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Understanding Multiple-Input Multiple-Output Active Noise Control from a Perspective of Sampling and Reconstruction

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Abstract—This paper formulates the multiple-input multiple-output active noise control as a spatial sampling and reconstruction problem. With the proposed formulation, the inputs from the reference microphones and the outputs of the anti-noise sources are regarded as spatial samples. We show that the proposed formulation is general and can unify the existing control strategies. Three control strategies, for instance, are derived from the proposed formulation and linked to different cost functions in the practical implementation. Finally, simulation results are presented to verify the effectiveness of our analysis.

I. INTRODUCTION

The active noise control (ANC) proposed in the early 20th century has gained rapid development in the last three decades to tackle noise problems, where passive noise control measures are relatively less efficient in terms of size, weight, volume and cost [1], [2]. The principal task of the ANC is to transmit an anti-noise wave with the same amplitude but opposite phase of the noise wave in order that the noise and anti-noise waves can cancel each other on the basis of the acoustic superposition [1], [2]. This definition is readily extended to the 3D space, where the ANC constructs an anti-noise field and results in reduced sound pressure levels (SPLs) in the control zones, *a.k.a.* the zones of quietness (ZoQs) [3].

The ANC can be categorized by the size of the ZoQs into the local ANC and the global ANC. The local ANC has been widely applied in many applications [4], [5], [6], [7]. A common trade-off of the local ANC is the increased SPLs outside the ZoQs [8], [9]. A thin duct, as one of the most well-known testbeds for different ANC techniques, is an example where the global ANC is feasible [10]. Just one anti-noise source is often sufficient, since the wave propagation in the thin duct is expressed by a 1D acoustic equation. However, when the diameter of the duct is larger than the noise wavelength, multiple anti-noise sources are necessary to cover the cross section of the duct. The global ANC is hence achievable beyond the near field of the anti-noise sources.

In a free field where there is no duct to regulate the propagation path of the noise wave, the global ANC is complicated to implement. One theoretical approach is to introduce a continuous layer of anti-noise sources to enclose the noise sources [11]. In practice, a sufficient number of discrete anti-noise sources are deployed, for example, to abate the transformer's noise [12]. It has been noted that the ANC

Fig. 1. Two configurations of the MIMO ANC, where $\times$ and $\circ$ indicate anti-noise sources and sensors, respectively.

involving multiple outputs can be essentially related to the waveform synthesis (WFS) [13], [14].

The driving signals of the anti-noise sources rely on certain observations of the noise field. Therefore, the ANC has to involve sensors too. Some of the sensors, namely the reference sensors, sample the noise field. The other sensors, namely the error sensors, monitor the ZoQs, either physically or virtually [15], [16]. The ANC consisting of multiple anti-noise sources and sensors are referred to as the multiple-input multiple-output (MIMO) ANC.

Figure 1 illustrates two configurations of the anti-noise sources and sensors when the MIMO ANC is applied. Due to the principle of reciprocity, either the outer zone or the inner zone can be the ZoQ. Assuming that noise sources exist in one zone, the MIMO ANC creates the ZoQ in the other zone. Therefore, the reference and error sensors are not specified in Fig. 1. The reference sensors are always those closer to the noise field, while the error sensors are the rest closer to the ZoQ. Figure 1(b) shows an opening on an infinite rigid plate. It can be understood as a segment of Fig. 1(a) when the circles are infinitely large or a short duct possesses a very large diameter. Moreover, the practical meaning of the configuration shown in Fig. 1(b) is of importance. Many recent ANC applications, such as the virtual sound barrier [18], active acoustic shielding [17], and noise canceling window [19], can be investigated using Fig. 1(b) for their physical limitations.

The following part of this paper presents the formulation of the MIMO ANC as a spatial sampling and reconstruction

problem. With the proposed formulation, the inputs from the reference microphones and the outputs of the anti-noise sources are regarded as spatial samples. We will show that different control strategies can be derived from the proposed formulation and the simulation results obey well with our analysis.

II. MIMO ANC FORMULATION

To facilitate this derivation, we first define that the anti-noise sources are distributed on a control surface. The control surface is then uniformly meshed with a very small grid size. Each anti-noise source is assumed to locate on a grid. The reference sensors are assumed to be distributed on the control surface too. It is noteworthy that in practice, the reference sensors have to be placed in front of the control surface to ensure the causality of the feedforward ANC when canceling the wideband noise. Hence, the noise sources are assumed to be tonal here.

The noise path model is defined as a matrix consisting of all the transfer functions from the control surface to the error sensors, which is written as

$$G_d = \begin{bmatrix} h_{11} & h_{21} & \cdots & h_{J1} \\ h_{12} & h_{22} & \cdots & h_{J2} \\ \vdots & \vdots & \ddots & \vdots \\ h_{1M} & h_{2M} & \cdots & h_{JM} \end{bmatrix}, \quad (1)$$

where $h_{i,j}$ is a complex number; J and M denote the numbers of the grids and error sensors, respectively. The placement of the error sensors is not specified at this step.

