Networked filtering with Markov transmission delays and packet disordering

Liu, Andong; Zhang, Wen-an; Chen, Bo; Yu, Li

2018

Liu, A., Zhang, W., Chen, B., & Yu, L. (2018). Networked filtering with Markov transmission delays and packet disordering. *IET Control Theory & Applications*, 12(5), 687-693.

<https://hdl.handle.net/10356/87745>

<https://doi.org/10.1049/iet-cta.2017.0755>

© 2018 Institution of Engineering and Technology. This paper was published in *IET Control Theory and Applications* and is made available as an electronic reprint (preprint) with permission of Institution of Engineering and Technology. The published version is available at: [<http://dx.doi.org/10.1049/iet-cta.2017.0755>]. One print or electronic copy may be made for personal use only. Systematic or multiple reproduction, distribution to multiple locations via electronic or other means, duplication of any material in this paper for a fee or for commercial purposes, or modification of the content of the paper is prohibited and is subject to penalties under law.

Downloaded on 26 Apr 2023 18:54:19 SGT

Networked filtering with Markov transmission delays and packet disordering

ISSN 1751-8644

Received on 13th July 2017

Revised 9th November 2017

Accepted on 11th December 2017

E-First on 17th January 2018

doi: 10.1049/iet-cta.2017.0755

www.ietdl.org

Andong Liu¹ ✉, Wen-an Zhang¹, Bo Chen², Li Yu¹¹Department of Automation, Zhejiang University of Technology, Hangzhou, People's Republic of China²School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore, Singapore

✉ E-mail: lad@zjut.edu.cn

Abstract: This is concerned with the filtering problem for a class of networked systems with Markov transmission delays and packet disordering. The transmission delays are considered to be a Markov chain with finite states, and a packet reordering mechanism is proposed to deal with the packet disordering. According to the original transition probability matrix of Markov delays, a new method is developed to calculate the reordered transition probability matrix. A sufficient condition is derived such that the filtering error system is exponentially stable in the mean-square sense. Moreover, an optimal filter is designed by solving a convex optimisation problem. An illustrative example is provided to show the effectiveness of the proposed filtering method.

1 Introduction

In networked systems, sensor data are often transmitted through wireless or wired network to remote units for real-time monitoring and control. However, not all of the state variables are always available for direct measurement by sensors in practical applications. Therefore, state estimation plays an important role in networked systems for information processing problems, and has aroused the concern and interest of many researchers (see [1–3], and the references therein).

Owing to the impact of network conflict and bandwidth limitation of communication network, time delay is unavoidable and may deteriorate the networked system performance [4–7]. Since the delays caused by the limitation of communication network always occur in a random way, the method of finite-state Markov chain has been widely used to model the random delays [8–10]. Meanwhile, Kalman filter is one of the most useful techniques to design the state estimator under random delays [11–13]. However, this method requires to know the statistical properties of noises exactly, which may not be possible in practise. To overcome these shortages, H_∞ filtering method has been widely used as an alternative approach [14–18]. Owing to multi-path routing or frequency hopping for communication network, packet disordering may occur in many network scenarios, especially for long time delay [19, 20]. However, the issue of packet disordering was not considered in the afore-mentioned estimation results.

It should be pointed out that if the packet disordering happens, the data packets to be transmitted are not necessary to follow the principle of ‘first send–first arrive’ as assumed in most existing works [21]. Control problems for networked systems with packet disordering have been extensively investigated (see [22–24], and the references therein), while the packet disordering issues are also intensively studied in the field of state estimation [25, 26]. Particularly, the filter design problem has been studied in [25] for networked systems with the phenomenon of Markov delays and packet losses. However, the expression of packet reordering mechanism, which can select the newest signal, was not presented in [25]. Although the logical expression was given in [26], the transition probability matrix after packet reordering was not presented, and the proposed results were derived by using the original transition probability matrix *before* packet reordering, which is unreasonable. Note that the reordered transition probability matrix was presented in [27], but it needs to find out all of feasible paths of delay during interval $[k - d, k]$, where k means

the k th sampling instant and d means the maximum delay bound, that is, complex and not intuitive. Moreover, the feasible path defined in [27] neglects the delay of time $k - d$ during interval $[k - d, k]$, which causes the result to be incorrect.

Motivated by the above analysis, it is significant to study the packet reordering and filter design problems for networked systems in the presence of random transmission delays. The random delays are considered as a Markov chain and may be longer than one sampling interval. The crucial contributions of this paper are listed as follows: (i) a general model of filtering error system with the phenomenon of Markov delays and packet disordering is presented by the proposed reordering mechanism. (ii) A new method to calculate the reordered transition probability matrix is presented based on the given transition probability matrix of random transmission delays. (iii) A sufficient condition of mean-square exponential stability is derived for the filtering error system, and the filter parameters are obtained by solving an optimisation problem. Finally, the effectiveness of the proposed method is verified by a numerical example.

Notations: R^n denotes the Euclidean space with n -dimensional. $l_2[0, \infty)$ is the space of summable sequences. $\underline{\lambda}(A)$ and $\bar{\lambda}(A)$ are the minimum and maximum eigenvalues of matrix A . $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation and $\text{Prob}\{\cdot\}$ is the occurrence probability of the event ‘ $\cdot$ ’. The superscript ‘ T ’ represents the transpose. The symbol ‘ $*$ ’ in a symmetric block matrix is used to describe the symmetry term. $\emptyset$ represents the empty set. $\|x\|$ refers to the Euclidean norm of vector x and $\|A\|_Q^2 = A^T Q A$ for matrices A and Q .

2 Problem formulation

The structure of the considered networked filtering system is shown in Fig. 1, where the filter is connected to the plant by a communication network. The model of the plant is described by the following linear system:

$$\begin{cases} x(k+1) = Ax(k) + Bw(k) \\ y(k) = Cx(k) \\ z(k) = Mx(k) \end{cases} \quad (1)$$

where $x(k) \in R^n$ and $y(k) \in R^m$ are the system state and measured output, respectively, $z(k) \in R^r$ is the signal to be estimated,

Fig. 1 Diagram for filtering problem with Markov delays

Fig. 2 An example of time diagram

$w(k) \in R^p$ is the disturbance signal belonging to $l_2[0, \infty)$, τ_k is the random sensor-filter transmission delay, and each packet is time stamped. A , B , C , M are constant matrices of appropriate dimensions. The filter has a fixed length buffer to store the most recent received packets from the sensor, and random delay τ_k is assumed to be a Markov chain which takes values in $\mathcal{N} = \{0, 1, \dots, d\}$ with given transition probability matrix $P = [p_{ij}]$. The transition probability p_{ij} is given by

$$p_{ij} = \text{Prob}\{\tau_{k+1} = j | \tau_k = i\} \quad (2)$$

where $p_{ij} \geq 0$, $\sum_{j=0}^d p_{ij} = 1$, $\forall i, j \in \mathcal{N}$.

