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Effect of spatial variability of shear strength parameters on critical slip surfaces of slopes

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Abstract: Critical slip surface (CSS) of slopes is useful information in system reliability analyses and risk assessments of slopes. This paper investigates the effect of spatial variability of shear strength parameters on the CSS distribution of a two-dimensional slope using a finite element stress-based slope stability method. The failure mechanics for typical CSSs in a heterogeneous slope are analyzed and distributions of CSS for various combinations of horizontal and vertical scales of fluctuation (SOFs) are evaluated. The results show that local failures may occur in statistically homogeneous slopes, which cannot be observed in deterministic slope stability analyses. The distribution range of CSS first increases and then decreases with an increasing scale of fluctuation. As the ratio of horizontal SOF to vertical SOF is large, local failures have entry points located at the top of a slope. By contrast, as the ratio of horizontal SOF to vertical SOF is relatively small, local failures have entry points located either at the top of a slope or on the slope surface. The findings facilitate a more accurate evaluation of the system reliabilities or risks of slopes.

Keywords: slope stability; critical slip surface; spatial variability; non-intrusive stochastic analysis

1. Introduction

Spatial variability is an inherent property of soil materials because of complex geological, environmental and physical effects. It is more realistic to consider the spatial variability of soil parameters in the design of geotechnical structures (e.g., Liu et al., 2015; Wang et al., 2016; Huang et al., 2017; Tan et al., 2017; Xiao et al., 2017). A large number of studies have been conducted to investigate the effect of spatial variability of soil parameters on the reliability or risk of slopes. For example, Griffiths et al. (2011) and Li et al. (2014) evaluated the reliability of infinite slopes in the presence of spatially autocorrelated shear strength parameters. Huang et al. (2013), Zhu et al. (2013), Li et al. (2016a, b), and Deng et al. (2017) studied the reliability or risk of two-dimensional slopes considering the spatial variability of soil parameters. Ji (2014), Xiao et al. (2016), Li et al. (2017) and Liu et al. (2017) conducted reliability or risk analyses of three-dimensional heterogeneous slopes.

However, rare studies focused on the uncertainty of the critical slip surface for a slope. The critical slip surface (CSS) refers to the slip surface with the minimum factor of safety in a slope. In reality, it is not possible to know the whole geotechnical condition of a slope, leading to an uncertainty in the size and location of the critical slip surface. The CSS plays a vital role in risk, system reliability analyses of slopes and design of site investigation schemes. An analysis of the distribution of CSS hence has wide applications as elaborated as follows. First, key failure modes or representative slip surfaces, which are important inputs in system reliability or risk analysis of slopes, are always determined according to the CSS of slopes (e.g., Li et al., 2013; Tang et al., 2013; Jiang et al., 2015; Zhang and Huang, 2016; Jiang et al., 2017). The analyses of the distribution of CSS in heterogeneous slopes help to determine more reasonable representative slip surfaces. Second, it is well known that the site investigations within the influential range (namely

the area occupied by the CSS) of a slope is more useful than that beyond the influential range (e.g., Li et al., 2016c; Deng et al., 2017). An evaluation of the distribution of CSS helps to design more effective site investigation schemes. Third, back analyses of slope failures are always performed to infer the knowledge (such as mean and standard deviation) of soil parameters (e.g., Zhang et al., 2010). It has been demonstrated by Hicks and Spencer (2010) that the scale of fluctuation of shear strength parameters significantly affects the scale or location of the CSS. Therefore, the size or location of a slope failure surface is useful information for inferring the scales of fluctuation of shear strength parameters in back analyses.

Albeit significant, a systematic analysis of the distribution of CSS for heterogeneous slopes is rarely conducted. Griffiths et al. (2011) and Li et al. (2014) investigated the critical depth of slopes, but the analyses focused merely on infinite slopes. Wang et al. (2010) evaluated the spatial variability of critical slip surfaces for a two-dimensional slope, but only considered the spatial variability in the vertical direction. Hicks and Spencer (2010) investigated the failure modes and reliability of three-dimensional heterogeneous slope. Hicks et al. (2014) further proposed one method to quantify the slide volume and slide length of three-dimensional heterogeneous slopes. These two studies provide a good basis for risk analysis of three-dimensional slopes. However, it is worth noting that a two-dimensional slope represents another type of slopes in reality and the CSS of a two-dimensional slope may display different distribution characteristics from these of one-dimensional and three-dimensional slopes. The distribution of CSS for two-dimensional slopes also deserves a thorough study.

