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Modeling, simulation and analysis of birefringent effects in plastic optics

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ABSTRACT

Plastic optics has been widely used in different application. They have been facing birefringent effects during manufacturing or during certain application. Finite element modeling of plastic optics in CAD interface is done along with experimental and theoretical comparison of the specimen with the help of solid mechanics and image processing. Low birefringence plastic optics is chosen for the experiment and varying load is applied to observe the characteristics both in experiment and simulation. Low birefringence polariscope was used to measure the birefringence in the plastic specimen. Birefringence is caused due to many effects like stress induced birefringence temperature induced due to thermal gradient and pressure during manufacturing. Here stress is induced on low birefringence specimen by two point compression loading and is compared on the base of solid mechanics, finite element modeling and image processing. The results were found to be similar and convincing.

Keywords: Birefringence, Plastic, Polariscope, polarization, photo elasticity, residual stress, retardation

1. INTRODUCTION

Plastic optics is prevalent in every field with its high application. Most of them are manufactured by injection molded optics while 3 D printing has also been delving research into the transparent and high quality plastic optics. Plastic optics has been widely used and need to be characterized^[1] for quality testing before launch in the market. Even though advancement in manufacturing of injection molded plastic optics^[2] is on progress, yet the limitation exist. Free mold shrinkage, temperature gradient^[3], stress induced distortion are the main hindrance causing birefringence^[4]. Here the plastic optics has been designed in CAD (Computer aided design) software based on Finite element modeling analysis and compared the birefringence to that of experiment and image processing. Stress induced birefringence is observed in CAD interface and later on calculation is done to observe the birefringence caused by stress. Birefringence due to applied stress at two points loading seems to be in linear fashion i.e. birefringence increases proportionally on increasing loading at two points. Since CAD doesn't have image module interface hence we calculated stress on the CAD and mapped birefringence.

2. THEORY

Modeling of Poly methyl methacrylate specimen has been done on CAD software by finite element method. The elastic, plastic characteristics and material density were carefully loaded on the material properties in CAD and the designed amount of force was applied same as on theoretical and experimental loading. Mechanical properties of the test specimen are given below,

Density = 1180 kg/m³

Bending strength= 134 MPa

Compressive strength = 165 MPa

Tensile strength = 62 MPa

Yield strength = 60 MPa

Shear Modulus = 1700 MPa

Young's Modulus= 2450 MPa

Poisson's ratio= 0.35

Material type: Disk shaped acrylic test specimen

Figure 1: Acrylic disk

Figure 2: Specimen on 2 point compression loading

Two point compressions loading was applied by properly fixing the boundary condition in abaqus as in the experiment. Uniform loading was applied on the specimen by keeping one point fixed on boundary condition and applied on other point with uniform load of 49, 98, 196 N and so on. The stress intensity distribution along the diameter was carefully observed in both experiment and simulation. The results were found to be similar. Further the results were compared with theoretical calculation; it seems to be of reasonable accuracy and precision. Hence, stress values were further mapped to birefringence values by using calibration method in polariscope^[5]. Calibration of photoelastic constant^[6] has to be done for mapping stress to suitable birefringence values. Photoelastic constant of the material is determined by calculating fringe constant of the material by applying a known amount of stress and observing retardation exhibited by the material. Working wavelength of the source light is of 550 nm.

$$f_{\sigma} = 2\pi h \frac{\sigma_1 - \sigma_2}{\Delta} \quad (1)$$

$$C = \frac{\lambda}{f_{\sigma}} \quad (2)$$

Thus, we can find the photoelastic constant of the material essential for mapping stress values to birefringence values. For calibration circular acrylic disk is used of diameter D and thickness h which is loaded as shown below

