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Abstract: In order to reduce radar interference on communication systems within shared bands, the radar transmit waveform forms frequency notches in the shared bands. However, because of the notches, the auto-correlation of the transmit waveform with frequency notches leads to high-range sidelobes. In order to achieve low-range sidelobes, the authors propose a mismatched filter. The mismatched filter for the transmit waveform with frequency notches has the same range sidelobes as the auto-correlation of the transmit waveform without frequency notches at the expense of a signal-to-noise ratio (SNR) loss. Simulation and analysis results demonstrate that the proposed mismatched filter always has low sidelobes at the cost of a SNR loss. In addition, simulation results show that the SNR loss is acceptable even for the deep notches (i.e. 20 dB), when the ratio of the waveform energy in the shared bands to the total waveform energy is moderate (i.e. <8%). However, the SNR loss increases rapidly with the depth of notches, when the ratio is large.

1 Introduction

The radio-frequency spectrum congestion problem is currently becoming a hot topic in both regulation and research due to a rapid increase in demand for a wide range of purposes, including communications, radio and television broadcasting, radio navigation, and radar [1, 2]. Many articles in the open literature have handled the problem of designing radar waveforms with frequency notches in order to reduce the interference levels on frequency overloaded systems [3–7].

The chirp-like waveform [3] can form notches in narrow bands, at the expense of little performance loss for the radar signal such as sidelobe increment. However, it tends to broaden the out-of-band spectrum because it preserves constant amplitude in the presence of in-band notches [2]. Moreover, the chirp-like waveform cannot form notches for wide bands which are used for the high data-rate communication services. In order to form wideband notches, most of existing methods are based on phase-coded waveforms [4–7]. The methods in [4–7] designed the code sequence during one radar pulse by limiting the energy of the radar waveforms in the shared bands and setting certain constraints for low correlation sidelobes. The phase-coded waveform with frequency notches generally comes with high-range sidelobes. To overcome this problem, the least-mean-square inverse filter [8] can be explored to reduce the range sidelobes. In addition, the mismatched filters based on the convex optimisation techniques in [9, 10] can be employed to reduce the range sidelobes and at the same time constrain the loss in the main peak gain.

In [11, 12], a convex optimisation method was proposed to jointly design the transmit and receive code sequences in order to maximise the radar signal-to-interference-plus-noise ratio (SINR) under the interference constraint and the similarity constraint. It can reduce the mutual interference between radar and communication. However, it involves the semidefinite programming (SDP) which has a computation complexity of $O(N^{3.5})$ where N is the number of codes and is related to the radar time-bandwidth product. Thus, it has a huge computation complexity for the high-resolution radar with a large time-bandwidth product. In practice, the propagation channel of the communication is changing. Thus, the code sequences need to be updated according to changes in the environment in order to suppress the communication interference on the radar. As a result,

the huge computational load of the method in [11, 12] may limit its usage from a practical point of view. Alternatively, the suppression of the communication interference on the radar can be achieved by an adaptive interference cancellation technique [13], which adaptively estimates the communication channel parameters in order to regenerate the communication signal for the suppression. However, this topic is not the focus of this paper. In this paper, we focus on how to reduce the radar interference on the communication services and simultaneously maintain low-range sidelobes for the radar receiver. It is worth noting that, the discrete phase-coded waveforms face the problem of high sidelobes of signal spectrum because of the phase discontinuity, which is indicated in [14]. The high sidelobes of the signal spectrum may cause inevitable interference to the communication systems of which the bands are adjacent to the radar band.

The linear frequency modulated (LFM) waveform is widely used in modern radar systems, since it can separately control pulse energy (through its duration) and range resolution (through its swept bandwidth) [14]. In order to reduce the radar interference on the shared bands, the technique for generating the LFM waveform with frequency notches was developed in [15]. However, the output of the matched filter for the waveform with frequency notches has high-range sidelobes due to the notches. The detailed sidelobe analysis of the output of the matched filter for the radar waveform with disjoint spectrum was given in [16].

In this paper, we develop a mismatched filter for the transmit waveform with frequency notches, in order to maintain low-range sidelobes. The proposed mismatched filter in the frequency domain is obtained by dividing the spectrum of the auto-correlation of the transmit waveform without frequency notches by the spectrum of the transmit waveform with frequency notches. Theoretically, the output of the mismatched filter for the transmit waveform with frequency notches is the same as the auto-correlation of the transmit waveform without frequency notches except it comes with a signal-to-noise ratio (SNR) loss. The methodology of the proposed mismatched filter is similar to the inverse filter which is widely used in the channel equalisation in communication systems [17]. However, it is important to point out that the conventional inverse filter in communication is designed to render the equalised channel frequency response flat over the signal bandwidth in order to eliminate the signal distortion caused by the channel propagation. In contrast, the product of the mismatched filter and

the transmit waveform with frequency notches in the frequency domain is equal to the Fourier transform of the auto-correlation of the transmit waveform without frequency notches. In [18], a code inverse filter was used to completely remove the range sidelobe for the binary phase-coded waveform. In [19], a least-square mismatched filter was employed to address the range straddling problem. Nevertheless, the proposed mismatched filter is different from those methods because of the different methodologies and research problems involved.

