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One-stop Measurement Model for Fast and Accurate Tensor Display Characterization

PHIL SURMAN¹, WANG SHIZHENG¹, YUAN JUNSONG¹, AND ZHENG YUANJIN^{1,*}

¹Nanyang Technological University, 50 Nanyang Drive, Singapore 639798

*Corresponding author: YJZHENG@ntu.edu.sg

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Many light field displays are fundamentally different to other displays in that they do not have quantized pixels, quantized angular outputs or a physical screen position; this can make definitions and characterization problematic. We have determined that it is more appropriate to express the spatial resolution in terms of spatial cut-off frequency rather than a physical distance as in the case of a display with actual quantized pixels; this concept is then extended to encompass angular resolution. The technique exploits the fact that when spatial resolution of a sinusoidal grating pattern is halved, its contrast ratio is reduced by a known proportion. An improved model, based on an earlier design concept has been developed, can not only measure spatial and angular cut-off frequencies but can also enable comprehensive characterization of the display. This provides fast and simple measurement with good accuracy, that does not use special equipment or the use of time-consuming subjective evaluations. The result of using the model to characterize images in a rapid and accurate manner validates the effectiveness of this technique. © 2017

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1. INTRODUCTION

Light field displays replicate the light pattern of the original scene using geometrical optics principles rather than those of light interference, as in holography that is far more complex to implement. When a scene is observed through a screen, every light ray passing through it can be specified by what is known as a four-dimensional plenoptic function [1]. The simplest way of specifying an individual ray is the X, Y position of the ray exiting the screen and the horizontal and vertical components of its angle. The term "light field" was first coined by Andrey Gershun in 1936 [2].

Multi-layer light field displays were originally developed in Russia in the early 2000's [3] and used neural networks to generate the images on a stack of transmission display panels; a product based on this, known as SmartrON [4] was marketed by Neurok Optics where two liquid crystal display (LCD) panels were cascaded.

This approach was further developed by the MIT Media Lab where there are various embodiments including tensor displays [5], high rank display (HR3D) and dynamic parallax barrier displays [6]. The high-rank 3D display uses a stack of semi-transparent LCD layers and can generate different light rays in each direction from the same point of the screen. Each LCD layer contains a different pattern and the whole stack can approximate

the light field of a given scene and re-create the scene with a hologram-like appearance.

Another type of light field display produces many discrete perspective views derived from many projectors. The images of these are projected on a vertically diffusing screen that can be either reflective or transmissive. The action of the diffusing screen is to convert the light from each projector lens vertically to enable vertical viewer movement. The Holografika display [7] is an example of this type and uses optical modules to provide multiple beams that converge and intersect in front of the screen to form real image "voxels", or diverge to produce virtual voxels behind the screen. Holografika also produces a front-illuminated cinema system with a reflective screen. The Institute for Creative Technologies at the University of Southern California has built a 110° field of view system employing 72 Texas Instruments DLP pico projectors [8].

Our previous work on the metrology of these displays [9] has led to a clearer understanding of how this should be tackled. The most significant finding was that the display behaves as a low-pass filter, thus indicating that its spatial resolution performance is best expressed in terms of cut-off frequency; as would be the case for any other type of filter. As an example, it is customary to specify a simple electrical low-pass filter in terms of its cut-off frequency and not its time constant.

Fig. 1. Image of Depth Object. Snellen 'E' pattern on stepped model where each step from left to right goes 5 mm further back Maximum resolution is on fourth column from left (15 mm behind front screen).

In this paper, cut-off frequency is used to specify spatial and angular frequency responses and, for consistency, depth of field is defined as the region over which the image resolution is greater or equal to half of the maximum value. Regarding the other parameters measured; the virtual screen position is not dependent on cut-off frequency, we found that viewing angle could not be measured readily and contrast ratio and magnifications are independent of cut-off frequency. The spatial and angular cut-off frequencies are given in cycles per meter and cycles per radian respectively.

Our early work was not only hampered by lack of clear definitions for parameters but also by the virtual model used. It was difficult to obtain measurements of sufficient accuracy due to coarse depth and spatial increments in the model dimensions and to the limited range of spatial frequencies. The deficiencies in the model are rectified in the improved version used in this study.

This paper is structured as follows: in the Theory of Section 2, justification for treating the display system as a low-pass filter is provided, along with the explanation that a filtered bar grating image is substantially a sinusoidal grating. In Section 3, the test model and its pattern are described. Section 4 discusses the particular challenges of measuring light field displays and goes on to describe the different screen resolutions involved. The production of the Reference Grating used for analysis and allowance for gamma is discussed in Section 5. The definitions of each of the parameters are given in Section 6 along with the measurement results obtained. Section 7 covers future work.

