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Thermo-mechanical Analysis of a Radiative Annular Fin

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Abstract. The thermo-mechanical reliability is a critical issue in a radiative fin. The radiative dissipation pertaining to enhance heat rejection from the fin surfaces. Due to the temperature gradient, the non-uniform thermal deformation occurs in the fin material. Thermal deformation may cause various defects such as crack propagation, creep, delamination, fatigue failure, erosion and warping. In this study, an analytical model for thermal stresses in an annular radiative fin is presented. The non-linear heat transfer equation with variable thermal parameters has been solved using the homotopy perturbation method (HPM). The comprehensive closed form solution for thermal stresses are obtained from the HPM based temperature field coupled with the classical theory of elasticity. The effects of radiation on the temperature field and thermal stress field are investigated. In addition to forward solution, an inverse analysis has been carried out for estimating the thermal parameters for a reference stress field.

INTRODUCTION

The extensive literature review using ScienceDirect database revealed that most of the fin analyses mainly deal with the temperature distribution and thermal performance [1, 2]. The further extension of those analyses such as the effect of heat transfer on the material behaviour was ignored. While, the thermal loading is responsible for the various mechanical failures such as fatigue failure, crack propagation, creep and delamination, etc. For an annular fin, the thermal stresses are developed due to a non-uniform temperature gradient. Recently, Mallick et al. [3] presented thermal stress analysis of a conductive-convective annular fin with variable thermal conductivity. Another aspect of the fin analysis is designing the thermo-mechanical parameters in a controlled stress field. In this regard, instead of expensive and rigorous material testing one can estimate the various thermal parameters for a reference stress field using the inverse analysis. The inverse analysis is mathematically ill-posed and gaining rapid popularity due to the computational advancement. Few researchers have reported the inverse study for estimating the thermo-physical parameters for a reference temperature field in the fin [2, 4].

The aim of this study is to present the effect of radiation on the thermal stresses in an annular fin and the inverse study for estimating the thermo-mechanical parameters for a reference stress field:

GOVERNING EQUATIONS

Consider an isotropic and homogeneous thin annular fin of a rectangular profile having an inner radius r_i , outer radius r_o and thickness t as shown in Fig.1. The base of the fin is attached to a surface with a fixed temperature, T_b , and the peripheral surfaces are exposed in the surrounding medium with an ambient temperature, T_a . The effective sink temperature for radiative heat transfer is taken as T_s . The tip of the thin fin is assumed to be adiabatic. The thermal conductivity and surface emissivity are assumed to be as a linear function of temperature and can be expressed as:

$$k = k_o \{1 + \kappa(T - T_a)\} \quad \text{and} \quad \varepsilon = \varepsilon_s \{1 + \lambda(T - T_s)\} \quad (1)$$

where, k_o is the thermal conductivity parameter at T_a and ε_s is the surface emissivity at the sink temperature T_s . The parameters κ and γ are designated as linear variations of thermal conductivity and surface emissivity respectively. The steady state energy balance equation for heat transfer and its boundary conditions are expressed as [3]:

$$t \frac{d}{dr} \left[k(T)r \cdot \frac{dT}{dr} \right] - 2hr(T - T_a) - 2\varepsilon(T)\sigma r(T^4 - T_s^4) = 0 \text{ with B.Cs } r = r_i : T = T_b \text{ and } r = r_o : \frac{dT}{dr} = 0 \quad (2)$$

where h stands for convective heat transfer coefficient and σ is the Stefan-Boltzmann constant. Neglecting the terms $\theta_a (= T_a/T_b)$ and $\theta_s (= T_s/T_b)$, the energy balance equation and the boundary conditions are generalized and simplified into the following dimensionless form:

$$\theta'' + \beta\theta\theta'' + \beta(\theta')^2 + \frac{1}{\xi}\theta' + \frac{\beta}{\xi}\theta\theta' - N^2\theta - M(\theta^4 + \lambda_R\theta_s^4) = 0 \text{ with B.Cs } \theta = 1 : \xi = 1 \text{ and } \theta' = 1 : \xi = R \quad (3)$$

where $(')$ and $('')$ are representing the differentiation and double differentiation with respect to ξ . The various non-dimensional entities used in Eq. (3) are as follows:

$$\theta = \frac{T}{T_b}, \theta_a = \frac{T_a}{T_b}, \xi = \frac{r}{r_i}, R = \frac{r_o}{r_i}, \beta = \kappa T_b, N^2 = \frac{2hr_i^2}{k_o t}, M = \frac{2r_i^2 \sigma \varepsilon_s T_b^3}{k_o t} \text{ and } \lambda_R = \lambda T_b \quad (4)$$