On the basis of the discussion in [22] and taking into account the phenomenon of packet disordering, it is known that the dynamics of the considered system are actually determined by $\{\tau_{k-d}, \dots, \tau_{k-1}, \tau_k\}$. Hence, we can use a mapping of $R^{d+1} \rightarrow R$: $[\tau_{k-d}, \dots, \tau_k] \rightarrow d_k$ to describe the system with packet disordering. The expression of d_k can be described as follows:

$$d_k = \min\{i | \tau_{k-i} - i \leq 0, i = 0, 1, \dots, d\} \quad (3)$$

and a time diagram of the signal transmission is shown in Fig. 2.

Considering the effect of packet disordering and external disturbance, system (1) can be reformulated as follows:

$$\begin{cases} x(k+1) = Ax(k) + Bw(k) \\ \tilde{y}(k) = Cx(k-d_k) + Dw(k) \\ z(k) = Mx(k) \end{cases} \quad (4)$$

where $\tilde{y}(k) \in R^p$ is the filter input after packet reordering and D is a constant matrix of appropriate dimension. It is worth noting that the external disturbance for filter input is considered as $w(k)$ for simplicity, but without loss generality.

Remark 1: Note that τ_k is the transmission delay from the sensor to the filter, and obeys a Markov probability distribution. Since packet disordering issue arises, the disordered packets will be initiative discarded, and the filter destination needs to reorder the packets. Thus, τ_k represents a disordering sequence, while d_k represents a reordered sequence.

To estimate the signal $z(k)$ in system (1), we consider the following filter:

$$\begin{cases} \tilde{x}(k+1) = A_f \tilde{x}(k) + B_f \tilde{y}(k) \\ \tilde{z}(k) = C_f \tilde{x}(k) + D_f \tilde{y}(k) \end{cases} \quad (5)$$

where $\tilde{x}(k) \in R^n$ is the filter state, $\tilde{z}(k) \in R^r$ is the estimated signal, A_f , B_f , C_f and D_f are filter parameter matrices to be determined.

Let $e(k) = z(k) - \tilde{z}(k)$ and $\eta(k) = [x^T(k) \quad \tilde{x}^T(k)]^T$. Then, the following filtering error system can be obtained:

$$\begin{cases} \eta(k+1) = \bar{A}\eta(k) + \bar{B}_1\eta(k-d_k) + \bar{B}_2w(k) \\ e(k) = \bar{M}\eta(k) + \bar{C}\eta(k-d_k) + \bar{D}w(k) \end{cases} \quad (6)$$

where

$$\bar{A} = \begin{bmatrix} A & 0 \\ 0 & A_f \end{bmatrix} \quad \bar{B}_1 = \begin{bmatrix} 0 & 0 \\ B_f C & 0 \end{bmatrix} \quad \bar{B}_2 = \begin{bmatrix} B \\ B_f D \end{bmatrix}$$

$$\bar{M} = [M \quad -C_f] \quad \bar{C} = [-D_f C \quad 0] \quad \bar{D} = -D_f D.$$

The objective of this paper is to calculate the reordered transition probability matrix based on the packet reordering mechanism (3), and design a filter (5) such that the filtering error system (6) is mean-square exponentially stable with a prescribed H_∞ performance γ .

Remark 2: It should be noted that the model of (3) is first given in [22] to deal with the phenomenon of packet disordering caused by time-varying delay. Let $\Pi = [\pi_{ij}]$ be the reordered transition probability matrix, it can be known from Proposition 1 in [22] that the reordered transition probability $\pi_{ij} = 0$ if $j > i + 1$. Thus, the analysis method in [22] cannot be directly applied to the random delay.

Remark 3: The filtering problems for networked systems with random delay and packet disordering were investigated in [25, 26]. However, the transition probability matrix after packet reordering was not presented, and the proposed results were derived by using the origin transition probability matrix before packet reordering. By the discussion of Remark 2, we know that the reordered transition probability $\pi_{ij} = 0$ if $j > i + 1$. Thus, it is necessary to calculate the reordered transition probability matrix. Although the computational method for reordered transition probability matrix was presented in [27], it needs to find out all of feasible paths which is complex and not intuitive. Moreover, the set of feasible path neglects the fact of τ_{k-d} , which causes the result to be incorrect.

To this end, we first introduce the following definitions:

Definition 1: For any given initial conditions $\eta(i)$ and τ_i , $i = 0, -1, \dots, -d$, the filtering error system (6) with $w(k) = 0$ is said to be exponentially stable in the mean-square sense if its solution satisfies $\mathbb{E}\{\|\eta(k)\|^2 | \eta(i), \tau_i, i = 0, -1, \dots, -d\} \leq \beta \alpha^k$, where $0 < \alpha < 1$ is the decay rate and $\beta > 0$ is a constant.

Definition 2: The filtering error system (6) is said to be exponentially stable in the mean-square sense with a prescribed H_∞ noise attenuation level γ , if under zero initial conditions, the following inequality:

$$\mathbb{E}\left\{\sum_{k=0}^{\infty} e^T(k)e(k)\right\} < \gamma^2 \sum_{k=0}^{\infty} w^T(k)w(k)$$

holds for all non-zero $w(k) \in l_2[0, \infty)$.

3 Main results

In this section, we first give the computational method for the reordered transition probability matrix Π of d_k . For simplicity, we denote $\tau_k = \delta_0$, $\tau_{k-1} = \delta_1$, $\dots$, $\tau_{k-d} = \delta_d$ for $d_k = r$. It is known that $\delta_0^d = [\delta_0, \delta_1, \dots, \delta_d]$ is a feasible path for $d_k = r$. Denote $\mathcal{S}$ be the set of the feasible path for $d_k = r$. Thus, $\{\delta_0^d\} \in \mathcal{S}$. Similarly, we can define $\sigma_0^{d+1} = [\sigma_0, \sigma_1, \dots, \sigma_{d+1}]$ be a feasible path for $\{d_{k+1} = s, d_k = r\}$ with $\tau_{k+1} = \sigma_0$, $\tau_k = \sigma_1$, $\dots$, $\tau_{k-d} = \sigma_{d+1}$. Denote $\mathcal{H}$ be the set of the feasible path for $\{d_{k+1} = s, d_k = r\}$. Thus, $\{\sigma_0^{d+1}\} \in \mathcal{H}$. Note that δ_i may have a variety of values. Denote $\delta_i \in \mathcal{D}_i = \{\delta_i(1), \delta_i(2), \dots, \delta_i(d_i)\}$, $\mathcal{D}_i \in \mathcal{N}$ for $\tau_{k-i} \leq d_i$, $d_i \leq d$. Then, the following lemma can be derived.

