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2016

Van Coile, R., Balomenos, G. P., Pandey, M. D., Caspeelee, R., Criel, P., Wang, L., et al. (2016). Computationally efficient estimation of the probability density function for the load bearing capacity of concrete columns exposed to fire, 2016 International Symposium of the International Association for Life-Cycle Civil Engineering, Delft, The Netherlands, 16-19 October 2016.

<https://hdl.handle.net/10356/88637>

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Computationally efficient estimation of the probability density function for the load bearing capacity of concrete columns exposed to fire

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ABSTRACT: Concrete columns are critical for the stability of structures in case of fire. In order to allow for a true Performance Based Design, the design should be based on considerations of risk and reliability. Consequently, the probability density function (PDF) which describes the load-bearing capacity of concrete columns during fire exposure has to be assessed. As second order effects can be very significant for columns, traditional probabilistic methods to determine the PDF become very computationally expensive. More precisely, for most current numerical calculation tools (e.g. Finite Element), the computational requirements are so high that traditional Monte Carlo simulations become infeasible for any practical application. In order to tackle this, a computationally very efficient method is presented and applied in this paper. The method combines the Maximum Entropy Principle together with the Multiplicative Dimensional Reduction Method, and Gaussian Interpolation, resulting in an estimation of the full PDF requiring only a very limited number of numerical calculations. Although the result is necessarily an approximation, it gives very good assessment of the PDF and it is a significant step forward towards true risk- and reliability-based structural fire safety.

1 INTRODUCTION

Fire is one of the most severe load conditions for structures, and it is generally accepted that the structural stability of (concrete) columns in case of fire exposure is of primary importance for the overall stability of the building. However, considering the low probability of a fully developed fire to occur and considering the large uncertainty with respect to the fire load and ventilation characteristics, it is at times questioned whether a significant fire rating of the structural elements makes sense economically.

Dependent on the considered performance criterion, structural stability should be ensured for a specified period of time, for example to allow for evacuation and fire-fighting rescue actions to be completed, or alternatively up to the point of complete burn-out of the fire.

Whatever the performance criterion, there will always be a (small) probability of premature failure. More precisely, it is exactly this probability of premature failure which has to be evaluated implicitly or explicitly in a Performance Based Design (PBD). In case of an implicit evaluation the PBD can be based on a demonstration that structural stability is maintained for a representative (conservative) design fire.

If such implicit evaluation is performed the true safety level remains unknown.

However, in order to rationally balance investments in structural safety (i.e. fire resistance, blast resistance, extreme wind load resistance...), an explicit evaluation of the safety level is required. Referring specifically to fire exposure, the annual probability P_f of fire induced structural failure is given by:

$$P_f = P_{f,\bar{f}} p_{\bar{f}} \quad (1)$$

where $P_{f,\bar{f}}$ = probability of structural failure given the occurrence of a fully developed fire; $p_{\bar{f}}$ = annual probability of a fully developed fire.

The annual probability $p_{\bar{f}}$ can be related to the probabilities of fire ignition and successful fire suppression, as specified in Handbook 5 of the Eurocodes (Holicky et al., 2005). Evaluating the probability $P_{f,\bar{f}}$ of structural failure given a fully developed fire is however less straightforward.

Ideally, the total structural behavior is considered taking into account the interaction between different parts of the structure by applying an advanced Finite Element Model (FEM). This approach will be time-consuming and expensive for many projects and necessarily requires a detailed case-specific analysis. Therefore, a faster element-based evaluation may be preferred, allowing for more general guidelines and

conclusions which can easily be transferred across projects.

Either way, the probability density function (PDF) describing the uncertain response of the total structure or of the structural element has to be determined implicitly or explicitly. Evaluating for example the PDF f_R of the load bearing capacity R during fire exposure, the probability of failure $P_{f,fi}$ can be calculated from Equation 2 with E the uncertain load on the structure or structural element. Widely accepted stochastic models for E can be found in (JCSS, 2007).

$$P_{f,fi} = \int_0^E \int_0^E f_E(E) f_R(R) dR dE \quad (2)$$

Traditionally, the PDF f_R is evaluated implicitly through Monte Carlo simulations, see for example (Sidibé et al., 2000), but this method is too time-consuming when applied in conjunction with complex computationally expensive models. Alternatively, more efficient methods as FORM or Latin Hypercube Sampling can be applied, but these respectively require a known or approximated analytical formulation for the resistance effect, or prior knowledge of the type of PDF describing f_R . Therefore, a practical and computationally efficient method is required.

