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Structure of non-maximally decimated filter bank derived by MSFGs

Luyun Wang, Ronggang Qi and Guoan Bi[✉]

Practical applications in digital communication need non-maximally decimated filter banks to provide oversampled baseband signals for other necessary operations. The non-maximally decimated filter banks allow the required prototype filter to have wider transitional band to significantly reduce the required computational complexity. This Letter presents a non-maximally decimated filter bank structure derived by using multi-rate signal flow graphs.

Introduction: Multi-rate filter banks have been widely used for various applications such as transmultiplexing in communications, speech, audio and image coding [1–3]. Although the popular maximally decimated filter banks (MDFBs) have been extensively studied [2–5], the non-maximally decimated filter banks (NMDFBs) have been reported to be useful for obtaining oversampled base-band signals needed for symbol timing estimation and clock tracking, carrier recovery, and equalisation [1]. Efficient implementation of NMDFBs has been investigated, and design techniques and applications of NMDFB have been reported, e.g. [6, 7]. The MDFB and NMDFB generally share similar structures consisting of polyphase filters, down-samplers and a fast Fourier transform (FFT). One main difference between them is that the MDFB generally requires very high filter order since ideally the transitional bandwidth of brick-wall prototype filter is zero, which virtually needs an infinite filter order. Although various methods of designing perfect reconstruction filter banks have been reported [2–5] to allow a non-zero transitional bandwidth, the choice of efficient filter design is quite limited because of the perfect reconstruction condition.

Fig. 1 *N*-channel signal spectrum and the generic filter bank ($M = N/2$)

Let us consider the filter needed for a NMDFB. Fig. 1 shows a signal consisting of N uniformly spaced channels to be separated and the multi-rate signal flow graph (MSFG) of a demultiplexer. The channel spacing is equal to the bandwidth (BW) of each channel and the input sampling frequency is $F_s = N \cdot BW$. The original N -channel signal is frequency-shifted so that channel 0 is centred at zero frequency. Assuming the decimation factor is $M = N/2$, the sampling frequency of each output channel is oversampled by a factor of 2. In this case, the passband and stopband edges are $F_{\text{pass}} \geq BW/2$ and $F_{\text{stop}} \geq (2N - M) \cdot BW/2$, respectively. Then the transitional bandwidth becomes $\Delta F = F_{\text{stop}} - F_{\text{pass}} = (N/M - 1) \cdot BW$. Because of the large transitional bandwidth, the order of the prototype filter can be reduced to significantly minimise the overall computational complexity of the NMDFB. Although the merits of NMDFB have brought much attention, the studies on the structure for efficient implementation have not been sufficiently reported yet. In [6], a filtering solution is presented for the NMDFB being implemented by an FFT based multi-branch polyphase filters. An oversampled modulated filter bank was also studied for perfect reconstruction [7].

In this Letter, we shall provide detailed steps of using MSFGs to derive an FFT based filter bank structure for the NMDFB.

Transformations of multi-rate operations: Three MSFG transformations that have not been reported in the literature are presented for deriving the FFT based structure of NMDFB. The first one is the polyphase implementation of modulation operation, $y(n) = x(n) \cdot \phi(n)$, shown on the left side of Fig. 2, where $\phi(n)$ is the modulating function. An M -branch

polyphase structure, shown on the right side of Fig. 2, equivalently performs the modulation operation. Each branch contains a decimated modulating function, $\phi(nM + i)$, between a pair of up- and down-samplers by a factor of M . The operation in each branch is defined as $y_i(n) = y(nM + i) = x_i(n) \cdot \phi_i(n)$, where $x_i(n) = x(nM + i)$ and $\phi_i(n) = \phi(nM + i)$. The proof of this transformation is given in [8].

Fig. 2 *Polyphase implementation of modulation operation*

The next transformation is frequency shift operation, as shown in Fig. 3. This transformation,

$$y(n) = (x(n) e^{-j\alpha n}) * h(n) = (x(n) * (h(n) e^{j\alpha n})) \cdot e^{-j\alpha n},$$

can be easily proved by using the definition of linear filter with simple mathematical manipulations.

Fig. 3 *Frequency shift and its equivalent operation*

The last transformation is the parallel data transfer, as shown in Fig. 4. On the left side, the sum of M parallelly delayed and up-sampled data passes a positive delay, z^{M-1} , before being re-distributed by a delay line into M parallel down-sampled outputs. This operation is equivalently a parallel M -path data transfer, as shown on the right side of Fig. 4. This equivalence can be easily proved by following the data movements according to the operations of up- and down-samplers in each branch in the time domain [8].