We have analysed the SNR loss due to the mismatched filter and find that the SNR loss is related to the notch depth, the ratio of the waveform energy in the shared bands to the total waveform energy. Simulations are conducted for the transmit LFM waveform with and without receiver weighting, and the non-linear frequency modulated (NLFM) waveform without receiver weighting. Simulation results show that the mismatched filter for the transmit waveform with frequency notches can approximately achieve the same sidelobes as those of the auto-correlation of the transmit waveform without frequency notches, with the trade-off of a SNR loss. Moreover, simulation results verify the analysis of the SNR loss and indicate that the SNR loss is smaller when the frequency notches are deployed near the edges of the radar band in the case that the receiver weighting or NLFM waveform is employed. In addition, simulation results show that when the ratio of the total notch bandwidth to the radar bandwidth is small ($\leq 8\%$), the SNR loss increases moderately against the notch depth increment and the mismatched filter can give an acceptable SNR loss of 3 dB at the notch depth of more than 20 dB. When the ratio is more than 16%, the SNR loss increases dramatically against the notch depth increment. Thus, depending on the practical parameters of the radar and shared bands, the SNR loss of the mismatched filter due to the same notch depth can be different. Furthermore, we compare the proposed mismatched filter with the convex optimisation method in [12] which explores the joint transmit–receive optimisation procedure [20] to jointly design the transmit phased-coded waveform and the receive filter to simultaneously fulfil the spectral compatibility requirement and maximise the SINR at the radar output. We note that the signal-dependent interference used in the convex optimisation method in [12] corresponds to the range sidelobes. Note that our work does not consider the communication and jammer interference on the radar. Thus, for a fair comparison, in the convex optimisation method, we ignore the interference caused by the communication systems and jammers on the radar and assume that the interference on radar receiver only include the white noise and the signal-dependent interference caused by the range sidelobes.

In Section 2, we introduce the background of the LFM and NLFM waveforms. In Section 3, we give a detailed analysis of the transmit waveform with frequency notches. In Section 4, we propose a mismatched filter for the transmit waveform with frequency notches to ensure low-range sidelobes. Section 5 verifies the effectiveness of the proposed mismatched filter by using numerical simulations and compares the proposed mismatched filter with the convex optimisation method. Lastly, conclusion is given in Section 6.

2 Background

We assume that the transmit waveform has a pulse duration of t_p and a bandwidth of B . Denote the transmit waveform without frequency notches as $s(t)$. We can express $s(t)$ as

$$s(t) = a(t)\exp(j\phi(t)), \quad \text{where } -\frac{t_p}{2} \leq t < \frac{t_p}{2} \quad (1)$$

where $a(t)$ and $\phi(t)$ represent the waveform amplitude and phase, respectively. Without loss of generality, we assume that the transmit signal has an energy of E , that is

$$\int_{-t_p/2}^{t_p/2} a^2(t) dt = E. \quad (2)$$

Different modulations of the amplitude and/or phase can be employed to generate different transmit waveforms such as LFM, NLFM, and hybrid waveforms. In general, in order to maintain the maximum transmit energy, the amplitude of the transmit waveform is a constant [21, 22]. For example, the transmit LFM and NLFM waveforms only modulate the phase. However, the hybrid method modulates the amplitude with a small amount of window as well as the phase in order to further reduce the sidelobes, at the expense of the reduction in the transmit energy.

In order to reduce the sidelobes, the matched filter at the receiver can be weighted/tapered. Denote the weighted matched filter in the time domain and frequency domain as $z(t)$ and $Z(f)$, respectively. $Z(f)$ is expressed as [23]

$$Z(f) = c_1 W(f) \odot S(f) \quad (3)$$

where $W(f)$ are the weights, and c_1 is a constant for normalising the weighted matched filter such that

$$\int_{-\infty}^{+\infty} |Z(f)|^2 df = 1 \quad (4)$$

The linearly weighted matched filter response is given by

$$R_{sz}(\tau) = \int_{-\infty}^{\infty} s(t) z^*(t - \tau) dt \quad (5)$$

Note that the weighted matched filter is a kind of mismatched filter. However, in order to distinguish from the proposed mismatched filter in Section 4, we name (5) as the auto-correlation of the transmit waveform when the receive weighting is applied. In this paper, we take LFM and NLFM as examples to verify the proposed mismatched filter.

2.1 LFM waveform

The LFM signal can be expressed as

$$s(t) = \sqrt{\frac{E}{t_p}} \exp(j\pi\gamma t^2), \quad \text{where } -\frac{t_p}{2} \leq t < \frac{t_p}{2} \quad (6)$$

where γ is the chirp rate and it is equal to B/t_p . Note that in (6), we consider the signal after down-conversion and thus we ignore the centre carrier frequency. From (6), we observe that the instantaneous frequency $f(t)$ of the LFM waveform varies linearly with time

$$f(t) = \gamma t. \quad (7)$$

2.2 NLFM waveform

The weighted matched filter causes a SNR loss due to the mismatching between the transmit waveform and the receive waveform with weighting. In order to avoid the SNR loss, the NLFM waveform can be employed to replace the receive weighting. The NLFM waveform achieves the low-range sidelobes by designing the chirp rate at each frequency in order to shape the signal's power spectral density (PSD). In this paper, we use the polynomial function to construct the NLFM waveform [24]. The baseband signal of an NLFM pulse can be represented as

$$s(t) = \sqrt{\frac{E}{t_p}} \exp(j\phi(t)), \quad \text{where } -\frac{t_p}{2} \leq t < \frac{t_p}{2} \quad (8)$$

where $\phi(t) = 2\pi(b_1 t + b_2 t^2 + b_3 t^3 + \dots + b_N t^N)$, b_i for $i = 1, 2, \dots, N$ are the polynomial coefficients, and N is the polynomial order. Given the desired shape of PSD such as Hamming, Hanning, and Taylor, the polynomial curve fitting is employed to find the polynomial coefficients.

