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Multiple input multiple output radar imaging based on multidimensional linear equations and sparse signal recovery

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Abstract: Multiple input multiple output (MIMO) radar forms large virtual aperture and improves the cross-range resolution of radar imaging. Sparse signal recovery algorithms can be used to improve image quality of target with sparse property in spatial domain. Conventional sparse signal recovery-based MIMO radar imaging method rearranges the received two-dimensional (2D) or 3D signals into a vector, then linear equations describing the relation between the received signal and the reflectivity of the scatterers are solved. However, this method occupies huge memory spaces and increases the computational load. In this study, by introducing synthetic codes, multidimensional linear equations of MIMO radar imaging are derived, which occupy less memory spaces and cost less computationally. A L_1 L_0 norms homotopy sparse signal recovery algorithm for multidimensional linear equations is used to recover the image. Simulation results verify the high efficiency of using multidimensional linear equations.

1 Introduction

Inverse synthetic-aperture radar (ISAR) imaging has been studied for almost four decades [1–3]. The range resolution of ISAR is obtained by transmitting wideband signals, while the cross-range resolution is obtained by the relative rotation between the target and the radar. However, the ISAR image is a two-dimensional (2D) projection of the target onto the range-Doppler plane. This is inadequate for target identification. So, 3D imaging methods using interferometric and antenna array techniques are proposed [4–8].

Multiple input multiple output (MIMO) radar has received much attention recently. Collocated MIMO radar transmits multiple independent signals and receives the scattered signals through multiple antennas, forms large virtual aperture and improves the spatial resolution [9–14]. By using the collocated MIMO radar, 2D and 3D images of the target can be obtained by processing one snapshot signal and thus motion compensation is avoided [15–19]. Some papers assumed that the transmitting codes are fully orthogonal [15, 16, 19, 20], although codes with orthogonal property for all delays do not exist. Some codes used in the literature are bipolar phase shift keying (BPSK) signals, stepped frequency signals [21], orthogonal frequency division multiplexing (OFDM) chirp, [22–24] etc. One drawback of these signals is the resultant high sidelobes. If conventional correlation method is used to process the data, then poor images will be produced [19]. In [24], beamforming is simultaneously used to mitigate range sidelobes.

Fortunately, a target, especially a man-made target, usually has only a few strong scatterers (sparse). Sparse signal recovery algorithms can thus be used to improve image quality. This has been shown in the 2D ISAR imaging [25, 26], SAR imaging [27] and MIMO radar imaging [20, 28].

A linear system usually can be expressed as

$$s = \Psi\alpha + n, \quad (1)$$

where s is the output vector, Ψ is the coefficient matrix, α is the input vector, n is the observation noise or/and modelling error. In many physical systems, such as ISAR, α is sparse. The sparse signal recovery method or compressive sensing (CS) utilises the a prior information of sparse property of the signal [29, 30].

Most of sparse signal recovery algorithms are based on the above linear equations expression. However, the data of many applications, such as MIMO radar imaging, are 2D or 3D. Assume that the received signal from a point scatterer can be expressed as a $u \otimes v^T$ form, where u and v are column vectors, and $\otimes$ expresses the outer product. In order to use linear equations expression, the multidimensional atom ($u \otimes v^T$) is rewritten as a 1D atom ($\text{vec}(u \otimes v^T)$), where vec expresses vectorisation operator), then the basis matrix Ψ can be formed [28, 31]. For MIMO radar imaging, the expressions of u and v can be obtained according to equation (4). The drawback of this method is that the dimensions of the coefficient matrix Ψ are huge, which occupies huge memory spaces and decreases computation efficiency.

For MIMO radar direction of arrival (DOA) estimation, under the assumption of transmitting orthogonal codes, after signal separation, the received signal can be expressed as a tensor [32]. However in radar imaging, there are multiple range units. Because the codes are not truly orthogonal, orthogonal assumption will introduce modelling error and degrade imaging performance. In this paper, we do not assume that the codes are orthogonal. On the other hand, we observe that the synthetic transmitting codes are spatial periodic after phase compensation, and the multidimensional atoms are variable separable in some imaging areas. The received signals can then be rearranged into a segment variables separable form. Finally, a $S = A\Gamma C$ form expression is obtained, where S is the receive signal, A and C are coefficient matrices (A is the receive array steering vector matrix and C is the code matrix), Γ is the unknown variable matrix (scatterers reflection coefficients matrix). The dimensions of A and C are much less than the dimensions of Ψ , where Ψ is formed by rewritten multidimensional atoms as 1D atoms. Then 2D CS methods can be directly used to recover the image. The memory and computation complexity are greatly reduced.

This paper is organised as follows. Multidimensional linear equations-based MIMO radar signal model is derived in Section 2. Section 3 discusses the L_1 L_0 norms homotopy algorithm. Comparison of memory and computational complexity is given in Section 4. Simulation results are presented in Section 5. Finally, Section 6 provides the conclusion.

2 MIMO radar multidimensional linear equations signal model

Assume that there are M transmit antennas and N receive antennas. Let P_m and Q_n denote the positions of the m th ($0 \leq m < M$) transmit antenna and the n th ($0 \leq n < N$) receive antenna, respectively. Let $\tilde{c}_m$ denote the transmit code of the m th transmit antenna. The code can be phase-modulated signal [33]. The baseband transmit waveform can be expressed as

$$\varphi_m(t) = \sum_l \tilde{c}_m(l)u(t - lT_0), \quad (2)$$

where $u(t) = \begin{cases} 1 & \text{if } -T_0/2 < t < T_0/2 \\ 0 & \text{others} \end{cases}$, T_0 is the subpulse duration, and t is the fast time.