Hence, this study investigates the distribution of critical slip surface for a two-dimensional heterogeneous slope. First, a finite element stress-based slope stability method is presented to evaluate the critical slip surface of a heterogeneous slope. Second, the failure mechanics for typical

critical slip surfaces are analyzed. Third, distributions of the CSS for different combinations of horizontal and vertical scales of fluctuation are compared and the effect of coefficient of variation (COV) of shear strength parameters on the CSS distribution is investigated. At last, the significant role of CSS distribution in reliability analyses of slopes is illustrated by evaluating the contribution of local failures to the probability of failure of slopes. The CSS distributions obtained in this study provide guidelines for selecting representative slip surfaces in system reliability analyses and risk assessments of two-dimensional slopes.

2. Method

In this study, the spatial variability of shear strength parameters is modeled as a lognormally distributed random field and a widely used spectral representation method is used to generate the random field (e.g., Popescu et al., 2005; Soubra et al., 2008). The critical slip surfaces of heterogeneous slopes are computed by a finite element stress-based slope stability analysis. An efficient non-intrusive stochastic analysis method is used to automatically evaluate the distribution of critical slip surfaces. The spectral representation method and non-intrusive stochastic analysis method are well known and are not presented herein. Details of the two methods can be respectively found in Shinozuka and Deodatis (1991, 1996) and Jiang et al. (2014). This section merely introduces the finite element stress-based slope stability method. This method is adopted because it overcomes the drawbacks of other widely used methods, such as finite element (or finite difference) shear strength reduction method (e.g., Cho, 2009a; Griffiths et al., 2009; Hicks et al., 2014) and the limit equilibrium method (e.g., Cho, 2009b). The finite element shear strength reduction (FESSR) method has many desirable features, such as being able to calculate the stresses, movement of slope and to realistically model complex conditions of a slope (such as nonlinear stress-strain behavior) (Duncan, 1996). But the method is computationally expensive and can only

calculate a shear zone rather than an explicit critical slip surface. The limit equilibrium (LE) method is less time-consuming but has to make some assumptions on the forces between slices of slopes. Furthermore, the LE method provides no strain information of slopes and the obtained stress distribution in slopes may not be realistic (Duncan, 1996). In comparison, the finite element stress-based slope stability method does not need to repeatedly compute the stress and hence is less time-consuming than the FESSR method. In addition, it could compute realistic stress distributions and an explicit critical slip surface.

The basic idea of the finite element stress-based method is first establishing the stress distribution in the ground using a finite element analysis and then evaluating the slope stability using the calculated stresses (GEO-SLOPE Ltd, 2008). For any potential slip surface, the associated factor of safety (FS) is evaluated by (1) integrating the available shear resistance (derived from the normal stresses applied to the slip surface) and mobilized shear stress along the slip surface and (2) dividing the total available shear resistance by the total mobilized shear stress. The critical slip surface is determined through the procedure as follows. (1) Generate many trial circular slip surfaces and find the slip surface with the minimum FS. (2) Optimize the slip surface with the minimum FS and obtain a non-circular critical slip surface. The second step is implemented because the heterogeneity of a slope always renders the CSS non-circular. This method has been successfully applied to slopes with spatially autocorrelated soil parameters (e.g., Jiang et al., 2014, 2015, 2017). In the current study, the slope stability is evaluated by the finite element stress-based method using a widely used commercial software, GeoStudio (GEO-SLOPE Ltd, 2008). More details of the method can be found in GEO-SLOPE Ltd (2008).