In this paper the PDF describing the maximum load bearing capacity of eccentrically loaded concrete columns during fire exposure is estimated using a computationally very efficient method. Firstly, the calculation method is presented for determining the maximum eccentric load P_{max} for a single deterministic column, taking into account second order effects. Subsequently, a methodology for estimating the PDF of P_{max} is presented, requiring only a very limited number of model evaluations.

The results can be applied in a Performance Based Design focusing on limiting the probability of failure of the columns during fire. Note that the part of the methodology focusing on estimating the PDF can be applied in conjunction with FEM models as well, as illustrated by Balomenos et al (2015) for normal design situations.

2 MAXIMUM ECCENTRIC LOAD FOR CONCRETE COLUMN DURING FIRE

2.1 General description and literature review

The considered resistance parameter is the load bearing capacity of concrete columns for (eccentric) vertical loads. The eccentricity of the vertical load can relate both to geometric imperfections and designed eccentricity of the soliciting load.

For specific situations analytical formulas can be applied to take into consideration second order effects allowing for straightforward reliability calculations, see for example Achenbach and Morgenthal

(2015) for concrete columns with pinned supports. However, in order to allow in future work to consider concrete frames or multi-story columns exposed to a fire contained on a single floor, a numerical tool has been developed based on the Direct Stiffness Method (DSM), (Melosh, 1963).

The main principles of the model are explained below, but considering the similarity of the models reference is made to (Kodur and Dwaikat, 2008) and (Prakash and Gaurav, 2015). The version of the model presented further assumes the thermal expansion is not being restrained. Work is currently being done on expanding the calculation tool to allow for situations where this is not the case.

2.2 Direct Stiffness Method for second order structural analysis

Consider a 2D structural frame consisting of a number of beams and columns. Each of the components of the frame can be idealized as a combination of small beam-elements whose response to forces acting in its end-nodes i^* and j^* can be studied in isolation. Due to these forces and an optional uniformly distributed load p acting perpendicular to the beam-element, the element deforms as visualized in Figure 1. The equilibrium of the beam-element in the local axis system of Figure 1 is defined in matrix format through Equations (3) – (12), based on (Vandepitte, 1979), with forces and displacements as indicated in Figure 1, N the axial compression force (>0) acting on the beam-element, l the length of the beam-element, EA the axial stiffness, EI the bending stiffness, Λ and Γ stability functions as defined in (Vandepitte, 1979) and given by (13) and (14), $\Pi = \Lambda + \Gamma$, ε and ς given by (15) and (16), and t the transpose operator. Since $N = X_i/2 - X_j/2$ can be initially unknown, the equations are solved iteratively. Furthermore, due to fire exposure the stiffness of the column is nonlinearly dependent on the fire duration, the force acting on the column and on the acting bending moment. Therefore, also the stiffness components are updated iteratively in accordance with the material models of EN 1992-1-2 (CEN, 2004) and a cross-sectional calculation model as in (Van Coile, 2015).

Figure 1. Deformation of an isolated beam-element and nodal forces acting in the end-nodes, based on (Caspele, 2015).

$$[F'] = [S'] [D'] + [F_0'] \quad (3)$$

$$[F'] = [X_i \ Y_i \ M_i \ X_j \ Y_j \ M_j]^t \quad (4)$$

$$[D'] = [u_i \ v_i \ \varphi_i \ u_j \ v_j \ \varphi_j]^t \quad (5)$$

$$[F_0'] = \begin{bmatrix} 0 & \frac{pl}{2} & \frac{pl^2}{\epsilon^2} \varsigma & 0 & \frac{pl}{2} & -\frac{pl^2}{\epsilon^2} \varsigma \end{bmatrix} \quad (6)$$