In order to decrease the length of the receive signal, the envelopes of the transmit signal and the receive signal should be adjusted. Denote O the reference point near the target, $|P_0O|$ and $|P_mO|$ are the distances between P_0 and O and between P_m and O , $T_m = ((|P_0O| - |P_mO|)/c)$ is the envelope adjustment time. Then the transmit signal from the m th transmit antenna can be expressed as $\varphi_m(t - T_m)e^{j2\pi ft}$, where f is the carrier frequency. Define $T_n = ((|Q_0O| - |Q_nO|)/c)$ as the envelope shift of the n th received signal. After demodulation and envelope shifting, the received back-scattered signal at receive antenna n from scatterer A can be expressed as

$$\begin{aligned} \tilde{s}_n(t) = \alpha_A \sum_{m=0}^{M-1} \varphi_m(t - T_m - T_n - \tau_{P_mA} - \tau_{Q_nA}) \\ \times \exp(-j2\pi f(\tau_{P_mA} + \tau_{Q_nA})), \end{aligned} \quad (3)$$

where α_A is the reflection coefficient, $\tau_{P_mA} = |P_mA|/c$ and $\tau_{Q_nA} = |Q_nA|/c$ are the delays from P_m to A and from Q_n to A , and c is the speed of light. In the far field, $T_m + T_n + \tau_{P_mA} + \tau_{Q_nA}$ can be approximated as $\tau_A = \tau_{P_0A} + \tau_{Q_0A}$.

Denote

$$\begin{aligned} \tilde{\mathbf{a}} = [e^{-j2\pi f\tau_{Q_0A}}, e^{-j2\pi f\tau_{Q_1A}}, \dots, e^{-j2\pi f\tau_{Q_{N-1}A}}]^T, \\ \tilde{\mathbf{b}} = [e^{-j2\pi f\tau_{P_0A}}, e^{-j2\pi f\tau_{P_1A}}, \dots, e^{-j2\pi f\tau_{P_{M-1}A}}]^T \end{aligned}$$

and

$$\tilde{\mathbf{s}}(t) = [\tilde{s}_0(t), \tilde{s}_1(t), \dots, \tilde{s}_{N-1}(t)]^T,$$

$\tilde{\mathbf{s}}(t)$ can be expressed as

$$\tilde{\mathbf{s}}(t) = \alpha_A \tilde{\mathbf{a}} \tilde{\mathbf{b}}^T \Phi(t - \tau_A), \quad (4)$$

where

$$\Phi(t) = [\varphi_0(t), \dots, \varphi_{M-1}(t)]^T.$$

After discrete sampling in the time domain, $\tilde{\mathbf{s}}(t)$ becomes a 2D matrix $\mathbf{S}$. Denote $\mathbf{s} = \text{vec}(\mathbf{S})$ and a 1D atom vector as $\boldsymbol{\psi}_A = \text{vec}(\tilde{\mathbf{a}} \tilde{\mathbf{b}}^T \Phi(t - \tau_A))$. The imaging areas are divided as grids. Basis matrix $\boldsymbol{\Psi} = [\boldsymbol{\psi}_A, \dots]$ and scatterer amplitudes vector $\boldsymbol{\alpha} = [\alpha_A, \dots]^T$ are then formed for all grids. Then the received discrete signal $\mathbf{s}$ can be expressed as $\mathbf{s} = \boldsymbol{\Psi} \boldsymbol{\alpha} + \mathbf{n}$ and imaging performed accordingly.

However, the dimension of the coefficient matrix $\boldsymbol{\Psi}$ is $N\mathbb{L} \times \mathbb{K}$, where $\mathbb{L}$ is the signal length of discrete-time domain $\tilde{s}_n(t)$, $\mathbb{K}$ is the number of grids of the imaging area. For a 3D imaging volume, the dimension of $\boldsymbol{\Psi}$ is huge. For example, for a standard 3D imaging

situation, we assume that there are $N = 20$ receive antennas, $\mathbb{L} = 250$, the number of grids in each dimension is 100, then the number of the elements of $\boldsymbol{\Psi}$ can be as large as 1.25×10^{10} . This takes up a large memory space and demands huge computing time. Furthermore, the expression of $\boldsymbol{\Psi}$ is dependent on the coordinates of the imaging region, and must be computed again when imaging new regions. This is undesirable.

We propose a multidimensional linear equations approach, which results in small coefficient matrices and at the same time the coefficient matrices are imaging regions independent. Denote

$$\tilde{\mathbf{a}}_0 = [e^{-j2\pi f\tau_{Q_0O}}, e^{-j2\pi f\tau_{Q_1O}}, \dots, e^{-j2\pi f\tau_{Q_{N-1}O}}]^T$$

and

$$\tilde{\mathbf{b}}_0 = [e^{-j2\pi f\tau_{P_0O}}, e^{-j2\pi f\tau_{P_1O}}, \dots, e^{-j2\pi f\tau_{P_{M-1}O}}]^T$$

as the reference vectors. Different from [28], in the new method, the signal is phase compensated first using $\tilde{\mathbf{a}}_0$. After phase compensation, the received signal can be expressed as

$$\mathbf{s}(t) =: \tilde{\mathbf{s}}(t) \odot \tilde{\mathbf{a}}_0^* = \alpha_A (\tilde{\mathbf{a}} \odot \tilde{\mathbf{a}}_0^*) (\tilde{\mathbf{b}} \odot \tilde{\mathbf{b}}_0^*)^T (\tilde{\mathbf{b}}_0 \odot \Phi(t - \tau_A)), \quad (5)$$

where $\odot$ denotes element-wise multiplication. The phase of the (n, m) th element of $(\tilde{\mathbf{a}} \odot \tilde{\mathbf{a}}_0^*) (\tilde{\mathbf{b}} \odot \tilde{\mathbf{b}}_0^*)^T$ is

$$2\pi f(\tau_{Q_nO} - \tau_{Q_0O} + \tau_{P_mO} - \tau_{P_0O}).$$

In far field, because [19]

$$\begin{aligned} |P_mO| + |Q_nO| - |P_mA| - |Q_nA| \\ \approx -2(|P_0A| - |P_0O|) \\ + (\overrightarrow{OA} - (\overrightarrow{OA}^T \mathbf{n}_0) \mathbf{n}_0)^T (\overrightarrow{P_0Q_0} + \overrightarrow{P_0P_m} + \overrightarrow{Q_0Q_n})/r, \end{aligned} \quad (6)$$

where $\mathbf{n}_0$ is the target line-of-sight unit vector, r is the distance from the radar to the target, the arrow $\rightarrow$ at the hat of OA expresses the vector from O to A . The geometric meaning using vector is clearer than using coordinates. $2\pi f(\tau_{Q_nO} - \tau_{Q_0O} + \tau_{P_mO} - \tau_{P_0O})$ can be expressed as

$$\varphi_0 + 2\pi (\overrightarrow{O'A}^T (\overrightarrow{P_0P_m} + \overrightarrow{Q_0Q_n})/\lambda r),$$

where

$$\overrightarrow{O'A} = \overrightarrow{OA} - (\overrightarrow{OA}^T \mathbf{n}_0) \mathbf{n}_0, \quad (7)$$

$\varphi_0 = -4\pi(|P_0A| - |P_0O|)/\lambda + 2\pi \overrightarrow{O'A}^T \overrightarrow{P_0Q_0}/\lambda r$ is a constant phase term and can be omitted.