3. Illustrative examples

In this section, one heterogeneous slope is first analyzed to illustrate the capacity of the finite element stress-based method in evaluating the critical slip surface of a heterogeneous slope. Second, the failure mechanics for typical CSSs in a heterogeneous slope are analyzed and distributions of CSS for various combinations of the horizontal and vertical SOF for cohesion are evaluated. Finally, parametric studies are conducted to explore the effect of coefficient of variation of cohesion on the distribution of CSS. The CSS distributions for a slope with spatially autocorrelated friction coefficient are also investigated. In all the slope stability analyses, a slope model with a size of $30\text{ m} \times 10\text{ m}$ as shown in Fig. 1 is used. The slope height is 5 m and the size of random field grid is $0.5\text{ m} \times 0.5\text{ m}$. The resulting grid density is equal to or larger than that adopted by Hicks and Spencer (2010), and Hicks et al. (2014). Hence, the accuracy of the stochastic analysis could be guaranteed.

(a) critical slip surface searched by GeoStudio (b) contour plot of shear strain rate calculated by FLAC3D
Fig. 1 Slope stability evaluated by different methods

3.1 Capacity of the finite element stress-based slope stability method

In this section, the capacity of the finite element stress-based slope stability method (FESSSM) is illustrated by comparing the critical slip surface evaluated from FESSSM with the shear strain rate contour calculated by a finite difference method. The latter is accomplished in a widely used commercial software, i.e., FLAC3D. The contour plot of shear strain rate evaluated from the finite difference method is widely used to display the sliding zone of geotechnical structures (e.g., Li et al., 2015; Shen and Fu, 2011). The two analyses merely consider the spatial variability of friction

coefficient and adopt the same soil parameters. In this study, the autocorrelation structure of shear strength parameters is represented by a widely used single exponential autocorrelation function (e.g., Jiang et al., 2017) given by Eq. 1.

$$\rho_{\ln}(\Delta x, \Delta y) = \exp\left(-\frac{2|\Delta x|}{\theta_x} - \frac{2|\Delta y|}{\theta_y}\right) \quad (1)$$

where $\rho_{\ln}(\Delta x, \Delta y) = \rho_{\ln}(x_i - x_j, y_i - y_j)$ is the autocorrelation coefficient between $\log(Z(x_i, y_i))$ and $\log(Z(x_j, y_j))$, in which $Z(x_i, y_i)$ and $Z(x_j, y_j)$ are two random variables in the random field and are respectively located at (x_i, y_i) and (x_j, y_j) , Z could be cohesion, c , or friction coefficient, f ; θ_x and θ_y respectively are the horizontal and vertical scales of fluctuation for the field of $\log(Z)$.

One random field realization for friction coefficient with the mean $\mu_f = 0.268$, coefficient of variation, $COV_f = 0.3$ and scales of fluctuation $\theta_{\ln f, x} = \theta_{\ln f, y} = 5$ m is plotted in Fig. 1(a). In Fig. 1(a), dark colors represent large values of friction coefficient while light colors represent small values of friction coefficient. The friction coefficient values in this realization are assigned to the corresponding regions in the finite element model and to the corresponding elements in the finite difference model. The remaining soil parameters are kept constant, i.e., cohesion = 5 kPa, elastic modulus = 8 MPa, Poisson's ratio = 0.33 and unit weight = 20 kN/m³. The same constitutive model (namely an elastic-perfectly plastic model with a Mohr-Coulomb yield criterion) and boundary conditions are adopted in the finite element and finite difference analyses. The horizontal movement in all the vertical boundaries are constrained and the nodes at the bottom boundary are fixed in all directions. Note that the shape and size of the finite difference elements are the same as that of the random field grids and the soil parameters for each element are assigned using a built-in programming language in FLAC3D, i.e., FISH (e.g., Li et al., 2015). The critical slip surface evaluated from the FESSM and shear strain rate contour calculated by the finite difference

method are plotted in Fig. 1. As shown, the critical slip surface is located in the area with high shear strain rate, indicating that the two analyses obtain consistent results. In addition, the obtaining critical slip surface is typically non-circular and automatically captures the weak soils (see Fig. 1(a)). Hence, the FESSSM is effective in searching out the non-circular critical slip surface of a heterogeneous slope.