2. THEORY

A. Display as a Low-pass Filter

The investigation of light field and multiview displays reveals that maximum resolution occurs at the screen, and decreases with increasing distance from the screen. A paper from TP Vision [10] shows a hyperbolic fall-off in an 8-view multiview display, indicating that in this case the resolution decreases in inverse proportion to the distance from the screen. Photographs in MIT's tensor display paper [11], show blurring, like photographs taken with a large aperture. The effect can be seen in Fig. 1 which is the image of a virtual model of a stepped object with 5 mm depth increments. The results were published in our paper [9] and further analysis has led to the improved specification of our current model.

In Fig. 1, it is seen that the display acts as a low-pass filter as the blurring on the Es appears to get worse gradually over several pattern size increments and with increasing distance from a virtual screen position. An interesting observation on the

Fig. 2. Image of Depth Object. Actual photograph of Snellen E on Test Object superimposed on Contrast Sensitivity Chart. Note the good match within red ellipse.

Fig. 3. Ringing. The figure on the left shows the low-pass filter-like characteristic of ringing. Lower right shows the Gibbs Phenomenon of ringing in a square wave function.

appearance of the Snellen E images is that they closely resemble a sinusoidal grating [12], as shown in Fig. 2 where a sample of the image of a displayed E is superimposed on a contrast sensitivity chart. This is a sinusoidal grating with contrast ranging from 0% to 100% in the vertical direction and the spatial frequency spanning many octaves in the horizontal direction.

Another manifestation of low-pass filter-like properties is the appearance of ringing in the images where a ripple effect is seen around step functions - edges in images. The photograph in Fig. 3 left shows this on the larger Snellen Es in the Tensor Display image of the depth object. It is also referred to as the Gibbs Phenomenon [13] and is illustrated in Fig. 3 lower right.

Considering the display as a low-pass filter provides a convenient means of describing the performance where there are no quantized pixels or quantized angles of ray direction. In Fig. 4, the typical spatial and angular frequency responses of the display are shown. The cut-off frequencies were obtained from measurements on the stepped depth model [9], but the actual shape of the plots is obtained from the shape of a typical 3 dB per octave filter as the object does not enable sufficiently precise measurements to be made; in this example, the spatial and angular cut-off frequencies (-3 Db intensity amplitude) are 560 cycles/meter and 50 cycles/radian, respectively.

Fig. 4. Cut-off Frequencies. (Top) Spatial cut-off frequency for 560 cs/m. (Lower) Angular cut-off frequency for 50 cs/radian.

Fig. 5. Fourier Series. Low-pass filtered square wave approximates to a sinusoidal function & 'DC component' as 3rd & higher harmonics can be neglected.

B. Low-pass Filtered Square Wave

A bar grating can be analyzed by treating the luminance along a line at right angles to the bars as a square wave function that is illustrated as a black line in Fig. 5. The same considerations as an electrical waveform apply here where any waveform can be represented by a series of sinusoidal waveforms whose frequencies are integer multiples of the fundamental frequency, otherwise known as the first harmonic. The other waveforms are referred to as the second harmonic, the third harmonic and so on. A square wave is made up of a sine wave at the first harmonic, and odd harmonics with amplitude decreasing with increasing frequency. In Fig. 5, the first harmonic is shown in red and the third, fifth and seventh harmonics in light grey; the higher harmonics are not shown. The blue plot is the sum of the first seven harmonics.

The Fourier series coefficients of a square wave function can be found from the following equation [13]:

$$f(\theta) = \frac{4}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n} \sin\left(\frac{\pi n \theta}{L}\right), \quad (1)$$

where θ is an arbitrary angle, n is an integer and L is the half pitch of the grating.

Table 1. Fourier Series Coefficients

Harmonic	A	B		
	Normalized coefficient	Attenuation 3 dB/octave	A x B	dB
1	1	1	1	0
3	-1/3	1/3	-1/9	9.5
5	1/5	1/5	1/25	14
7	-1/7	1/7	-1/49	16.9

This gives a first harmonic whose peak-to-peak value is $4/\pi$ times the peak-to-peak value of the square wave function of the grating pattern, as shown in Fig. 5. In our analysis, we are interested in the cut-off frequency; that is where the amplitude of the output is halved. This is approximately where the contrast ratio of the first harmonic is halved.

In Table 1, Column A shows the Fourier series coefficients normalized to the amplitude of the first harmonic, and Column B is the approximate attenuation of the filtering action of the screen. The column on the right is the combined reduction of each harmonic. This shows that the third harmonic is attenuated by almost 10 dB and consequently can be neglected for the purposes of comparing the filtered grating image with a sinusoidal grating. This is borne out by the good matching seen in Fig. 2. There is also a so-called 'DC component' in an electrical waveform, which in the case of an image is a constant gray level.