Lemma 1: Denote $\delta_i \in \mathcal{D}_i = \{\delta_i(1), \delta_i(2), \dots, \delta_i(d_i)\}$, $\mathcal{D}_i \in \mathcal{N}$, $\delta_i(j) \cap \mathcal{D}_i \delta_i(j) = \emptyset$, $i = 1, 2$, $\mathcal{L}(\delta_i \in \mathcal{D}_i) = \Lambda_i$, Λ_i is a diagonal matrix with element

$$\Lambda_i(l+1, l+1) = \begin{cases} 1 & l \in \mathcal{D}_i \\ 0 & \text{else} \end{cases} \quad l = 0, 1, \dots, d.$$

If the following equation holds:

$$\mathcal{F}(\delta_1 \in \mathcal{D}_1, \delta_2 \in \mathcal{D}_2) = \sum_{\delta_1 \in \mathcal{D}_1} \sum_{\delta_2 \in \mathcal{D}_2} p_{\delta_1, \delta_2} \quad (7)$$

then one has

$$\mathcal{F}(\delta_1 \in \mathcal{D}_1, \delta_2 \in \mathcal{D}_2) = E^T \mathcal{L}(\delta_1) P \mathcal{L}(\delta_2) E \quad (8)$$

where E is a vector of all ones of appropriate dimension.

Proof: We first consider (7) that

$$\begin{aligned} & \sum_{\delta_1 \in \mathcal{D}_1} \sum_{\delta_2 \in \mathcal{D}_2} p_{\delta_1, \delta_2} \\ &= p_{\delta_1(1), \delta_2(1)} + p_{\delta_1(1), \delta_2(2)} + \dots + p_{\delta_1(1), \delta_2(d_2)} \\ &+ \sum_{\delta_1 \in \mathcal{D}_1 \setminus \delta_1(1)} \sum_{\delta_2 \in \mathcal{D}_2} p_{\delta_1, \delta_2} \\ &= E^T \text{diag}\{\underbrace{0, \dots, 0, 1}_{\delta_1(1)}, \underbrace{0, \dots, 0}_{d - \delta_1(1)}\} P \text{diag}\{\underbrace{0, \dots, 0, 1}_{\delta_2(1)}, \underbrace{0, \dots, 0}_{d - \delta_2(1)}\} E \\ &+ E^T \text{diag}\{\underbrace{0, \dots, 0, 1}_{\delta_1(1)}, \underbrace{0, \dots, 0}_{d - \delta_1(1)}\} P \text{diag}\{\underbrace{0, \dots, 0, 1}_{\delta_2(2)}, \underbrace{0, \dots, 0}_{d - \delta_2(2)}\} E \\ &\vdots \\ &+ E^T \text{diag}\{\underbrace{0, \dots, 0, 1}_{\delta_1(1)}, \underbrace{0, \dots, 0}_{d - \delta_1(1)}\} P \text{diag}\{\underbrace{0, \dots, 0, 1}_{\delta_2(d_2)}, \underbrace{0, \dots, 0}_{d - \delta_2(d_2)}\} E \\ &+ \sum_{\delta_1 \in \mathcal{D}_1 \setminus \delta_1(1)} \sum_{\delta_2 \in \mathcal{D}_2} p_{\delta_1, \delta_2} \\ &= E^T \text{diag}\{\underbrace{0, \dots, 0, 1}_{\delta_1(1)}, \underbrace{0, \dots, 0}_{d - \delta_1(1)}\} P \Lambda_2 E \\ &+ \sum_{\delta_1 \in \mathcal{D}_1 \setminus \delta_1(1)} \sum_{\delta_2 \in \mathcal{D}_2} p_{\delta_1, \delta_2} \\ &= E^T \Lambda_1 P \Lambda_2 E \end{aligned} \quad (9)$$

Then, the proof is completed. $\square$

Corollary 1: The following two equations are true:

$$\mathcal{F}(\delta_1 \in \mathcal{D}_1, \dots, \delta_r \in \mathcal{D}_r) = \sum_{\delta_1 \in \mathcal{D}_1} \dots \sum_{\delta_r \in \mathcal{D}_r} (p_{\delta_1, \delta_2} \dots p_{\delta_{r-1}, \delta_r})$$

$$\mathcal{F}(\delta_1 \in \mathcal{D}_1, \dots, \delta_r \in \mathcal{D}_r) = E^T \left(\prod_{j=1}^{r-1} \mathcal{L}(\delta_j) P \mathcal{L}(\delta_{j+1}) \right) E$$

Proof: By using the similar method in Lemma 1, the above results can be easily obtained. Thus, the proof is omitted. $\square$

Theorem 1: Given a scalar d and a matrix P , the reordered transition probability π_{rs} of d_k can be calculated as follows:

$$\pi_{rs} = \text{Prob}\{d_{k+1} = s | d_k = r\} = \frac{\pi_s}{\pi_r} \quad (10)$$

where

$$\pi_r = E^T \left(\prod_{j=0}^r \mathcal{L}(\delta_{j+1}) P \mathcal{L}(\delta_j) \right) E \quad (11)$$

with $\{\delta_{r+1} \in \mathcal{N}, \delta_r \leq r, \delta_j > j, j = r-1, \dots, 0 | r \in \{1, \dots, d\}\}$, $\{\delta_1 \in \mathcal{N}, \delta_0 \leq 0 | r = 0\}$ and

$$\pi_s = E^T \left(\prod_{j=0}^{r+1} \mathcal{L}(\sigma_{j+1}) P \mathcal{L}(\sigma_j) \right) E \quad (12)$$

with $\{\sigma_{r+2} \in \mathcal{N}, \sigma_{r+1} \leq r, \sigma_r > r-1, \dots, \sigma_{s+1} > s, \sigma_s > s-1 | \sigma_s \leq s, \sigma_{s-1} > s-1, \dots, \sigma_0 > 0 | 0 < s < r+1\}$, $\{\sigma_2 \in \mathcal{N}, \sigma_1 \leq 0, \sigma_0 \leq 0 | r = 0, s = 0\}$, $\{\sigma_{r+2} \in \mathcal{N}, \sigma_{r+1} \leq r, \sigma_r > r-1, \dots, \sigma_1 > 0, \sigma_0 \leq 0 | r \neq 0, s = 0\}$, $\{\sigma_{r+2} \in \mathcal{N}, \sigma_{r+1} \leq r, \sigma_r > r, \dots, \sigma_0 > 0 | s = r+1\}$, $\{\sigma_i \in \emptyset, i = 0, \dots, r+2 | s > r+1\}$.