3.2 Effect of spatial variability of cohesion on the distribution of critical slip surface

In this subsection, the suitable number of Monte Carlo simulation is first determined through a parametric study. Second, the failure mechanics for typical CSSs are analyzed. Third, the CSS distributions for different combinations of horizontal scale of fluctuation and vertical scale of fluctuation are compared. At last, the effect of COV of cohesion on the CSS distribution is studied. For simplicity, this subsection merely focuses on the spatial variability of cohesion. The remaining parameters are constant and take the same value as that adopted in section 3.1 (the friction coefficient is fixed at its mean value, i.e., 0.268). The same slope model as that in section 3.1 is used.

3.2.1 Determining the number of Monte Carlo simulation

In this section, the random field parameters for the cohesion are set to be mean $\mu_c = 5$ kPa, coefficient of variation $COV_c = 0.5$, scales of fluctuation $\theta_{lc,x} = 25$ m, and $\theta_{lc,y} = 2.5$ m. The values of COV_c , $\theta_{lc,x}$, and $\theta_{lc,y}$ respectively lie within the corresponding typical ranges summarized by Phoon and Kulhawy (1999). 500, 1000, and 2000 realizations of cohesion random fields are respectively simulated. The distribution of critical slip surfaces for these realizations are evaluated and plotted in Fig. 2. In Fig. 2, the CSS for the deterministic analysis using the mean value of cohesion is denoted by a thick line while the CSSs for random field realizations are denoted by thin lines. As shown, there are no big differences between the distributions of CSS for

1000 and 2000 realizations. Hence, the number of random field realizations is set to 1000. In addition, the critical slip surfaces are a cluster of curves with different locations and sizes. Local or deep-seated overall failures may occur if the spatial variability is considered. By contrast, a deterministic analysis can only produce an overall slope failure. It is worth noting that local slip failures did occur in reality (e.g., Canuti et al., 2005; Du et al., 2013; Li et al., 2009). This fact highlights the importance of considering spatial variability in slope stability analyses.

Fig. 2 Critical slip surfaces for different numbers of random field realizations

3.2.2 Failure mechanics for typical critical slip surfaces

In this section, the failure mechanics for typical critical slip surface are analyzed. Different realizations of cohesion random field and associated CSSs are plotted in Fig. 3. These realizations are extracted from the Monte Carlo simulation in section 3.2.1. In Fig. 3, light colors represent small values of cohesion while dark colors represent large values of cohesion. The typical realizations in Fig. 3 have different characteristics of weak zone distributions and are divided into five categories as follows.

(1) The slope is relatively homogeneous and has no obvious weak zones. Typical realizations of this category are plotted in Figs. 3(a, b). For this case, the failure mode of the slope is an overall

failure and the critical slip surface is located around the critical slip surface obtained from a deterministic analysis using the mean value of cohesion. In addition, the CSS is shallow when the cohesion is relatively small (see Fig. 3(a)) while the CSS is deep when the cohesion is relatively large (see Fig. 3(b)).

Fig. 3 Critical slip surfaces for typical realizations of a random field with $\theta_{\text{inc},x} = 25$ m and $\theta_{\text{inc},y} = 2.5$ m as $COV_c = 0.5$

(2) The weak zone is located at the upper area of the slope and is relatively large (see Fig. 3(c)). The associated failure mode is a local failure and the location of the CSS is consistent with the location of the weak zone. The reason is that the FS for the local slip surface running across the weak zone is smaller than that for a large slip surface representing an overall failure.

(3) The weak zone is located at the upper area of the slope and is relatively small (see Fig. 3(d)). The failure mode for this case is an overall failure and the location of CSS is not consistent with the location of the weak zone. The reason is explained as follows. The load of a slope is a

component of the gravity of the slope, which is proportional to the volume of the slide mass. The shear resistance of the slope consists of two parts, friction angle part (i.e., friction coefficient $\times$ normal stress applied to a slip surface) and cohesion part (i.e., cohesion $\times$ area of a slip surface). The frictional angle part is also proportional to the volume of slide mass but the cohesion part is proportional to the area of a slip surface. Obviously, as the size of a slip mass decreases, the decreasing rate of the associated slip surface is smaller than that of the slip volume. In other words, if the shear strength parameter keeps constant and the size of sliding mass decreases, the load of a slope decreases faster than the resistance of the slope. To achieve the same factor of safety (i.e., the same decreasing rate of the load and resistance of a slope), the shear strength has to decrease simultaneously with the size of a slip surface. For a relatively large slip surface as shown in Fig. 3(c), the cohesion in the weak zone is small enough to generate an FS smaller than the FS for an overall slip surface. However, if the slip surface continues to decrease to a level as shown in Fig. 3(d), the FS for the local slip surface is larger than the FS for overall slip surface. The reason is that the cohesion in the weak zone cannot further decrease for a given COV_c . Hence, overall failure happens in the case of Fig. 3(d).