In relation the amplitude of the square wave, the peak-to-peak amplitude of the first harmonic is 1.273 times that of the square wave. At the spatial cut-off frequency of the filter, the amplitude of the first harmonic is halved and the DC component is unaffected. The cut-off frequency can be found from where the contrast is equal to $1.273/2 = 0.637$ (Fig. 6) as this represents a halved amplitude.

It should be noted that the 3rd and higher order coefficients are neglected purely on the grounds of their reduced value and their greater attenuation, not on the basis of subjective assessments that are reliant upon contrast sensitivity. It is shown in Fig. 15 that the only human judgement required is the comparison between maximum gray levels and minimum gray levels, which is considerably more robust.

3. TEST MODEL

A. Method

In our first series of measurements, lengthy user tests on subjects were carried out on the image of the object in Fig. 7 top. In addition to being time-consuming, some doubts existed as to the accuracy of the procedures existed, particularly in the areas of image resolution and viewing angle. Also, it was not possible to obtain a value for the all-important angular resolution. However, the previous results and procedures have proved invaluable in informing the direction of further work in this area.

The procedure provides the opportunity of enabling fast and sufficiently accurate measurements to be made without having to carry out lengthy trials on subjects as all the measurements can be obtained by analyzing photographs of the displayed image.

Fig. 6. Contrast. The contrast at the cut-off frequency is 0.637 of the maximum contrast. In this case the maximum and minimum contrasts are 1 and 0, respectively.

B. Test Pattern

Examination of the results from the original stepped model show that the 5 mm depth increments used are too large to obtain accurate angular measurements as 5 mm is significant compared to the depth of field. Additionally, the steps between the pitch of adjacent Es of $2^{1/6}$, which is a factor of 1.122, can be eliminated with the use of a continuously variable pattern pitch. The Es are no longer necessary as subjective measurements are not required; the Es were used is it demanded a choice of orientation of the E that the subjects had to record. It was also found that Es in their normal orientation yielded over double the amount of incorrect results than Es that are rotated 90° . This is a useful finding as it indicates that a grating with vertical bars might perform better than one with horizontal bars; it also has the advantage that it can be used with a horizontal parallax only version of the Tensor Display where horizontal bars would not produce images that required any processing.

Evaluation of the stepped model indicates that a range of grating pitch from 0.5 mm to 8 mm is sufficient to carry out the measurements we require. A continuously varying pitch from 0.5 mm to 8 mm can be achieved by increasing the pitch by $2^{1/16}$ between adjacent bars. This is 16 cycles/octave giving a width of 171 mm, and the height is set at 150 mm (Fig. 7 lower).

To eliminate the steps, the grating is tilted forward to give a depth of 30 mm at the upper edge and -30 mm at the lower edge. There are black and white squares in each corner for the measurement of the minimum and maximum 'black' and 'white' levels. Red dots are used as markers to indicate the positions of the 1, 2 and 4 mm pitch positions.

4. SCREEN DEFINITIONS AND UNITS

A. Challenges

Unlike traditional displays, some light field displays do not have pixels as such at the effective screen plane; for example, in a multi-layer display the effective screen position is generally located away from one of the display screen layers, and in a multi-projector display, although the projectors will be focused on a vertically diffusing screen, it is unlikely that the pixels from the different projectors will be exactly aligned with each other.

In these displays, the display devices themselves have discrete pixels but these are not seen by the viewers. This poses two problems; first, where is the effective screen located, and

Fig. 7. Depth Objects. (Top) Original stepped object. (Lower) New object with sloping surface and logarithmic bar grating.

second, what is its resolution?

The realization that the display system behaves as a low-pass filter whose has led the authors to a very convenient measurement technique that uses what we refer to as the 'One-stop Measurement Model' that can be used for the measurement of up to eight parameters. It comprises a simple bar grating with a geometrical progression in pitch across its width. This is placed on a surface that is rotated around its central horizontal axis to give a range of substantially continuous spatial frequency and depth values.

Another important parameter that applies to three-dimensional (3D) displays is angular resolution; this does not apply to two-dimensional displays where the light emitted by a pixel in any direction remains substantially unchanged. The higher the angular resolution, the greater is the depth of field (DOF). Angular resolution measurement is not as straightforward as that of spatial resolution, and this is made more difficult as it is not quantized in a multi-layer display. For example, in a multi projector display, the angular resolution can be determined simply from the spacing between the projector centers and their distance from the screen.

