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Tan, Eng Leong; Yang, Zaifeng

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3-D non-uniform time step locally one-dimensional FDTD method

Zaifeng Yang[✉] and Eng Leong Tan

A 3-D non-uniform time step locally one-dimensional finite-difference time-domain (NUTS LOD-FDTD) method is presented. Unlike conventional and the other LOD-FDTD methods, NUTS LOD-FDTD method employs NUTS for different periods during the simulation. The update equation of NUTS LOD-FDTD method with arbitrary time step is given. Numerical results are provided to compare with conventional LOD-FDTD method and trade-off between efficiency and accuracy for different NUTSs.

Introduction: Finite-difference time-domain (FDTD) method is widely used to solve various electromagnetic wave propagation problems [1]. However, the time step has to be chosen no larger than Courant–Friedrichs–Lewy (CFL) limit. On the other hand, unconditionally FDTD methods such as alternating-direction-implicit FDTD (ADI-FDTD) [2, 3], leapfrog ADI-FDTD [4] and split-step/locally one-dimensional FDTD (LOD-FDTD) [5–7] methods can use time step larger than CFL limit and thus improve the computational efficiency compared with explicit FDTD method. The LOD-FDTD method is more computationally efficient than ADI-FDTD method. Nevertheless, more numerical errors would be introduced especially when larger time step is used for higher efficiency [8]. To reduce the numerical error for LOD-FDTD method, higher-order methods such as [7, 9] are further developed, but additional treatment procedures may be required. Therefore, one has to make a compromise between efficiency (larger time step) and accuracy (less numerical errors). To the best of the authors' knowledge, almost all the unconditionally stable FDTD methods employ uniform time step during the whole simulation. There is no investigation on LOD-FDTD method using non-uniform time step (NUTS) to trade-off between efficiency and accuracy before.

This Letter presents a 3-D NUTS LOD-FDTD method. Unlike conventional and the other LOD-FDTD methods, NUTS LOD-FDTD method employs NUTS for different periods during the simulation. The update equation of NUTS LOD-FDTD method with arbitrary time step will be given. Numerical results will be provided to compare with conventional LOD-FDTD method and trade-off between efficiency and accuracy for different NUTSs.

NUTS LOD-FDTD method: Given that Δt_n is the time step from the n th time index to the $(n+1)$ th time index, the update equation of the NUTS LOD-FDTD method can be written as

$$\left(I - \frac{\Delta t_n}{2} \mathbf{A}\right) \mathbf{u}^{n+1/2} = \left(I + \frac{\Delta t_n}{2} \mathbf{A}\right) \mathbf{u}^n \quad (1a)$$

$$\left(I - \frac{\Delta t_n}{2} \mathbf{B}\right) \mathbf{u}^{n+1} = \left(I + \frac{\Delta t_n}{2} \mathbf{B}\right) \mathbf{u}^{n+1/2}, \quad (1b)$$

where $\mathbf{A}$ and $\mathbf{B}$ are the two split matrices for LOD-FDTD method. Note that Δt_n is usually larger than the time step for CFL condition Δt_{CFL} . Δt_n can be arbitrary and different for each n . To update the field components using NUTS LOD-FDTD method from $\mathbf{u}^0$ to $\mathbf{u}^{n_{\text{max}}}$, the total time T_{total} used for computation is

$$\begin{aligned} T_{\text{total}} &= \Delta t_1 + \Delta t_2 + \dots + \Delta t_{n_{\text{max}}} = \sum_{n=1}^{n_{\text{max}}} \Delta t_n \\ &= \text{CFLN}_1 \Delta t_{\text{CFL}} + \dots + \text{CFLN}_{n_{\text{max}}} \Delta t_{\text{CFL}} = \Delta t_{\text{CFL}} \sum_{n=1}^{n_{\text{max}}} \text{CFLN}_n \quad (2) \end{aligned}$$

where the CFL number is $\text{CFLN}_n = \Delta t_n / \Delta t_{\text{CFL}}$. Unlike the previous LOD-FDTD methods that constant time step is applied and the $\text{CFLN} = \Delta t / \Delta t_{\text{CFL}}$ remains the same during the whole simulation, NUTS LOD-FDTD method can use smaller time step for certain periods, e.g. when the magnitudes of the field components are quite large or change dramatically during source excitation period.