Proof: On the basis of the conditional probability, one has

$$\begin{aligned} \frac{\pi_s}{\pi_r} &= \text{Pr}\{d_{k+1} = s | d_k = r\} \\ &= \frac{\text{Pr}\{d_{k+1} = s, d_k = r\}}{\text{Pr}\{d_k = r\}} \\ &= \frac{\sum_{\sigma_0^{d+1} \in \mathcal{N}} \text{Pr}\{\sigma_0, \sigma_1, \dots, \sigma_{d+1}\}}{\sum_{\delta_0^d \in \mathcal{N}} \text{Pr}\{\delta_0, \delta_1, \dots, \delta_d\}} \end{aligned} \quad (13)$$

Since $d_k = r$, it can be known from (3) that $\delta_r - r \leq 0$, $\delta_j - j > 0, j = 0, \dots, r-1$ and $\delta_j \in \mathcal{N}, j = r+1, \dots, d$. Thus, π_r can be derived as follows when $r > 0$:

$$\pi_r = \sum_{\delta_d \in \mathcal{N}} \dots \sum_{\delta_{r+1} \in \mathcal{N}} \sum_{\delta_r \leq r} \sum_{\delta_{r-1} > r-1} \dots \sum_{\delta_0 > 0} (p_{\delta_d, \delta_{d-1}} \dots p_{\delta_1, \delta_0}) \quad (14)$$

Owing to $\sum_{j=0}^d p_{ij} = 1$, (14) can be rewritten as follows:

$$\pi_r = \sum_{\delta_{r+1} \in \mathcal{N}} \sum_{\delta_r \leq r} \sum_{\delta_{r-1} > r-1} \dots \sum_{\delta_0 > 0} (p_{\delta_{r+1}, \delta_r} \dots p_{\delta_1, \delta_0}) \quad (15)$$

By using the similar method of (9) and according to Corollary 1 with the fact that $\Lambda_j^2 = \Lambda_j$, one has

$$\pi_r = E^T \left(\prod_{j=0}^r \mathcal{L}(\delta_{j+1}) P \mathcal{L}(\delta_j) \right) E \quad (16)$$

with $\{\delta_{r+1} \in \mathcal{N}, \delta_r \leq r, \delta_j > j, j = r-1, \dots, 1, 0 | r \in \{1, \dots, d\}\}$.

On the other hand, $\delta_0 = 0$ and $\delta_j \in \mathcal{N}, j = 1, \dots, d$ if $r = 0$. Then, it is easy to obtain that

$$\pi_r = \sum_{\delta_1 \in \mathcal{N}} \sum_{\delta_0 \leq 0} p_{\delta_1, \delta_0} \quad (17)$$

According to Lemma 1, (17) can be rewritten as

$$\pi_r = E^T \mathcal{L}(\delta_1) P \mathcal{L}(\delta_0) E \quad (18)$$

with $\{\delta_1 \in \mathcal{N}, \delta_0 \leq 0 | r = 0\}$. Combining (16) and (18) implies (11).

Since $d_{k+1} = s$, it is clear that $\sigma_s - s \leq 0$, $\sigma_j - j > 0, j = 0, \dots, s-1$ and $\sigma_j \in \mathcal{N}, j = s+1, \dots, d+1$. On the basis of the discussion in Remark 1, it is known that $\pi_s = 0$ if $s > r+1$. Then, we concern about $s \leq r+1$. If $0 < s < r+1$, one has $\sigma_{j+1} - j > 0, j = 0, \dots, r-1$ because $d_k = r$. Thus, π_r can be derived as follows:

$$\begin{aligned}
\pi_s &= \sum_{\sigma_{r+2} \in \mathcal{N}} \sum_{\sigma_l \cap \sigma_{l+s+1} \in \mathcal{N}} \sum_{\sigma_r \sigma_{r+1} \leq r \cap \sigma_{l+s} \in \mathcal{N}} \\
&\quad \sum_{\sigma_r > r-1 \cap \sigma_{l+s-1} \in \mathcal{N}} \cdots \sum_{\sigma_{r+2-l} > r+1-l \cap \sigma_{s+1} \in \mathcal{N}} \\
&\quad \sum_{\sigma_{r+1-l} > r-l \cap \sigma_s \leq s} \sum_{\sigma_{r-l} > r-1-l \cap \sigma_{s-1} > s-1} \\
&\quad \cdots \sum_{\sigma_1 > 0 \cap \sigma_1 > 1} \sum_{\sigma_0 > 0} (p_{\sigma_{r+2}, \sigma_{r+1}} \cdots p_{\sigma_1, \sigma_0}) \\
&= \sum_{\sigma_{r+2} \in \mathcal{N}} \sum_{\sigma_{r+1} \leq r} \sum_{\sigma_r > r-1} \\
&\quad \cdots \sum_{\sigma_{r+2-l} > r+1-l} \sum_{\sigma_{r+1-l} > r-l} \sum_{\sigma_s \leq s} \sum_{\sigma_{s-1} > s-1} \\
&\quad \cdots \sum_{\sigma_1 > 1} \sum_{\sigma_0 > 0} (p_{\sigma_{r+2}, \sigma_{r+1}} \cdots p_{\sigma_1, \sigma_0}) \\
&= E^T \left(\prod_{j=0}^{r+1} \mathcal{L}(\sigma_{j+1}) P \mathcal{L}(\sigma_j) \right) E
\end{aligned} \tag{19}$$

with $\{\sigma_{r+2} \in \mathcal{N}, \sigma_{r+1} \leq r, \sigma_r > r-1, \dots, \sigma_{s+1} > s, \sigma_s > s-1 \cap \sigma_s \leq s, \sigma_{s-1} > s-1, \dots, \sigma_0 > 0 \mid 0 < s < r+1\}$, $l = r-s+1$. Similarly, we can also obtain that

$$\begin{aligned}
\pi_s &= \sum_{\sigma_2 \in \mathcal{N}} \sum_{\sigma_1 \leq 0} \sum_{\sigma_0 \leq 0} (p_{\sigma_2, \sigma_1} p_{\sigma_1, \sigma_0}) \\
&= E^T \left(\prod_{j=0}^1 \mathcal{L}(\sigma_{j+1}) P \mathcal{L}(\sigma_j) \right) E
\end{aligned} \tag{20a}$$

with $\{\sigma_2 \in \mathcal{N}, \sigma_1 \leq 0, \sigma_0 \leq 0 \mid r=0, s=0\}$

$$\begin{aligned}
\pi_s &= \sum_{\sigma_{r+2} \in \mathcal{N}} \sum_{\sigma_{r+1} \leq r} \sum_{\sigma_r > r-1} \\
&\quad \cdots \sum_{\sigma_1 > 0} \sum_{\sigma_0 \leq 0} (p_{\sigma_{r+2}, \sigma_{r+1}} \cdots p_{\sigma_1, \sigma_0}) \\
&= E^T \left(\prod_{j=0}^{r+1} \mathcal{L}(\sigma_{j+1}) P \mathcal{L}(\sigma_j) \right) E
\end{aligned} \tag{20b}$$