$$[S'] = \begin{bmatrix} S_1' & S_2' \\ S_3' & S_4' \end{bmatrix} \quad (7)$$

$$[S_1'] = \begin{bmatrix} \frac{EA}{l} & \frac{M_i + M_j}{l^2} & 0 \\ \frac{M_i + M_j}{l^2} & \frac{4EI}{l^3} \Pi - \frac{N}{l} & \frac{2EI}{l^2} \Pi \\ 0 & \frac{2EI}{l^2} \Pi & \frac{2EI}{l} \Gamma \end{bmatrix} \quad (8)$$

$$[S_2'] = \begin{bmatrix} -\frac{EA}{l} & -\frac{M_i + M_j}{l^2} & 0 \\ -\frac{M_i + M_j}{l^2} & -\frac{4EI}{l^3} \Pi + \frac{N}{l} & \frac{2EI}{l^2} \Pi \\ 0 & -\frac{2EI}{l^2} \Pi & \frac{2EI}{l} \Lambda \end{bmatrix} \quad (9)$$

$$[S_3'] = [S_2']^t \quad (10)$$

$$[S_4'] = \begin{bmatrix} \frac{EA}{l} & \frac{M_i + M_j}{l^2} & 0 \\ \frac{M_i + M_j}{l^2} & \frac{4EI}{l^3} \Pi - \frac{N}{l} & -\frac{2EI}{l^2} \Pi \\ 0 & -\frac{2EI}{l^2} \Pi & \frac{2EI}{l} \Gamma \end{bmatrix} \quad (11)$$

$$[D'] = [u_i \ v_i \ \varphi_i \ u_j \ v_j \ \varphi_j]^t \quad (12)$$

$$\Lambda(\epsilon) = \frac{\epsilon(\epsilon - \sin \epsilon)}{2(2 - 2 \cos \epsilon - \epsilon \sin \epsilon)} \quad (13)$$

$$\Gamma(\epsilon) = \frac{\epsilon(\sin \epsilon - \epsilon \cos \epsilon)}{2(2 - 2 \cos \epsilon - \epsilon \sin \epsilon)} \quad (14)$$

$$\epsilon = l \sqrt{\frac{N}{EI}} \quad (15)$$

$$\varsigma = \left(1 - \frac{\frac{\epsilon}{2}}{\operatorname{tg} \frac{\epsilon}{2}} \right) \quad (16)$$

The matrix equations given above for a single beam-element are established separately for all beam-elements constituting the structural frame (or in casu 100 beam elements making up the full-length concrete column). Subsequently the matrices are transformed to a global axis system and combined in a global version of equation (3), i.e. equation 3 without the primes indicating the local axes. For given beam-element stiffness values and frame boundary conditions the matrix equations can be solved using traditional linear algebra. However, additional iterations are performed as the compression force N and the stiffness EI and EA are updated for each beam-element based on the obtained forces and displacements of the previous calculation step. This procedure is repeated until convergence is obtained.

The performance of the developed DSM tool for assessing second-order deflections in concrete columns with a pinned bottom support and roll top support exposed to fire has been validated by comparing calculation results for a given column axial load P and eccentricity e with results obtained independently in (Wang et al., 2015). Although a different calculation method is used, the difference in calculation results is very small at convergence.

To evaluate the load bearing capacity of a concrete column for a given eccentricity, the maximum axial load P_{max} for which convergence is obtained has to be determined. For larger P the lateral deflection of the column results in a second order bending moment at the mid-section of the pinned column which is larger than the bending moment capacity (for this given axial load P), resulting in failure of the column. Determining P_{max} is again done iteratively by step-wise improving the estimate for P_{max} . Note that a computational precision of 1 kN has been applied.

3 COMPUTATIONALLY EFFICIENT METHOD FOR ESTIMATING THE PDF

The DSM model introduced above can be used for probabilistic calculations by considering the stochastic distributions of input variables. While a calculation loop for traditional Monte Carlo simulations can be implemented easily, the total number of model evaluations for Monte Carlo simulations is very high (dependent on the required precision: order of

magnitude 10^4) and the required computation time quickly becomes impractical.

