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Practical relay code design based on protograph codes

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Abstract: The low-density parity-check (LDPC) code design for three-terminal (namely, source, relay, and destination) relay network while considering decode-and-forward protocol is studied. Numerous works have been done on LDPC relay code design with parity bits forwarding approach, where additional parity bits are generated at the relay and forwarded to the destination. Most of the previous works assume that the forwarded parity bits are received perfectly at the destination and hence ignore the impact of relay–destination channel. This assumption is unrealistic in practical/finite-length codeword scenario. In this study, a protograph based LDPC code design is proposed while lifting the above unrealistic assumption. A Gaussian approximated density evolution is presented for the proposed scheme, which considers that the overall codeword experiences source–destination and relay–destination channels. For various designed codes with different rates, the authors show that the asymptotic thresholds of the designed codes are very close to the corresponding capacity bounds. Asymptotically, our designed code gives similar decoding threshold compared to the existing optimised irregular LDPC codes, while protograph codes are simpler to optimise and encode than the irregular LDPC codes. More importantly, proposed code performs better than the existing LDPC-based relay codes for finite-length scenarios.

1 Introduction

Three-terminal (namely, source, relay, and destination) relay network, where the relay helps the communication between the source and the destination terminals, has attracted much attention due to its ability to enhance the reliability and robustness of wireless communication. Different protocols for relay network have been previously studied, among which decode-and-forward (DF) is the most prominent one [1, 2]. With DF protocol, the relay decodes source's information and then forwards side information to the destination. The destination then attempts to decode the source's information based on the signals obtained from the source and relay terminals. Among different research problems of DF relay network, design of error correction codes has attracted a lot of attention.

Low-density parity-check (LDPC) codes are potential candidate codes for DF relay network due to their low-complexity decoding and capacity approaching properties. A lot of work have been done on the LDPC code design for relay channel, such as [3–14], which are based on generation of additional parity bits at the relay and then forward them to the destination. Based on the forwarding strategy, these works can be divided into two streams: (i) protected relaying and (ii) unprotected relaying. In the protected relaying scheme [3–12], the generated additional parity bits are protected through a separate point-to-point (P2P) error correcting code and error-free decoding of additional parity bits at the destination is assumed. Thus, the asymptotic analytical tools (such as density evolution or EXIT chart) in [3–12], which are used to design relay code, do not take into account the impact of relay–destination channel. However, this assumption is not valid for practical scenario; more precisely when the codeword length is finite. On the other hand, the unprotected relaying strategy [13, 14] lifts this assumption and does not encode the generated additional parity bits with a separate P2P error correcting code. The asymptotic analysis of this relaying strategy considers that the variable nodes experience two different channels (namely, source–destination and relay–destination channels) and one joint decoding is performed at the destination based on all received signals. Thus, the unprotected relaying strategy is more practical, since the assumption of the error-free decoding of the extra parity bits is removed. Motivated

from the above facts, in this paper, we focus on the unprotected relaying strategy presented in [13, 14].

Capacity approaching LDPC code design for the unprotected relaying strategy is first investigated in [13], where irregular LDPC codes are utilised. However, finding of those capacity approaching irregular LDPC relay codes requires extensive and complicated optimisation process. Later, [14] utilised spatially coupled LDPC (SC-LDPC) codes to design capacity approaching relay code, where capacity approaching relay codes are obtained by solving simple optimisation problem. Although, spatially coupled based relay codes exhibit superior asymptotic performance, their finite-length performance is very poor. Moreover, the relay codes of [13, 14] suffer from high encoding complexity. The aim of this work is to design relay code based on unprotected relaying strategy, which offers: (i) threshold close to the capacity, (ii) simple encoding, (iii) tractable optimisation procedure, and (iv) good finite-length performance.