with $\{\sigma_{r+2} \in \mathcal{N}, \sigma_{r+1} \leq r, \sigma_r > r-1, \dots, \sigma_1 > 0, \sigma_0 \leq 0 \mid r \neq 0, s=0\}$

$$\begin{aligned}
\pi_s &= \sum_{\sigma_{r+2} \in \mathcal{N}} \sum_{\sigma_{r+1} \leq r} \sum_{\sigma_r > r} \\
&\quad \cdots \sum_{\sigma_1 > 1} \sum_{\sigma_0 > 0} (p_{\sigma_{r+2}, \sigma_{r+1}} \cdots p_{\sigma_1, \sigma_0}) \\
&= E^T \left(\prod_{j=0}^{r+1} \mathcal{L}(\sigma_{j+1}) P \mathcal{L}(\sigma_j) \right) E
\end{aligned} \tag{20c}$$

with $\{\sigma_{r+2} \in \mathcal{N}, \sigma_{r+1} \leq r, \sigma_r > r, \dots, \sigma_0 > 0 \mid s=r+1\}$. This completes the proof. $\square$

Remark 4: It is worth noting that the reordered transition probability $\pi_{rs} = 0$ if $s > r+1$, which means the next moment of transition state cannot be more than $r+1$. Thus, it can be seen from (10) that π_s equals to 0 if $s > r+1$. In this situation, we define the sets of $\sigma_i, i=0, \dots, r+2$ are all empty sets $\{\sigma_i \in \emptyset, i=0, \dots, r+2 \mid s > r+1\}$. Then, one has $\mathcal{L}(\sigma_i) = 0$ which infers $\pi_s = 0$.

Remark 5: From Theorem 1, it only needs to determine the values of $\mathcal{L}(\delta_i), i=0, \dots, r+1$ and $\mathcal{L}(\sigma_j), j=0, \dots, r+2$ to calculate the reordered transition probability matrix Π . By giving the models r and s , it is easy to determine $\mathcal{L}(\delta_i)$ and $\mathcal{L}(\sigma_j)$ by the model of (3). Therefore, the proposed method in Theorem 1 does not need to find the feasible path during d sampling interval and is more convenient to calculate Π by comparing with [27].

Proposition 1: For any $r, r \in \{0, 1, \dots, d\}$, $\sum_{s=0}^d \pi_{rs} = 1$.

Proof: To avoid confusion, we redefine the set of the feasible path for $d_k = r$ and $\{d_{k+1} = s, d_k = r\}$ as $\mathcal{G}_r$ and $\mathcal{H}_s$, respectively. Then, we have $\mathcal{G}_0 = \{\delta_1 \in \mathcal{N}, \delta_0 \leq 0 \mid r=0\}$, $\mathcal{G}_r = \{\delta_{r+1} \in \mathcal{N}, \delta_r \leq r, \delta_j > j, j=r-1, \dots, 0 \mid r \in \{1, \dots, d\}\}$, $\mathcal{H}_0 = \{\sigma_{r+2} \in \mathcal{N}, \sigma_{r+1} \leq r, \sigma_r > r-1, \dots, \sigma_1 > 0, \sigma_0 \leq 0 \mid r \neq 0, s=0\}$, $\mathcal{H}_s = \{\sigma_{r+2} \in \mathcal{N}, \sigma_{r+1} \leq r, \sigma_r > r-1, \dots, \sigma_{s+1} > s, \sigma_s > s-1 \cap \sigma_s \leq s, \sigma_{s-1} > s-1, \dots, \sigma_0 > 0 \mid 0 < s < r+1\}$, $\mathcal{H}_{r+1} = \{\sigma_{r+2} \in \mathcal{N}, \sigma_{r+1} \leq r, \sigma_r > r, \dots, \sigma_0 > 0 \mid s=r+1\}$.

On the other hand, $\sum_{s=0}^d \pi_{rs} = \sum_{s=0}^d \pi_s / \pi_r$ for any $r, r \in \{1, \dots, d\}$. Then, the set of the feasible path for all $s, s=0, 1, \dots, d$ can be determined as $\mathcal{H}_0^d = \cup_{s=0}^d \mathcal{H}_s = \cup_{s=0}^{r+1} \mathcal{H}_s = \{\sigma_{r+2} \in \mathcal{N}, \sigma_{r+1} \leq r, \sigma_r > r-1, \dots, \sigma_1 > 0, \sigma_0 \in \mathcal{N}\}$. By comparing with $\mathcal{G}_r$ and $\mathcal{H}_0^d$, it is known that δ_i and σ_{i+1} have the same values for $i=0, \dots, d+1$. Thus, $\mathcal{L}(\sigma_1) = \mathcal{L}(\delta_0), \dots, \mathcal{L}(\sigma_{d+2}) = \mathcal{L}(\delta_{d+1})$. Note that $\sigma_0 \in \mathcal{N}$ in $\mathcal{H}_0^d$, we have $P \mathcal{L}(\sigma_0) E = E$. Therefore

$$\sum_{s=0}^d \pi_{rs} = \frac{\sum_{s=0}^d \pi_s}{\pi_r} = \frac{E^T \left(\prod_{j=1}^{r+1} \mathcal{L}(\sigma_{j+1}) P \mathcal{L}(\sigma_j) \right) E}{E^T \left(\prod_{j=0}^r \mathcal{L}(\delta_{j+1}) P \mathcal{L}(\delta_j) \right) E} = 1.$$

By using the similar method, we have $\sum_{s=0}^d \pi_{rs} = 1$ for $r=0$. The proof is completed. $\square$

Theorem 2: Given scalars $\alpha < 1, \gamma$ and d , if there exist matrices $\tilde{A}_f, \tilde{B}_f, C_f, D_f, P_2(r), Q_2, G_1, G_2, G_3$ and symmetric matrices $P_1(r) > 0, P_3(r) > 0, r=0, 1, \dots, d, Q_1 > 0, Q_3 > 0$ such that

$$\begin{bmatrix} \Omega_1 & 0 & 0 & \Omega_2 & \bar{M}^T \\ * & -\alpha^d Q & 0 & \Omega_3 & \bar{C}^T \\ * & * & -\gamma^2 I & \Omega_4 & \bar{D}^T \\ * & * & * & \Omega_5 & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{21}$$