Recently a computationally very efficient method has been developed at Waterloo University by Zhang (2013) which allows estimating the PDF of a model output variable Y , function of n stochastic input variables x_l , using only a very limited set of model evaluations. The method is known as the (Fractional-Moment) Maximum Entropy Multiplicative Dimensional Reduction Method (ME-MDRM) and makes an unbiased estimation of the PDF by using the criterion of maximum entropy and reduces the overall computational requirements through Multiplicative Dimensional Reduction and Gaussian Interpolation.

While the method will be applied further to the evaluation of the resistance effect for concrete columns during fire exposure, a more general description of the method will be pursued here for clarity (i.e. considering a general model output variable Y).

3.1 Maximum Entropy estimation of the PDF

The maximum-entropy estimation is considered to be the mathematically correct procedure for avoiding biases with respect to the unknown PDF type or shape (Jaynes, 1957) and results in a mathematical formulation of the PDF $f_Y(y)$ as given by Equation (17), (Novi Inverardi and Tagliani, 2003), where m is the order of the estimation, and the coefficients λ_i and exponents α_i are determined by an optimization procedure.

$$\hat{f}_Y(y) = \exp\left(-\lambda_0 - \sum_{i=1}^m \lambda_i y^{\alpha_i}\right) \quad (17)$$

The optimization procedure consists of minimizing Equation (18), which is equivalent to minimizing the Kullback-Leibler divergence between the true and the estimated PDF (Zhang, 2013). In Equation (18) $M_Y^{\alpha_i}$ is the α_i^{th} sample moment of Y as given mathematically by Equation (19) for a sample of N model realizations y_k . Furthermore, λ_0 is normalization constant and is given by Equation (20).

$$\text{Minimize: } \lambda_0 + \sum_{i=1}^m \lambda_i M_Y^{\alpha_i} \quad (18)$$

$$M_Y^{\alpha_i} = \frac{\sum_{k=1}^N y_k^{\alpha_i}}{N} \quad (19)$$

$$\lambda_0 = \log \left[\int_Y \exp\left(-\sum_{i=1}^m \lambda_i x^{\alpha_i}\right) dy \right] \quad (20)$$

The minimization of Equation (18) can be done with readily available optimization algorithms. Dependent on the algorithm used, the result may be highly sensitive to the starting solution. Therefore it

is recommended to apply a Monte Carlo simulation or preferably Latin Hypercube Sampling (LHS) for the starting solution and to limit the exponents α_i without loss of generality to real numbers in the range $[-2; 2]$. The computational requirements of this Monte Carlo or LHS in the optimization calculations are small relative to those for evaluating the (computationally expensive) model describing Y . Furthermore, while these random starting solutions will result in a different final result in every calculation run, the differences are found to be negligible. Note that the differences in optimization result can be avoided when using a modified Latin Hypercube Sample where the mid-node of every probability interval is taken, see (Novak et al., 2014) for an example of this type of modified LHS.

In principle the estimation order m can be chosen freely, but while a higher estimation order results in a better match with respect to the input data it may also introduce spurious relationships for necessarily limited sets of input data. For the concrete column PDF's described further, $m = 4$ has been applied.

3.2 Multiplicative Dimensional Reduction

The Maximum Entropy PDF estimation described above can theoretically be applied to a dataset obtained through Monte Carlo simulations. This allows to obtain an analytical expression for the PDF, but the computational requirements related to the large number of model evaluations would not have been reduced significantly – possibly a slight reduction would be possible as less data points would be required for the Maximum Entropy PDF estimation than for a direct evaluation of the PDF by a histogram.

Zhang (2013) however proposes to use multiplicative dimensional reduction (MDR) when calculating the sample moments $M_Y^{\alpha_i}$. Denoting with $\mathbf{x}$ the vector of the n stochastic variables x_l , and with h the modelled relationship, the MDR approximates the model response $h(\mathbf{x})$ by:

$$Y = h(\mathbf{x}) \approx h_0^{1-n} \prod_{l=1}^n h_l(x_l) \quad (21)$$

with h_0 the model response when all n stochastic input variables are set equal to a fixed value c_l , and h_l the unidimensional cut functions as defined by:

$$h_l(x_l) = h(c_1, \dots, c_{l-1}, x_l, c_{l+1}, \dots, c_n) \quad (22)$$

Generally the mean value μ_l is used for the fixed (reference) value c_l , but as shown further it has computational benefits to use the median value $\hat{\mu}_l$ instead.