Denote

$$\overrightarrow{P'_0P'_m} = \overrightarrow{P_0P_m} - (\overrightarrow{P_0P_m}^T \mathbf{n}_0) \mathbf{n}_0, \quad (8)$$

and

$$\overrightarrow{Q'_0Q'_n} = \overrightarrow{Q_0Q_n} - (\overrightarrow{Q_0Q_n}^T \mathbf{n}_0) \mathbf{n}_0, \quad (9)$$

where $\overrightarrow{P'_0P'_m}$ and $\overrightarrow{Q'_0Q'_n}$, the projection of $\overrightarrow{P_0P_m}$ and $\overrightarrow{Q_0Q_n}$ on the plane perpendicular to the line-of-sight of the radar, can be explained as equivalent arrays. Because $\overrightarrow{O'A}^T \perp \mathbf{n}_0$, we have

$$\overrightarrow{O'A}^T \overrightarrow{P_0P_m} = \overrightarrow{O'A}^T \overrightarrow{P'_0P'_m} \quad (10)$$

and

Fig. 1 Projection relationship of antennas and scatterers

$$\overrightarrow{O'A} \overrightarrow{Q_0 Q_n} = \overrightarrow{O'A} \overrightarrow{Q_0' Q_n'} \quad (11)$$

If the transmit codes are phase compensated before transmitting, that is the transmitting codes are $\tilde{\mathbf{b}}_0^* \odot \Phi(t)$, then after denoting

$$a_n = e^{j2\pi \overrightarrow{O'A} \overrightarrow{Q_0' Q_n'} / \lambda r} \quad (12)$$

and

$$b_m = e^{j2\pi \overrightarrow{O'A} \overrightarrow{P_0' P_m'} / \lambda r}, \quad (13)$$

$s(t)$ can be expressed as

$$s(t) = \alpha_A \mathbf{a}^T \Phi(t - \tau_A). \quad (14)$$

For a 1D antenna array, the geometric relationship discussed above is shown in Fig. 1. $\overrightarrow{O'A} \overrightarrow{P_0' P_m'}$ and $\overrightarrow{O'A} \overrightarrow{Q_0' Q_n'}$ are the projections of $\overrightarrow{O'A}$ on the new equivalent arrays, which are perpendicular to the target line-of-sight. Denote matrix $\mathbf{S}$ as the discrete-time expression of $s(t)$. The details on how $\mathbf{S}$ can be expressed as the AFG form for two specific MIMO radar configurations are discussed as follows.

2.1 2D imaging using 1D MIMO radar

The geometry of linear array MIMO radar and the division of the imaging area are shown in Fig. 2, where M equivalent transmit antennas $\{P_m\}$ and N equivalent receive antennas $\{Q_n\}$ are evenly deployed on the X -axis (cross-range). Y is the range axis. The inter-element distances between the equivalent transmit antennas and between the equivalent receive antennas are d_t and d_r , respectively. Denote x as the coordinate of $\overrightarrow{O'A}$ in the X -axis. Then we have

$$a_n(x) = e^{j(2\pi x n d_r / \lambda r)}, \quad (15)$$

and

$$b_m(x) = e^{j(2\pi x m d_t / \lambda r)}. \quad (16)$$

According to (14) the receive signal at the n th receive antenna can be expressed as

$$s_n(t) = \alpha_A e^{j2\pi x n d_r / (\lambda r)} \times \sum_m e^{j2\pi x m d_t / (\lambda r)} \varphi_m(t - \tau_A). \quad (17)$$

Let the inter-element distance of the transmit antennas be larger than the inter-element distance of the receive antennas and satisfy $d_t = N d_r$, then the virtual array forms a full array. Note that the

Fig. 2 Imaging field division for 1D MIMO array radar

unambiguous distances in the X -axis are $W = (\lambda/d_r)r$ [8]. Then $e^{j2\pi x m d_t / (\lambda r)} = e^{j2\pi x m N / W}$. A key observation is that $e^{j2\pi x m N / W}$ is a periodic function of x with period of W/N , by defining

$$c(x, t) = \sum_{m=0}^{M-1} e^{j2\pi x m N / W} \varphi_m(t), \quad (18)$$

then $c(x, t)$ is also a periodic function of x . Function $c(x, t)$ can be regarded as a synthetic code at x .

In Fig. 2, as an example, there are three receive antennas, the unambiguous window $[0, W]$ is divided into $N = 3$ parts (because $c(x, t)$ is a periodic function with period of W/N). In each part $(iW/N, (i+1)W/N)$, we assume that it is divided as M_1 (as an example, $M_1 = 4$ in Fig. 2) strips, where M_1 is larger or equal to M (number of transmit antennas). Then there are NM_1 strips in the imaging region. The value of M_1 is decided by the cross-range resolution of the image expected by the designer. So there are M_1 independent synthetic codes. Let all $\{x\}$, having the same synthetic code, belong to one set. Then the grids with the same pattern in Fig. 2 belong to one set. Denote Ω_{m_1} as the index set defined as $\Omega_{m_1} = \{(nM_1 + m_1): n = 0, 1, \dots, N - 1, m_1 \in [0, M_1 - 1]\}$. Denote Δ as the inter-grid distance in the X -axis: $\Delta = W/(NM_1)$. For simplicity, we also use Ω_{m_1} to express the region with cross-range coordinates of $\{p\Delta\}$ $p \in \Omega_{m_1}$.