This paper proposes relay code design based on protograph LDPC codes while considering unprotected relaying strategy. The rationales behind choosing protograph LDPC code are that protograph codes are easy to encode and optimisation can be performed systematically [15]. A Gaussian approximated density evolution analysis for the proposed relay coding scheme is presented. A simple design procedure is shown to obtain capacity approaching relay code through the protograph LDPC code. We show that the thresholds of our designed codes of different rates are very close to the theoretical limit. For asymptotic case, proposed code is slightly better than the existing optimised irregular LDPC code, while slightly worse than the spatially coupled based relay code. On the other hand, proposed code significantly outperforms existing irregular and spatially coupled relay codes for finite-length scenario.

The rest of the paper is organised as follows. In Section 2, we describe the relay system model, code design principle and the traditional protograph LDPC codes. In Section 3, we propose the design of the protograph relay code for unprotected relaying strategy and present a Gaussian approximated density evolution analysis. Some sample designed codes and their asymptotic and finite-length performances are presented in Section 4. In this section, we also present a comparison between our proposed relay
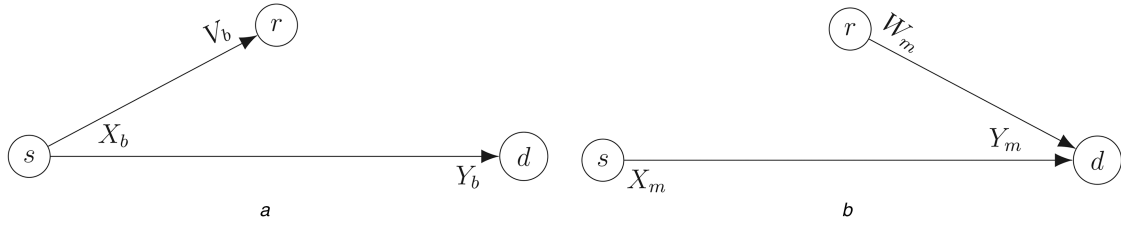
Fig. 1 *Half-duplex relay communication with two successive phases*
(a) BC phase, *(b)* MAC phase
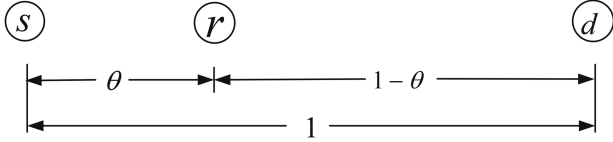
Fig. 2 *Simplified straight line representation of three-terminal relay network*

code and the existing relay codes. Finally, we conclude the paper in Section 5.

2 Backgrounds

2.1 Half-duplex relay communication

2.1.1 System model.: A three-terminal relay communication system, consists of a source s, a relay r, and a destination d, and is depicted in Fig. 1. In this paper, we consider a time-division half-duplex relay model, where communications between terminals take place in two successive phases with normalised time duration t and $1 - t$. In the first phase, known as broadcast (BC) phase, the source transmits to both relay and the destination terminals. In the second phase, known as multiple-access (MAC) phase, both source and relay simultaneously transmit to the destination terminal. In this phase, a perfect synchronisation is assumed. Both phases are depicted in the left and right parts of Fig. 1, respectively. In Fig. 1, X_p and W_p are the transmitted signals from the source and relay terminals, respectively, where subscript $p = b$ for the BC phase and $p = m$ for the MAC phase. Similarly, V_p and Y_p are the received signals at the relay and destination terminals, respectively. With these notations, the received signals at BC and MAC phases can be written as

$$V_b = h_{sr}X_b + N_{r_b},$$
$$Y_b = h_{sd}X_b + N_{d_b},$$
$$Y_m = h_{sd}X_m + h_{rd}W_m + N_{d_m},$$

where h_{ij} is the channel coefficient between terminal $i \in \{s, r\}$ and terminal $j \in \{r, d\}$; N_{d_b} and N_{d_m} are the noise realisations at the destination in BC and MAC phases, respectively; and N_{r_b} is the noise realisation at the relay in BC phase. All the noises are assumed to be zero mean and unit variance Gaussians. With a total power P_T, we consider a constraint Θ, which satisfies following power allocation relationship:

$$\Theta: tP_{s_b} + (1 - t)\left(P_{s_m} + P_{r_m}\right) \leq P_T,$$

where $P_{s_b} = E[X_b^2]$ is the transmit power from the source terminal at the BC phase; $P_{s_m} = E[X_m^2]$ and $P_{r_m} = E[W_m^2]$ are the transmit powers from the source and relay terminals, respectively, at the MAC phase.