The DOF specifies the distance between the near and far limits of acceptable image quality in front of, and behind the screen. As the effective screen is a virtual one, similar techniques for determining spatial and angular resolutions are investigated for characterizing DOF.

The other parameters of contrast, depth magnification and area magnification are straightforward and do not pose undue difficulty. Absolute luminance and color measurements are not covered in this paper and require special equipment. The method proposed here only requires an un-calibrated camera in conjunction with the model we have developed.

B. Definitions and Units

As the displays being measured do not behave in the same way as a conventional pixelated display, different definitions for their parameters must be used. In this section, each of the eight parameters mentioned in the previous section is defined and their units given. This not a case of overturning previous definitions as we are not comparing like with like, and to the best of the authors' knowledge, there is currently no definitive way of expressing the parameters of this display type.

The conclusion from these findings is that the virtual screen

Table 2. Screen Resolution Type, Description and Units

Resolution type	Description	Units
Native resolution	The resolution of the actual display panel.	mm
Screen resolution	The resolution of the virtual screen.	cs/m
Image resolution	The resolution of the image that appears in space either in front of, or behind the virtual screen.	cs/m
Perceived resolution	The resolution as seen by the eye.	cs/deg

position can be considered as the position at which the maximum resolution occurs.

At this point, it will be helpful to clarify to the reader the definitions of the various resolutions involved in this investigation. The resolution of the actual display panel is referred to as the native resolution and is generally expressed in units of millimeters. The resolution of the virtual screen is simply referred to as the *screen resolution* and is given in cycles per meter. If other 3D display types are to be included within these definitions then it should be noted that for a slanted lenticular multiview display, the screen resolution in the horizontal direction is set by width of the lenses.

The resolution of the image that appears in space either in front of, or behind the virtual screen is termed the *image resolution* and is also expressed in cycles per meter. The resolution as seen by the eye is the *perceived resolution* which is in cycles per degree. This figure, which is obviously dependent on viewing distance, is useful as it can be used to determine whether the display resolution is likely to have a perceptible effect. The resolution required for 20/20 vision is 30 cycles per degree; however, hyper-acuity [14] can provide higher resolution for some observers. The screen resolutions are summarized in Table 2.

The cut-off frequency is the frequency at which the output halves; in terms of decibels (dB), that is 3 dB. Clearly, cut-off spatial frequency is more appropriate for this type of display as a conventional display with discrete pixels cannot display a higher spatial frequency than that set by double the pixel pitch. A display that behaves as a low-pass spatial filter can display higher frequencies than the cut-off frequency, but with reduced output.

As the spatial resolution is not quantized, then neither is the angular resolution. In a conventional 3D display, the angular resolution in radians is the screen 'pixel' width (lens width for slanted lenticular) divided the distance from the screen at which the resolution halves. Similarly, as the tensor display does not have a pixel width as such, neither does the angular resolution have set angular increments, but has a cut-off frequency expressed in cycles per radian. In the same way as spatial resolution, a higher angular resolution than the cut-off frequency can be displayed, but with reduced contrast.

The spatial and angular resolutions are expressed in terms of their 3 dB cut-off frequencies so it is consistent to also express DOF and viewing angle in the same way. DOF is the distance, on axis, between the half screen resolution planes in front of, and behind the screen. This is expressed in millimeters. The

Fig. 8. Reference Sinusoidal Grating. This is for 0.637 contrast. In practice, the contrast will be less as black and white levels are not 0, 0, 0 and 255, 255, 255.

Fig. 9. Gamma Correction. (L) Non-linear characteristic, matches eye sensitivity to shade differences. (R) Grayscale RGB from increments of 10. in RGB. The gamma value of 2.2 is used to plot the curve.

viewing angle is difficult to define and requires further work.

Contrast can be obtained from the greyscale values of black and white image areas of the test model. If the gamma values of the system and the camera used are known then the luminance contrast ratio can be calculated. The depth magnification is the ratio the test model image depth to its actual depth and area magnification is the ratio of X and Y dimensions on the image of the model to the model's 'actual' (in fact, virtual) dimensions.

All the parameter definitions in this section are summarized in Table 1 and the spatial resolutions in Table 2.

5. REFERENCE GRATING

A. Introduction

The procedure described here assumes that measurements are made on images on the monitor purely by using comparison with a reference pattern. This is derived from measurement of the displayed 'black' and 'white' levels the corners of the patterns. The method can be explained by referring to Fig. 2 where a sample of grating of known contrast ratio and mean

Fig. 10. Depth Object Image. Spatial resolution, angular resolution, DOF and FOV can be determined from the points K, L, M and N.

gray level is moved around the test object image until a match is found for the desired contrast mean gray level and spatial frequency. Although this might appear to be a subjective method as it requires a judgement on the part of a human operative, it should be noted that human visual system can distinguish between slight differences in gray level. This is the reason for 10-bit greyscale being preferable to 8-bit greyscale.