Let x_d denote the noise signal vector with the size of J by 1. Based on the Huygens' principle, x_d is a discrete representation of the noise field passing through the control surface. The error signal vector before the MIMO ANC is operated is written as

$$D = G_d x_d. \quad (2)$$

Furthermore, the anti-noise path model is defined as a matrix consisting of all the transfer functions from the anti-noise sources to the error sensors, which is written as

$$G_s = \begin{bmatrix} s_{11} & s_{21} & \cdots & s_{K1} \\ s_{12} & s_{22} & \cdots & s_{K2} \\ \vdots & \vdots & \ddots & \vdots \\ s_{1M} & s_{2M} & \cdots & s_{KM} \end{bmatrix}, \quad (3)$$

where $s_{i,j}$ is a complex number and K denotes the number of the anti-noise sources. Since the anti-noise sources are distributed on the control surface, G_s is obviously a subset of G_d .

If the configuration of the anti-noise sources is defined by a sampling matrix

$$S_K = \begin{bmatrix} 0 & 0 & 1 & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & \cdots & 0 & 1 & 0 & \cdots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & 0 & 1 & 0 & 0 \end{bmatrix}, \quad (4)$$

the anti-noise path model can be related to the noise path model by

$$G_s = G_d S_K^H, \quad (5)$$

where H denotes the conjugate transpose. S_K is a K by J matrix, having only one non-zero unity element in each row; and O is an all-zero row-wise vector, which is introduced for the ease of notation.

Similarly, the reference sensors provide a limited number of observations of the noise field passing through the control surface. The outputs of the reference sensors form the reference signal vector x_s with the size of L by 1. Here, L is the number of the reference sensors. Another sampling matrix S_L with the size of L by J being defined similar to S_K yields

$$x_s = x_d S_L^H. \quad (6)$$

The principal task of the MIMO ANC is to find out the driving signals of the anti-noise sources. Let y denotes the driving signal vector with the size of K by 1. The error signal vector after the MIMO ANC is operated is written as

$$E = D + G_s y = D - G_s W x_s, \quad (7)$$

where W is the control matrix. As aforementioned, the driving signals of the anti-noise sources rely on the reference signal vector, *i.e.* $y = -W x_s$.

The cost function of the ANC usually adopts the l^2 -norm form as

$$\min \|G_d x_d - G_s W x_s\|_2^2. \quad (8)$$

Thus, the optimum solution of W is given by

$$W^* = G_s^\dagger G_d x_d x_s^\dagger, \quad (9)$$

where $\dagger$ denotes the pseudo inverse.

We propose an alternative way to solve (9), that is, to treat (9) as two decoupled problems, which are

$$\min \|G_d - G_s W_1\|_2^2 \quad (10)$$

and

$$\min \|x_d - W_2 x_s\|_2^2. \quad (11)$$

The optimum solutions of W_1 and W_2 are written respectively as

$$W_1^* = G_s^\dagger G_d \quad (12)$$

and

$$W_2^* = x_d x_s^\dagger. \quad (13)$$

Obviously, $W^* = W_1^* W_2^*$ is valid.

The optimization problem in (11) is essentially a sampling and reconstruction problem. The sampling process is to obtain the subset x_s from the whole set x_d . The reconstruction process is to use linear combinations of the subset elements to generate the whole set. Therefore, the solution to (11) should be equivalent to the interpolation formula in the sampling theorem.

For a quick validation, we consider that there are $L = 8$ reference sensors in a uniform linear array (ULA) as illustrated in Fig. 1(b). The spacing between neighboring sensors is 0.12

Fig. 2. An example of W_2^* that consists of 8 *sinc* functions shifted by the locations of the reference sensors.

m. The opening size is thus set as 0.96 m. Considering a noise frequency of 1000 Hz at an arbitrary incidence angle, (13) is an over-determined solution to (11). Figure 2 shows W_2^* under the aforementioned conditions. A series of *sinc* functions are obtained. There are some truncation effects due to the finite aperture [20]. When the configuration of the reference sensors is not so regular as the ULA, finding W_2^* is a similar task to finding the suitable sampling kernel [21].

The optimization problem in (10) is another sampling and reconstruction problem. Due to the involvement of the noise and anti-noise path models, the samples have been filtered. Although the anti-noise sources can be configured in the same pattern as the reference sensors, the sampling process implied by (10) is no longer regular. Practically, this means that W_1^* needs extra cares to solve as it is associated with the placement of the error sensors.

III. CONTROL STRATEGIES

In this section, we will show that the placement of the error sensors, which also means a specified cost function, leads to a determined control strategy in the MIMO ANC. For convenience, we assume the same number of the anti-noise sources and reference sensors, *i.e.* $L = K$. Since they are both uniformly distributed on the control surface, each anti-noise source should collocate with a reference sensor.

