of the cylinder falling velocity ($t/T = 0.00 \sim 0.25$). At the same time, the lower separation point moves downward rapidly with the formation of an oblique jet at $t/T = 0.05$, and reaches a maximum negative angle $\theta = -164^\circ$ at $t/T = 0.181$ when a vortex is formed by the lower shear separation from the cylinder surface. Between $t/T = 0.25 \sim 0.5$, the cylinder's speed starts to decrease, causing the stagnation point to move downward gradually (refer to Fig. 17(c)). The lower separation point also moves backward in the upstream direction, and the oblique jet moves downstream with the growth of the vortex. The symmetry manifested in Figs. 17(a)-(c) is consistent, suggesting that the experimental measurements are accurate. For example, at the $u_0 = 18.76$ cm/s case, the maximum stagnation point occurs at $t/T = 0.236$, and 0.736 , whose interval corresponds to

half a cycle (see Fig. 17(a)). Likewise, the maximum lower separation point happens at $t/T = 0.181$, which is very close to half a cycle ahead of the maximum upper separation point at $t/T = 0.667$. It is also worth mentioning that the maximum values of the stagnation point and two separation points occur at different phases (t/T) for different u_0 .

Fig. 18 shows the routes undertaken by two pairs of vortices after they are shed from the cylinder ($u_0 = 14.92$ and 18.76 cm/s). It specifically displays the paths of the center of these vortices as each of them moves downstream. The vortex center point is taken to be the location where the positive or negative magnitude of the vorticity is highest. The data reveal that the path of the vortex center associated with the larger approach velocity ($u_0 = 18.76$ cm/s) is confined to locations along the x -axis for $1.6 < x/D < 4.0$ regardless of whether the vortex is generated from the upper or lower shear layer. Moreover, the upper and lower vortices may encroach into the opposite side of the cylinder. The “intrusion” is due to the presence of the oblique jets caused by cylinder vibration, as was discussed previously.

Fig. 19 shows the dimensionless circulation $|\Gamma|/u_0 D$ for the stationary and vibrating cylinder for $u_0 = 14.92$ and 18.76 cm/s. To ensure that the vortices have separated from the shear layer in the computation of $|\Gamma|/u_0 D$, the data points start from $x/D \approx 2.5$ for the stationary case. For $x/D < 2.0$, the circulation associated with the vibrating cylinder increases with distance. As the vortex moves away from the cylinder ($x/D > 2.0$), dissipation occurs and the strength of the circulation decreases. In locations beyond $x/D > 8$, the difference of circulation between the stationary and vibrating cases is no longer significant. The data in the figure also show that the magnitude of $|\Gamma|$ is expectedly directly proportional to V_r but its influence on the decay rate does not appear to be as significant.

From the comparisons of the velocity fields, vortex paths, and vortex dissipation between the stationary and vibrating cases, the oblique jets that intrude into the formation zone have obviously a pervasive influence. The characteristics of the jet such as its strength,

period and range depend on the cylinder vibration. To understand the properties of the oblique jets induced by different vibration amplitudes, a dimensionless velocity u_{jm}/u_0 is introduced to characterize the strength of the oblique jet, where u_{jm} = maximum absolute velocity in the jet flow at $y = 0$. Fig. 20 shows the variation of u_{jm}/u_0 with A^* . Similar to the behavior of $\bar{u}_{y=0}/u_0$ in Fig. 11(b), u_{jm}/u_0 also increases with A^* for $0.262 \leq A^* \leq 0.61$. Due to the small response amplitude, no apparent oblique jet is observed when $A^* \leq 0.262$. Moreover, the relationship between u_{jm}/u_0 and A^* is fitted to the following equation

$$u_{jm}/u_0 = 0.34 \ln(A^*) + 1.597, \quad \text{for } 0.262 \leq A^* \leq 0.61 \quad (4)$$

with a goodness-of-fit R^2 value of 0.92. The results reveal that the jet strength increases with the response amplitude. The jet may have influence on the drag and lift forces, which are outside the scope of this study. Moreover, if there is another structure in the wake of the cylinder, the jet will also have an influence on it. These interesting effects are worth investigating in a future work.

4. Conclusions

The vortex structure and turbulence statistics in the wake of a freely-vibrating cylinder were studied experimentally using visualization techniques and a high resolution PIV. The emphasis was on the understanding of the mechanism of the wake and the generation and evolution of the vortex associated with a vibrating cylinder. The experiments also included stationary cylinders for comparison. The main findings are summarized as follows:

1. Vibration of the cylinder results in the formation of oblique upward and downward jets that have not been mentioned in previous studies. These jets are highly dependent on V_r and have many important implications. The impinging location of the jet shifts farther downstream with increasing V_r and its strength also increases with V_r .

2. Because of the oblique jet, the size of the affected wake zone in proximity of the cylinder ($x/D < 5.01$) is much wider for the vibrating case compared to the stationary case.
3. The mean and fluctuating velocities are closely related to the oblique jets, which cause the turbulence intensity to be significantly higher when the cylinder is vibrating compared to the stationary condition.
4. Results show that $\bar{u}_{y=0}$ approaches a constant value $\bar{u}_c$ downstream of the recirculation zone. For the common situation whereby $A^* > 0.262$, $\bar{u}_c/u_0$ increases with A^* , and a linear empirical equation, Eq. (2) is proposed.
5. The variations of the locations of the stagnation point and two separation points during vibration were studied. As may be expected, there exists apparent phase shifts in the movements of the stagnation and separation points for different u_0 .
6. The oblique jets supply additional energy to the vortices up until $x/D = 2.0$, resulting in the circulation parameter $|I|/u_0D$ achieving its peak value around $x/D = 2.0$, beyond which $|I|/u_0D$ decays with distance.