For any x belongs to Ω_{m_1} , $x = (m_1 + pM_1)\Delta$, where p is an integer. Because $e^{j2\pi x m N / W} = e^{j2\pi (m_1 + pM_1)\Delta m N / (NM_1\Delta)} = e^{j2\pi m_1 m / M_1}$, the synthetic code corresponding to Ω_{m_1} is

$$c_{m_1}(t) = \sum_m e^{j2\pi m_1 m / M_1} \varphi_m(t). \quad (19)$$

From now, the imaging region has been divided as M_1 sets $\{\Omega_{m_1}, m_1 = 0, 1, \dots, M_1 - 1\}$. The signal received from a scatterer located at $((m_1 + pM_1)\Delta, c\tau/2)$ can be expressed as

$$s(t) = \alpha_A \mathbf{a}(m_1 + pM_1) c_{m_1}(t - \tau), \quad (20)$$

where the n th element of $\mathbf{a}(m_1 + pM_1)$ is $e^{j2\pi n(m_1 + pM_1)/(NM_1)}$.

The characteristic of the above expression is that $\mathbf{a}(m_1 + pM_1)$ is only related to the cross-range index $m_1 + pM_1$ and has no relation with delay τ . At the same time, $c_{m_1}(t - \tau)$ in set Ω_{m_1} is only related to τ and has no relation with the cross-range index. This means that the expression of (20) is down-range cross-range separable.

Denote

$$\mathbf{A}_{m_1} = [\mathbf{a}(m_1), \mathbf{a}(m_1 + M_1), \dots, \mathbf{a}(m_1 + (N - 1)M_1)], \quad (21)$$

$c_{m_1}(k)$ the discrete sampling version of $c_{m_1}(t)$. Then the code matrix can be expressed as (22), where K is the length of code $\mathbf{c}_{m_1}$.

$$\mathbf{C}_{m_1} =$$

$$\begin{bmatrix} c_{m_1}(1), & c_{m_1}(2), & \dots & c_{m_1}(K), & 0, & 0, & 0 \\ 0, & c_{m_1}(1), & c_{m_1}(2), & \dots, & c_{m_1}(K), & 0, & 0 \\ \vdots, & \vdots, & \vdots, & \vdots, & \vdots, & \vdots, & \vdots \\ 0, & \dots, & 0, & c_{m_1}(1), & c_{m_1}(2), & \dots, & c_{m_1}(K) \end{bmatrix} \quad (22)$$

The length of the rows of $\mathbf{C}_{m_1}$ is the number of range units of the region Ω_{m_1} . Then the 2D discrete signal received from region Ω_{m_1} can be expressed as

$$\mathbf{S}_{m_1} = \mathbf{A}_{m_1} \mathbf{\Gamma}_{m_1} \mathbf{C}_{m_1}, \quad (23)$$

where $\mathbf{\Gamma}_{m_1}$ is the scatterer reflection matrix on region Ω_{m_1} . Denote T the number of range units the target occupied, the dimension of $\mathbf{A}_{m_1}$, $\mathbf{\Gamma}_{m_1}$ and $\mathbf{C}_{m_1}$ are $N \times N$, $N \times T$ and $T \times (K + T)$, respectively. Then the signal received from the entire imaging region can be expressed as

$$\mathbf{S} = \sum_{m_1=0}^{M_1-1} \mathbf{A}_{m_1} \mathbf{\Gamma}_{m_1} \mathbf{C}_{m_1}. \quad (24)$$

Denote $\mathbf{\Psi} = [\mathbf{C}_0^T \otimes \mathbf{A}_0, \dots, \mathbf{C}_{M_1-1}^T \otimes \mathbf{A}_{M_1-1}]$, $\mathbf{s} = \text{vec}(\mathbf{S})$ and $\mathbf{a} = [\text{vec}(\mathbf{\Gamma}_0)^T, \dots, \text{vec}(\mathbf{\Gamma}_{M_1-1})^T]^T$, then (24) can be expressed as a linear equations with the form as expressed in (1).

By denoting $\mathbf{A} = [\mathbf{A}_0, \mathbf{A}_1, \dots, \mathbf{A}_{M_1-1}]$,

$\mathbf{\Gamma} = \text{diag}(\mathbf{\Gamma}_0, \dots, \mathbf{\Gamma}_{M_1-1})$ and $\mathbf{C} = [\mathbf{C}_0^T, \mathbf{C}_1^T, \dots, \mathbf{C}_{M_1-1}^T]^T$, then $\mathbf{S}$ can be expressed as

$$\mathbf{S} = \mathbf{A} \mathbf{\Gamma} \mathbf{C}, \quad (25)$$

which is 2D linear equations (tensor equations).

In the above expression, the matrix $\mathbf{A}$ is not the conventional Fourier matrix. However, by adjusting the orders of the columns of $\mathbf{A}$, $\mathbf{A}$ can become a conventional Fourier matrix. Then fast Fourier transform (FFT) can be used in computing $\mathbf{\Gamma}$ when the number of columns of $\mathbf{A}$ is a power of 2 of an integer. Following is one method on how to adjust the order of $\mathbf{\Gamma}$ and $\mathbf{C}$ such that the coefficient matrix $\mathbf{A}$ is a Fourier matrix (with $M_1 = 4$ as an example).