A. Far-Field Optimization

Firstly, there is a case when (10) can have the same solution as (11). That is to place all the error sensors in the far-field with the equal angular separation. In this case, the cost function is the sum of the squared error signals in the far-field. The solution to (10) is simply given by

$$W_1^* = W_2^{*H}. \quad (14)$$

Thus, the optimum control matrix is calculated as

$$W_F = W_2^{*H} W_2^*. \quad (15)$$

This MIMO ANC is a fully coupled $K \times K \times J$ system. When the number of the anti-noise sources is large, the computational complexity is overwhelming. However, the optimum

Fig. 3. Sound pressure level distribution (a) when the noise incidence angle is 30° ; (b) with the MIMO ANC optimized for the far-field noise reduction; (c) with the MIMO ANC optimized for the noise incidence angle of 0° ; (d) with the MIMO ANC optimized for the summed far-field noise reduction.

control matrix W_F ensures the noise reduction performance regardless of the noise incidence angle [15].

B. Decentralized Far-Field Optimization

The decentralized control matrix can be achieved when the noise source location is pre-defined. Presuming the the noise incidence angle is 0° , all the reference sensors should ideally have the same output. Using this presumption as a constraint, the optimum control matrix is constructed as

$$W_D = \text{Diag}\{W_2^{*H} I_{J,1}\}, \quad (16)$$

where $\text{Diag}\{\cdot\}$ is a function to create a diagonal matrix from a vector; and $I_{J,1}$ is an all-one vector with the size of J by 1. Since W_D is a diagonal matrix, the driving signal of an anti-noise source relies only on the collocated reference sensor. In practice, W_D is often obtained when the MIMO ANC is trained for a fixed far-field noise source [22], [23].

C. Summed Far-Field Optimization

An alternative approach to reduce the computational complexity is to reduce the size of the error signal vector. Therefore, the mixed error approach is considered [17]. The cost function is changed to the square of the sum of the error signals. This cost function can result in a much simplified algorithm. The noise reduction performance is still maintained if every error sensor always provides a similar output level.

With our proposed formulation, the optimum control matrix using the mixed error approach is written as

$$W_M = \text{Diag}\{I_{1,K} W_F\}, \quad (17)$$

where $I_{1,K}$ is an all-one vector with the size of 1 by K .

Fig. 4. Noise reduction performance of the MIMO ANC using the far-field optimization.

IV. SIMULATION RESULTS

Acoustic simulations are carried out using k-Wave, which is an open-sourced MATLAB toolbox [24], [25], [26]. A 2 m by 2 m room is modeled with the grid size of 0.01 m. A window is modeled at the bottom of the room, as shown in Fig. 3(a). The thickness of the window is 0.1 m and the opening is 0.96 m. $K = 8$ anti-noise sources and $L = 8$ reference sensors are configured at the window opening like a ULA with the inter-element spacing of 0.12 m. The noise field is rendered as a plane wave, while the noise frequency and incidence angle vary from 100 Hz to 3000 Hz and from 0° to 90° , respectively.

An example case is shown in Fig. 3, when the noise frequency is 1000 Hz and incidence angle is 30° . The SPL inside the room reaches about 80 dB. After the fully coupled MIMO ANC is operated, the SPL is reduced to less than 40 dB globally. The decentralized MIMO ANC achieves about 20 dB noise reduction, which is less efficient than the fully coupled MIMO ANC.

Figure 4 shows the overall noise reduction performance of the fully coupled MIMO ANC in the root mean square (RMS) sense. The room is divided into two evaluation zones, the upper half (far-field) and lower half (near-field). In the upper half of the room, below the Nyquist sampling frequency, W_F ensures the constant noise reduction performance across the noise incidence angles. The noise reduction performance is compromised in the lower half of the room, since the control matrix is not optimized for the near-field condition [23].

Figure 5 shows the overall noise reduction performance of the decentralized MIMO ANC. When the noise incidence angle is 0° , the noise reduction achieved by W_D is slightly better than the fully coupled solution W_F . This is because that the assumption hold by W_D is fulfilled when the noise incidence angle is 0° , while W_F is the optimal solution for different incidence angles obtained by the RMS calculation.

Figure 6 is the result when W_M is adopted. This is similar to the result of W_D , in both the far-field and near-field.

Moreover, in the plot of the near-field noise reduction, there is an obvious inflection point in between 1000 Hz and 3000 Hz where the decreasing trend of the noise reduction with respect to the frequency is suddenly changed. This inflection frequency appears in coincidence with the aliasing frequency of the ULA. A detailed wavenumber-based explanation can be found in [27], which implies that the above observation depends on the number of the secondary sources.

V. CONCLUSIONS

This paper derives the control matrix of the MIMO ANC from a spatial sampling and reconstruction problem. Three control strategies have been unified under the proposed formulation for the open window type of ANC systems. Observations made in the simulation results of this paper are in good agreement with the experiment results of several previous works. The proposed formulation is hence validated and expected to accelerate the process of finding the physical limitations when the MIMO ANC is deployed in a specified application.

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Fig. 5. Noise reduction performance of the MIMO ANC using the decentralized far-field optimization.

Fig. 6. Noise reduction performance of the MIMO ANC using the summed far-field optimization.

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