3 Transmit waveform with frequency notches

Denote the radar transmit waveform with frequency notches in the time domain and frequency domain as $x(t)$ and $X(f)$, respectively. The radar transmit waveform with frequency notches can be expressed as

$$X(f) = \frac{S(f) - \sum_{i=1}^M \alpha_i S_{\text{null}}^i(f)}{N_d} \quad (9)$$

where α_i is a constant for the i th shared band, M is the number of the shared bands, and

$$S_{\text{null}}^i(f) = \begin{cases} S(f), & \text{when } f_L^i \leq f \leq f_H^i \\ 0, & \text{otherwise} \end{cases} \quad (10)$$

Note that f_L^i and f_H^i are the lowest and highest frequencies of the i th shared band, respectively. N_d is used to ensure that the transmit power after notches is unchanged and equal to E . Thus, N_d is given by

$$N_d = \text{sqr}t\left(\frac{1}{E} \int_{-\infty}^{+\infty} \left|S(f) - \sum_{i=1}^M \alpha_i S_{\text{null}}^i(f)\right|^2 df\right) \quad (11)$$

where $|\cdot|$ and $\text{sqr}t(\cdot)$ represent the absolute value and the square root of a number, respectively. According to (10) and (11), we have

$$N_d^2 = \frac{1}{E} \left(\int_{f \in B_{NN}} |S(f)|^2 df + \sum_{i=1}^M \int_{f \in B_N^i} (1 - \alpha_i)^2 |S(f)|^2 df \right) \quad (12)$$

where B_N^i is the shared band from f_L^i to f_H^i and B_{NN} is the band excluding the shared bands. Since

$$\int_{f \in B_{NN}} |S(f)|^2 df + \sum_{i=1}^M \int_{f \in B_N^i} |S(f)|^2 df = E \quad (13)$$

we have

$$N_d^2 = 1 + \sum_{i=1}^M ((1 - \alpha_i)^2 - 1) \frac{E_N^i}{E} \quad (14)$$

where

$$E_N^i = \int_{f \in B_N^i} |S(f)|^2 df. \quad (15)$$

Denote the notch depth as $NL(i)$ in dB for the i th shared band. $NL(i)$ is the difference between the magnitude in the non-notch frequency and the one in the notch frequency. According to (9) and (10), we calculate the notch depth as

$$NL(i) = \left| 20 \log_{10}^{1 - \alpha_i} \right| \text{dB}. \quad (16)$$

Define the time-domain signals of $S_{\text{null}}^i(f)$ as $s_{\text{null}}^i(t)$. As shown in (10), $S_{\text{null}}^i(f)$ is the same as $S(f)$ when $f_L^i \leq f \leq f_H^i$. According to the principle of stationary phase, when the instantaneous frequency of the waveform is a monostatic function against the time and the time-bandwidth product is moderately large (≥ 100), we obtain [22]

$$S(f) = \sqrt{\frac{2\pi}{\phi''(t)}} s(t) \exp\left(j\left(-2\pi f t + \frac{\pi}{4}\right)\right) \quad (17)$$

where $\phi''(t)$ is the second derivative of $\phi(t)$, the time t can be calculated according to

$$f = \frac{\phi'(t)}{2\pi} \quad (18)$$

where $\phi'(t)$ is the first derivative of $\phi(t)$.

According to the relationship of $s(t)$ and $S(f)$ as shown in (17), we get

$$s_{\text{null}}^i(t) \simeq \begin{cases} s(t), & \text{when } t_L^i \leq t \leq t_H^i \\ 0, & \text{otherwise} \end{cases} \quad (19)$$

where t_L^i and t_H^i are corresponding to f_L^i and f_H^i , respectively.

According to (9) and (19), we get

$$x(t) \simeq \frac{s(t) - \sum_{i=1}^M \alpha_i s_{\text{null}}^i(t)}{N_d}. \quad (20)$$

From (6), (19), and (20), we observe that the instantaneous frequency of the designed radar waveform has the approximately same relationship with respect to time as that of the transmit waveform without notches. It should be noted that because of the notches, the sidelobe of the matched filter for the transmit waveform with frequency notches is higher than the one for the transmit waveform without notches. This motivates us to develop a mismatched filter in order to reduce the sidelobes.

4 Proposed mismatched filter

Denote the mismatched filter for $x(t)$ as $y(t)$. Then, the output of the mismatched filter is equal to the cross-correlation between $x(t)$ and $y(t)$, denoted as $R_{xy}(\tau)$

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} x(t) y^*(t - \tau) dt. \quad (21)$$

Note that the cross-correlation of two signals has a Fourier transform equal to the product of the two signals' spectrums [14], that is

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} X(f) Y^*(f) \exp(j2\pi f \tau) df. \quad (22)$$

Based on (22), we have

$$R_{sz}(\tau) = \int_{-\infty}^{\infty} S(f) Z^*(f) \exp(j2\pi f \tau) df. \quad (23)$$

Based on (22) and (23), we develop the mismatched filter at the receiver in order to make $X(f)Y^*(f)$ to be proportional to $S(f)Z^*(f)$. Thus, we express the mismatched filter $Y(f)$ as

$$Y(f) = \frac{1}{N_m} \frac{S^*(f)Z(f)}{X^*(f)}. \quad (24)$$

In addition, N_m is a constant and it is used to normalise the mismatched filter to guarantee that the gain of the mismatched filter is equal to 1 and thus the noise power after this filter is unchanged. Thus, N_m is given by

$$N_m = \text{sqr}t\left(\int_{-\infty}^{+\infty} \left|\frac{S^*(f)Z(f)}{X^*(f)}\right|^2 df\right). \quad (25)$$

From (22)–(25), we observe that

$$R_{xy}(\tau) = \frac{1}{N_m} R_{sz}(\tau). \quad (26)$$

From (26), we conclude that the output of the mismatched filter for the transmit waveform with frequency notches is the same as the auto-correlation of the transmit waveform without frequency

Fig. 1 LFM waveform with frequency notches when the lowest frequency of the first notch is equal to -8 MHz

(a) Magnitude spectrum of the frequency-domain waveform, (b) Magnitude of the time-domain waveform

notches when the receive weighting is applied, except that the output of the mismatched filter is attenuated by a constant.