Eq. (3) is solved using the homotopy perturbation method (HPM) as [4],

$$\begin{aligned} \theta = 1 + (1 - \beta) \{ N^2 + M(1 + \lambda_R) \} & \left\{ \frac{(\xi - 1)^2}{2} - (R - 1)(\xi - 1) \right\} - \{ N^2 + M(1 + \lambda_R) \} (1 + \beta) \left[\frac{(\xi - 1)^2}{2} - \{ \ln \xi - 1 \} \xi R \right] + \\ & \{ N^2 - M(4 + 5\lambda_R) \} \{ N^2 + M(1 + \lambda_R) \} \left\{ \frac{(\xi - 1)^4}{24} - (R - 1) \frac{(\xi - 1)^3}{6} \right\} + (1 + \beta) \{ N^2 + M(1 + \lambda_R) \} R + \\ & \{ N^2 + M(1 + \lambda_R) \} \left[(1 + \beta) \{ (R - 1) - R \ln R \} + \{ N^2 - M(4 + 5\lambda_R) \} \frac{(R - 1)^3}{3} \right] (\xi - 1) \end{aligned} \quad (5)$$

FIGURE 1. Geometry of an annular fin

THERMAL STRESSES

Neglecting the body force and inertia, the stress equation of equilibrium for thin axisymmetric thin annular fin can be expressed as [3],

$$r \frac{d\sigma_r}{dr} + \sigma_r - \sigma_\phi = 0 \quad (6)$$

where σ_r and σ_ϕ are the radial and tangential stresses induced due to thermal loading only. The stress-strain-temperature relationship following classical theory of elasticity yields,

$$\begin{bmatrix} \sigma_r \\ \sigma_\phi \end{bmatrix} = \frac{E}{1 - \nu^2} \begin{bmatrix} 1 & \nu \\ \nu & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_r \\ \varepsilon_\phi \end{bmatrix} - \frac{E\alpha T}{1 - \nu} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (7)$$

where ε_r and ε_ϕ are the radial and tangential strain, ν is the Poisson's ratio, α is the co-efficient of linear thermal expansion, and E is the Young's modulus. Strain-displacement relations in the axisymmetric polar coordinate system are,

$$\varepsilon_r = \frac{\partial u_r}{\partial r} \quad \text{and} \quad \varepsilon_\phi = \frac{u_r}{r}. \quad (8)$$

On Substitution of Eq. (8) into Eq. (7) and using the Eq. (6), the displacement, u_r can be expressed as,

$$u_r = \frac{(1+\nu)\alpha}{r} \int_{r_i}^{r_o} Trdr + C_1 r + \frac{C_2}{r} \quad (9)$$

where C_1 and C_2 are the constants of integration. The constants, C_1 and C_2 are evaluated from the traction free boundary conditions ($\sigma_r = 0$ at $r = r_i$ and r_o) at outer and inner edges and using the Eqs. (7), (8) and (9) as,

$$C_1 = \frac{(1-\nu)\alpha}{r_o^2 - r_i^2} \int_{r_i}^{r_o} Trdr \quad \text{and} \quad C_2 = \frac{(1+\nu)\alpha r_i^2}{r_o^2 - r_i^2} \int_{r_i}^{r_o} Trdr \quad (10)$$

Consequently, the closed form solution for thermal stresses are,

$$\sigma_r = -\frac{\alpha E}{r^2} \int_{r_i}^r Trdr + \frac{\alpha E}{r_o^2 - r_i^2} \left(1 - \frac{r_i^2}{r^2}\right) \int_{r_i}^{r_o} Trdr, \quad \text{and} \quad \sigma_\phi = -\alpha ET + \frac{\alpha E}{r^2} \int_{r_i}^r Trdr + \frac{\alpha E}{r_o^2 - r_i^2} \left(1 + \frac{r_i^2}{r^2}\right) \int_{r_i}^{r_o} Trdr \quad (11)$$