Considering Equations (21) and (22), the k^{th} moment of the stochastic model response Y is given by Equation (23), with $E[.]$ the expectance operator.

$$\begin{aligned}
E[Y^k] &= E[(h(\mathbf{x}))^k] \\
&\approx h_0^{k(1-n)} \prod_{l=1}^n E[(h_l(x_l))^k] \\
&= h_0^{k(1-n)} \prod_{l=1}^n \int_{x_l} (h_l(x_l))^k f_{x_l}(x_l) dx_l
\end{aligned} \tag{23}$$

In principle the product of integrals in (23) can be translated into a product of sample moments similar to (19), allowing to split the Monte Carlo of the model response in n smaller Monte Carlo's for the cut functions. This option is however not elaborated further as combining MDR with Gaussian interpolation results in a very significant additional reduction in computational requirement.

3.3 Gaussian Interpolation

The k^{th} moment for the l^{th} cut function can be approximated by Gaussian quadrature. In its most basic form, Gaussian quadrature approximates the integration of a function $g(z)$ over the entire domain of a standard normally distributed variable Z by a weighted sum of a limited number of well-chosen evaluation points z_j , as mathematically given by Equation (24) with ϕ the standard normal PDF, L the number of Gauss integration points (for most cases 5 integration points is sufficiently accurate), and w_j the associated Gauss weights (Zhang, 2013).

$$\int_{-\infty}^{\infty} g(z) \phi(z) dz \approx \sum_{j=1}^L w_j g(z_j) \tag{24}$$

For $L = 5$ the Gauss points z_j and associated weights w_j are given in Table 1.

Table 1. Gauss integration points z_j and associated Gauss weights w_j for $L = 5$.

j	z_j	w_j
1	-2.85697	0.011257
2	-1.35563	0.222076
3	0	0.533333
4	1.35563	0.222076
5	2.85697	0.011257

For non-standard normal distributed variables X , Equation (24) can be generalized to (25), with F_x^{-1} the inverse cumulative density function of X . This operation reduces the precision of the approximation, but avoids having to considering different quadrature integration types dependent on the type of probability density function describing X .

$$\begin{aligned}
\int_x g(x) f_x(x) dx &= \int_{-\infty}^{\infty} g(F_x^{-1}(\Phi(z))) \phi(z) dz \\
&\approx \sum_{j=1}^L w_j g(F_x^{-1}(\Phi(z_j)))
\end{aligned} \tag{25}$$

Considering the k^{th} power of the cut function h_l and the probability density functions f_{x_l} as specific situations of equation (25), the application of Gaussian quadrature for the evaluation of equation (23) is straightforward, resulting in an approximation for the moment $M_Y^{\alpha_i}$ by:

$$\begin{aligned}
M_Y^{\alpha_i} &\approx [h_0^{1-n}]^{\alpha_i} \prod_{l=1}^n \sum_{j=1}^L w_j [h_l(F_{x_l}^{-1}(\Phi(z_j)))]^{\alpha_i} \\
&= [h_0^{1-n}]^{\alpha_i} \prod_{l=1}^n \sum_{j=1}^L w_j y_{j,l}^{\alpha_i}
\end{aligned} \tag{26}$$

where $y_{j,l}$ is the model realization for the j^{th} Gauss point in the l^{th} cut function:

$$h_l(F_{x_l}^{-1}(\Phi(z_j))) = y_{j,l} \tag{27}$$

In summary, equation (26) replaces equation (19). Consequently, the estimation of the full PDF describing Y is obtained by considering 1 model evaluation for h_0 and nL model evaluations for the cut functions. Note that calculating a different power α_i in the minimization procedure of equation (18) does not require new model evaluations. Furthermore, if the number of Gauss integration points L is uneven, one of the Gauss points z_j equals 0, resulting in one of the Gauss points equal to the median $\hat{\mu}_l$. This further reduces the number of required model calculations to $1 + n \cdot (L-1)$ if the fixed values c_l in the MDR are chosen equal to the median. Consequently, when considering 5 Gauss integration points, the total number of model evaluations required for applying the total ME-MDRM for estimating the PDF describing Y is $1 + 4n$.