Figure 3: Compression disk modeling

Figure 4: Schematic diagram of disk compression

Vertical and horizontal stresses are present along y axis and x axis also known as principal stresses, shear stresses vanishes due to symmetry of the material. Here σ_2 is positive, while σ_1 is negative, so $\sigma_2 = \sigma_x$ and $\sigma_1 = \sigma_y$, thus $\sigma_1 - \sigma_2 \geq 0$. By theory of photo elasticity

$$\sigma_1 = \frac{2P}{\pi h D} \left(\frac{1 - \nu^2}{1 + \nu^2} \right), \quad (3)$$

$$\sigma_2 = - \frac{6P}{\pi h D} \frac{(1 + \nu^2)(1 - \frac{1}{3}\nu^2)}{(1 + \nu^2)^2}, \quad (4)$$

Where $\zeta=x/R=2x/D$. Maximum difference $\sigma_1 - \sigma_2$ occurs at the center along the horizontal diameter at $\zeta=0$. At this point, Theoretical value of stress at the mid-point of the specimen,

$$\sigma_1 - \sigma_2 = \frac{8P}{\pi h D} \quad (5)$$

Combining this result with basic photoelastic relation gives,

$$\frac{Nf_\sigma}{h} = \sigma_1 - \sigma_2 = \frac{8P}{\pi h D} \quad (6)$$

$$f_\sigma = \frac{8P}{\pi D N} \quad (7)$$

Where $N = \frac{\Delta}{2\pi}$, so

$$f_\sigma = \frac{16P}{\Delta D} \quad (8)$$

Thus we can find out the fringe constant of the material for mapping stress values to birefringence values both in theoretical and simulation results.

Hence theoretical and simulation stress values were mapped to birefringence values with the aid of photo elasticity based on grey field imaging. Schematic diagram of low birefringence polariscope is shown below. Circularly polarized light passes through acrylic disk and resulting polarization image is analyzed by phase shift algorithm.

Figure 5: Schematic diagram of Low Birefringence Polariscope

Light transmits through polarizer, quarter wave plate, specimen and analyzer represented by (U, V) as follows:

$$\begin{bmatrix} U \\ V \end{bmatrix} = J_A J_M J_Q J_P a e^{j\omega t} \quad (9)$$

Here J_P, J_Q, J_M and J_A represents Jones vector for polarizer, quarter wave plate, material and analyser respectively. By matrix multiplication,

$$\begin{Bmatrix} U \\ V \end{Bmatrix} = \frac{\sqrt{2}}{2} \begin{bmatrix} 0 & 0 \\ \cos\alpha & \sin\alpha \end{bmatrix} \begin{bmatrix} \cos\Delta/2 - i\cos 2\beta \sin\Delta/2 & -i\sin 2\beta \sin\Delta/2 \\ -i\sin 2\beta \sin\Delta/2 & \frac{\cos\Delta}{2} + i\cos 2\beta \sin\Delta/2 \end{bmatrix} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} ae^{j\omega t} \quad (10)$$

$$= \frac{\sqrt{2}}{2} \begin{bmatrix} 0 \\ \left(\sin \frac{\Delta}{2} \cos(\alpha - 2\beta) + \sin\alpha \cos \frac{\Delta}{2} \right) + i \left(\sin \frac{\Delta}{2} \sin(\alpha - 2\beta) + \cos\alpha \cos \frac{\Delta}{2} \right) \end{bmatrix} ae^{j\omega t} \quad (11)$$

Here α is angular position of analyzer, Δ is phase difference from birefringent sample, β is orientation of fast axis of light from sample, a is the amplitude of circularly polarized light and ω is temporal angular frequency of the wave.

Intensity of light emerging from a sample is given as

$$I = |V|^2 = \frac{a^2}{2} (1 + \sin\Delta \sin 2(\alpha - \beta)) \quad (12)$$

Intensity received by detector is,

$$I = I_a + I_{ca} \cos 2\alpha + I_{sa} \sin 2\alpha, \quad (13)$$

Where,

$$\begin{aligned} I_a &= \frac{a^2}{2} \\ I_{ca} &= -\left(\frac{a^2}{2} \sin\Delta\right) \sin 2\beta \\ I_{sa} &= \left(\frac{a^2}{2} \sin\Delta\right) \cos 2\beta \end{aligned}$$

I_a is average light intensity, I_{ca} is cosine component of light intensity and I_{sa} is sine component of light intensity received by detector.