Then, system (6) is mean-square exponentially stable with a prescribed H_∞ performance γ , and the filter parameters are obtained as $A_f = (\tilde{A}_f G_3^{-1})^T, B_f = (\tilde{B}_f G_3^{-1})^T, C_f$ and D_f , where $\mu = 1 + d(1 - \underline{\pi}), \underline{\pi} = \min \{\pi_{rr}, r \in \mathcal{N}\}$

$$\begin{aligned}
\bar{P}(r) &= \sum_{s=0}^d \pi_{rs} P(s) \quad P(r) = \begin{bmatrix} P_1(r) & P_2^T(r) \\ P_2(r) & P_3(r) \end{bmatrix} \\
\Omega_1 &= \mu Q - \alpha P(r) \quad \Omega_5 = \bar{P}(r) - G^T - G \\
\Omega_2 &= \begin{bmatrix} A^T G_1 & A^T G_2 \\ \tilde{A}_f & \tilde{A}_f \end{bmatrix} \quad \Omega_3 = \begin{bmatrix} C^T \tilde{B}_f & C^T \tilde{B}_f \\ 0 & 0 \end{bmatrix} \\
\Omega_4 &= [B^T G_1 + D^T \tilde{B}_f \quad B^T G_2 + D^T \tilde{B}_f] \\
Q &= \begin{bmatrix} Q_1 & Q_2^T \\ Q_2 & Q_3 \end{bmatrix} \quad G = \begin{bmatrix} G_1 & G_2 \\ G_3 & G_3 \end{bmatrix}.
\end{aligned}$$

Proof: We first concern about the mean-square exponential stability of system (6) with $w(k) = 0$. To achieve the goal, we define the following Lyapunov–Krasovskii functional:

$$V(k, d_k) = \sum_{l=1}^3 V_l(k, d_k) \tag{22}$$

with $V_1 = \|\eta(k)\|_{\tilde{P}(d_k)}^2, V_2 = \sum_{i=k-d_k}^{k-1} \alpha^{k-i-1} \|\eta(i)\|_{\tilde{Q}}^2, V_3 = (1 - \underline{\pi}) \sum_{j=-d+1}^{-1} \sum_{i=k+j}^{k-1} \alpha^{k-i-1} \|\eta(i)\|_{\tilde{Q}}^2$. Let $d_k = r, d_{k+1} = s$. By taking expectation on the difference of V , we have

$$\mathbb{E}\{V(k+1, d_{k+1}) - \alpha V(k, d_k) | \mathcal{M}_k\} \leq \xi^T(k) \Phi \xi(k) \quad (23)$$

where $\xi^T(k) = [\eta^T(k), \eta^T(k-r)]$, $\mathcal{M}_k = \{\tau_k, \dots, \tau_{k-d}\}$

$$\Phi = \begin{bmatrix} \mu Q - \alpha P(r) + \bar{A}^T \bar{P}(r) \bar{A} & \bar{A}^T \bar{P}(r) \bar{B}_1 \\ * & \bar{B}_1^T \bar{P}(r) \bar{B}_1 - \alpha^d Q \end{bmatrix}.$$

By the definition of $P(r)$, Q , G , Ω_1 , Ω_2 , Ω_3 , Ω_4 and Ω_5 , and using Lemma 2.3 in [28], it is known that the following inequality:

$$\bar{\Phi} < 0 \quad (24)$$

is equivalent to (21), where $\bar{\Phi} = \text{diag}\{\mu Q - \alpha P(r), -\alpha^d Q, -\gamma^2 I\} + [\bar{M} \ \bar{C} \ \bar{D}]^T [\bar{M} \ \bar{C} \ \bar{D}] + [\bar{A} \ \bar{B}_1 \ \bar{B}_2]^T \bar{P}(r) [\bar{A} \ \bar{B}_1 \ \bar{B}_2]$. In view of (24), the following inequality:

$$\mathbb{E}\{V(k+1, d_{k+1}) - \alpha V(k, d_k) | \mathcal{M}_k\} < 0 \quad (25)$$

is held for all non-zero $\xi(k)$ by Schur complement lemma. Then, the following inequality:

$$\mathbb{E}\{V(k, d_k) | \mathcal{M}_k\} < \dots < \alpha^k V(0, d_0) \quad (26)$$

can be derived by (25). In view of (22) and (26), we can verify that the following inequality is satisfied by two positive scalars β_1 and β_2 :

$$\beta_1 \mathbb{E}\{\|\eta(k)\|^2 | \mathcal{M}_k\} \leq \mathbb{E}\{V(k, d_k) | \mathcal{M}_k\} < \beta_2 \alpha^k \|\eta(0)\|^2 \quad (27)$$

where $\beta_1 = \min\{\lambda(P(i)) | i \in \mathcal{N}\}$, $\beta_2 = \max\{\bar{\lambda}(P(i)) | i \in \mathcal{N}\} + \bar{\lambda}(Q)(d + ((d-1)/2)(1 - \underline{\alpha}))$. Then, we have $\mathbb{E}\{\|\eta(k)\|^2 | \mathcal{M}_k\} < (\beta_2/\beta_1) \alpha^k \|\eta(0)\|^2$. Consequently, we can conclude from Definition 1 that system (6) is mean-square exponentially stable with a decay rate α under the condition of $w(k) = 0$.

We now consider the H_∞ performance function as follows $J_T = \sum_{k=0}^T \{\mathbb{E}\{e^T(k)e(k)\} - \gamma^2 w^T(k)w(k)\}$. For any non-zero $w(k) \in l_2[0, +\infty)$ and zero initial condition, one has $J_T < \sum_{k=0}^T \{\xi^T(k) \bar{\Phi} \xi(k)\}$, where $\xi^T(k) = [\eta^T(k), \eta^T(k-r), w^T(k)]$. By (24), we know that $J_T < 0$ for any T . Let $T \rightarrow \infty$, we can conclude from Definition 2 that system (6) is mean-square exponentially stable with a prescribed H_∞ performance γ . Thus, the proof is completed. $\square$

Remark 6: By solving the feasibility problem (21) in Theorem 2, we can obtain the filter gain in (5). It should be pointed out that the LMI condition (21) is convex in the scalar γ^2 . Therefore, the optimal γ can be obtained by solving the following minimisation problem:

$$\min \beta \text{ s.t. (21) with } \beta = \gamma^2 \quad (28)$$

4 Illustrative example

In this section, an example of continuous stirred tank reactor (CSTR) is given to show the effectiveness of our proposed method. According to the linearised model of CSTR in [29], we assume that the measurement output in this paper is the reactor temperature ϑ due to the fact of easy measure. ϑ is measured by temperature sensor, and thus the outputs are transmitted to remote filter by sensor via communication channel. The objective of this paper is to design a filter to estimate the concentration C_B by using ϑ . The sensor measures the reactor temperature ϑ with sampling period $T = 0.5$ min. Discretising the system with sampling period T and taking into account disturbances, we obtain system (4) with the following parameters:

$$A = \begin{bmatrix} 0.503 & 0.016 & 0.019 \\ 0.219 & 0.559 & -0.001 \\ 0.97 & 0.959 & 0.606 \end{bmatrix} \quad B = \begin{bmatrix} 0.15 \\ -0.2 \\ 0.12 \end{bmatrix}$$

$$C = [0 \ 0 \ 1] \quad D = 1 \quad M = [0 \ 1 \ 0],$$

$w(k)$ represents external disturbances. It should be noted that the control input is removed here for filtering problem and model parameters can be found in Table 1 in [29].