Supplemental Data

Videos 1 and 2 are available online.

Acknowledgments

The authors gratefully acknowledge the financial support provided by Singapore Maritime Institute (SMI) and EMAS AMC, under the SMI Deepwater Technology R&D Program.

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Figure

Fig. 1. Schematic diagrams of experimental setup and coordinate system.

Fig. 2 Illustration of different shadow regions resulting from projecting the laser from two angles

Fig. 3 Convergence test of PIV measurements comparing with ADVP data for $u_0 = 16.32$ cm/s at $x/D = 11$: (a) horizontal mean velocity profiles; (b) profiles of $(\overline{u'u'})^{0.5} / u_0$.

Fig. 4 Plot of A^* against V_r .

Fig. 5 Flow field visualization for vibrating cylinder with $V_r = 6.66$ ($u_0 = 18.76$ cm/s) and $A^* = 0.61$.

(a) $u_0 = 14.92$ cm/s, stationary cylinder

(c) $u_0 = 18.76$ cm/s, stationary cylinder

(b) $u_0 = 14.92$ cm/s, vibrating cylinder

(d) $u_0 = 18.76$ cm/s, vibrating cylinder

Fig. 6 Comparison of mean velocity vector between stationary and vibrating cylinders for $u_0 = 14.92$ and 18.76 cm/s.

Fig. 7 Profiles of $\bar{u}$ and $\bar{v}$ at different x/D for $u_0 = 18.76$ cm/s.

Fig. 8 Comparison of turbulence flow fields between stationary and vibrating cylinder with $u_0 = 14.92$ cm/s. (a)-(c) stationary case; (d)-(f) vibrating case ($V_r = 5.30$, $A^* = 0.25$).

Fig. 9 Comparison of turbulence flow fields between stationary and vibrating cylinder with $u_0 = 18.76$ cm/s. (a)-(c) stationary case; (d)-(f) vibrating case ($V_r = 6.66$, $A^* = 0.61$).

Fig. 10 Comparison of profiles for (a) $\sqrt{u'^2}/u_0$, (b) $\sqrt{v'^2}/u_0$ and (c) $-\overline{u'v'}/u_0^2$ between stationary and vibrating cylinder with $u_0 = 18.76$ cm/s.

Fig. 11 (a) Variation of $\bar{u}_{y=0}/u_0$ with x/D and (b) relationship between $\bar{u}_c/u_0$ and A^* .

Fig. 12 Comparison of PIV measured results of velocity field and vorticity contour under different sampling rates for vibrating cylinder, with $u_0 = 18.76$ cm/s ($V_r = 6.66$, $A^* = 0.61$).

Fig. 13 Comparison of PIV measured results of dimensionless circulation during vortex shedding process under different sampling rates for vibrating cylinder with $u_0 = 18.76$ cm/s ($V_r = 6.66$, $A^* = 0.61$).

Fig. 14 Phase-averaged velocity fields for $u_0 = 14.92$ cm/s ($V_r = 5.30$, $A^* = 0.25$).

Fig. 15 Phase-averaged velocity fields for $u_0 = 18.76$ cm/s ($V_r = 6.66$, $A^* = 0.61$).

Fig. 16 Estimation of the stagnation and separation points from streamlines

Fig. 17 Time-history of locations of the stagnation and separation points for the vibrating cases. (a) stagnation point; (b) upper separation points; (c) lower separation points.

Fig. 18 Path of vortex shedding for $u_0 = 14.92$ and 18.76 cm/s.

Fig. 19 Comparison of non-dimensional circulation of shedding vortex between stationary and vibrating cylinder for $u_0 = 14.92$ and 18.76 cm/s.

Fig. 20 Relationship between u_{jm}/u_0 and A^* .

Table 1 Values of u_0 and the corresponding V_r, A^*, f_p and Re

u_0 (cm/s)	13.49	13.57	14.18	14.92	15.25	15.46	15.97	16.40	17.04	17.22	17.44	18.29	18.76
V_r	4.79	4.82	5.03	5.30	5.41	5.49	5.67	5.82	6.05	6.11	6.19	6.49	6.66
A^*	0.004	0.03	0.21	0.25	0.31	0.33	0.38	0.39	0.48	0.51	0.53	0.55	0.61
f_p (Hz)	1.40	1.35	1.40	1.41 (1.56) ^a	1.40	1.430	1.46	1.48	1.50	1.52 (1.91) ^a	1.53	1.56	1.64 (2.15) ^a
Re	2698	2714	2836	2984	3050	3092	3194	3280	3408	3444	3488	3658	3752

^a Values in parentheses refer to vortex shedding frequency for stationary condition

Video

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Video

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Video Stills

Video 1 Phase-averaged velocity fields for $u_0 = 14.92$ cm/s ($V_r = 5.30$, $A^* = 0.25$).

Video 2 Phase-averaged velocity fields for $u_0 = 18.76$ cm/s ($V_r = 6.66$, $A^* = 0.61$).