In this paper, we assume that the interference is white noise and does not include the communication signal which shares the radar band. Under this assumption, we take $R_{sz}(\tau)$ as a reference and define the SNR loss due to the mismatched filter as the ratio of the peak voltage of $|R_{xy}(\tau)|^2$ to the one of $|R_{sz}(\tau)|^2$ [25]. We define the SNR loss caused by the mismatched filter by Loss_{MM} . According to (26), we observe

$$\text{Loss}_{MM} = -10\log_{10}(N_m^2). \quad (27)$$

According to (9) and (24), we have

$$\begin{aligned} \frac{S^*(f)Z(f)}{X^*(f)} &= N_d[w(f) \odot S(f) \\ &+ \sum_{i=1}^M \left(\frac{1}{1-\alpha_i} - 1 \right) w(f) \odot S_{\text{null}}^i(f)] \end{aligned} \quad (28)$$

and

$$\begin{aligned} \int \left| \frac{S^*(f)Z(f)}{X^*(f)} \right|_2^2 df &= N_d^2 \left[\int_{f \in B_{NN}} |Z(f)|_2^2 df \right. \\ &+ \left. \sum_{i=1}^M \frac{1}{(1-\alpha_i)^2} \int_{f \in B_{N^i}} |Z(f)|_2^2 df \right] \end{aligned} \quad (29)$$

According to (4), we have

$$\sum_{i=1}^M \int_{f \in B_{N^i}} |Z(f)|_2^2 df + \int_{f \in B_{NN}} |Z(f)|_2^2 df = 1 \quad (30)$$

$$\int \left| \frac{S^*(f)Z(f)}{X^*(f)} \right|_2^2 df = N_d^2 \left(1 + \sum_{i=1}^M \left(\frac{1}{(1-\alpha_i)^2} - 1 \right) F_N^i \right) \quad (31)$$

where $F_N^i = \int_{f \in B_{N^i}} |Z(f)|_2^2 df$.

According to (14) and (31), we get

$$\begin{aligned} N_m^2 &= \left(1 + \sum_{i=1}^M \left((1-\alpha_i)^2 - 1 \right) \frac{E_N^i}{E} \right) \\ &\times \left(1 + \sum_{i=1}^M \left(\frac{1}{(1-\alpha_i)^2} - 1 \right) F_N^i \right). \end{aligned} \quad (32)$$

According to (27) and (32), we have

$$\begin{aligned} \text{Loss}_{MM} &= -10\log_{10} \left(1 + \sum_{i=1}^M \left((1-\alpha_i)^2 - 1 \right) \frac{E_N^i}{E} \right) \\ &-10\log_{10} \left(1 + \sum_{i=1}^M \left(\frac{1}{(1-\alpha_i)^2} - 1 \right) F_N^i \right). \end{aligned} \quad (33)$$

From (33), we observe that the SNR loss depends on the notch depth, and the ratio of the waveform energy in the shared bands to the total waveform energy. In addition, we observe that the SNR loss reduces with the reduction of F_N^i . Thus, we can expect that when the weights in the notch frequencies at the receiver are smaller, the corresponding mismatched filter has a smaller SNR loss. This observation is verified in Section 5.1.2.

It is noted that the SNR loss in (33) is caused by the proposed mismatched filter and it is referred to $|R_{sz}(\tau)|^2$. The total SNR loss of the output of the mismatched filter with receive weighting should be referred to $|R_{ss}(\tau)|^2$. Thus, the SNR loss of the output of the mismatched filter with receive weighting includes the SNR loss caused by the proposed mismatched filter and the one caused by the receive weighting, that is

$$\text{Loss} = \text{Loss}_{MM} + \text{Loss}_{RW} \quad (34)$$

where Loss_{RW} is the SNR loss caused by the receive weighting and it is defined as the ratio of the peak voltage of $|R_{sz}(\tau)|^2$ in the presence of receive weighting to the one of $|R_{ss}(\tau)|^2$ in the absence of receive weighting.

5 Simulation results for different waveforms

In this section, we show the transmit waveform with frequency notches and its mismatched filter, and provide the outputs of the mismatched filter and matched filter for the transmit waveform with frequency notches. In particular, the output of the mismatched filter is the cross-correlation between the transmit waveform with frequency notches and the mismatched filter, while the output of the matched filter is the auto-correlation of the transmit waveform with frequency notches. For the auto/cross-correlation, the mainlobe width is the first null-to-null width and the rest are the sidelobes, unless stated otherwise. For comparisons, we also provide the auto-correlation of the transmit waveform without notches. The integrated sidelobe ratio (ISLR) is defined as the ratio of the energy in the sidelobes to that in the mainlobe. The peak sidelobe ratio (PSLR) is defined as the ratio of the sidelobe peak to the mainlobe peak.