4 EXAMPLE APPLICATION AND RESULTS

Reliability calculations for concrete columns subjected to eccentric loads have been performed in (Sidibé et al., 2000) and (Achenbach, 2015) using Monte Carlo simulations and FORM. Here the combined DSM and ME-MDRM calculation tools introduced above are applied to the concrete column given in Table 2, requiring only 25 model evaluations for estimating the PDF of the resistance effect. Once the PDF is known, evaluating the probability of failure given by Equation (2) can be done for different load distributions f_E using analytical or numerical integration. This evaluation of failure probabilities will be pursued in a follow-up study.

4.1 Load bearing capacity for given eccentricity e

The load bearing capacity P_{max} is evaluated for an eccentricity $e_{nom} = 5$ mm. Results obtained through the ME-MDRM are compared with a histogram of 10,000 Monte Carlo simulations in Figure 2, illustrating the excellent performance of the method for correctly capturing the shape of the PDF. Furthermore, Figure 2 gives a lognormal approximation of P_{max} for which the parameters μ and σ have been estimated by the 10,000 Monte Carlo simulations. As the precision of the parameter estimating using 10,000 samples is excellent for the given number of simulations, Figure 2 clearly shows that a priori assuming a lognormal approximation would not result in a correct description of the shape of the PDF. Furthermore, as illustrated by Figure 3 the lognormal approximation results in an unsafe estimate of the occurrence rate of low values for P_{max} . Admittedly the ME-MDRM result deviates as well from the observed cumulative frequency for low P_{max} , but the deviation is conservative and the approximation is excellent for probabilities as small as 0.005, indicating for example that the ME-MDRM would very accurately predict a characteristic value for P_{max} corresponding with a 99.5% confidence level.

The lognormal approximation can also be evaluated by estimating the parameters of the lognormal distribution through Equation (26) and the 25 MDRM model results, since the first order and second order moment can be approximated by (26) when α_i is given a value of 1 and 2 respectively. While the first order moment is a direct estimate of the mean value μ , the standard deviation σ is approximated by equation (28).

$$\hat{\sigma}_Y = \sqrt{(M_Y^{\alpha=2}) - (M_Y^{\alpha=1})^2} \quad (28)$$

Although the application of the Maximum Entropy principle is clearly beneficial for an improved estimate of the PDF, the application of MDRM and Gaussian quadrature (MDRM Gauss) in itself gives an almost perfect estimate of the parameters for the lognormal distribution. This is illustrated by the excellent match of the two lognormal approximations with each other, both in Figure 2 and in Figure 3. The underlying estimations of the mean μ and standard deviation σ of P_{max} are given in Table 3 for the MCS and MDRM Gauss respectively. Although not the intent of this paper, this reduced version of the methodology would in itself be very beneficial for studies where currently the parameters of an a-priori chosen PDF are estimated using for example Latin Hypercube Sampling.

Table 2. Variable name, symbol and dimension, distribution, mean value μ and coefficient of variation V for the investigated concrete column

Variable name & symbol			Dist.	μ	V
20°C concrete compressive strength f_c [MPa] ($f_{ck} = 55$ MPa)			LN	78.6	0.15
20°C reinforcement yield stress f_y [MPa] ($f_{yk} = 500$ MPa)			LN	581.4	0.07
Concrete cover c [mm]			B	25	0.2
Compressive strength reduction factor at elevated temperature k_{fc} [-]			B	*	**
Yield stress reduction factor at elevated temperature k_{fy} [-]			B	*	**
Reinforcement area A_s [mm ²] (4Ø32mm)			N	3217	0.02
Column width z			DET	300	-
Vertical load eccentricity e			DET	e_{nom}	-
Column height h			DET	4	-

with LN and N the lognormal and normal distribution, B a Beta distribution in the range $[\mu - 3\mu V; \mu + 3\mu V]$

* nominal value EN 1992-1-2 (CEN, 2004)

** temperature dependent as given in (Van Coile et al. 2015)

Figure 2. ME-MDRM result for PDF of P_{max} for $e = 5$ mm at 60 minutes ISO 834 and comparison with histogram of 10,000 MCS and lognormal approximation (parameters respectively through MCS and through MDRM and Gaussian quadrature)

Table 3. Mean μ and standard deviation σ for P_{max} estimated respectively through the MCS and the MDRM Gauss.