Fig. 8 is an example of a reference sinusoidal grating where the black RGB level is 0, 0, 0 and the white level is 255, 255, 255. In practice, the displayed black and white levels will have lower difference and this is allowed for as described in the following section.

B. Gamma Correction

As areas in the computer-generated object that should be completely black and completely white are not displayed as such in the image, correction must be made for this. However, the relationship between greyscale level and luminance on the screen is not linear in order to match the response of the human eye where differences between smaller increments in grey level can be detected at lower luminance levels. This relationship is shown in Fig. 9 where the screen luminance is plotted against grey scale values from 0 to 255, and is approximately a power function with the power denoted by the symbol γ .

Strictly speaking, gamma correction of the system and camera must be allowed for if the measurements are to be made accurately and without special equipment. We will assume that the camera gamma matches that of the system. It is also assumed that the actual image on the grating is substantially sinusoidal as discussed in Section 2.B.

Although sRGB is employed in the system used for the measurements, we use the approximation of gamma = 2.2 here in order to simplify the explanation of why non-linearity might have to be accounted for. In Fig. 9, the Y_W and Y_B that are the values that the display delivers for pure white and pure black respectively are measured. This is carried out with a visual comparison using a test area of known greyscale values, as illustrated in the right section of Fig. 9 where the increments are in steps of 10 grey levels.

Equations (2) to (4) are used to calculate the gray levels on the Reference Grating. Although the approximation of using γ is used here, it should be borne in mind that the purpose of using

Fig. 11. The model covers 4 octaves of spatial frequency (0.5 to 8 mm pitch) and depths from -15 to 15 mm. There are 'black' and 'white' areas in each corner for calibrating the reference sinusoidal grating.

the Reference Grating is to obtain the half-resolution boundary of Fig. 16 right. Even if this boundary was drawn by eye, reasonably accurate cut-off frequency results would still be obtained. When the camera and system gammas are matched, the sinusoidal grating shown by the monitor is similar to the display under test. To match this pattern accurately, the test comparison image should show a sinusoidally varying luminance profile. This is performed in three stages:

(I) the Y_W and Y_B values are measured and transferred to the y axis of Fig. 9 by raising them to the γ -th power to give values of L_W and L_B respectively (Equation (2)); the mean value can be calculated from this.

(II) the maximum and minimum values of L_H and L_L respectively for the decrease in contrast equivalent to halving the resolution are determined (Equation (3)) and amplitude A determined.

(III) the function for calculating the value of luminance Y for an arbitrary angle θ is used to convert this back to grey scale by applying the $\frac{1}{\gamma}$ th power (Equation (4)). Note that the y -axis values in Fig. 9 are of luminance in arbitrary units.

$$B = \frac{Y_W^\gamma + Y_B^\gamma}{2}. \quad (2)$$

$$A = \frac{0.637(Y_W^\gamma + Y_B^\gamma)}{2}. \quad (3)$$

$$Y = (A \sin \theta + B)^{\frac{1}{\gamma}}. \quad (4)$$

The luminance of the 'black' and 'white' levels are determined by drawing a square on the screen and superimposing this over the photograph at the regions of the black and white squares. The grey level of the square is then adjusted so that it exactly matches the square under it (Fig. 14); this provides the Y_W and Y_B values. Equations (2) (3) and (4) show that the reference pattern can be drawn from these two values alone. This pattern can then be either stretched or shrunk to match its pitch to the appropriate regions of the image.

Fig. 12. Image Depth. The image does not appear in front of the nearest screen to the viewer. The magnification in the z-direction is 1.20 (taken from the red mean line). '100' and '200' refer to the distance from the center at 1m from screen.

The Reference Grating is made by calculating the grey levels according to Equation (4) and using these to color the cells of an Excel spreadsheet. This block of cells is saved as a drawing and superimposed on the photograph of the image. For all the measurements, the bars in the pattern remain vertical, and the pattern width adjusted as necessary.

Before this process, the photograph is de-colored by setting its saturation to 0% as this makes the matching process far easier to perform. For our measurements, the result of this was to change the RGB values of the 'white' reference region in the corner from 132, 149, 132 (color) to 142, 142, 142 (gray), which will have a negligible effect on the plotted position of the half-resolution boundary of Fig. 16 right.