5.1 Transmit LFM waveform with notches

5.1.1 Without receive weighting: We assume that $B = 100$ MHz, $t_p = 10 \mu\text{s}$, and the sampling rate f_s is equal to $8B$. There are two shared bands and each band is 8 MHz. $f_L^1 = -8$ MHz, $f_H^1 = 0$ MHz, $f_L^2 = 8$ MHz, and $f_H^2 = 16$ MHz, $\alpha_1 = \alpha_2 = 0.5$ (i.e. $NL = 6.02$ dB) according to (16).

We design the mismatched filter when the matched filter does not have a taper, that is, $Z(f) = S(f)$. The transmit waveform with frequency notches is illustrated in Fig. 1. In addition, according to (24), the corresponding mismatched filter is shown in Fig. 2. As shown in Fig. 1, we observe that the transmit waveform with frequency notches in time domain is consistent with (20).

The outputs of the mismatched and matched filters and the auto-correlation of the LFM waveform are shown in Fig. 3. From Fig. 3, we observe that the output of the mismatched filter for the LFM waveform with frequency notches and the auto-correlation of the LFM waveform nearly overlap. For the mismatched filter, the ISLR is equal to -9.67 dB, which is consistent with the ISLR (-9.69 dB) of the auto-correlation of the LFM waveform. In Fig. 3b, the SNR loss of the mismatched filter is equal to 1.16 dB which is consistent with the value calculated by (33). In addition, its PSLR is equal to -13.42 dB, which is equal to that of the LFM waveform. Note that in Fig. 3, the small difference between the output of the mismatched filter and the auto-correlation of the LFM

Fig. 2 Mismatched filter for the LFM waveform with frequency notches when there is no receive weighting and the lowest frequency of the first notch is equal to -8 MHz

(a) Magnitude spectrum of the frequency-domain mismatched filter, (b) Magnitude of the time-domain mismatched filter

Fig. 3 Outputs of the mismatched and matched filters for the LFM waveform with frequency notches and the auto-correlation of the LFM waveform when there is no receive weighting and the lowest frequency of the first notch is equal to -8 MHz

(a) Full version, (b) Zoom-in version

Table 1 Performance of the outputs of the mismatched and matched filters for the LFM waveform with frequency notches and the auto-correlation of the LFM waveform when there is no receive weighting and the lowest frequency of the first notch is equal to -8 MHz

	SNR loss, dB	ISLR, dB	PSLR, dB	Mainlobe width, μs
mismatched filter	-1.16	-9.67	-13.42	0.0200
matched filter	0	-5.17	-9.30	0.0175
LFM waveform	0	-9.69	-13.42	0.0200

waveform is due to the fact that the transmit waveform with notches has discontinuities at the edges of the rectangular notches and (22) is implemented in the discrete-frequency domain.

The matched filter for the LFM waveform with frequency notches does not have SNR loss. However, because of two 8 MHz notches, its ISLR is equal to -5.17 dB which is 4.52 dB higher than that of the LFM waveform. In addition, its PSLR is equal to -9.30 dB , which is 4.12 dB higher than that of the LFM waveform. The mainlobe widths of the mismatched and matched filter are both equal to $0.0200\text{ }\mu\text{s}$, which is equal to $2/B$. The matched filter has a mainlobe width of $0.0175\text{ }\mu\text{s}$, which is slightly smaller than $2/B$. For clarification, we have summarised the observations from Fig. 3 in Table 1.

The cross-ambiguity function of the mismatched filter, and the ambiguity functions of the matched filter and LFM waveform are shown in Fig. 4.

From Fig. 4, we observe that for the mismatched filter, matched filter, and the LFM waveform, the SNR losses due to the Doppler effect are small. The cross-ambiguity function of the mismatched filter is similar to the ambiguity function of the LFM waveform except an additional SNR loss due to the mismatched processing. The matched filter has higher sidelobes than those of the

Fig. 4 Cross-ambiguity and ambiguity functions when there is no receive weighting and the lowest frequency of the first notch is equal to -8 MHz

(a) Cross-ambiguity function of mismatched filter, (b) Ambiguity function of LFM waveform, (c) Ambiguity function of matched filter

Table 2 Performance of the outputs of the mismatched and matched filters for the LFM waveform with frequency notches and the auto-correlation of the LFM waveform when the receive Hamming weighting is applied and the lowest frequency of the first notch is equal to -8 MHz

	SNR loss, dB	ISLR, dB	PSLR, dB	Mainlobe width, μs
mismatched filter	-3.08	-23.87	-28.82	0.0300
matched filter	-0.86	-7.19	-14.94	0.0250
LFM waveform with receive weighting	-0.80	-24.55	-28.81	0.0300

mismatched filter at around zero delay. In the following, the ambiguities of the mismatched and matched filters and the LFM waveform are similar to the ambiguities in Fig. 4. Thus, in the following, we omit the simulation results of the ambiguity functions.

5.1.2 With receive weighting: In this subsection, we design the mismatched filter when the receiver is weighted, that is, $Z(f) = W(f) \odot S(f)$. Herein, $W(f)$ is the Hamming window. Other simulation parameters are the same as those in Section 5.1.1. The performance of the mismatched and matched filters, and the auto-correlation of the LFM waveform when the receive Hamming weighting is applied is given in Table 2.