The stress fields are now expressed in the following non-dimensional form:

$$\bar{\sigma}_r = -\frac{\chi}{\xi^2} \int_1^\xi \theta \xi d\xi + \frac{\chi(\xi^2 - 1)}{(R^2 - 1)\xi^2} \int_1^R \theta \xi d\xi, \quad \text{and} \quad \bar{\sigma}_\phi = -\chi\theta + \frac{\chi}{\xi^2} \int_1^\xi \theta \xi d\xi + \frac{\chi(\xi^2 + 1)}{(R^2 - 1)\xi^2} \int_1^R \theta \xi d\xi \quad (12)$$

where $\bar{\sigma}_r = \frac{\sigma_r}{E}$, $\bar{\sigma}_\phi = \frac{\sigma_\phi}{E}$ and $\chi = \alpha T_b$.

The von Mises stress in this problem can be expressed as,

$$\bar{\sigma}_{VM} = \sqrt{\bar{\sigma}_r^2 - \bar{\sigma}_r \bar{\sigma}_\theta + \bar{\sigma}_\theta^2} \quad (13)$$

The details of all the non-dimensional parameters are described in the nomenclature.

RESULTS AND DISCUSSION

In this section, the results of heat transfer on the temperature distribution and its effect on thermal stresses are discussed. Unless mentioned otherwise, the thermal parameters $\beta = 0.3$, $N_c = 0.6$, $M = 0.1$, $\lambda_R = 0.5$, $\chi = 1.0$ and $R = 2$ are considered throughout the analysis. In order to demonstrate the accuracy of the present HPM based closed form solution, the results are compared with the numerical solution based on finite element method (FEM). Table 1 compares the HPM results with FEM results for temperature distribution, radial stress distribution and the tangential stress distribution. The results revealed a good agreement that confirmed the correctness of the present HPM based closed form solution for temperature distribution and thermal stresses. The maximum percentage of errors are found to be 0.2224%, 2.1041% and 6.2012% for temperature, radial stress and tangential stress respectively.

We now examine the effect of radiation on the temperature field and stress field. Fig.2a illustrates the temperature distribution for non-radiative fin and radiative fin with different variable radiative parameters. It can be seen the higher tip temperature is observed in the non-radiative fin. Further, the tip temperature decreases with the increase of variable radiative parameters. This result is quite obvious as the radioactive fin continuously dissipates heat from fin surfaces to the surrounding. On the other hand, the radiation adversely affects the fin material due to high thermal stresses as shown in Fig. 2b – d. The magnitude of radial stress, tangential stress and the von-Mises stress increases with the increase of radiative parameter. The temperature gradient increases with the increase of radiative parameter, result in the increase of thermal stresses. The radial stress behaves as compressive, and the maximum value lies about at the one-third of the radial distance from the base of the fin. The mixed behaviour (tensile and compressive) of tangential stress can be seen in Fig.2c. The magnitude of maximum tangential stress is found to be at the bore of the fin, and it is about eight times of the maximum value of the compressive stress. Thus, the tangential stress is more pronounced in an annular fin as compared to radial stress and the chance of mechanical failure may occur due to tangential stress. Fig. 2d shows the variation of von-Mises stress and its maximum value lie at the bore. The analysis of von-Mises result is important for the failure analysis of a fin made of ductile materials.

TABLE 1. Validation of HPM results with the FEM results for temperature field and stress fields for $\lambda_R = 0$.