We assume that the bound of the delay is 1.5 min. Then, one has $d = 3$ which means that there is a maximum of four measurements arriving at filter during a sampling interval and $d_k \in \{0, 1, 2, 3\}$. First, we choose the following transition probability matrix:

$$P = \begin{bmatrix} 0.4 & 0.4 & 0.1 & 0.1 \\ 0.4 & 0.2 & 0.2 & 0.2 \\ 0.5 & 0.2 & 0.15 & 0.15 \\ 0.4 & 0.35 & 0.15 & 0.1 \end{bmatrix}.$$

To show the usefulness and advantage of the proposed computational method for transition probability matrix Π by comparing with the one in [27], we choose $r = 2$ and $s = 1$ as a simple example to illustrate. Since $r = 2$, it is known from (3) that the set of feasible paths for π_r is $\{\delta_3 \in \mathcal{N}, \delta_2 \leq 2, \delta_1 > 1, \delta_0 > 0\}$. If $s = 1$, one has $\{\sigma_4 \in \mathcal{N}, \sigma_3 \in \mathcal{N}, \sigma_2 \in \mathcal{N}, \sigma_1 \leq 1, \sigma_0 > 0\}$ by (3). On the other hand, there is $\{\sigma_4 \in \mathcal{N}, \sigma_3 \leq 2, \sigma_2 > 1, \sigma_1 > 0, \sigma_0 > 0\}$ due to $r = 2$. Thus, the set of feasible paths for π_s is $\{\sigma_4 \in \mathcal{N}, \sigma_3 \leq 2, \sigma_2 > 1, \sigma_1 = 1, \sigma_0 > 0\}$. By the definition of $\mathcal{L}$ and $\delta_0 > 0$, one has $\mathcal{L}(\delta_0) = \text{diag}\{0, 1, 1, 1\}$. Similarly, we can obtain the values of $\mathcal{L}(\delta_i), i = 1, 2, 3$ and $\mathcal{L}(\sigma_j), j = 0, 1, \dots, 4$ as follows:

$$\begin{aligned} \mathcal{L}(\delta_1) &= \text{diag}\{0, 0, 1, 1\}, \mathcal{L}(\delta_2) = \text{diag}\{1, 1, 1, 0\}, \\ \mathcal{L}(\delta_3) &= \text{diag}\{1, 1, 1, 1\}, \mathcal{L}(\sigma_0) = \text{diag}\{0, 1, 1, 1\}, \\ \mathcal{L}(\sigma_1) &= \text{diag}\{0, 1, 0, 0\}, \mathcal{L}(\sigma_2) = \text{diag}\{0, 0, 1, 1\}, \\ \mathcal{L}(\sigma_3) &= \text{diag}\{1, 1, 1, 0\}, \mathcal{L}(\sigma_4) = \text{diag}\{1, 1, 1, 1\}. \end{aligned}$$

According to (10)–(12), It is easy to get the results of π_{rs} . However, the method proposed in [27] needs to find out all feasible paths of π_r and π_s . In this example, we know that the feasible paths of π_r and π_s are 72 and 24, which infer the complicated and cumbersome proposed method in [27]. By Theorem 1, we can obtain the transition probability matrix Π for the reordered sequence as follows:

$$\Pi = \begin{bmatrix} 0.4 & 0.6 & 0 & 0 \\ 0.4234 & 0.3193 & 0.2573 & 0 \\ 0.4273 & 0.3 & 0.15 & 0.1227 \\ 0.4278 & 0.2889 & 0.1833 & 0.1 \end{bmatrix}.$$

On the basis of the given transition probability matrix P , we choose a delay sequence. Fig. 3 shows the distribution of the chosen delay sequence τ_k , where the asterisk and the circle denote time delay and the time of packet disordering occurred, respectively. According to the time stamp of packets stored in the buffer and using the reordering rule in (3), the reordered sequence d_k is shown in Fig. 3.

It shows from Fig. 3 that the number of packet disordering phenomenon is 14 during the given interval. The distribution of delay sequence τ_k is $\{27, 12, 14, 7\}$ for $\tau_k = 0, 1, 2, 3$, whereas the distribution of reordered sequence d_k is $\{27, 23, 9, 1\}$ for $d_k = 0, 1, 2, 3$. It can be concluded that packet disordering seriously affects the probability distribution and the filter to select the input signal.

Fig. 3 Distribution of τ_k and d_k

Fig. 4 Estimated signal $\hat{z}$ and estimation error

In what follows, we choose the zero initial state and $\alpha = 0.97$. By solving the optimisation problem (28), we obtain optimal $\gamma^* = 0.213$ and filter gain matrices as follows:

$$A_f = \begin{bmatrix} -0.296 & -0.632 & 0.037 \\ 0.655 & 0.902 & -0.08 \\ -0.292 & -0.225 & 0.363 \end{bmatrix} \quad B_f = \begin{bmatrix} -0.196 \\ 0.204 \\ -0.238 \end{bmatrix}$$

$$C_f = [0.198 \quad -0.843 \quad -0.14] \quad D_f = -0.034.$$

Then, we choose the following disturbance signals to show the performance of the designed filter $w(k) = 2e^{-0.2k}\sin(0.6k)$, if $0 < k \leq 20$; otherwise, $w(k) = 0$. The simulations are shown in Fig. 4. It is concluded from Fig. 4 that the designed filter has a good estimation performance. In addition, when there is no disturbance, the estimation error can converge to zero. The H_∞ performance of the networked CSTR is given as follows. By computation, one obtains $\sum_{i=0}^{50} e^2(i) = 0.0028$, $\sum_{i=0}^{50} w^2(i) = 2.078$, and thus $\sqrt{\sum_{i=0}^{50} e^2(i) / \sum_{i=0}^{50} w^2(i)} = 0.0365 < \gamma^*$ which demonstrates the effectiveness of the designed H_∞ filter.

5 Conclusion

The exponential filtering problem has been studied for a class of networked systems with random delays and packet disordering. A Markov chain has been used to model the random transmission delays. A new method has been presented to calculate the reordered transition probability matrix. A sufficient condition of mean-square exponential stability has been derived for the filtering error system with a prescribed H_∞ performance, and the optimal filter parameters have been designed by solving a convex optimisation problem. The validity of the proposed filtering approach has been verified by an example.