The ISLR and PSLR of mismatched filter are, respectively, equal to -23.87 and -28.82 dB , which approaches the ISLR (-24.55 dB) and PSLR (-28.81 dB) of the auto-correlation of the LFM waveform in the presence of the receive Hamming weighting, respectively. The matched filter for the LFM waveform with frequency notches has an ISLR of -7.19 dB and a PSLR of -14.94 dB which are much higher than those of the LFM waveform with receive weighting. On the other hand, because of the receive weighting, the mismatched filter has a higher SNR loss of 3.08 dB , and both the matched filter and the LFM waveform have SNR losses, which are equal to 0.86 and 0.80 dB , respectively. Note that the SNR loss of mismatched filter is 2.28 dB higher than that of the matched filter, which is consistent with the value calculated by (33). Lastly, we observe that the mainlobes are also broadened because of the receive weighting. By comparing Tables 1 and 2, we observe that the mainlobe widths for the mismatched filter and the LFM waveform in the presence of receive weighting are broadened up to 1.5 times of those in the absence of receive weighting.

Table 3 Performance of the outputs of the mismatched and matched filters for the LFM waveform with frequency notches and the auto-correlation of the LFM waveform when the receive Hamming weighting is applied and the lowest frequency of the first notch is equal to $-B/2$

	SNR loss, dB	ISLR, dB	PSLR, dB	Mainlobe width, μ s
mismatched filter	-0.95	-23.96	-28.77	0.0300
matched filter	-0.71	-16.43	-25.05	0.0350
LFM waveform with receive weighting	-0.80	-24.55	-28.81	0.0300

Fig. 5 SNR loss of the output of the mismatched filter with receive weighting against the notch depth

We set the starting frequency of the first notch in the transmit waveform at $-B/2$, i.e. -50 MHz. Thus, $f_L^1 = -50$ MHz, $f_H^1 = -42$ MHz, $f_L^2 = -34$ MHz, and $f_H^2 = -26$ MHz. The performance of the mismatched and matched filters is summarised in Table 3. By comparing Tables 2 and 3, we can see that the main difference is that the mismatched filter reduces the SNR loss to 0.95 dB because the transmit waveform deploys the notches at $-B/2$ around which the window weights are smaller. This result is consistent with the observation from (33). Therefore, in order to reduce the SNR loss due to the mismatched filter, we can manage the radar band in the way that the shared bands are located near the edges of the radar band.

In order to convey the impact of the total notch bandwidth, notch depth, and notch starting frequency on the SNR loss, we consider four scenarios for the notches: (i) *scenario 1*: one notch with a bandwidth equal to 4 MHz; (ii) *scenarios 2*: one notch with a bandwidth equal to 8 MHz; (iii) *scenarios 3*: two notches, where each notch has a bandwidth equal to 8 MHz and the separation between the two notches is 8 MHz, i.e. $f_L^2 - f_H^1 = 8$ MHz; (iv) *scenarios 4*: three notches, where the first two notches are the same as those in scenario 3 and the third notch has a bandwidth equal to 3 MHz and its separation from the second notch is 10 MHz, i.e. $f_L^3 - f_H^2 = 10$ MHz. According to (33), we can calculate the SNR loss against the notch depth and the starting frequency.

First, we fix the starting frequency of the first notch for each scenario as -50 MHz. Then, we plot the SNR loss of the output of the mismatched filter with receive weighting against the notch depth, which is shown in Fig. 5. From Fig. 5, by comparing the curves for the four scenarios, we observe that the SNR loss increases as the total notch bandwidth increases. In particular, we observe that for scenarios 1 and 2, the SNR loss increases moderately when the notch depth goes deeper, and the 3 dB SNR losses happen at the notch depths of 25 and 20.5 dB, respectively. In contrast, for scenarios 3 and 4, the SNR loss increases dramatically when the notch depth goes deeper, and the 3 dB SNR losses happen at the notch depths of 12.5 and 10.2 dB, respectively. Since the one-way radar signal propagation loss is inversely proportional to the square of the propagation range, the 6 dB notch depth means that the minimum standoff range between the radar and communication can be reduced up to two times, and 12 dB notch depth can reduce the minimum standoff range up to four

Fig. 6 SNR loss of the output of the mismatched filter with receive weighting against the starting frequency of the first notch

Table 4 Performance of the outputs of the mismatched and matched filters for the NLFM waveform with frequency notches and the auto-correlation of the NLFM waveform when there is no receive weighting and the lowest frequency of the first notch is equal to $-B/2$

	SNR loss, dB	ISLR, dB	PSLR, dB	Mainlobe width, μ s
mismatched filter	-0.62	-26.74	-41.14	0.040
matched filter	0	-18.69	-27.41	0.043
NLFM waveform	0	-30.36	-42.14	0.040

times. Thus, the shallow notch contributes to reduce the minimum standoff range.

Second, we fix the notch depth as 6.02 dB for all the notches in the four scenarios. Then, we plot the SNR loss against the starting frequency of the first notch according to (33), which is shown in Fig. 6. Note that in order to ensure that all the notch bands are within the radar band, the starting frequency is limited from $-B/2$ MHz to $((B/2) - (f_H^M - f_L^1))$ MHz, where M is the notch number and f_H^M is the ending frequency of the last notch and f_L^1 is the starting frequency of the first notch. Thus, the four scenarios have different ending frequency ranges. However, in order to accommodate the curves for the four scenarios in the same plot, we add an arbitrary large number for SNR losses at the unavailable starting frequencies from $((B/2) - (f_H^M - f_L^1))$ to $B/2$ in the four scenarios. As a result, in Fig. 6, the sudden increases are corresponding to the endings of the applicable starting frequencies for the scenarios 2–4.

In Fig. 6, we observe that the SNR loss for each scenario has a similar shape as the inverse of the receive Hamming window and is smaller near the edges of the radar band. In addition, the largest SNR losses for scenarios 1–4 are increasing with the increment of the total notch bandwidth and they are equal to 1.66, 2.31, 3.11, and 3.22 dB, respectively.