Denote $\tilde{\mathbf{A}}_1 = [\mathbf{a}(0), \mathbf{a}(1), \dots, \mathbf{a}(NM_1 - 1)]$ the conventional Fourier matrix, where

$$\mathbf{a}(m) = [1, e^{j2\pi m/(NM_1)}, \dots, e^{j2\pi m(N-1)/(NM_1)}]^T.$$

The natural order reflection coefficient matrix is

$$\begin{bmatrix} \alpha_{0,1} & \alpha_{0,2} & \dots \\ \alpha_{1,1} & \alpha_{1,2} & \dots \\ \alpha_{2,1} & \alpha_{2,2} & \dots \\ \alpha_{3,1} & \alpha_{3,2} & \dots \\ \alpha_{4,1} & \alpha_{4,2} & \dots \\ \alpha_{5,1} & \alpha_{5,2} & \dots \\ \alpha_{6,1} & \alpha_{6,2} & \dots \\ \alpha_{7,1} & \alpha_{7,2} & \dots \\ \vdots & \vdots & \vdots \end{bmatrix}, \quad (26)$$

where rows express the cross-range (X -direction), columns express the down-range (Y -direction). According to the parameters of the MIMO radar shown in Fig. 2, a new matrix $\tilde{\mathbf{\Gamma}}$ is defined as (where $M_1 = 4$ is assumed)

$$\tilde{\mathbf{\Gamma}}_1 = \begin{bmatrix} \alpha_{0,1} & 0 & 0 & 0 & \alpha_{0,2} & 0 & 0 & 0 & \cdot \\ 0 & \alpha_{1,1} & 0 & 0 & 0 & \alpha_{1,2} & 0 & 0 & \cdot \\ 0 & 0 & \alpha_{2,1} & 0 & 0 & 0 & \alpha_{2,2} & 0 & \cdot \\ 0 & 0 & 0 & \alpha_{3,1} & 0 & 0 & 0 & \alpha_{3,2} & \cdot \\ \alpha_{4,1} & 0 & 0 & 0 & \alpha_{4,2} & 0 & 0 & 0 & \cdot \\ 0 & \alpha_{5,1} & 0 & 0 & 0 & \alpha_{5,2} & 0 & 0 & \cdot \\ 0 & 0 & \alpha_{6,1} & 0 & 0 & 0 & \alpha_{6,2} & 0 & \cdot \\ 0 & 0 & 0 & \alpha_{7,1} & 0 & 0 & 0 & \alpha_{7,2} & \cdot \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdot \end{bmatrix}, \quad (27)$$

and a new code matrix is defined as

$$\tilde{\mathbf{C}}_1 = \begin{bmatrix} c_0(1) & c_0(2) & \dots & c_0(K) & 0 & 0 & 0 \\ c_1(1) & c_1(2) & \dots & c_1(K) & 0 & 0 & 0 \\ c_2(1) & c_2(2) & \dots & c_2(K) & 0 & 0 & 0 \\ c_3(1) & c_3(2) & \dots & c_3(K) & 0 & 0 & 0 \\ 0 & c_0(1) & c_0(2) & \dots & c_0(K) & 0 & 0 \\ 0 & c_1(1) & c_1(2) & \dots & c_1(K) & 0 & 0 \\ 0 & c_2(1) & c_2(2) & \dots & c_2(K) & 0 & 0 \\ 0 & c_3(1) & c_3(2) & \dots & c_3(K) & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix}. \quad (28)$$

Then we have

$$\mathbf{S} = \tilde{\mathbf{A}}_1 \tilde{\mathbf{\Gamma}}_1 \tilde{\mathbf{C}}_1. \quad (29)$$

Equations (25) and (29) are both multidimensional linear equations. The difference is that $\tilde{\mathbf{A}}_1$ in (29) is a normal Fourier transform matrix and fast FFT can be used to save computation when NM_1 is a power of 2 of an integer.

2.2 3D imaging signal model using cross-array MIMO radar

The geometry of a cross-array MIMO radar is as shown in Fig. 3, where the equivalent transmit array and the equivalent receive array are deployed evenly on the Y - and X -axes, respectively. While the original transmit and receive arrays are perpendicular to each other, because the equivalent transmit array and the equivalent receive array are projections of the original arrays on a plane perpendicular to the line-of-sight, the two equivalent arrays usually could not be perpendicular to each other. So we do not assume that the Y - and X -axes in Fig. 3 are perpendicular. The range direction is the Z -axis. Note that the unambiguous distances in the X - and Y -axes are $W_x = (\lambda/d_x)r$ and $W_y = (\lambda/d_y)r$, respectively, where d_x and d_y are the inter-element distances of the equivalent receive antennas and equivalent transmit antennas. Denote x the projection of $\overrightarrow{O'A}$ on the equivalent transmit array direction, $x = \overrightarrow{O'A}^T \overrightarrow{P'_0 P'_m} / \|\overrightarrow{P'_0 P'_m}\|$, y the projection of $\overrightarrow{O'A}$ on the equivalent receive array direction, $y = \overrightarrow{O'A}^T \overrightarrow{Q'_0 Q'_n} / \|\overrightarrow{Q'_0 Q'_n}\|$. According to the definitions of a_n and b_m , $\mathbf{a}$ and $\mathbf{b}$ can be expressed as

$$\mathbf{a} = [1, e^{j2\pi x/W_x}, \dots, e^{j2\pi x(N-1)/W_x}]^T \quad (30)$$

and

$$\mathbf{b} = [1, e^{j2\pi y/W_y}, \dots, e^{j2\pi y(M-1)/W_y}]. \quad (31)$$

Assume that W_x and W_y are divided into $\mathcal{N}$ and $\mathcal{M}$ parts and $(x, y) = (\tilde{n}W_x/\mathcal{N}, \tilde{m}W_y/\mathcal{M})$. Then $s(t)$ can be expressed as

Fig. 3 Imaging field division for cross-array MIMO radar

$$s(t) = \alpha_A \begin{pmatrix} 1 \\ e^{j2\pi \frac{\tilde{m}}{\mathcal{N}}} \\ \vdots \\ e^{j2\pi(N-1)\frac{\tilde{m}}{\mathcal{N}}} \end{pmatrix} \begin{pmatrix} 1 \\ e^{j2\pi \frac{\tilde{m}}{\mathcal{M}}} \\ \vdots \\ e^{j2\pi(M-1)\frac{\tilde{m}}{\mathcal{M}}} \end{pmatrix}^T \Phi(t - \tau_A). \quad (32)$$

Denote $\mathbf{c}_{\tilde{m}}(t) = [1, e^{j2\pi \frac{\tilde{m}}{\mathcal{M}}}, \dots, e^{j2\pi(M-1)\frac{\tilde{m}}{\mathcal{M}}}]^T \Phi(t)$, we can see that scatterers with the same coordinate $\tilde{m}$ (on the Y-axis) have the same synthetic code $\mathbf{c}_{\tilde{m}}(t)$. Generally speaking, $\mathbf{c}_{\tilde{m}_2}(t) \neq \mathbf{c}_{\tilde{m}_1}(t)$ when $\tilde{m}_2 \neq \tilde{m}_1$. So the imaging volume can be divided as $\mathcal{M}$ parts along the Y-direction. This is shown in Fig. 3. The imaging volumes with the same pattern have the same synthetic code.