For numerical illustration, we assume that the source, relay, and destination terminals are located on a straight line as shown in Fig. 2. We normalise the distance between the source and destination terminals to unity and denote θ as the normalised distance between source and relay terminals. With path loss exponent α, we define the channel gain g_{ij} between terminal $i \in \{s, r\}$ and terminal $j \in \{r, d\}$ as

$$g_{ij} = h_{ij}^2 = \left|\frac{1}{(\eta_{ij})^\alpha}\right|,$$

where η_{ij} is the normalised distance between terminal i and terminal j.

2.1.2 Channel coding for unprotected relaying.: In this subsection, we describe the code design principle for unprotected relaying strategy illustrated in [13, 14]. In the BC phase, the source first encodes its information bits using an error correcting code $\mathscr{C}_b$ of rate R_b to generate a codeword $\mathbf{c}_b$ of length n_b and then broadcasts to the relay and destination terminals. We assume that the relay can correctly decode the codeword. However, the destination cannot recover the codeword $\mathbf{c}_b$ due to the worst source–destination channel quality. In the MAC phase, both source and relay terminals generate a codeword segment $\mathbf{c}_m$ of length n_m and transmit simultaneously to the destination terminal. Codeword segment $\mathbf{c}_m$ is generated in a manner that $\mathbf{c}_b$ and $\mathbf{c}_m$ form an extended codeword $\mathbf{c} = [\mathbf{c}_b \, \mathbf{c}_m]$ of length $n = n_b + n_m$, and the extended codeword $\mathbf{c}$ is decodable at the destination. Let γ_b be the signal-to-noise ratio that corresponds to codeword $\mathbf{c}_b$, and γ_m be the signal-to-noise ratio that corresponds to codeword segment $\mathbf{c}_m$ at the destination. Since noises are assumed to be zero mean and unit variance Gaussians, γ_b and γ_m can be written as

$$\gamma_b = g_{sd}P_{s_b} \quad \text{and} \quad \gamma_m = \left(\sqrt{g_{sd}P_{s_m}} + \sqrt{g_{rd}P_{r_m}}\right)^2.$$

In summary, the aim of the above relay code design is to jointly design codeword $\mathbf{c}_b$ and codeword segment $\mathbf{c}_m$ such that $\mathbf{c}_b$ is decodable at the relay, while the destination can successfully decode the extended codeword $\mathbf{c}$.

2.2 Protograph LDPC codes

The concept of constructing LDPC codes by connecting small graphs, so called *protographs*, was first introduced by Thorpe in [16]. Later, design of capacity approaching protograph LDPC codes is demonstrated in [15, 17–20]. A large LDPC graph can be obtained by performing copy and permutation procedure on the protograph. Assume that we have a protograph of size $u \times v$, i.e. the protograph consists of u check nodes and v variable nodes, which can be represented as a base matrix $\mathbf{B}$ of equal size $u \times v$. The value of each entry of $\mathbf{B}$ represents the number of edges that exist between the corresponding check node and variable node. The Tanner graph of an optimised rate-$\frac{1}{2}$ protograph LDPC code is shown in Fig. 3, where the filled circle nodes are transmitted variable nodes, unfilled circle node is untransmitted/punctured variable node, and rectangular nodes are check nodes. The base matrix of the protograph LDPC code shown in Fig. 3 is given by

$$\mathbf{B}_{0.5} = \begin{pmatrix} 1 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 3 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 2 & 2 & 1 & 1 \\ 0 & 2 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}. \tag{1}$$