6. DEFINITION AND MEASUREMENT

All the measurements, except those for viewing angle, can be made from the analysis of a single captured image of the displayed image of the test object. At first sight, this image (Fig. 11) appears rather meaningless; however, when only the region of interest is considered, its interpretation becomes clearer. By having a grating pitch from 0.5 mm to 8 mm, the pattern allows for a considerable range of possible screen resolutions and system magnifications to be covered. Fig. 10 is a drawing of the image in the region of the relevant spatial cut-off frequencies from which the first five parameters in Table 3 can be determined. The positions J, K, L and M are found by moving the Reference Grating over the image and finding matches according to the procedures in the following subsections.

The measurements commenced by calibrating the image in terms of depth, magnification in the x-y plane and grayscale level. Depth was obtained by first determining the direction where the maximum resolution is obtained as it is found in practice that this is not necessarily on the central axis. Maximum resolution occurs when the leftmost part of the resolvable pattern is closest to the left side of the screen. A grating bar that spans the complete height of the screen is chosen and tape markers are attached to the screen to indicate the position at the bottom, 1/3 of the way up, 2/3 of the way up and at the top of the bar. The image magnification is obtained by measuring the size the image and dividing it by the dimensions of the virtual object. Z magnification and area magnification are used for the calculation of the first five parameters.

Fig. 13. Viewing Angle. The angle over which the resolution is greater or equal to half the resolution at the centre, measured across the central axis.

At 1 m from the screen, the maximum resolution occurs 125 mm to left of the central axis. With the observation position shifted 100 mm to the right, the position of the edge of the bar was marked on the tape at each height by the observer's assistant. The process was repeated for the position 200 mm to the right of the maximum resolution position. The lateral shifts obtained are given in columns 2 and 3 of the table in Fig. 12. The results in columns 4 and 5 are calculated simply by applying the principle of similar triangles. To obtain greater accuracy, the mean of the results from 100 and 200 mm shifts are shown in column 6.

In Fig. 12, the red mean line is used to determine the calibration values and the depth magnification. It was found that the bars either shifted to the right in relation to the screen, or not at all; thus the image only exists at the plane of the screen or behind it. This is consistent with our earlier findings.

Having gathered the basic data to calibrate the captured image, the next stage is to plot the half resolution curve and obtain measurements from it. As the contrast ratio is relatively low, we will assume that the gamma curve can be considered as being sufficiently linear for this application and can be approximated by a straight line. This gives the grayscale values of 124 and 60 for maximum and minimum levels respectively. These values are derived from the measured 'black' and 'white' levels of 42 and 142 respectively and finding 0.637 of the amplitude, but with the same mean value (this assumes that the gamma curve is substantially linear over the region). Instead of constructing a new grating, it is simpler to use a pre-prepared sinusoidal grating and adjust its brightness and contrast until a good match is found, as illustrated in Fig. 15. The actual pattern shown in the lower half of Fig. 15 can then be moved in the X and Y directions as necessary to match the pattern pitch on the image it is overlaying.

Table 3. Parameter Definitions and Units

Parameter	Definition	Units
Screen position	The Z coordinate at which the screen resolution is maximum.	mm
Spatial resolution	The spatial cut-off frequency of the screen resolution.	cs/m
Angular resolution	The angular cut-off frequency of the screen.	cs/rad
Depth of field	The distance between the near and far points where, on axis, the screen spatial resolution halves.	mm
Viewing angle	The angle where, over the width of the central horizontal axis, the screen spatial resolution halves.	deg.
Contrast ratio	The difference between the displayed black and white levels of the original scene.	Dimensionless
Z magnification	The magnification between a depth in the image and the corresponding object depth.	Dimensionless
Area magnification	The magnification between X and Y in the image and the corresponding distance in the object.	Dimensionless

In Fig. 16 left, it can be seen that six small Reference Gratings have been overlaid on the image at the half spatial resolution boundary. Good matches are obtained between the gratings and the image as can be seen at the centers of the yellow circles. Although it is difficult to see, the inconsistencies below the circles are the lower ends of the Reference Gratings and not defects in the displayed pattern.

Although, in principle, only the points K and L are required, it is better to plot a curve for increased accuracy as some quite large variations can be seen. From the plot in Fig. 16 right the points K and L can be used to determine; the screen position, spatial cut-off frequency, angular cut-off frequency and depth of field. The point M of Fig. 10 is not plotted as the image is not clear in this region; also, it has no depth so it is not a region that should be used for presenting images.

The definitions and results are as follows:

A. Screen Position

Definition: The Z coordinate where screen resolution is maximum.

This is found from position K in Fig. 16 right where the Reference Grating is used to determine the position furthest to the left where the contrast ratio has a value of 0.634 ($2/\pi$). The screen position is the depth at which this occurs. Note that this depth is obtained using the calibration described in Section 6.G. Positions above and below the central axis must also be checked as it is possible that the optimum viewing position is not on the central axis.