Fig. 4 *Equivalence of parallel data transfer*

Filter bank structure derived by MSFGs: Let us consider a digital demultiplexing system, as shown in Fig. 1, whose input is an N -channel signal. Each branch performs operations of frequency shift, low pass filtering and down-sampling. Assume each output channel is oversampled by a factor of 2, i.e., $M = N/2$. With a series of MSFG transformations, we show the main steps of deriving a desirable structure for an N -channel NMDFB. Starting with the operations along branch k in Fig. 1, the modulation operation becomes an M -branch polyphase structure based on Fig. 2, as shown on the left side of dashed line in Fig. 5. The cascade of a filter and a down-sampler on the right side of Fig. 5 can be replaced by an $N/2$ -branch polyphase structure [3].

By applying the transformation in Fig. 4 for the operations between the solid lines in Fig. 5, we obtain Fig. 6 which contains an $N/2$ -branch structure. Each branch performs down-sampling by a factor of $N/2$ and filtering whose input and output are associated with a twiddle factor, respectively. Following Fig. 3, the multiplication by the twiddle factor, W_2^{-nk} , can be implemented by multiplying the same twiddle factor with the output of a modified filter whose impulse response is $W_2^{nk} h_j(n)$. With a two-branch polyphase implementation of the modified filter, the

operations along a branch, between the dashed lines in Fig. 6, can be performed by the structure in Fig. 7. The twiddle factor, $W_N^{k(N/2+j)}$, along the lower branch is the result of combining $(-1)^k$ and W_N^{jk} . Then the twiddle factors are moved inside the summation of the two branches in Fig. 7b.

Fig. 5 Transformation of a demultiplexer branch

Fig. 6 Multi-branch structure

Fig. 7 Transformation of the branch operations in Fig. 6

The branch filter in Fig. 6 is implemented by two-channel polyphase filters, as shown in Fig. 7a. Then the two twiddle factors are moved inside the summation to obtain Fig. 7b. Based on Fig. 2, the twiddle factor, $(W_2)^{-nk}$, is performed by a two-branch implementation that contains down-sampling, twiddle factor rotation and up-sampling, as shown between the dashed lines in Fig. 7c. Fig. 7c is obtained by grouping the branches associated with the same twiddle factors and re-allocating the common twiddle factors outside the summations. It is noted that the twiddle factors in Fig. 7c are not related to the time index, they can be moved freely at any location in the figure. To further simplify presentation, the operations between the dashed lines in Fig. 7c are represented by a 2×2 permutation, as shown in Fig. 7d. Although this function contains delays, down-samplers, up-samplers and summations, it is quite simple. Table 1 lists the input/output sequences of this function, which can be proved according to Fig. 7.

Table 1: 2×2 permutation

n	0	1	2	3	4	5	6	...
x_0	$x_{0,0}$	$x_{0,1}$	$x_{0,2}$	$x_{0,3}$	$x_{0,4}$	$x_{0,5}$	$x_{0,6}$	...
x_1	$x_{1,0}$	$x_{1,1}$	$x_{1,2}$	$x_{1,3}$	$x_{1,4}$	$x_{1,5}$	$x_{1,6}$	...
y_0	0	$x_{0,0}$	$x_{1,0}$	$x_{0,2}$	$x_{1,2}$	$x_{0,4}$	$x_{1,4}$	...
y_1	0	0	$x_{0,1}$	$x_{1,1}$	$x_{0,3}$	$x_{1,3}$	$x_{0,5}$	...

By combining the filter operations and the 2×2 permutation, included in the dashed square, as one functional block, we obtain Fig. 8a, which is for channel k only. In this figure, the input is distributed by a delay line and then down-sampled by a factor of $N/2$ before being filtered. The outputs of the filtering are summed after rotation operation. This is the final structure for a single branch in Fig. 5 after all the transformation steps mentioned above.

Fig. 8 FFT-based structure of the N-channel demultiplexer

Substituting the operations along all the branches in Fig. 1 with Fig. 7d, we obtain the structure of the entire filter bank. It is seen that these twiddle factors are exactly those used for an N -point FFT. However, they are not in the required order for an FFT operation. Therefore, an intermediate process is needed to rearrange the required input order of an FFT operation. The entire structure of the NMDFB is given in Fig. 8b which uses $N/2$ polyphase filters and a reordering process before the FFT operation. The delay line at the input of Fig. 8a is performed by a $1:N/2$ commutator in Fig. 8b.

Conclusions: By manipulating the MSFGs, a computationally efficient structure is transformed into an FFT based NMDFB. It has a regular structure and interconnections for an easy hardware implementation. The proposed structure has been verified through simulations.

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One or more of the Figures in this Letter are available in colour online.

Luyun Wang (School of EEA, City Institute of Dalian University of Technology, Dalian, People's Republic of China)

Ronggang Qi (Broadlands, VT iDirect, Inc., USA)

Guoan Bi (Nanyang Technological University, Singapore, Singapore)

✉ E-mail: egbi@ntu.edu.sg

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