Asymptotically, above protograph LDPC code exhibits a threshold of 0.41 dB over binary input additive white Gaussian
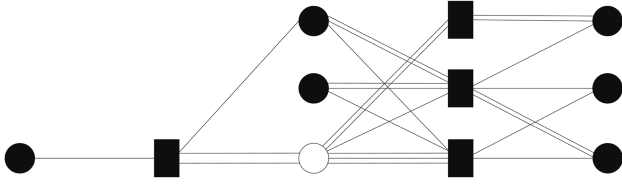
Fig. 3 *Optimised rate- 1/2 protograph LDPC code. Filled circle, unfilled circle, and rectangular nodes represent transmitted variable node, untransmitted/punctured variable node, and check node, respectively*

noise channel. To obtain a large LDPC code of size $n_c \times n_v$ from a protograph, first we have to copy the protograph M times such that $n_c = Mu$ and $n_v = Mv$, where M is called lifting factor [21]. Then connecting the edges by permutation between the same edge type of the M replicated protographs will result in the required LDPC codes.

3 Proposed relay code design

For a given relay network setting (i.e. predefined total power P_T, power allocation scenario, and relay position), we now present a relay code design with protograph LDPC code while considering unprotected relaying strategy. At first an optimised protograph LDPC code $\mathcal{C}_b$ is chosen such that the relay terminal can successfully decode source's encoded message for a given source–relay channel and power allocation scheme. The optimisation procedure for code $\mathcal{C}_b$ is same as the traditional protograph LDPC code design for P2P channel. In the BC phase, the source uses lifted version of protograph code $\mathcal{C}_b$ to encode the information bits and the resultant codeword c_b is transmitted to the relay and destination. Let B_b be the base matrix that corresponds to code $\mathcal{C}_b$. We also denote v_b, u_b, and v_p as the number of variable, check, and punctured nodes, respectively, of the code $\mathcal{C}_b$. Thus, the rate of the code becomes $R_b = (v_b - u_b)/(v_b - v_p)$, and the length of codeword c_b becomes $n_b = M(v_b - v_p)$.

In the MAC phase, an extended protograph code is formed based on the code $\mathcal{C}_b$ to generate the codeword segment c_m. We construct the extended protograph code in the following systematic manner. We start by forming a temporary protograph base matrix B_{temp} by adding a row and a column with B_b, which result in

$$B_{\text{temp}} = \begin{pmatrix} & & & 0 \\ & & & 0 \\ & B_b & & \vdots \\ & & & 0 \\ & & & 0 \\ \hline p_1 & p_2 & \cdot & \cdot & p_{v_b} & q_1 \end{pmatrix}, \quad (2)$$

Algorithm 1. Protograph-based relay code design for unprotected relaying.:

1: Choose a protograph code $\mathcal{C}_b$ with base matrix B_b that enables the relay to decode the codeword c_b for a given source–relay channel and power allocation scenario.
2: Set $B_{\text{new}} \leftarrow B_b$.
3: Add a row and column with B_{new} and form B_{temp}.
4: Perform search operation to obtain values of the added row such that resultant protograph code provides lowest threshold. Set B_o as the corresponding base matrix of the resultant protograph code.
5: If the threshold of the resultant protograph code is higher that the predefined total power, set $B_{\text{new}} \leftarrow B_o$ and go to step 3.

where $p_i \in \mathbb{N}_0$ for $i \in \{1, 2, ..., v_b\}$ and $q_1 > 0$. This addition of row and column is equivalent to the addition of a variable node and a check node, respectively, with the Tanner graph of code $\mathcal{C}_b$. We perform a search operation to find the values of p_i and $q_1 > 0$, for