(4) The weak zone is located at the lower part of the slope and the failure mode is an overall failure (Fig. 3(e)). The reason for the overall failure mode is that the FS for a local slip surface in the weak zone is larger than that for an overall slip surface.

(5) The weak zone is located below the toe of the slope and the failure mode is a deep-seated overall failure (see Fig. 3(f)).

From the above analyses, it can be clearly seen that the failure mode for a slope with a relatively large value of $\theta_{lc,x} / \theta_{lc,y}$ is an overall failure in most cases. Although local failures

may happen in certain cases, the associated critical slip surfaces always have entry points (the uppermost points of slip surfaces) located at the top of the slope.

3.2.3 Distributions of critical slip surface for different combinations of $\theta_{\ln c, x}$ and $\theta_{\ln c, y}$

In this subsection, the distributions of CSS for different combinations of $\theta_{\ln c, x}$ and $\theta_{\ln c, y}$ are compared. Seven cases are considered, namely $\theta_{\ln c, x} = \theta_{\ln c, y} = 1$ m; $\theta_{\ln c, x} = \theta_{\ln c, y} = 2.5$ m; $\theta_{\ln c, x} = \theta_{\ln c, y} = 5$ m; $\theta_{\ln c, x} = \theta_{\ln c, y} = 25$ m; $\theta_{\ln c, x} = \theta_{\ln c, y} = \text{infinite}$; $\theta_{\ln c, x} = 10$ m, $\theta_{\ln c, y} = 1$ m; and $\theta_{\ln c, x} = 12.5$ m, $\theta_{\ln c, y} = 2.5$ m. The seven cases can be divided into two categories, namely $\theta_{\ln c, x} = \theta_{\ln c, y}$, and $\theta_{\ln c, x} > \theta_{\ln c, y}$. In the fifth case, the cohesion can be viewed as a random variable. The CSS distributions for the seven sets of scales of fluctuation are plotted in Fig. 4. In Fig. 4, the critical slip surface for the deterministic analysis using the mean value of cohesion is denoted by a thick line while the CSSs for random field realizations are denoted by thin lines. As shown in Figs. 4(a-e), the distribution range of CSS first increases and then decreases with an increasing scale of fluctuation. This phenomenon can be explained as follows. First, the cases with the scales of fluctuation smaller than the influence zone (namely the cases as shown in Figs. 4(a-c)) are considered. The influence zone refers to the zone covered by the CSS of a slope (e.g., Li et al., 2016c). For the studied slope, the size of the influence zone is approximately equal to $20 \text{ m} \times 8 \text{ m}$ (see Figs. 2 and 4). As the scale of fluctuation is small, the size of a continuous weak zone is small. For this case, a potential slip surface is likely to run across both weak and strong soil layers and the average value of cohesion along a potential slip surface is close to the input mean value. Hence, the critical slip surface is located around the CSS for the deterministic analysis. By contrast, a small or large potential slip surface for relatively large SOFs is highly likely to run across continuous weak soils and be the critical slip surface. Hence, when the SOFs are smaller than the

influence zone, the distribution range of CSS is larger for large SOFs than that for small SOFs. This phenomenon can also be observed by comparing the CSSs in Fig. 2(b) and Fig. 4(g). Second, the cases with the scales of fluctuation larger than the influence zone (namely the cases as shown in Figs. 4(d-e)) are considered. Compared with the case of $\theta_{\text{inc},x} = \theta_{\text{inc},y} = 5$ m in Fig. 4(c), these cases have relatively small distribution ranges of CSS. The reason is that, for these large values of scale of fluctuation, the whole slope area is almost homogeneous. The small weak zone as shown in Fig. 3(c) and large weak zone as shown in Fig. 3(f) are not likely to occur.