5.2 Transmit NLFM waveform with notches

We designed the NLFM waveform according to (8) to achieve a magnitude spectrum with the Hamming window shape. There is no weighting in the receiver. The other simulation parameters are the same as those used to produce Table 3. The performance of the mismatched and matched filters, and the auto-correlation of the NLFM waveform are given in Table 4. By comparing Tables 3 and 4, we observe that the ISLR and PSLR for the mismatched and matched filters are further reduced. However, the mainlobes are correspondingly broadened. In addition, because of the absence of receive weighting, the outputs of matched filter and the auto-correlation of NLFM do not have SNR loss. In addition, the output of the mismatched filter has a smaller SNR loss of 0.62 dB.

Note that because of the absence of the receive weighting, the total SNR loss of the output of the mismatched filter is the same as the one caused by the mismatched filter as shown in (33).

Fig. 7 Magnitude spectrum of the transmit waveform given by the convex optimisation method and the one used in the mismatched filter
(a) Full version, (b) Zoom-in version

Fig. 8 Magnitude spectrum of the receive filter in the convex optimisation method and the mismatched filter
(a) Full version, (b) Zoom-in version

5.3 Compare with the convex optimisation method in [12]

5.3.1 LFM waveform is used for the transmit waveform design:

In the following, we give the simulation parameters for the convex optimisation method. The convex optimisation method jointly designs the transmit waveform and the receive filter by using the convex optimisation. It has a huge computation complexity due to the SDP involved. In order to optimise a small number of the phases, in this simulation, we change $B = 0.75$ MHz, $t_p = 200$ μ s. Therefore, for the convex optimisation method, the number of codes to be optimised is equal to 150. We use the discrete phase-coded waveform generated from the LFM waveform as the reference waveform c_0 .

We note that the signal-dependent interference used in the convex optimisation method in [9] corresponds to the range sidelobes. However, our work does not consider the communication and jammer interference on the radar. Thus, for a fair comparison, in the convex optimisation method, we ignore the interference caused by the communication systems and jammers on the radar and assume that the interference on radar receiver only include the white noise and the signal-dependent interference caused by the range sidelobes. As given in [12], we also set the noise power $\sigma_l = 0$ dB and the signal-dependent interference power $\beta_k = 8$ dB.

We assume $f_L^1 = -0.23$ MHz, $f_H^1 = -0.17$ MHz, $f_L^2 = -0.11$ MHz, and $f_H^2 = -0.045$ MHz. Note that the ratio of each shared band to the radar band is the same as that in Section 5.1.1. We set the maximum allowed radar interference on the shared bands to be $E_f = 0.035$ in order to get a notch depth of 6.02 dB (which is calculated as the ratio of the energy in the shared bands to that in the bands adjacent to the shared bands). According to the interference/similarity achievable region, we set the similarity level to be $\varepsilon = 0.0535$. The iteration number is set to be 20. We assume that each subpulse/chip has a rectangular shape in the time domain as illustrated in [14]. Recall that the sample rate used in the previous sections is $8B$. Thus, to ensure a fair and accurate comparison, for the transmit and receive code sequences obtained by the convex optimisation method, we should repeat the

Fig. 9 Output of the receive filter in the convex optimisation method and the output of the mismatched filter
(a) Full version, (b) Zoom-in version

copies of each code eight times. As a result, in the following figures, the results of the convex optimisation method are obtained based on the repeated transmit and receive code sequences. Herein, we compare the convex optimisation method when the discrete LFM waveform is used as the reference waveform to the proposed mismatched filter for the LFM waveform with notches in the presence of receive weighting.

Fig. 7 shows the magnitude spectrum of the transmit waveform given by the convex optimisation method and the one used in the proposed mismatched filter. From Fig. 7, we observe that the convex optimisation method can form two notches which are similar to the ones used in the proposed mismatched filter. However, from Fig. 7, we observe that the out-of-band spectrum of the convex optimisation waveform decays much more slowly than those of the LFM-based waveform. This is because the convex optimisation method designs the discrete waveform. This phenomenon is caused by the phase discontinuity of the discrete transmit waveform, as indicated in [14]. It is worth mentioning that the high out-of-band spectrum of the convex optimisation waveform may cause serious interference to the communication systems of which the bands are adjacent to the radar band.

Fig. 8 shows the magnitude spectrum of the receive filter in the convex optimisation method and the proposed mismatched filter when the frequency-domain receive Hamming weighting is applied. Interestingly, the receive filter in the convex optimisation method has a similar shape as that of the mismatched filter. Both of them have larger amplitude in the shared bands compared to the other radar band. Therefore, it might be necessary for the receive filter to maintain larger amplitude at the shared bands in order to ensure low sidelobes at the receive filter output.

Fig. 9 shows the output of the receive filter in the convex optimisation method (i.e. the cross-correlation of the transmit waveform and the receive filter) and the output of the mismatched filter. Fig. 9 indicates that the convex optimisation method has ISL and PSL equal to -18.03 and -27.97 dB, respectively, which are about 1 dB higher than those of the mismatched filter (ISL = -19.24 dB and PSL = -28.78 dB). On the other hand, the convex optimisation method has a SNR loss equal to 0.62 which is 1.86 dB smaller than that of the mismatched filter (2.48 dB). In addition, the 3 dB mainlobe of the receive filter in the convex optimisation is 0.83 μ s, which is 1.8 times narrower than that of the mismatched filter with receive weighting (Herein, we use 3 dB mainlobe width instead of the first null-to-null mainlobe width because the first null-to-null mainlobe width of the convex optimisation method is very wide as shown in Fig. 9b). We summarise the above-mentioned observations in Table 5.