which resultant protograph code gives a lowest asymptotic threshold (in terms of total power), provided that variable nodes (except the punctured nodes) of code $\mathcal{C}_b$ experience source–destination channel and newly added variable node experiences relay–destination channel. With a predefined power allocation strategy, we define the threshold of the protograph relay code as the lowest value of total power P_T, for which the belief propagation decoding of the code asymptotically converges. In the following subsection, we provide the asymptotic analysis to find the threshold of a protograph relay code, while the overall codeword experiences source–destination and relay–destination channels. To limit the search space, we restrict $p_i \in \{0, 1, 2\}$ and $q_1 \in \{1, 2\}$, which simplifies the optimisation problem greatly. Once the search is performed, if the threshold of the resultant protograph code is higher than the given total power, we add another variable and check node with the resultant protograph code and perform similar search operation. We repeat this process until the threshold of the resultant protograph code is below than the given total power. The overall code design procedure is summarised in Algorithm 1. Once the final protograph is obtained after the above design procedure, the lifted version of the added variable nodes constructs codeword segment c_m. Let v_m be the number of variable and check nodes that are added with the protograph code $\mathcal{C}_b$. Thus, the length of the codeword segment c_m becomes $n_m = Mv_m$ and the normalised time duration of BC phase becomes $t = (v_b - v_p)/(v_b + v_m - v_p)$. For a final code with effective rate R_e, we denote the resultant base matrix as B_{R_e}, where $R_e = (v_b + v_m - u_b)/(v_b + v_m - v_p)$.

Density evolution

We utilise density evolution technique for the asymptotic analysis of the proposed relay code provided that the codeword experiences source–destination and relay–destination channels. We consider the multi-edge type density evolution technique with Gaussian approximation [22, 23], where the messages passed between variable and check nodes are scalar quantity, namely mean of the log-likelihood ratio (LLR) message. For any base matrix of the proposed relay code structure, we consider the following notations in the density evolution.

- $x^{(\ell)}(i, j)$ denotes the mean of LLR message outgoing from the ith variable node to the jth check node at iteration ℓ.
- $y^{(\ell)}(i, j)$ denotes the mean of LLR message outgoing from the jth check node to the ith variable node at iteration ℓ.
- $\mathcal{C}_i$ be the set of check nodes that are connected with variable node $i \in \{1, 2, ..., v_b + v_m\}$.
- $\mathcal{V}_j$ be the set of variable nodes that are connected with check node $j \in \{1, 2, ..., u_b + v_m\}$.
- e_{ij} be the number of connection between variable node i and check node j.
- $L^{(\ell)}(i)$ be the total mean of LLR message of variable node $i \in \{1, 2, ..., v_b + v_m\}$ at iteration ℓ.

We get the following density evolution update equations for the presented protograph relay code:

- Initialisation:

  ○ For $i \in \{1, 2, ..., v_b\}$:

$$x^{(0)}(i, j) = \begin{cases} 0, & \text{if node } i \text{ is punctured} \\ 4\gamma_b, & \text{otherwise} \end{cases}$$

  ○ For $i \in \{v_b + 1, v_b + 2, ..., v_b + v_m\}$: $x^{(0)}(i, j) = 4\gamma_m$.

- Check nodes update:

$$y^{(\ell)}(i, j) = \phi^{-1}\left[1 - \frac{\prod_{k \in \mathcal{V}_j}\left(1 - \phi\left(x^{(\ell)}(i, k)\right)\right)^{e_{ik}}}{1 - \phi\left(x^{(\ell)}(i, j)\right)}\right],$$

where (see equation below)

- Variable nodes update:

$$x^{(\ell)}(i, j) = \Gamma_i + \sum_{k \in \mathscr{C}_i} e_{ik} y^{(\ell-1)}(i, k) - y^{(\ell-1)}(i, j),$$

where

$$\Gamma_i = \begin{cases} 0, & \text{if variable node } i \text{ is punctured} \\ 4\gamma_b, & \text{otherwise} \end{cases}$$

for $i \in \{1, 2, \ldots, v_b\}$ and $\Gamma_i = 4\gamma_m$ for $i \in \{v_b + 1, v_b + 1, \ldots, v_b + v_m\}$.

- $L^{(\ell)}(i)$ calculation:

$$L^{(\ell)}(i) = \Gamma_i + \sum_{k \in \mathscr{C}_i} e_{ik} y^{(\ell-1)}(i, k).$$

The asymptotic threshold P_{T}^*, in terms of total power, can be calculated by

$$P_{\mathrm{T}}^* = \min \left\{ P_{\mathrm{T}} : \lim_{\ell \to \infty} L^{(\ell)}(i) = \infty, \ \forall i \right\}.$$

4 Numerical results

4.1 Design examples and their performance

We now present few design examples of the optimised protograph relay codes, which are designed based on the relay code structure described in the earlier section. We fix the relay at $\theta = 0.25$ with path loss exponent $\alpha = 2$, and allocate the total power P such that $P_{s_b} = P_{\mathrm{T}}$, $P_{s_m} = P_{r_m} = \frac{P_{\mathrm{T}}}{2}$. Although equal power allocation is considered for simplicity, the proposed relay code design is applicable for any arbitrary power allocation scenario.