Fig. 4 Critical slip surfaces for random fields with different combinations of $\theta_{\text{inc},x}$ and $\theta_{\text{inc},y}$

Furthermore, the CSS distributions for cases with small values of $\theta_{\ln c,x} / \theta_{\ln c,y}$ are compared with the cases with relatively large value of $\theta_{\ln c,x} / \theta_{\ln c,y}$. As shown in Figs. 4(a-f), the local critical slip surfaces for cases with relatively small values of $\theta_{\ln c,x} / \theta_{\ln c,y}$ (namely 1 or 5) may have entry points located on the slope surface. This finding is significantly different from the cases with $\theta_{\ln c,x} / \theta_{\ln c,y} = 10$ as shown in Fig. 2(b) and Fig. 4(g), where local critical slip surfaces enter into the slope merely at the top of the slope. To explore the reason, one typical realization of the random field with $\theta_{\ln c,x} = 12.5$ m and $\theta_{\ln c,y} = 2.5$ m and the associated CSS are plotted in Fig. 5. As shown in Fig. 5, the FS for the local slip surface is smaller than that for a large slip surface. This is different from the case illustrated in Fig. 3(e). In Fig. 3(e), although the weak zone is also located at the lower area of the slope, the weak zone is not large enough to produce a local slip surface with the minimum FS. The reason is that when $\theta_{\ln c,x} / \theta_{\ln c,y}$ is large, the weak zone has a layered shape, which limits the height of the weak zone. By contrast, as $\theta_{\ln c,x} / \theta_{\ln c,y}$ is relatively small, weak soils in different depths are more likely to connect and the resulting weak zone reaches a larger size.

Fig. 5 Critical slip surfaces for one typical realization of a random field with $\theta_{\ln c,x} = 12.5$ m and $\theta_{\ln c,y} = 2.5$ m as $COV_c = 0.5$

3.2.4 Distributions of critical slip surface for a relatively small COV_c

The effect of coefficient of variation of cohesion is investigated in this section. The CSS distributions with various COV_c s and different combinations of $\theta_{\ln c,x}$ and $\theta_{\ln c,y}$ are plotted in Fig.

6. As shown in Figs. 6(a-d), when the COV of shear strength is small, there are no big differences among the CSS distributions for various combinations of $\theta_{\ln c, x}$ and $\theta_{\ln c, y}$. The reason is that the CSS is always located around the CSS for the deterministic analysis no matter where the weak zone is. In other words, the COV of shear strength is too small to produce a local or very large CSS. In addition, as shown in Figs. 6(d-g) and Fig. 2(b), the distribution range of CSS increases with an increasing COV_c . In other words, local or very large CSS are more likely to happen as the COV of shear strength is large. The reason is quite obvious and not repeated herein.

Fig. 6 Critical slip surfaces for various combinations of horizontal and vertical scales of fluctuation and various COV_c s

3.3 Effect of spatial variability of friction coefficient on the distribution of critical slip surface

This section investigates the effect of spatial variability of friction coefficient on the distribution of slip surface. As the friction coefficient normally has a smaller COV than the cohesion (e.g., Phoon and Kulhawy, 1999), the COV for the friction coefficient, COV_f , is set to be 0.3 and 0.1. The mean of friction coefficient is set to be 0.268. Figs. 7(a-b) plot the CSS distributions for slopes with spatially autocorrelated friction coefficient. The scales of fluctuation of friction coefficient is set to be $\theta_{\ln f, x} = 25$ m and $\theta_{\ln f, y} = 2.5$ m. The other parameters are kept constant and are set to values the same as the case in section 3.1. As shown, as the COV_f decreases, the distribution of CSS also decreases significantly. No local failures happen as $COV_f = 0.1$. The phenomena are the same as that for cohesion and the reasons are not repeated herein. Figs. 7(c-d) plot the CSS distributions for slopes with spatially autocorrelated cohesion and friction coefficient. The COV s for cohesion and friction coefficient are respectively set to be 0.5 and 0.3. The scales of fluctuation for the two parameters are assumed to be the same and have values of $\theta_x = 25$ m and $\theta_y = 2.5$ m. The correlation coefficient of the two parameters is set to be -0.5. As shown, when the spatial variabilities of both cohesion and friction coefficient are considered, the CSS distribution display similar phenomena as the case with only spatially autocorrelated cohesion. In other words, when the ratio of horizontal scale of fluctuation to vertical scale of fluctuation is large, local CSSs are highly likely to enter the slope from the top of slope. By contrast, when the ratio of horizontal scale of fluctuation to vertical scale of fluctuation is relatively small, local failures have an entry point located either at the top of the slope or on the slope surface. The CSS distributions for other cases exhibit similar trends as that shown in Figs. 4 and 6. These cases are not repeated herein.