	μ [kN]	σ [kN]
MCS	1639.51	212.73
MDRM Gauss	1639.04	215.08

Figure 3. ME-MDRM result for CDF of P_{max} for $e = 5$ mm at 60 minutes ISO 834 and comparison with cumulative frequency of 10,000 MCS and lognormal approximation (parameters respectively through MCS and through MDRM and Gaussian quadrature)

4.2 Maximum eccentricity for given vertical load

An alternative application of the DSM calculation tool allows to evaluate the maximum allowable eccentricity e_{max} for a given column load P . The calculation methodology itself is identical as discussed above for determining P_{max} for given e .

Results of 10,000 Monte Carlo simulations are compared in Figure 4 and Figure 5 with the ME-MDRM estimation and with two lognormal approximations: one for which the parameters are estimated from the MCS, and one for which the parameters are estimated based on Equations (26) and (28).

Again it is observed that the Maximum Entropy estimation of the PDF results in a very good estimate of the overall shape of the PDF. On the other hand, a priori assuming a lognormal distribution does not give a good match with the observed histogram.

Evaluating Figure 5, the occurrence rate of low e_{max} is much better estimated through the ME-MDRM, although the result is admittedly non-conservative compared to the observed histogram.

As mentioned above, the lognormal approximation can be estimated very efficiently by applying the MDRM in conjunction with Gaussian interpolation (MDRM Gauss). The values obtained for the mean μ and standard deviation σ estimated through the MCS and the MDRM Gauss are given in Table 4, again illustrating the excellent approximation by the MDRM Gauss methodology. While it is recommended to apply the Maximum Entropy principle for an improved estimation of the true PDF, this alternative way of estimating parameters can have significant practical applicability as well.

Figure 4. ME-MDRM result for PDF of e_{max} for $P = 1290$ kN at 60 minutes ISO 834 and comparison with histogram of 10,000 MCS and lognormal approximation (parameters respectively through MCS and through MDRM and Gaussian quadrature)

Figure 5. ME-MDRM result for CDF of e_{max} for $P = 1290$ kN at 60 minutes ISO 834 and comparison with cumulative frequency of 10,000 MCS and lognormal approximation (parameters respectively through MCS and through MDRM and Gaussian quadrature)

Table 4. Mean μ and standard deviation σ for e_{max} estimated respectively through the MCS and the MDRM Gauss.

	μ [mm]	σ [mm]
MCS	79.22	17.44
MDRM Gauss	79.86	17.58

5 DISCUSSION AND CONCLUSIONS

In order to allow for true risk and reliability-based decision making for structural fire safety, the structural reliability of concrete columns during fire exposure has to be evaluated. However, even evaluating the behavior of a single deterministic column is not easy. Due to the strong non-linearity of the column behavior and due to second order effects, advanced computationally expensive calculation tools

need to be used, especially if interaction with floorplates or local fire exposure of continuous columns is to be modelled. As the evaluation of a single column can already be computationally expensive, performing Monte Carlo simulations for reliability analysis becomes untenable. Therefore, more efficient methods are required.

More efficient methods do currently exist, but many improve calculation efficiency by postulating a distribution type for the model output (as for example the load bearing capacity for concrete columns during fire exposure), this postulation may often prove to be unsafe.

A straightforward and very efficient calculation methodology has been introduced by Zhang (2013) for making an unbiased estimate of the probability density function (PDF) describing a model output. The methodology is known as the (Fractional-Moment) Maximum Entropy Multiplicative Dimensional Reduction Method (ME-MDRM) and has been applied in this paper for evaluating the PDF describing the maximum eccentric load on a fire exposed concrete column.