Numerical results: An air-filled $80 \times 40 \times 40 \text{ mm}^3$ cavity meshed with uniform cells of 1 mm is simulated to demonstrate the accuracy and efficiency of the proposed NUTS LOD-FDTD method for various NUTSs compared with conventional LOD-FDTD method for various

CFLN. The L^2 -norm error of the electric field E_z is averaged at ten observation points with respect to explicit FDTD results from 0 to $T_{\text{total}} = 3.85 \text{ ns}$. The time step of the explicit FDTD method is set to Δt_{CFL} . The locations of the ten observation points are: (45, 25, 20), (35, 15, 20), (50, 30, 20), (30, 10, 20), (45, 25, 25), (35, 15, 15), (50, 20, 25), (35, 10, 30), (40, 20, 30) and (40, 20, 25). A line current source of differentiated Gaussian pulse along the z -direction: $(t - t_0)e^{-(t-t_0)^2/\tau^2}$ ($\tau = 200 \text{ ps}$, $t_0 = 3\tau$) is used to excite the electromagnetic fields through the centre of the cavity.

Table 1 shows the comparison between conventional LOD-FDTD method for various CFLN and NUTS LOD-FDTD method for various CFLN_n with different NUTSs. Different time steps for NUTS LOD-FDTD method are expressed in terms of CFLN_n according to (2). The L^2 -norm errors compared with explicit FDTD method and the iterations counts are calculated for both methods. For NUTS LOD-FDTD method, CFLN_n with NUTS for two different periods [ξ , ζ] are used to simulate the example. ξ is the CFLN used for source excitation period from 0 to 1.155 ns, whereas ζ is the CFLN used for the rest of the simulation until $T_{\text{total}} = 3.85 \text{ ns}$.

Table 1: Comparison between conventional LOD-FDTD method for various CFLN and NUTS LOD-FDTD method for various CFLN_n with different NUTSs

Method	Conventional LOD-FDTD				
CFLN	1	2	4	6	8
L^2 -norm error	0.023	0.054	0.123	0.206	0.317
Iterations	2000	1000	500	333	250
Method	NUTS LOD-FDTD				
CFLN _n	[2, 4]	[2, 6]	[2, 8]	[4, 6]	[4, 8]
L^2 -norm error	0.057	0.063	0.072	0.129	0.139
Iterations	650	533	475	383	325
Efficiency gain compared with CFLN = 1	3.077	3.752	4.211	5.222	6.154

It can be observed from Table 1 that the L^2 -norm errors of NUTS LOD-FDTD method with CFLN_n [ξ , ζ] lie between the ones of conventional LOD-FDTD method for CFLN = ξ and ζ (mostly close to CFLN = ξ which bears higher accuracy). Furthermore, the efficiency gains are calculated by the counts of iterations for different schemes compared with conventional LOD-FDTD method for CFLN = 1. Take NUTS LOD-FDTD for CFLN_n [4, 8] as an example, the executed iterations are 325 with an efficiency gain of 6.154 and the L^2 -norm error is 0.139. The L^2 -norm error is just a little more than the one for CFLN = 4 (i.e. 0.123) and much less than the one for CFLN = 8 (i.e. 0.317) using conventional LOD-FDTD method. With only slight increase of 0.016 for the L^2 -norm error for CFLN_n [4, 8] compared with CFLN = 4, the iteration count is decreased from 500 to 325, which is 1.538 times faster. Similarly, the iteration is 475 for CFLN_n [2, 8] using NUTS LOD-FDTD method. It is less than 500 for CFLN = 4 using conventional LOD-FDTD method, which means that CFLN_n [2, 8] is more efficient than CFLN = 4. Moreover, the L^2 -norm error for CFLN_n [2, 8] is also less than the one for CFLN = 4. This result shows that the proposed NUTS LOD-FDTD method for CFLN_n [2, 8] is not only more computationally efficient, but also more accurate than conventional LOD-FDTD method for CFLN = 4.

Fig. 1 Electric field E_z at observation point (40, 20, 25) with respect to time using explicit FDTD, LOD-FDTD for CFLN = 4, 8 and NUTS LOD-FDTD with CFLN_n [4, 8] method

Fig. 1 shows the electric field E_z at observation point (40, 20, 25) with respect to time using explicit FDTD, LOD-FDTD for CFLN = 4, 8 and NUTS LOD-FDTD with CFLN_n [4, 8] method. It can be seen that the magnitude of the electric field E_z varies largely when the source is excited, so finer time step used for this period based on NUTS LOD-FDTD can soothe the errors which may otherwise be caused by larger time step.

Conclusion: This Letter has presented a 3-D NUTS LOD-FDTD method. Unlike conventional and the other LOD-FDTD methods, NUTS LOD-FDTD method employs NUTS for different periods during the simulation. The update equation of NUTS LOD-FDTD method with arbitrary time step has been given. Numerical results have been provided to compare with conventional LOD-FDTD method and trade-off between efficiency and accuracy for different NUTSs.

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One or more of the Figures in this Letter are available in colour online.

Zaifeng Yang and Eng Leong Tan (School of Electrical and Electronic Engineering, Nanyang Technological University, 639798 Singapore, Singapore)

✉ E-mail: zyang007@e.ntu.edu.sg

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