ξ	Temperature			Radial Stress			Tangential Stress		
	HPM	FEM	Error (%)	HPM	FEM	Error (%)	HPM	FEM	Error (%)
1	2	2.000	0	0	0.0000	0	-52.312	-51.920	0.7492
1.1	1.949	1.949	0.0308	-3.839	-3.761	2.0230	-33.022	-32.351	2.0323
1.2	1.905	1.907	0.0787	-5.628	-5.510	2.1041	-18.227	-17.850	2.0696
1.3	1.869	1.871	0.1266	-6.141	-6.014	2.0653	-6.8825	-6.776	1.5463
1.4	1.839	1.843	0.1685	-5.872	-5.754	2.0101	1.7308	1.623	6.2012
1.5	1.816	1.819	0.2019	-5.140	-5.050	1.7477	8.1274	7.879	3.0532
1.6	1.797	1.801	0.2151	-4.160	-4.089	1.7012	12.688	12.383	2.2480
1.7	1.783	1.787	0.2224	-3.073	-3.021	1.6893	15.704	15.400	1.9344
1.8	1.774	1.778	0.2198	-1.977	-1.944	1.6725	17.401	17.133	1.5389
1.9	1.769	1.773	0.2186	-0.938	-0.923	1.6035	17.962	17.737	1.2529
2	1.767	1.771	0.2150	0	-0.057	0	17.536	17.320	1.2333

FIGURE 2. Comparison of radiative and non-radiative fin and the effect of radiative parameters on non-dimensional temperature field and stress fields.

Next, three thermo-geometrical parameters, N_c , M and λ_R are inversely estimated for a preassigned stress field. For this purpose, HPM based closed form solution of tangential stress field is taken as forward solution. The same thermal parameters as mentioned above are considered in the forward solution. Fig. 3a illustrates the variation of function values of the inverse analysis using differential evolution (DE) method. It can be seen that the function value gradually converges with the simulation time. Fig. 3b-e depict the variation of inverse parameters for different

runs. The values of the parameters are continuously changing with the simulation time and finally get stable. Table 2 shows the inversely estimated parameters for different runs.

FIGURE 3. Variation of (a) cost function, (b) thermo-geometric parameter, N_c , (c) radiative parameter (M), (d) coefficient for the variation of radiation (λ), and (e) non-dimensional coefficient of thermal expansion (χ).

TABLE 2. Inversely estimated various thermal parameters at different runs.

Run (Measurement points)	N_c	M	λ	χ	Cost function	Time (sec)	LB (N_c, N, λ, χ) UB (N_c, N, λ, χ)
Forward	0.6	0.1	0.5	300	----	----	-----
R1 (25)	0.7045	0.1349	0.3456	0.754	0.0072	89.9	{ 0.2, 0.08, 0.2, 300 0.9, 0.2, 0.8, 400
R2 (100)	0.7303	0.1074	0.7320	0.709	0.0005	200.17	{ 0.2, 0.08, 0.2, 300 0.9, 0.2, 0.8, 400
R3 (25)	0.5157	0.0896	0.4097	1.304	0.0072	90.74	{ 0.2, 0.08, 0.2, 350 0.6, 0.2, 0.8, 400
R4 (100)	0.5460	0.1036	0.3336	1.169	0.0005	202.7	{ 0.2, 0.08, 0.2, 350 0.6, 0.2, 0.8, 400

Now, the stress fields are reconstructed using the inversely estimated parameters and compared with the forward solution in Fig. 4. A very good agreement is observed. However, the temperature field estimated from the inverse parameters are completely different from the forward temperature field. This result suggests that the thermal efficiency can be improved in a controlled thermal stress through proper tuning of fin the parameters.

FIGURE 4. Variation of (a) cost function, (b) thermo-geometric parameter, N_c , (c) radiative parameter (M), (d) coefficient for the variation of radiation (λ), and non-dimensional coefficient of thermal expansion(χ).

CONCLUSIONS

The HPM based closed form solutions for temperature field and stress fields of a conductive-convective-radiative annular fin made of isotropic materials are successfully derived. The radiation increases the temperature gradient, and thus can adversely affect the fin materials by increasing the thermal stresses. The heat transfer can be enhanced in a controlled stress field by inversely estimating the proper combinations of thermo-mechanical parameters. The present analysis provides a technical significance in fin designing.

REFERENCES

1. A.D. Kraus, et al., *Extended Surface Heat Transfer* (John Wiley & Sons, New York, 2001).
2. R. Das and B. Kundu, *Int. J. Heat Mass Transf.* **124**, 331-343 (2018).
3. A. Mallick, et al., *J. Therm. Stresses* **38**, 110-132 (2015).
4. A. Mallick, et al., *Heat Mass Transf.* **53**, 1037-1049 (2017).