Measurement: From Fig. 16 right, the vertical white line tangential to the boundary represents the maximum resolution, and the point K, where it touches the boundary, is the screen position which is on the 0 mm depth line.

B. Spatial Resolution

Definition: The spatial cut-off frequency of the screen resolution.

Spatial resolution R in cycles/metre is given by:

$$R = \frac{1000}{G}. \quad (5)$$

Where:

G = pitch at cut-off frequency (mm)

Measurement: The effective screen resolution is the spatial cut-off frequency at point K, which is obtained from the grating bar position and the image magnification as follows: $1000 / (2^{11/16} \times 1.12) = 554.4 \text{ cs/m}$ (equivalent to pitch of 1.95mm).

C. Angular Resolution

Definition: The angular cut-off frequency of the screen.

This is obtained from points L and M that are located at the spatial resolution line representing the pitch $2G$; that is where the spatial resolution is halved. The width of the Reference Grating is doubled so that its spatial frequency is halved and its pitch matches the image distance $2G$; it is then shifted horizontally to position N where the pitch matches. It is subsequently moved vertically until its contrast matches the image; this gives point L. The Reference Grating is then moved downwards to find the point M where the contrast again matches (if this is possible).

As the angular resolution was found to be non-symmetrical due to the image being unable to be presented in front of the screen nearest screen, it is more representative of the performance to measure a near and a far angular resolution (A_N and A_F respectively) and these are expressed in cycles/radian as:

$$A_N = \frac{1}{\arctan \frac{2G}{D_N}} \approx \frac{D_N}{2G}. \quad (6)$$

$$A_F = \frac{1}{\arctan \frac{2G}{D_F}} \approx \frac{D_F}{2G}. \quad (7)$$

Where:

D_N = near half-resolution distance (mm)

D_F = far half-resolution distance (mm)

Measurement: The angular cut-off frequency is given approximately by:

$A_F = 17.5/1.95 \text{ cs/rad.}$ (equivalent to an angle of 6.4°)

$A_N = 0 \text{ cs/rad.}$ as the image does not appear in front of the virtual screen.

D. Depth of Field

Definition: The distance between the near and far points where, in the direction of the maximum resolution, the screen spatial resolution halves.

Fig. 14. 'Black' and 'white' grayscale levels are found from superimposing gray squares on test area in corner and adjusting for invisibility as shown in this figure.

The depth of field (DOF) is the distance in millimeters between the near half-resolution distance from the screen and the far half-resolution distance from the screen.

$$DOF = D_N + D_F. \quad (8)$$

Measurement: In Fig. 16 (right) the vertical distance between K and L corresponds to a **depth of field of 17.5 mm**. This is the complete depth of field as the image does not appear in front of the closest screen.

E. Viewing Angle

Definition: The angle where, over the width of the central horizontal axis, the screen spatial resolution halves.

As the viewer moves either side from the optimum viewing position, the leftmost point of maximum resolution should move from K to N. We need to determine the angle over which the resolution is within half the maximum; therefore, it is where the expanded Reference Grating used for measuring angular resolution matches the image pattern at the screen position, as shown in Fig. 13 and where the viewing angle V in degrees is given by:

$$V = \arctan \frac{D_R}{O} + \arctan \frac{D_L}{O}. \quad (9)$$

Where:

O = viewing distance

D_R = distance to the right where spatial resolution drops to half

D_L = distance to the left where spatial resolution drops to half

A series of photographs is taken at the viewing distance and an analysis of the positions of the maximum resolution is used to determine by interpolation the position of the X coordinates where the spatial resolution is halved.

Measurement: Viewing angle could not be determined on the images available as the boundary becomes ill-defined as the observation point moves away from the optimum position 125 mm to the left of the central axis. In Fig. 17, it is seen that there is a well-defined boundary at the optimum viewing position but this does not proceed in an orderly fashion and a measurement cannot be obtained.

Fig. 15. Calibrating Reference Sinusoidal grating. The maxima and minima of the lower sinusoidal pattern are matched with the grayscale values obtained from the 'black/white' levels and the contrast ratio for half spatial resolution.

E.1. Contrast Ratio

Definition: The difference between the displayed black and white levels of the original scene.