Fig. 7 Critical slip surfaces when spatial variability of friction coefficient is considered

Note: Figs. 7(a, b) only consider the spatial variability of friction coefficient while Figs. 7(c, d) consider the spatial variability of both cohesion and friction coefficient.

4. Discussion – significance of CSS distribution in reliability analyses of slopes

To illustrate the significance of identifying the distribution of critical slip surface in reliability analyses of slopes, this section analyzes the number of local critical slip surfaces and local failures in Monte Carlo simulations. The local critical slip surface refers to a critical slip surface with an entry point or an exit point on the slope surface, namely $10 \text{ m} < x_{\text{Entry}} < 20 \text{ m}$ or $10 \text{ m} < x_{\text{Exit}} < 20 \text{ m}$, where x_{Entry} and x_{Exit} respectively are the horizontal coordinates of the entry (uppermost) and exit (lowermost) points of one critical slip surface. The local failures denote the cases with local critical slip surfaces and factors of safety less than 1. The local critical slip surfaces and local failures are analyzed because existing system reliability analyses of slopes tend to neglect or put little weight on the failure mode corresponding to a local failure (e.g., Li et al., 2013). The local critical slip surfaces are classified into two categories in this study. The first category has an entry point located at the top of the slope ($x_{\text{Entry}} < 10 \text{ m}$) and an exit point on the slope surface ($10 \text{ m} < x_{\text{Exit}} < 20 \text{ m}$). The second category has an entry point located on the slope surface, namely

10 m < x_{Entry} < 20 m. For illustration, Fig. 8 plots the distributions of the two types of local CSSs for two cohesion random fields investigated in section 3.2.1 and 3.2.3, namely the random fields with $\theta_{\text{inc},x} = 25$ m, $\theta_{\text{inc},y} = 2.5$ m and $\theta_{\text{inc},x} = 5$ m, $\theta_{\text{inc},y} = 5$ m. These distributions are extracted from Fig. 2(b) and Fig. 4(c). The case of $\theta_{\text{inc},x} = 25$ m, $\theta_{\text{inc},y} = 2.5$ m has only one subfigure because all the local CSSs have an entry point located at the top of the slope.

(a) all the local critical slip surfaces for $\theta_{\text{inc},x} = 25$ m and $\theta_{\text{inc},y} = 2.5$ m (b1) local critical slip surfaces with entry points located at the top of the slope for $\theta_{\text{inc},x} = 5$ m and $\theta_{\text{inc},y} = 5$ m

(b2) local critical slip surfaces with entry points located on slope surfaces for $\theta_{\text{inc},x} = 5$ m and $\theta_{\text{inc},y} = 5$ m

Fig. 8 Local critical slip surfaces for various combinations of $\theta_{\text{inc},x}$ and $\theta_{\text{inc},y}$

Table 1 summarizes the numbers and percentages of local critical slip surfaces and local failures for the two cohesion random fields. In Table 1(a), the percentage of each type of local CSS is the ratio of the number of associated realizations to the total number of realizations for one random field, i.e., 1000. In Table 1(b), the percentage of each type of local failure is the ratio of the number of associated realizations to the total number of failures. As shown in Table 1(a), local critical slip surfaces in both cases take up a large percentage, i.e., 29.8% and 32.7%. In addition, the percentages of local failures are further higher, i.e., 43.8% and 46.1% (see Table 1(b)). These phenomena indicate that local failure mode may contribute a large part to the probability of failure

of a slope and should be well considered in system reliability analyses of slopes. Another important observation is that the local failures with entry points on the slope surface contribute 5.6% of the failures. Although this percentage is high, the failure mode corresponding to the local failures with entry points located on the slope surface is rarely considered in system reliability analyses of slopes. This may lead to an overestimation of the reliability of a slope when the ratio of horizontal scale of fluctuation to vertical scale of fluctuation is relatively small.