Denote $\mathbf{\Gamma}_{\tilde{m}}$ as the scatterers' reflectivity matrix at the $\tilde{m}$ th region (along the Y-axis), where the row and column express the indices in the X- and Z-directions, respectively. Denote

$$\bar{\mathbf{A}}_2 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & e^{j2\pi/\mathcal{N}} & \dots & e^{j2\pi(\mathcal{N}-1)/\mathcal{N}} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & e^{j2\pi(N-1)/\mathcal{N}} & \dots & e^{j2\pi(N-1)(\mathcal{N}-1)/\mathcal{N}} \end{pmatrix}_{N \times \mathcal{N}}. \quad (33)$$

Let $\mathbf{c}_{\tilde{m}}$ express the discrete version of $\mathbf{c}_{\tilde{m}}(t)$. Let $\mathbf{C}_{\tilde{m}}$ have the same expression of (22), and then the signal received from the $\tilde{m}$ th region can be expressed as

$$\mathbf{S}_{\tilde{m}} = \bar{\mathbf{A}}_2 \mathbf{\Gamma}_{\tilde{m}} \mathbf{C}_{\tilde{m}}. \quad (34)$$

The total receive signal can be expressed as

$$\mathbf{S} = \sum_{\tilde{m}=0}^{\mathcal{M}-1} \bar{\mathbf{A}}_2 \mathbf{\Gamma}_{\tilde{m}} \mathbf{C}_{\tilde{m}}. \quad (35)$$

Denote $\bar{\mathbf{\Gamma}}_2 = [\mathbf{\Gamma}_0, \mathbf{\Gamma}_1, \dots, \mathbf{\Gamma}_{\mathcal{M}-1}]$ and

$\bar{\mathbf{C}}_2 = [\mathbf{C}_0^T, \mathbf{C}_1^T, \dots, \mathbf{C}_{\mathcal{M}-1}^T]^T$, $\mathbf{S}$ can be expressed as

$$\mathbf{S} = \bar{\mathbf{A}}_2 \bar{\mathbf{\Gamma}}_2 \bar{\mathbf{C}}_2, \quad (36)$$

which is a 2D linear equation. It should be noted that $\bar{\mathbf{\Gamma}}_2$ is not a diagonal matrix, but a full dense matrix, which saves the story space.

From the expressions of $\mathbf{A}$ and $\bar{\mathbf{A}}_2$, we can see that they are only related with the antenna array configurations, and have no relation with the coordinates of the imaging reference point. This is different from the matrix $\mathbf{\Psi}$ in [28].

The imaging field divisions in Figs. 2 and 3 are with respect to the coordinate systems formed by the equivalent antennas and the line-of-sight (range). The imaging grids in the original coordinate system for the 1D MIMO radar case (Section 2.1) is shown in Fig. 4. For the 3D case, the situation is similar.

Fig. 4 Imaging field division for 1D MIMO array

It should be noted that after obtaining x and y , the scatterer's position vector $\overrightarrow{OA}$ can be obtained according to Ma *et al.* [19].

3 L₁ L₀ norms homotopy sparse signal recovery algorithm for multidimensional linear equations

Similar to the L₁ L₀ norms homotopy algorithm for linear equations, the L₁ L₀ norms homotopy algorithm for multidimensional linear equations (MD L₁L₀) can be designed. Define $G_{\sigma}(\mathbf{\Gamma}) = \sum_m \sum_n g_{\sigma}(\mathbf{\Gamma}(m, n))$, where $g_{\sigma}(\alpha) = 1 - e^{-|\alpha|/\sigma}$. The sparse signal recovery criterion can be described as

$$\begin{aligned} \hat{\mathbf{\Gamma}} &= \lim_{\sigma \rightarrow \sigma_{\min}} \arg \min_{\mathbf{\Gamma}} G_{\sigma}(\mathbf{\Gamma}) \\ &\text{subject to } \|\mathbf{S} - \mathbf{A}\mathbf{\Gamma}\mathbf{C}\|_F < \varepsilon, \end{aligned} \quad (37)$$

where $\|\cdot\|_F$ is the Frobenius norm of a 2D matrix, ε is the noise and model error. The final MD L₁L₀ algorithm can be expressed as [34, 35]:

Multidimensional linear equations L₁ L₀ norms homotopy algorithm (MD L₁L₀)

- Initialisation:
 - 1) Define constants F_0, F_1, J, ε .
 - 2) Let $\hat{\mathbf{\Gamma}}_0$ be equal to the minimum Frobenius norm solution of $\mathbf{S} = \mathbf{A}\mathbf{\Gamma}\mathbf{C}$, obtained by $\hat{\mathbf{\Gamma}}_0 = \mathbf{A}^{\dagger} \mathbf{S} \mathbf{C}^{\dagger}$, where $\dagger$ expresses pseudo inverse.
 - 3) Choose a suitable decreasing sequence for $\{\sigma_j\}$, $[\sigma_1, \dots, \sigma_J]$.
- For $j = 1, \dots, J$:
 - 1) Minimise the function $G_{\sigma_j}(\mathbf{\Gamma})$ on the feasible set $\{\mathbf{\Gamma} : \|\mathbf{S} - \mathbf{A}\mathbf{\Gamma}\mathbf{C}\|_F < \varepsilon\}$ using the revised steepest descent gradient algorithm (followed by projection onto the feasible set):
 - a) Let $\mathbf{\Gamma} = \hat{\mathbf{\Gamma}}_{j-1}$
 - b) Let δ be gradient of $G_{\sigma_j}(\mathbf{\Gamma})$ and