2.1 Four Step phase shifting method

For Phase shifting photo elasticity^[5, 7], intensity at different phases are recorded and analyzed for determining principal stress and direction. The average light intensity received is constant represented by I_a throughout whole experiment, Phases shifting approach is applied to extract the unknowns α, β . Fours step phase shifting and their image intensity with corresponding equations as shown below

Table 1: Four step phase shifting and their corresponding equation

Steps	Analyzer orientation (α)	Image intensity
1	0	$I_1 = I_a (1 - \sin\Delta \sin 2\beta)$
2	$\pi/4$	$I_2 = I_a (1 + \sin\Delta \cos 2\beta)$
3	$\pi/2$	$I_3 = I_a (1 + \sin\Delta \sin 2\beta)$
4	$3\pi/2$	$I_4 = I_a (1 - \sin\Delta \cos 2\beta)$

Retardation value of specimen and its orientation are as follows

$$\Delta = \frac{1}{2I_a} ((I_3 - I_1)^2 + (I_4 - I_2)^2)^{1/2} \quad (14)$$

$$\beta = \frac{1}{2} \tan^{-1} \left(\frac{I_1 - I_3}{I_4 - I_2} \right) \quad (15)$$

Thus retardation and direction of principal stress at the specimen is received by the detector. Hence we can find out the retardation values exhibited by specimen and further map to birefringence values,

$$\text{Birefringence} = n_1 - n_2 = \frac{\Delta\lambda}{2\pi h}, \quad (16)$$

Where h is thickness of the specimen

Thus stress values from theoretical and simulation values were mapped into birefringence values and compared with the experiment based on polarization of light with the aid of image processing.

3. RESULTS AND DISCUSSION

The acrylic specimen was under loading of 0, 5, 10, 20, 30, 40 kg and so on. Finite element modeling of specimen is done in CAD software abaqus and the stress values were applied as in the experiment and found to have similarity in the result. The boundary condition and the experimental parameters were meticulously inserted in the program and were found to have following result. Stress distribution will be compared with experimental and theoretical values and then will be mapped to birefringence values. Similarity in stress values gives the clue that materials have similar birefringence values.

Simulation results

Figure 6: Stress simulation of specimen on CAD at different loads

Experimental results

Figure 7: Experimental stress of specimen at different loads by photoelasticity

The stress values were found to be the same along the whole material with 88 percent of accuracy compared to theoretical and experimental results. The stress along the vertical diameter were plotted for theoretical, experimental and simulation. The results were found to be quite convincing and matching despite nuance discrepancies as shown in figure below.

Figure 8: Comparison of methods on stress values at different points along diameter

The stress values were taken along the vertical diameter in the center line and found to be of same nature i.e. dumbbell shaped. It is maximum at the center and found to be reduced along the edges. The von mises stress was calculated by finite element modeling in simulation. Similarly photoelastic concept was applied to determine the stress on the specimen. Hence stress results from finite element modeling, photoelasticity and solid mechanics^[8] was compared and graph was plotted as shown in the above figure representing the same nature along vertical diameter.

Birefringence mapping

Figure 9: Intrinsic birefringence (left) Intrinsic birefringence plot (center) Comparison of birefringence plot (right)

Before finding actual birefringence values intrinsic birefringence value was calculated. Intrinsic birefringence is caused due to residual stress^[9, 10] in the specimen itself. The residual stress is calculated by photoelastic method by knowing the retardation values in the specimen. The residual stress are further mapped to birefringence which is intrinsic birefringence obtained on the material. Residual stress is ideally absent in theoretical and modelling because there material is manufactured and supposed to be stress free but doesn't happens so. This residual stress are subtracted from stress induced loading of 5 kg, 10kg, 20 kg and so on to get the actual birefringence of the material . We can see the birefringence values more at end and constant almost zero in the inner side of the material. Though it is hard to get birefringence free material i.e. stress free material, this study help us to know the birefringence nature of the specimen on theoretical and finite element modeling and simulating their results with their own limitations. Furthermore experimental stress analysis based on grey field photo elasticity^[11] with the aid of image processing helps to determine low level birefringence^[12] of the material. Birefringence mapping is done with stress images obtained through photo elasticity. Photoelastic constant was found with the aid of experiment and multiplied to stress values to get the birefringence at different position. The birefringence values by different methods exhibited same properties along vertical diameter of the specimen at center point. Once f_{σ} is found out experimentally we can know the value of photoelastic constant of the material. The governing equation is