6 Acknowledgments

The authors thank the anonymous reviewers for their valuable suggestions to improve the quality of this PAPER. This work was supported partly by the National Natural Science Foundation of China under Grant No. 61403344, the Hong Kong RGC under the Project CityU 111613, the Hong Kong Scholars Program under grant no. XJ2015040 and the Zhejiang Provincial Natural Science Foundation of China under grant no. LY17F030019.

7 References

- [1] Sandberg, H., Amin, S., Johansson, K.H.: 'Cyberphysical security in networked control systems: an introduction to the issue', *IEEE Control Syst.*, 2015, **35**, pp. 20–23
- [2] Deshmukh, S., Natarajan, B., Pahwa, A.: 'State estimation over a lossy network in spatially distributed cyber-physical systems', *IEEE Trans. Signal Process.*, 2014, **62**, pp. 3911–3923
- [3] Jin, D., Zeng, W., Su, H.S., et al.: 'Distributed estimation and control of mobile sensor networks based only on position measurements', *IET Control Theory Appl.*, 2017, **11**, pp. 1627–1633
- [4] Zhang, L., Gao, H., Kaynak, O.: 'Network-induced constraints in networked control systems – a survey', *IEEE Trans. Ind. Inf.*, 2013, **9**, pp. 403–416
- [5] Zhang, X.M., Han, Q.L., Yu, X.: 'Survey on recent advances in networked control systems', *IEEE Trans. Ind. Inf.*, 2013, **12**, pp. 1740–1752
- [6] Zhang, D., Shi, P., Wang, Q.G., et al.: 'Analysis and synthesis of networked control systems: a survey of recent advances and challenges', *ISA Trans.*, 2017, **66**, pp. 376–392
- [7] Zhang, X.M., Han, Q.L.: 'Event-triggered dynamic output feedback control for networked control systems', *IET Control Theory Appl.*, 2014, **8**, pp. 226–234
- [8] Liu, Y., Wang, Z., He, X., et al.: ' H_∞ filtering for nonlinear systems with stochastic sensor saturations and Markov time-delays: the asymptotic stability in probability', *IET Control Theory Appl.*, 2016, **10**, pp. 1706–1715
- [9] Yang, X., Feng, Z., Feng, J., et al.: 'Synchronization of discrete-time neural networks with delays and Markov jump topologies based on tracker information', *Neural Netw.*, 2017, **85**, pp. 157–164
- [10] Liu, M., Ho, D.W.C., Niu, Y.: 'Stabilization of Markovian jump linear system over networks with random communication delay', *Automatica*, 2009, **45**, pp. 416–421
- [11] Chen, B., Ho, D.W.C., Zhang, W., et al.: 'Networked fusion estimation with bounded noises', *IEEE Trans. Autom. Control*, 2017, **62**, pp. 5415–5421
- [12] Schenato, L.: 'Optimal estimation in networked control systems subject to random delay and packet drop', *IEEE Trans. Autom. Control*, 2008, **53**, pp. 1311–1317
- [13] Han, C., Zhang, H., Fu, M.: 'Optimal filtering for networked systems with Markovian communication delays', *Automatica*, 2013, **49**, pp. 3097–3104
- [14] Yang, R., Shi, P., Liu, G.P.: 'Filtering for discrete-time networked nonlinear systems with mixed random delays and packet dropouts', *IEEE Trans. Autom. Control*, 2011, **56**, pp. 2655–2660
- [15] Mathiyalagan, K., Su, H., Shi, P., et al.: 'Exponential H_∞ filtering for discrete-time switched neural networks with random delays', *IEEE Trans. Cybern.*, 2015, **45**, pp. 676–687
- [16] Zhang, D., Shi, P., Zhang, W.A., et al.: 'Energy-efficient distributed filtering in sensor networks: a unified switched system approach', *IEEE Trans. Cybern.*, 2017, **47**, pp. 1618–1629
- [17] Li, H., Shi, Y.: 'Robust H_∞ filtering for nonlinear stochastic systems with uncertainties and Markov delays', *Automatica*, 2012, **48**, pp. 159–166
- [18] Zhang, X.M., Han, Q.L.: 'Network-based H_∞ filtering using a logic jumping-like trigger', *Automatica*, 2013, **49**, pp. 1428–1435
- [19] Zhang, K., Li, X.R., Zhu, Y.: 'Optimal update with out-of-sequence measurements', *IEEE Trans. Signal Process.*, 2005, **53**, pp. 1992–2004
- [20] Zhang, C., Hu, J., Qiu, J., et al.: 'Event-triggered nonsynchronized H_∞ filtering for discrete-time T-S fuzzy systems based on piecewise Lyapunov functions', *IEEE Trans. Syst. Man Cybern. Syst.*, 2017, **47**, pp. 2330–2341
- [21] Zhao, Y.B., Liu, G.P., Rees, D.: 'Actively compensating for data packet disorder in networked control systems', *IEEE Trans. Circuits Syst. Express Briefs*, 2010, **57**, pp. 913–917
- [22] Liu, A., Zhang, W.A., Yu, L., et al.: 'New results on stabilization of networked control systems with packet disordering', *Automatica*, 2015, **52**, pp. 255–259
- [23] Cloosterman, M.B.G., Hetel, L., Van de Wouw, N., et al.: 'Controller synthesis for networked control systems', *Automatica*, 2010, **46**, pp. 1584–1594
- [24] Li, J., Joo, E.M., Yu, H.: 'Sampling and control strategy: networked control systems subject to packet disordering', *IET Control Theory Appl.*, 2016, **10**, pp. 674–683
- [25] Zhang, X.M., Han, Q.L.: 'Network-based H_∞ filtering for discrete-time systems', *IEEE Trans. Signal Process.*, 2012, **60**, pp. 956–961
- [26] Song, H., Yu, L., Zhang, W.A.: ' H_∞ filtering of network-based systems with random delays', *Signal Process.*, 2009, **89**, pp. 615–622
- [27] Liu, A., Yu, L., Zhang, W.A.: 'One-step receding horizon H_∞ control for networked control systems with random delay and packet disordering', *ISA Trans.*, 2011, **50**, pp. 44–52
- [28] Han, F., Feng, G., Wang, Y.: ' H_∞ filter design of networked nonlinear systems with communication constraints via T-S fuzzy dynamic models'. 2013 American Control Conf., 2013, pp. 6406–6411

- [29] Liu, A., Zhang, W.A., Yu, L., *et al.*: 'Moving horizon estimation for multi-rate systems'. Proc. 54th IEEE Conf. Decision Control, 2015, pp. 6850–6855