To design optimised protograph relay code, we first set the code $\mathscr{C}_b$ as the rate-$\frac{1}{2}$ protograph code $\boldsymbol{B}_{0.5}$ shown in Section 2. Following the proposed design procedure, we produce following relay codes:

$$\boldsymbol{B}_{0.428} = \begin{pmatrix} & & & & & & 0 \\ & & \boldsymbol{B}_{0.5} & & & & 0 \\ & & & & & & \vdots \\ & & & & & & 0 \\ & & & & & & 0 \\ \hline 1 & 1 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\boldsymbol{B}_{0.375} = \begin{pmatrix} & & & & & & & 0 \\ & & & \boldsymbol{B}_{0.428} & & & & 0 \\ & & & & & & & \vdots \\ & & & & & & & 0 \\ & & & & & & & 0 \\ \hline 0 & 2 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\boldsymbol{B}_{0.333} = \begin{pmatrix} & & & & & & & 0 \\ & & & \boldsymbol{B}_{0.375} & & & & 0 \\ & & & & & & & \vdots \\ & & & & & & & 0 \\ & & & & & & & 0 \\ \hline 0 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

$$\boldsymbol{B}_{0.3} = \begin{pmatrix} & & & & & & & 0 \\ & & & \boldsymbol{B}_{0.333} & & & & 0 \\ & & & & & & & \vdots \\ & & & & & & & 0 \\ & & & & & & & 0 \\ \hline 0 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

$$\boldsymbol{B}_{0.273} = \begin{pmatrix} & & & & & & & 0 \\ & & & \boldsymbol{B}_{0.3} & & & & 0 \\ & & & & & & & \vdots \\ & & & & & & & 0 \\ & & & & & & & 0 \\ \hline 0 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\boldsymbol{B}_{0.25} = \begin{pmatrix} & & & & & & & 0 \\ & & & \boldsymbol{B}_{0.273} & & & & 0 \\ & & & & & & & \vdots \\ & & & & & & & 0 \\ & & & & & & & 0 \\ \hline 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

4.2 Remarks on code design for different θ

A straightforward way to adjust with different θ is to vary the transmitted power and the power allocation scheme such that the resultant γ_b and γ_m match with γ_b and γ_m for $\theta = 0.25$. However, for fixed power allocation scheme and transmitted power, the protograph LDPC code needs to be redesigned for different θ based on the resultant γ_b and γ_m. Note that a higher θ will result in lower code rate for BC phase, however higher code rate for MAC phase. On the other hand, a lower θ will result in higher code rate for BC phase, however lower code rate for MAC phase. Thus, there exists a trade-off between θ and the code rate of the designed LDPC code. This trade-off is well characterised in [24, Chapter 9, Fig. 9.6].

For the above designed codes, first we show the asymptotic (i.e. density evolution) performance. The rates and thresholds of the above mentioned protograph codes are summarised in Table 1. Along with the total power threshold (P_{T}^*), we provide the threshold in terms of E_b/N_0 (dB), defined by $(E_b/N_0)_{\mathrm{TH}} = 10\log_{10} P_{\mathrm{T}}^*/2R_e$. For a given effective rate R_e, the corresponding capacity bound [13] is also shown in terms of E_b/N_0, dB denoted as $(E_b/N_0)_{\mathrm{CB}}$. Note that this capacity bound is calculated for the corresponding phase duration t with equal power allocation scenario. We observe that the designed codes exhibit thresholds within 0.45 dB from the respective capacity bound.