$$f_{\sigma} = \frac{\lambda}{C} \quad (17)$$

Wavelength of light used is of green color (550 nm) hence we can find out photoelastic constant of the material. So stress values are mapped into birefringence values once photoelastic constant of the material is determined. Birefringence values at different loads are shown in the figure below. Birefringence can be defined by following equation

$$Br = n_1 - n_2 = C(\sigma_1 - \sigma_2) \quad (18)$$

Photoelastic constant of the material is found to be $2.74 \times 10^{-12} \text{ m}^2/\text{N}$

Figure 10: Birefringence mapping at different loads

Figure 11: Birefringence plot along vertical diameter

Figure 12: Birefringence comparison of different methods at different loads

Figure 13: Birefringence versus load linear trend line (Experimental)

$$y = 6E-06x - 4E-06$$

(19)

Birefringence obtained due to stress is of linear nature as we can see from the figure above linear equation defines the characteristics of birefringence induced at various loads.

4. ANALYSIS

Intrinsic birefringence is caused on specimen during manufacturing because of thermal gradient, pressure flow discontinuity and many other reasons. Here, we explore intrinsic birefringence of acrylic specimen with the aid of photo elasticity and subtracted stress induced birefringence from intrinsic birefringence to get the actual stress induced birefringence. Birefringence is ideally absent theoretically and on modelling and simulation since there is not still defined parameter and their values to simulate. So, birefringence at certain points due to load might be deceiving if we neglect intrinsic birefringence caused during manufacturing. Photoelastic constant value found by photoelastic method was much useful for mapping stress values to birefringence values. Since PMMA (Poly methyl methacrylate) material is of isotropic nature, so we considered photoelastic constant to be same. Moreover we calculated at different point and was found to be almost the same averaged the value and implemented for mapping. Stress induced birefringence was obtained at various loads on two point compression loading. The birefringence was found to be maximum at the center with increasing values on increasing load. The birefringence is entirely dependent on stress applied at the specimen. The birefringence value is maximum at edges during no load but once load is applied the values will be maximum at center point of the specimen. The birefringence nature is found to be dumbbell shaped during loading. The birefringence increases to some extent during loading until the material is elastic afterward it shows plastic behavior. Once it crosses the elastic limit birefringence begin increasing towards Increase of birefringence on varying load is of linear nature though some values do not fit due to experimental errors. Hence we can validate the result from experimental based on photo elasticity and finite element modeling simulation and theoretical results based on solid mechanics.

5. CONCLUSION

Thus we are able to model the plastic optics in the commercial software and compare the simulated results (finite element modeling) both theoretically (based on solid mechanics) and experimentally (based on photo elasticity). Since there is no

image module in abaqus we need to find the photoelastic constant from photoelastic principle and implement that to on simulated and theoretical results to map out the birefringence. Increasing load increases birefringence on the specimen as justified in the figure above. This study helps us to know the stress induced birefringence on acrylic materials and its response at various loading. Whenever load increases after certain limit the material begin to show plastic behavior. Moreover it helps us to know the photoelastic constant of the acrylic material. It was almost constant at various points and at different loading. Maximum birefringence occurs at the center of specimen during two point loading also depend on inherent properties. Hence system is capable of non-destructive rapid measurement of stress induced birefringence. Validation of the birefringence values are done with the help of finite element modeling and theoretical method via stress mapped birefringence. Stress induced birefringence measurement can be very helpful tool in industry level which is validated both by solid mechanics and finite element modeling. This study will be quite useful for quality testing where slight or large loading is applied on acrylic specimen. We are able to know the stress values at certain points and their deformation response on varying load. Hence quality of specimen can be assessed by measuring birefringence of the specimen

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