$$\mu = \min(\max(|\mathbf{\Gamma}|)/F_0, \sigma_j/F_1)$$
 - c) For every element of $\mathbf{\Gamma}$, let

$$\Gamma_{m,n} \leftarrow \Gamma_{m,n} - \frac{\Gamma_{m,n}}{|\Gamma_{m,n}|} \times \min(|\Gamma_{m,n}|, |\mu \sigma_j \delta_{m,n}|)$$
 - d) For $\mathbf{\Gamma}$ such as in (27), let the $\mathbf{\Gamma}$ satisfy the structure of (27).
 - e) If $\|\mathbf{S} - \mathbf{A}\mathbf{\Gamma}\mathbf{C}\|_F > \varepsilon$, project $\mathbf{\Gamma}$ back into the feasible set: $\mathbf{\Gamma} \leftarrow \mathbf{\Gamma} + \mathbf{A}^{\dagger}(\mathbf{S} - \mathbf{A}\mathbf{\Gamma}\mathbf{C})\mathbf{C}^{\dagger}$
 - 2) Set $\hat{\mathbf{\Gamma}}_j = \mathbf{\Gamma}$
- Final answer is $\hat{\mathbf{\Gamma}} = \hat{\mathbf{\Gamma}}_J$.

F_0, F_1 control the step size of searching. σ_1 is chosen as $16 \max(\hat{\Gamma}_0)$ [28]. σ_J should be chosen as less than the minimum value of the sparse solution.

The algorithm in this paper only includes one loop; however, there are two loops in [34, 35]. The solution of the inner loop in [34] only provides the initial guess for the outer loop. In this version, we omit the inner loop, at the same time decrease the steps between σ_j and σ_{j+1} and increase the J . Using one loop can increase the efficiency of some software such as Matlab, at the same time keep the algorithm's performance invariant. This paper's algorithm belongs to the smoothed ℓ_0 norm class algorithm. Compared with the existing multidimensional smoothed ℓ_0 norm algorithm, we proposed a modified kernel cost function $g_\sigma(\alpha) = 1 - e^{-|\alpha|/\sigma}$. The kernel cost function of the smoothed ℓ_0 norm algorithm is $f_\sigma(\alpha) = 1 - e^{-|\alpha|^2/(2\sigma^2)}$. It has been proven in [28] that when σ approaches positive infinity, $F_\sigma(\alpha) = \sum_n f_\sigma(\alpha(n))$ approximates to ℓ_2 norm. However, $G_\sigma(\alpha)$ approximates to ℓ_1 norm. This result shows that for any choice σ , the new $\ell_1 \ell_0$ norms homotopy algorithm searches for sparse solution. For detailed analysis, see [28, 34, 36].

4 Comparison of memory needs and computational complexity

For the original linear equations signal model in [28], because the elements of the coefficient matrix Ψ are not regular, no matter which algorithms are used to solve it, the coefficient matrix Ψ must be computed and stored. For multidimensional linear equations signal model derived in this paper, the coefficient matrices $\tilde{A}_1, \tilde{A}_2, \tilde{C}_1$ and $\tilde{C}_2$ are regular. $\tilde{A}_1$ and $\tilde{A}_2$ are Fourier matrices. We compare the two signal models in memory needed and computation load.

The L_1L_0 algorithm needs to compute the gradient of the cost function. There is no difference in computing the gradient of $G_{\sigma_j}(\Gamma)$ whether Γ is a matrix or a vector. So the difference is on the computation of the projection step. In this analysis, we will use the following standard results. Assume that the dimensions of two matrices A and B are $M \times P$ and $P \times N$, the dimension of a vector α is $P \times 1$, then AB needs MPN multiplications, and $A\alpha$ needs MP multiplications.

4.1 1D MIMO radar case

For convenient, we define the parameters again. Assume that there are M transmitters and N receivers, the codes length is K , the imaging target has T conventional range resolution units, $d_t = Nd_t$, and $M_1 = M\zeta_1$ where ζ_1 ($\zeta_1 \geq 1$) is the ratio of super resolution. Assume that the super resolution ratio in the range direction is ζ_2 ($\zeta_2 \geq 1$). According to the definitions of A_{m1} and C_{m1} , the dimensions of A_{m1} and C_{m1} are $N \times N$ and $T\zeta_2 \times (K + T)$. Then the dimensions of A and C in (25) are $N \times (MN\zeta_1)$ and $TM\zeta_1\zeta_2 \times (K + T)$, whereas the dimension of Ψ using the method of [28] is $N(K + T) \times (MN\zeta_1T\zeta_2)$. It can be seen that the sizes of A and C are much less than Ψ . The number of elements of Ψ is $(K + T)T\zeta_2$ times that of A and N^2 times that of C .

It is obvious that the computation of $\Psi^\dagger$ is more expensive than the computation of $A^\dagger$ and $C^\dagger$. The computation of $\Psi^\dagger(s - \Psi a)$ needs $2N(K + T)MNT\zeta_1\zeta_2$ multiplications, while the computation of $A^\dagger(S - A\Gamma C)C^\dagger$ needs $2TM^2N^2\zeta_1^2\zeta_2 + 2N(K + T)TM\zeta_1\zeta_2$ multiplications. Here when computing $A\Gamma C$, $A\Gamma$ is implemented first. Denote $\beta = S - A\Gamma C$, when computing $A^\dagger\beta C^\dagger$, $\beta C^\dagger$ is implemented first.

The ratio of the above matrices multiplications computational loads is

$$\begin{aligned} & \frac{2(TM^2N^2\zeta_1^2\zeta_2 + TMN(K + T)\zeta_1\zeta_2)}{2MNT(K + T)\zeta_1\zeta_2} \\ &= \frac{MN\zeta_1 + (K + T)}{N(K + T)} \\ &= \frac{M\zeta_1}{K + T} + \frac{1}{N}. \end{aligned} \quad (38)$$

Because the number of the transmit antennas cannot be too large, that is M is a small value, $(K + T)$ (codes length plus target length) will generally be larger than $M\zeta_1$. Generally speaking, the computation of multidimensional method is less than that of the 1D method.