The finite-length performances of the above codes are presented in terms of bit error rate (BER) and frame error rate (FER) in Fig. 4, where solid lines represent BER while dashed lines represent FER. In the figure, E_b/N_0 is defined by $E_b/N_0 = 10\log_{10} P_{\mathrm{T}}/2R_e$. In the simulation, a lifting factor of $M = 4000$ is used for all codes, which results in $n_b = 24000$ and $n_m = 4000, 8000, 12000, 16000, 20000, 24000$ for codes $\boldsymbol{B}_{0.428}, \boldsymbol{B}_{0.375}, \boldsymbol{B}_{0.333}, \boldsymbol{B}_{0.3}, \boldsymbol{B}_{0.273}, \boldsymbol{B}_{0.25}$, respectively. The results are averaged over 10000 frame transmissions and maximum number of iteration for the belief propagation decoding is set to 250. Note that, all codes are randomly constructed and BPSK modulation is employed in the simulation. We observe that at BER $= 10^{-5}$, the finite-length performance shows a gap of less than 0.8 dB from the respective threshold and upto BER $= 10^{-5}$, no error-floors were observed.

4.3 Comparison with existing LDPC relay codes

In the following, we present performance comparison between the proposed relay code with the existing unprotected relaying code shown in [13, 14]. Similar to the setting of [13, 14], we consider

Table 1 Thresholds and capacity bounds of designed protograph codes with different rates

R_e	t	P_T^*	$(E_b/N_0)_{TH}$, dB	$(E_b/N_0)_{CB}$, dB	Gap dB
0.428	0.857	0.768	−0.474	−0.815	0.341
0.375	0.75	0.563	−1.241	−1.601	0.360
0.333	0.667	0.431	−1.896	−2.202	0.306
0.3	0.6	0.349	−2.347	−2.668	0.321
0.273	0.545	0.295	−2.673	−3.035	0.362
0.25	0.5	0.256	−2.907	−3.329	0.422
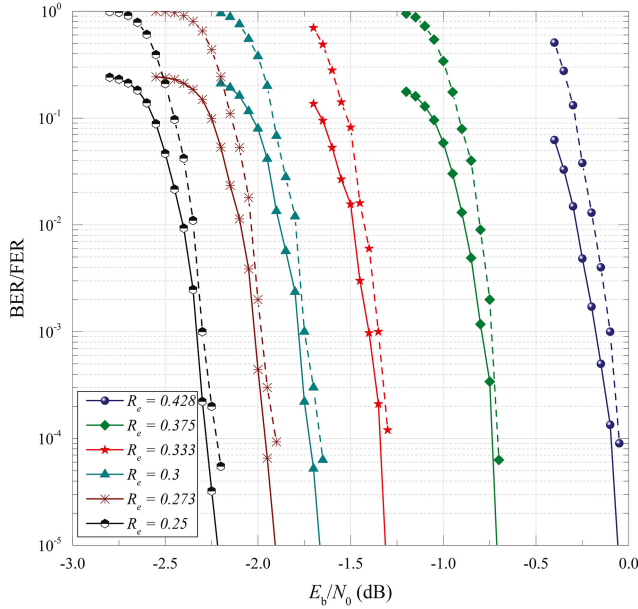
Fig. 4 *BER and FER performance of the codes shown in Table 1. Solid lines represent BER while dashed lines represent FER. All codes are simulated with $n_b = 24000$*
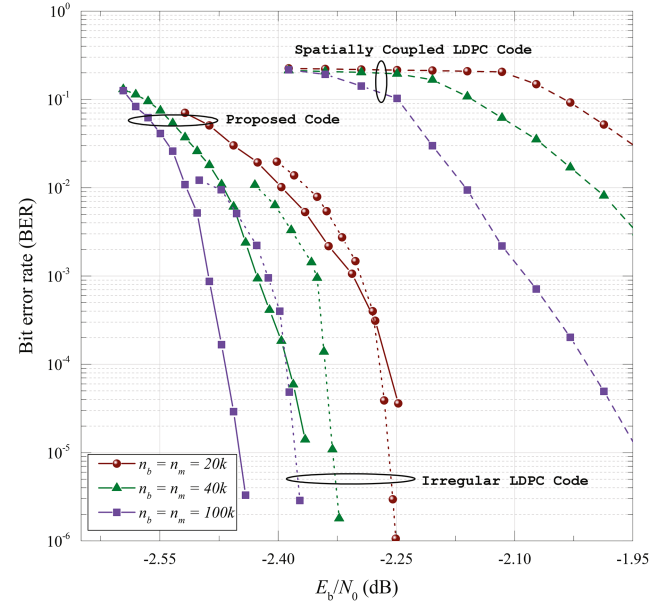
Fig. 5 *BER performance comparison between the proposed relay code and relay codes presented in [13, 14]. Solid line represents our code, dotted line represents the code of [13], and dashed line represents the code of [14]*