Table 1 Number and percentage of local critical slip surfaces and local failures

(a) Number and percentage of local critical slip surfaces

	Local CSSs with $x_{\text{Entry}} < 10$ m and $10 \text{ m} < x_{\text{Exit}} < 20$ m		Local CSSs with $10 \text{ m} < x_{\text{Entry}} < 20$ m		All local CSSs		All CSSs	
	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage
$\theta_{\text{Inc},x} = 25$ m, $\theta_{\text{Inc},y} = 2.5$ m	298	29.8%	0	0.0%	298	29.8%	1000	100%
$\theta_{\text{Inc},x} = 5$ m, $\theta_{\text{Inc},y} = 5$ m	291	29.1%	36	3.6%	327	32.7%	1000	100%

(b) Number and percentage of local failures (cases with local critical slip surfaces and FSs < 1.0)

	Local failures with $x_{\text{Entry}} < 10$ m and $10 \text{ m} < x_{\text{Exit}} < 20$ m		Local failures with $10 \text{ m} < x_{\text{Entry}} < 20$ m		All local failures		All failures	
	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage
$\theta_{\text{Inc},x} = 25$ m, $\theta_{\text{Inc},y} = 2.5$ m	99	43.8%	0	0.0%	99	43.8%	226	100%
$\theta_{\text{Inc},x} = 5$ m, $\theta_{\text{Inc},y} = 5$ m	72	40.5%	10	5.6%	82	46.1%	178	100%

Note: x_{Entry} = horizontal coordinate of the entry (or uppermost) point of a critical slip surface; x_{Exit} = horizontal coordinate of the exit (or lowermost) point of a critical slip surface

5. Conclusions

This paper investigates the distribution of critical slip surfaces in the presence of spatially autocorrelated shear strength parameters. A non-intrusive stochastic analysis method, which combines a spectral representation method and a finite element stress-based method, is adopted to evaluate the distribution of CSS for a heterogeneous slope. The mechanics for different locations

of CSS in a heterogeneous slope are analyzed and distributions of CSS for various combinations of the horizontal and vertical scales of fluctuation are compared. The following conclusions are drawn based on the analyses.

(1) A slope stability analysis ignoring the spatial variability of shear strength parameters can only produce an overall failure. However, if the spatial variability of shear strength is considered, the critical slip surface could be a local one. This phenomenon agrees with some landslide case histories, where local failures happen in statistically homogeneous slopes.

(2) The spatial variability of shear strength parameters has a significant influence on the distribution characteristic (i.e., location and scale) of critical slip surface. When no apparent weak zone exists, the failure mode of a slope is an overall failure. When a weak zone is large and located above the toe of a slope, local failure happens and the location of critical slip surface is consistent with that of the weak zone. However, if a weak zone is on a small scale, the failure mode is an overall failure with the CSS not located within the weak zone. As last, if the weak zone is located below the toe of a slope, the failure mode is a deep-seated overall failure.

(3) When the scale of fluctuation is smaller than the influence zone of a slope, the distribution range of critical slip surface increases with an increasing scale of fluctuation for shear strength parameters. However, if the scale of fluctuation exceeds the size of influence zone, the distribution range of CSS decreases with an increasing scale of fluctuation for shear strength parameters.

(4) As the ratio of the horizontal scale of fluctuation to the vertical scale of fluctuation is large, local failures have an entry point located at the top of a slope. By contrast, when the ratio of horizontal scale of fluctuation to the vertical scale of fluctuation is relatively small, a local failure may have an entry point located either at the top of a slope or on the slope surface. The failure mode corresponding local failures with entry points located on the slope surface is rarely

considered in system reliability analyses of slopes. This may lead to an overestimation of the reliability when the ratio of horizontal scale of fluctuation to vertical scale of fluctuation is relatively small.

(5) The distribution range of critical slip surface increases with an increasing coefficient of variation of shear strength parameters. Local failure does not happen if the coefficient of variation is small.

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