4.2 Cross-array MIMO radar 3D imaging case

We also assume that there are M transmit antennas and N receive antennas. $\mathcal{N} = \zeta_1N$ and $\mathcal{M} = \zeta_2M$. The dimension of Ψ using method of [28] is $N(K + T) \times (MNT\zeta_1\zeta_2\zeta_3)$, where ζ_1, ζ_2 and ζ_3 are the two cross-range and one down-range super-resolution ratios. However, the dimensions of $\tilde{A}_2$ and $\tilde{C}_2$ in (36) are $N \times \mathcal{N}$ and $MT\zeta_2\zeta_3 \times (K + T)$, respectively. Obviously, the memory and computations using multidimensional linear equations signal model is much less than that of using 1D signal model.

5 Simulation results

5.1 2D imaging using 1D MIMO array

In this simulation, the carrier frequency is 10 GHz, six transmit and eight receive antennas are evenly located in the X -axis with inter-element distances of 75 and 9.375 m, respectively. Six BPSK modulated signals with length of 100 are transmitted. The bandwidth is 150 MHz. The target, located in the $[0, 1]$ direction, is shown in Fig. 5a. The amplitudes of the scatterers belong to set $\{1, 1.5, 2\}$. The distance between the target and the radar is 15 km. The output signal-to-noise ratio (SNR) is defined as $SNR = 10\log_{10}(\alpha^2MNK/\sigma_n^2)$, where K is the length of the code (100), M and N are the number of transmit and receive antennas, α is the amplitude of a scatterer, σ_n^2 is the noise variance. The output SNR of the smallest scatterer is 30 dB ($\sigma_n = 1.0954$). According to the MIMO radar parameters, the range and cross-range resolutions at the target position using conventional method are all 1 m. The distances between sampling grid points at X - and Y -axes are all chosen as 0.5 m. The dimensions of A_{m1} and C_{m1} in (21) and (22) are respectively 8×8 and 86×144 , $M_1 = 12$. So the dimension of Ψ is $(144 \times 8) \times (86 \times 8 \times M_1) = 1152 \times 8256$. The dimensions of A and C in (25) are 8×96 and 1032×144 , respectively. We can see that the matrix sizes using multidimensional linear equations are much smaller compared with that using linear equations.

For comparison, the reconstructed image using conventional correlation method is shown in Fig. 5b. The images obtained using 1D linear equations and multidimensional linear equations are shown in Figs. 5c and d, respectively, where the parameters are chosen as $J = 1000$, $F_0 = 20$, $F_1 = 20$, $\sigma_J = 0.1$ and $\varepsilon = 0$. It can be seen that the image using the correlation method has high sidelobes, while the images using 1D L_1L_0 and MD L_1L_0 methods are similar to that of the original target. The computation times using the 1D linear equations signal model algorithm and 2D linear equations signal model algorithm are 37 and 4.68 s, respectively.

The imaging quality decreases with decreasing SNR. Nevertheless, the performance of the proposed CS method is still more superior than the conventional correlation method. Due to the limitation on number of figures, the results are not shown.

5.2 Simulation 2: 3D imaging using 2D cross MIMO array

Same as simulation 1, the carrier frequency and the bandwidth are respectively 10 GHz and 150 MHz. The MIMO array is a 48×48 cross-array. Forty-eight random BPSK codes with length 150 are transmitted. The target is 20 km away from the MIMO radar and located in the $[1, 1, 1]$ -direction. The inter-element distance of the

Fig. 5 Simulation results using 1D MIMO array

(a) Original target, (b) Reconstructed image using conventional correlation method, (c) Using 1D linear equations signal model, (d) Using multidimensional linear equations signal model (simulation 1)

Fig. 6 Original target

(a) Projection on XY-plane, (b) Projection on YZ-plane, (c) Projection on XZ-plane, (d) 3D image (simulation 2)

transmit array and the receive array are all 12.5 m. The $\mathcal{M}$ and $\mathcal{N}$ are both four times the numbers of the transmit and the receive antennas. The sampling distance in down-range is 0.5 m.

The dimensions of the coefficient matrices $\bar{\mathbf{A}}_2$ and $\bar{\mathbf{C}}_2$ are respectively 48×192 and 11136×179 ($11136 = 58$ (range units) $\times 192$ synthetic codes). The dimension of Ψ using the method of [28] is $(48 \times 179) \times (48 \times 48 \times 29 \times 4 \times 4 \times 2) = 8592 \times 2138112$. Due to the huge memories needed, the conventional 1D-based algorithm cannot be implemented in a general personal computer. Obviously, the matrices sizes using multidimensional linear equations signal model are much smaller compared with that using the 1D linear equations signal model. Parameters in the multidimensional linear equations L_1 L_0 norms homotopy algorithm are $\varepsilon = 0$, $J = 625$, $F_0 = 20$ and $F_1 = 20$. Fig. 6 shows the original image of the target. The images using the correlation method and MDL_1L_0 homotopy algorithm are shown in Figs. 7 and 8. Again, it can be seen that the images using correlation method have high sidelobes and the sidelobes using the MDL_1L_0 homotopy algorithm are low. The MDL_1L_0 homotopy algorithm costs 112 s. The codes in the simulations are not designed specially. The simulations show

Fig. 7 Image obtained using correlation method

(a) Projection on XY-plane, (b) Projection on YZ-plane, (c) Projection on XZ-plane, (d) 3D image (simulation 2)

Fig. 8 Image obtained using the MDL_1L_0 method

(a) Projection on XY-plane, (b) Projection on YZ-plane, (c) Projection on XZ-plane, (d) 3D image (simulation 2)

that the algorithm is useful for general codes, even the property of the codes is not perfect.

6 Conclusion

In this paper, by introducing the synthetic codes concept, multidimensional linear equations signal models for linear-array and cross-array MIMO radar imaging are derived. Compared with 1D linear equations signal model, the coefficient matrices of multidimensional linear equations have smaller sizes, which occupy less memory spaces and have lower computational load. Multidimensional linear equations based L_1 L_0 norms homotopy sparse signal recovery algorithm is used to form the image in two situations. The quality of the images obtained by using the MDL_1L_0 algorithm is better than the conventional correlation-based algorithm.

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8 References

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