Table 2 Threshold comparison between the irregular LDPC, SC-LDPC, and proposed protograph relay codes

Relay codes	R_e	P_T^*	$(E_b/N_0)_{TH}$, dB	$(E_b/N_0)_{CB}$, dB	Gap, dB
optimised irregular code [13]	0.25	0.257	−2.90	−3.329	0.429
SC-LDPC code [14]	0.243	0.233	−3.19	−3.35	0.16
proposed code	0.25	0.256	−2.907	−3.329	0.422

equal power allocation and equal time sharing scenario with $\theta = 0.25$, $\alpha = 2$. We compare the codes for $R_b = 0.5$ and $R_e = 0.25$. An asymptotic performance comparison between the proposed code and relay codes of [13, 14] is summarised in Table 2. We observe that our code provides similar threshold as the optimised irregular LDPC relay code presented in [13], while gives worse threshold than the SC-LDPC relay code presented in [14]. However, as shown in [14], SC-LDPC relay code does not provide good finite-length performance for low or moderate length of codewords. In Fig. 5, we present the BER performance comparison between our protograph relay code and relay codes presented in [13, 14]. We show the results for three codeword length $n_b = n_m = \{20000, 40000, 100000\}$. Solid, dotted, and dashed lines represent the proposed, optimised irregular LDPC, and SC-LDPC relay codes, respectively. From the results, we observe that the finite-length performance of our relay code completely outperforms both optimised irregular LDPC and SC-LDPC relay codes for all cases. For $n_b = n_m = 100000$, our designed relay code gives 0.05 and 0.5 dB better performance at BER = 10^{-5} than the relay codes presented in [13, 14], respectively. The observation of

superiority of the proposed protograph relay codes aligns with the performance of protograph codes, irregular codes, and SC-LDPC codes for traditional P2P communication. The finite-length performance of irregular codes approaches the asymptotic threshold at very large codeword length (SC-LDPC codes requires much larger codeword than irregular codes), while protograph codes can approach the asymptotic threshold at relatively lower codeword length [15, 25].

5 Conclusion

In this paper, design of protograph based relay code is shown while considering realistic unprotected relaying scheme. With two different channel (source–destination and relay–destination channel) realisations at the destination, we have presented a generalised Gaussian approximated density evolution analysis for the relay coding scheme. For a wide range of code rates, we have shown that the density evolution thresholds of the designed codes approach their corresponding theoretical limits. Compared to the existing optimised relay codes, our designed protograph relay code provides a similar threshold, while protograph codes are simpler to

$$\phi(z) = \begin{cases} 1 - \dfrac{1}{\sqrt{4\pi z}} \displaystyle\int_{-\infty}^{\infty} (\tanh \tfrac{f}{2})\, e^{-(((f-z)^2)/4z)}\, df, & \text{if } z > 0 \\ 1, & \text{if } z = 0 \end{cases}$$

encode. More importantly, our designed code outperforms the existing LDPC based relay codes for finite-length scenarios. In future it will be interesting to investigate the proposed unprotected relaying strategy for fading channel scenario. Note that the threshold based design criteria presented in the paper is not applicable for fading scenario. It is also worth mentioning that the optimised protograph code, designed for AWGN channel, may not be able to retain their advantages over channels with memory. Thus, modification may require in the code design for fading channel.

6 References

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