Next, we increase the notch depth to 9 dB. The performance of the proposed mismatched method and convex optimisation method is summarised in Table 6. By comparing Tables 5 and 6, we observe that the convex optimisation method can form a deeper notch with a slightly increased SNR loss, while the SNR loss of the proposed mismatched filter increases more obviously. On the other hand, the proposed mismatched filter always gives better sidelobes than the convex optimisation method.

Table 5 Performance of the output of the receive filter in the convex optimisation method when the discrete LFM waveform is used as the reference waveform and the output of the mismatched filter for the LFM waveform with notches in the presence of receive weighting, when the notch depth is equal to 6.02 dB

	SNR loss, dB	ISLR, dB	PSLR, dB	3 dB mainlobe width, μ s
mismatched filter with receive weighting	-2.48	-19.24	-28.78	1.5
convex optimisation method	-0.62	-18.03	-27.97	0.83

Table 6 Performance of the output of the receive filter in the convex optimisation method when the discrete LFM waveform is used as the reference waveform and the output of the mismatched filter for the LFM waveform with notches in the presence of receive weighting, when the notch depth is equal to 9 dB

	SNR loss, dB	ISLR, dB	PSLR, dB	3 dB mainlobe width, μ s
mismatched filter with receive weighting	-4.24	-17.24	-28.69	1.5
convex optimisation method	-0.93	-15.03	-24.44	0.83

5.3.2 NLFM waveform is used for the transmit waveform design: We change the reference waveform used in the convex optimisation method as the discrete NLFM waveform. Accordingly, we change $E_f = 0.0333$ and $\epsilon = 0.0903$ to ensure the notch depth in each shared band to be 6.02 dB. Other parameters are the same as those used to produce Table 5. We compare the convex optimisation method when the discrete NLFM waveform is used as the reference waveform to the proposed mismatched filter for the NLFM waveform with notches.

The performance of the output of the receive filter in the convex optimisation method and the output of the mismatched filter is given in Table 7. By comparing Tables 5 and 7, we observe that, when the discrete NLFM is used as the reference waveform instead of the discrete LFM, the receive filter of the convex optimisation method has a SNR increment of 0.5330 dB and an ISL increment of 1.6 dB. In contrast, it has a PSL reduction of 1.37 dB and its mainlobe width is unchanged. Overall, the convex optimisation method has better performance when the discrete LFM waveform is used as the reference waveform instead of the discrete NLFM waveform.

On the other hand, from Table 7, we observe that compared with the mismatched filter, the convex optimisation method has a smaller SNR loss and a narrower 3 dB mainlobe width, at the cost of higher ISL and PSL.

6 Conclusion

In this paper, we have analysed the transmit waveform with frequency notches and proposed a mismatched filter to reduce the correlation sidelobes of the transmit waveform with frequency notches. The output of the mismatched filter for the transmit waveform with frequency notches has similar range sidelobes as the auto-correlation of the transmit waveform without frequency notches, with a trade-off of a SNR loss. The SNR loss of the proposed mismatched filter is derived. The analytical results show that the SNR loss depends on the notch depth, and the ratio of the waveform energy in the shared bands to the total waveform energy. The simulation results are consistent with the theoretical analysis. In addition, it is illustrated that the SNR loss of the mismatched filter is reduced when the notches are deployed near the edges of

Table 7 Performance of the output of the receive filter in the convex optimisation method when the discrete NLFM waveform is used as the reference waveform and the output of the mismatched filter for the NLFM waveform with notches, when the notch depth is equal to 6.02 dB

	SNR loss, dB	ISLR, dB	PSLR, dB	3 dB mainlobe width, μ s
mismatched filter with receive weighting	-1.442	-18.93	-33.88	1.95
convex optimisation method	-1.153	-16.43	-29.34	0.83

the radar band, in the case that the receive weighting or NLFM waveform is employed. Therefore, in practice, we can manage the radar band such that the shared bands are allocated at the edges of the radar band in order to reduce the SNR loss caused by the proposed mismatched filter.

Furthermore, simulation and analytical results illustrate that the proposed mismatched filter has an acceptable SNR loss for the deep notch depth when the ratio of the total notch bandwidth to the radar bandwidth is small (no more than 8%). However, its SNR loss increases rapidly with the increment of the notch depth, when the ratio is large. Nevertheless, in this case, it still can give an acceptable SNR loss for a shallow notch depth such as 10 dB. It is worth noting that the 6 dB notch depth at the shared bands implies that the minimum standoff range between the radar and communication using the shared bands [26, 27] can be reduced by half. Thus, even the shallow notch depth contributes to reduce the minimum standoff range when the radar interference on communication is considered.

We have compared the mismatched filter with the convex optimisation waveform. Interestingly, similar to the proposed mismatched filter, we observe that the receive filter of the convex optimisation method has larger spectral magnitudes on the notch bands than those on the other bands. In addition, the convex optimisation method can provide deeper notches at the cost of a slightly increased SNR loss and moderately increased range sidelobes, compared with the mismatched filter. However, the convex optimisation method faces the challenge of high computational complexity, which may limit its usage from a practical point of view. In addition, the out-of-band spectrum of its transmit code sequence decays much slowly, which interferes the communication systems of which the bands are adjacent to the radar band.

For the spectrum sharing between the radar and communication, the communication interference on the radar is unavoidable and thus research efforts are certainly required to cancel it in order to ensure good radar performance. Note that the communication interference on the radar is changing with time. This challenge can be solved by using the adaptive interference cancellation technique to remove the communication interference on the radar.

7 References

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