Contrast ratio is the ratio between the luminance for 'white' (Y_W) divided by the luminance for 'black' (Y_B). As the luminance is proportional to the greyscale to the power of gamma, the contrast ratio CR is found as follows:

$$CR = \left(\frac{Y_W}{Y_B} \right)^\gamma. \quad (10)$$

Greyscale is found by superimposing gray squares on to an image the 'black' and 'white' region on a photograph of the test object and adjusting their gray level until they are least perceptible as illustrated in Fig. 14. The image is first adjusted to 0% saturation so the result is not affected by any residual color. The numbers in the figure denote the gray levels of 142 for 'white' and 42 for 'black', so allowing for gamma correction, the perceived contrast ratio is given by $CR = (142/42)^{2.2}$. This gives a **contrast ratio of 14.6:1**. This value is obtained by applying the approximate relationship of $\gamma = 2.2$; however, this should suffice for most applications of the results of these measurements where meaningful values for spatial and angular parameters are generally of greater importance.

F. Z Magnification

Definition: The magnification between a depth in the image and the corresponding depth in the object.

The depth of screen is found by marking the front screen where the edge of the pattern is seen from the dead-ahead position. The depth is obtained simply by noting the lateral displacement and the observation distance and then applying similar triangles.

Measurement: From the plot in Fig. 12, it is found that for an object depth of -10 mm, the apparent depth is 12.0 mm so **the depth magnification is 1.20**. The depth magnification for positive object depths is zero.

G. Image Magnification

Definition: The magnification between X and Y in the image and the corresponding distances in the object.

Measurement: The image magnification is found by measuring the width of the grating in the image and dividing it by the grating width on the object, which in this case is 171 mm. The

Fig. 16. Plotting Half Resolution Boundary. *Left:* reference sinusoidal gratings overlaid on image. *Right:* Plot from contrast matching points.

image grating width of 192 mm gives an **image magnification of 1.12**.

Table 3 is a summary of all the parameter descriptions and units used and Table 4 is a summary of all the results with brief comments on these. Fig. 18 is a flow chart showing the procedure for finding all the parameters with the exception of viewing angle.

7. FUTURE WORK

Whilst this model is based on our earlier development of a stepped version and we have incorporated new techniques and parameter measurements, it appears that this version now requires fewer improvements. The most obvious improvement is that of making the ranges of spatial frequency and depth more in line with typical display performance. On the assumption that future improvements will be in the areas of depth of field and spatial resolution, the grating pitch range could be altered so that it covers, for example, 0.25 to 4 mm. The redundant region from 4 to 8 mm is over half the total grating area and represents an unacceptably poor display quality.

The depth range of the object must be increased as it can be seen In Fig. 16 that the half resolution boundary is very close to the top edge of the grating. The depth of field of current multi-layer light field displays is still relatively poor and it is anticipated that research will be carried out into extending it. One possible cause of this is possible insufficient density of capture cameras and if this is the case, the addition of intermediate synthesized views might provide a solution. For the model, this enhanced capability is provided by simply tilting the grating at a greater angle.

If spatial resolution can be increased considerably then one potential problem could be that of aliasing with the camera pixels and the display screen pixels; however, with current performance, this is not yet a problem.

The inability to measure the viewing angle may not lie in the display performance, but in the way viewing angle has been defined. It is reasonable to use the cut-off frequency concept for angular and spatial bandwidths as these relate to periodic

Fig. 17. Viewing Angle. *Left:* At viewing angle with best performance, the 1/2 resolution boundary is well defined. *Right:* away from optimum viewing direction, boundary is ill-defined.

Fig. 18. Flow chart showing procedure for finding all parameters with the exception of viewing angle.

Table 4. Summary of results with Comments

No.	Parameter	Value	Unit	Comments
1	Screen Position	0	mm	The effective screen position is at the plane of the front LCD screen.
2	Spatial Cut-off Frequency	554.4	cs/m	This is for grating pitch of 1.95 mm.
3	Angular Cut-off Frequency	8.8	cs/m	Equivalent to 6.4° .
4	Depth of Field	17.5	mm	This is the total DOF that is all behind the front screen.
5	Viewing Angle	-	-	Cannot be measured on these images.
6	Contrast Ratio	14.6:1	-	This appears to vary over the area of the images.
7	Depth Magnification	1.20	-	This is behind the screen only - it is zero in front of the screen.
8	Image Magnification	1.12	-	-

functions; however, there is only one viewing field so it is likely that a less demanding criterion than that of halving resolution is more appropriate.

8. CONCLUSIONS

The model has proved a considerable improvement over our previous one for two principal reasons. First, time-consuming and subjectively affected user trials are eliminated and replaced by a far faster and simpler method that, although requires some human judgement at some stages, it is one at which the human visual system excels – that of detecting shade differences. Second, the use of a sloping logarithmic grating gives the continuous depth and pitch ranges necessary for accurate angular resolution measurement. The time taken to analyze the results is well under a tenth of that required previously.

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