In order to validate the methodology, Monte Carlo simulations can be used as a benchmark for the estimated PDF and CDF. However, the computational requirements of a traditional Finite Element model would be too high to allow for crude Monte Carlo simulations (or any other Monte Carlo type validation). Therefore, a matrix based calculation tool has been developed, applying the concepts of the Direct Stiffness Method. The model has been validated by comparing calculation results with those obtained by Wang et al. (2015). While the computational requirements for the Monte Carlo simulations (MCS) were still high, 10,000 MCS have been applied for validating the ME-MDRM PDF estimates.

Based on the presented results, it is concluded that the combined DSM and ME-MDRM models allow for a practically feasible probabilistic evaluation of the resistance effect for eccentrically loaded concrete columns exposed to fire. The low computational requirements of the method make practical applications feasible, resulting in a very significant step forward towards a true risk and reliability-based structural fire safety design.

Furthermore, the applicability of the ME-MDRM method for structural fire safety has been illustrated, opening up the possibility of applications in conjunction with more computationally expensive commercial Finite Element models.

6 REFERENCES

- Aachenbach, M., Morgenthal, G. 2015. Reliability of Reinforced Concrete Columns Subjected to Standard Fire. *Proceedings of CONFAB 2015*, 02-04/09, Glasgow, UK.
- Balomenos, G.P., Genikomsou, A.S., Polak, M.A., Pandey, M.D. 2015. Efficient method for probabilistic finite element analysis with application to reinforced concrete slabs. *Engineering Structures* 103: 85-101.
- Caspeele, R. 2015. Structural Analysis III. *Lecture notes*. Ghent University, Ghent, Belgium.
- CEN. 2004. EN 1992-1-2: Eurocode 2: Design of concrete structures – Part 1-2: General rules – Structural fire design. *European Standard*.
- Holický, M., Materna, A., Selacek, G., Schleich, J.-B., Arteaga, A., Sanpaolesi, L., Vrouwenvelder, T., Kovse, I., Gulvanessian, H. 2005. *Implementation of Eurocodes: Handbook 5: Design of Buildings for the Fire Situation*. Leonardo Da Vinci Pilot Project CZ/02/B/F/PP-134007.
- Jaynes, E. 1957. Information theory and statistical mechanics. *Phys. Rev.* 106: 620-630.
- JCSS. 2007. Probabilistic Model Code. The Joint Committee on Structural Safety.
- Kodur, V.K.R., Dwaikat, M.B. 2008. A numerical model for predicting the fire resistance of reinforced concrete beams. *Cement and Concrete Composites* 30: 431-443
- Melosh, R.J. (1963). Basis for derivation of matrices for the direct stiffness method. *AIAA Journal* 1: 1631-1637.
- Novak, D., Vorechovsky, M., Teply, B. (2014). FReET: Software for the statistical and reliability analysis of engineering problems and FReET-D: Degradation module. *Advances in Engineering Software* 72: 179-192.
- Novi Inverardi, P.L., Tagliani, A. 2003. Maximum Entropy Density Estimation from Fractional Moments. *Communications in Statistics – Theory and Methods* 32: 327-345.
- Prakash, P.R., Srivastava, G. (2015). Development of a matrix method based framework for the thermos-mechanical analysis of RCC frames. *Proceedings of PROTECT 2015*, 28-30/06, East Lansing, USA.
- Sidibé, K., Duprat, F., Pinglot, M., & Bourret, B. 2000. Fire Safety of reinforced concrete columns. *ACI Struct J* 97: 642-647.
- Van Coile, R. 2015. Reliability-Based Decision Making for Concrete Elements Exposed to Fire. *Doctoral dissertation*. Ghent University, Ghent, Belgium.
- Wang, L., Caspeele, R., Van Coile, R., Taerwe, L. 2015. Extension of tabulated design parameters for rectangular columns exposed to fire taking into account second order effects and various fire models. *Structural Concrete* 16: 17-35.
- Zhang, X. 2013. Efficient Computational Methods for Structural Reliability and Global Sensitivity Analyses. *Doctoral dissertation*. University of Waterloo, Waterloo, Canada.